Multi-User Massive MIMO And Physical Layer Network Coding

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***Abstract*—In this paper, we investigate the feasibility of applying Physical Layer Network Coding (PNC) in Multi-user Massive MIMO systems. In addition, we investigate the performance benefits of the joint Massive MIMO and PNC scheme. PNC has the potential of increasing capacity of a wireless system as the number of timeslots required for end-to-end communication reduces. We adopted a scheme that transforms the channel between a massive-antenna relay and a multitude of multi-antenna user terminals, with a Sum-Difference (SD) matrix. Through Log-Likelihood Ratio (LLR), PNC is achieved by deriving the network coded symbols from the estimates of the SD symbols at the massive-antenna relay node. The equalization matrix for the estimation is based on the SD transformed channel coefficient matrix. The error performance of the proposed joint Massive MIMO and PNC is evaluated through extensive simulation results. It is shown that joint Massive MIMO and PNC performs significantly better than Massive MIMO without PNC for QPSK modulation.**

***Index Terms*—Massive MIMO, Physical Layer Network Coding, Log-Likelihood Ratio, Sum-Difference Matrix, Log-Sum Exponent Approximation, BER, SNR, ZF, MMSE.**

I. Introduction

The next generation of wireless network access must focus and provide solutions for not only future capacity constraints but also existing challenges, such as network coverage, reliability, energy efficiency, latency and spectral efficiency. Some of these challenges can be addressed by using Massive MIMO and Beamforming [1] [2]. As an extension of conventional MIMO, Massive MIMO exploits large array of antennas in the region of 100's and even 1000's. The massive array of antennas strengthen the capability of spatial multiplexing of many user terminals in the same time-frequency resource, which yields higher channel capacity and higher throughput gains. Besides the spatial multiplexing gains, Massive MIMO also benefits from spatial diversity by improving link reliability. The large number of antennas leads to extra degree of freedom, by which Massive MIMO can harness the available spatial resources to improve spectral efficiency. In addition, and with the aid of beamforming, Massive MIMO can suppress interference and extend coverage by directing energy to desired terminals only. Based on these benefits, Massive MIMO is now a compelling physical layer technology for the next generation of wireless access and considered one of the key enablers in 5G [3].

Besides Massive MIMO, another promising physical layer technique that has the potential to address some of the above-mentioned challenges is Physical Layer Network Coding (PNC). PNC applies Network Coding technique at the Physical Layer [4]. Network Coding is a widely known network layer technique that generates outgoing bit stream as a function of incoming bit streams at intermediate nodes. Evidently, the processing technique is at the bit level. In a relay system, where nodes communicate through a central entity, the bit streams combine at the relay by XOR'ing. If the XOR'ed bit stream or network coded data stream is forwarded, the receiving nodes perform a similar XOR operation using a copy of the bit stream transmitted previously, to retrieve those sent from the other nodes. In wireless access networks, the physical layer processes modulated data streams in the form of symbols, and therefore, application of Network Coding in this layer requires a different approach. However, the wireless medium, inherently, superimposes the electromagnetic waves received from multiple transmitting nodes. This superposition, often inferred as interference, is a means to Network Coding at the physical layer. Because the constellation of the superimposed symbols go out-of-range as opposed to the constellations used by the transmitting nodes, mapping the superimposed symbols to the constellation from the transmitting nodes is essential - referred to as PNC Mapping. Only when PNC mapping algorithm produces unambiguous network coded symbols, which are comprehensible at the transmitting nodes, will PNC mapping deemed successful. In essence, interference is not a deterrent in PNC, and this is a very important characteristic to leverage on. The toleration of interference in PNC can be viewed as leading to capacity boost, as time slots required to have end-to-end communication in a relay system is reduced.

Massive MIMO and PNC are two distinct physical layer techniques that complement each other on the part that they differ. The combination of Massive-MIMO and PNC, hereafter, referred to as joint Massive MIMO and PNC, can reduce the impact of multi-user interference in Massive MIMO systems. Massive MIMO still suffers from intra-cell and inter-cell interference, albeit the enormous benefits it comes with. Signal processing techniques such as Zero Forcing (ZF), Minimum-Mean Square-Error (MMSE) and Maximum Ratio (MR) are some of the notable linear detectors that suppress interference

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in Massive MIMO systems, but they are sub-optimal [5] and require high SNRs to effectively null interference. Joint Massive MIMO and PNC can reduce SNR requirements, save up to 50% resources and increase throughput gains in end-to-end wireless communication.

While, intuitively, Massive MIMO and PNC can be used as complementary approaches, a comprehensive study to investigate the practicality and the benefits of the joint PNC and Massive MIMO is yet to be done. Previous works focus primarily on 2×2 MIMO and PNC. For example, [6] proposed a scheme that combines MIMO and PNC, but it was limited to a 2×2 Single-user MIMO system, [7] extended this to a 4×4 Multi-user MIMO PNC system, [8] [9] and [10] investigated PNC and Network-MIMO or Coordinated Multipoint, and although introduced concepts to optimally generating unambiguous network coded messages, they focused on single-antenna user terminals. However, none of these recent works produced any result for Multi-user combined Massive MIMO and PNC.

In this paper, we present an approach for joint PNC and Massive MIMO. We adopted a scheme that transforms the channel between a massive-antenna relay or base station (BS) and a multitude of multi-antenna user terminals, with a Sum-Difference (SD) matrix. Through Log-Likelihood Ratio (LLR), PNC is achieved by deriving the network-coded symbols from the estimates of the SD symbols at the massive-antenna relay. The equalization matrix for the estimation is based on the SD-transformed channel coefficient matrix. We also present the error performance of our joint Massive MIMO and PNC evaluated through extensive simulation results. We summarize our contributions as follows:

- We show that PNC is possible in Massive MIMO systems with a scheme that transforms the channel matrix to sum and difference of the channel gains - implicitly grouping input symbols from multiple users that are paired for Network Coding.
- We demonstrate how PNC mapping can be achieved through an approximated Log-Likelihood Ratio based on estimation of sum-difference symbols of the input source symbols at the BS.
- We evaluate the error performance of this scheme over varying dimensions of the Multi-user Massive MIMO system using QPSK modulation.

II. Joint Massive-MIMO and PNC System

A. System Model

We consider the uplink (UL) of a single-cell Massive MIMO system in which N multi-antenna User Equipment (UEs), each with up to K antennas, communicating through a central M-antenna base station in the same time-frequency resource. The corresponding system model is depicted in Fig.1. In fulfillment of the number of antenna requirements for Massive MIMO, it is assumed that $M \geq 64$, $M \gg N$, $M \gg K$, where $\gg$ denotes far greater than. Let $h(n)_{m,k} \in \mathbb{C}$, for $n = 1, \ldots, N$, $m = 1, \ldots, M$ and $k = 1, \ldots, K$, denotes

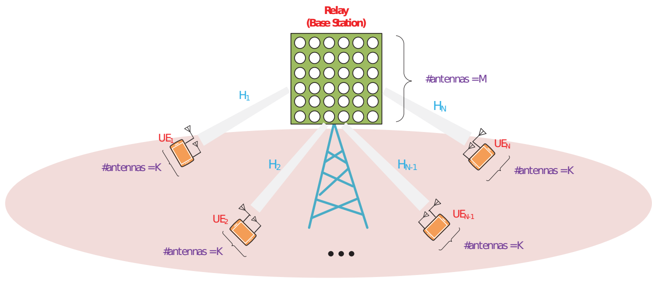

Fig. 1. A Multi-user Massive MIMO System Model.

the complex channel gain between m^{th} antenna of BS and k^{th} antenna of the n^{th} UE. Then, the $M \times K$ channel matrix between the n^{th} UE and the BS is given as

$$\boldsymbol{H}(n) = \begin{bmatrix} h(n)_{1,1} & h(n)_{1,2} & \ldots & h(n)_{1,k} \\ h(n)_{2,1} & h(n)_{2,2} & \ldots & h(n)_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ h(n)_{m,1} & h(n)_{m,2} & \ldots & h(n)_{m,k} \end{bmatrix}_{\mathrm{M\times K}} . \quad (1)$$

The complete channel matrix, $\boldsymbol{H}$, between all the M antennas at the BS and the K antennas of all the N UEs in the multi-user system model, as depicted in Fig.1, can be formulated as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}(1) & \boldsymbol{H}(2) & \ldots & \boldsymbol{H}(N) \end{bmatrix}_{\mathrm{M\times L}}, \quad (2)$$

where $L = K \times N$. The input-output relation over the channel $\boldsymbol{H}$ can, hence, be expressed as

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{z}, \quad (3)$$

where $\boldsymbol{r} \in \mathbb{C}^{M\times1}$ is the received symbols vector, $\boldsymbol{z} \in \mathbb{C}^{M\times1}$ the AWGN at the antennas of the BS, with zero mean and variance of σ^2, i.e. $\mathcal{CN}(0, \sigma^2)$, and $\boldsymbol{s} \in \mathbb{C}^{L\times1}$, is the transmitted symbols from all the UEs. The transmitted symbols vector, $\boldsymbol{s}$, can further be expressed as

$$\boldsymbol{s} = \begin{bmatrix} \boldsymbol{s}(1) & \boldsymbol{s}(2) & \ldots & \boldsymbol{s}(n) \end{bmatrix}^{\mathrm{T}}_{1\times \mathrm{L}}, \quad (4)$$

where $\boldsymbol{s}(n) = \begin{bmatrix} s(n)_1 & s(n)_2 & \ldots & s(n)_k \end{bmatrix}^{\mathrm{T}}_{1\times\mathrm{K}}$ represents a vector of k symbols from the n^{th} UE.

In [6], Zhang et al., presented a detection and network coding scheme in a 2×2 MIMO relay system, where the relay extracts the summation and difference of the two end packets from the 2 sources, and then, converts them to the network-coded form. The summation and difference are achieved through linear transformation of the transmit symbols vector by a sum-difference (SD) matrix $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. With the received symbols not changing, a linear transformation of the transmit symbols vector by an SD matrix equally leads to the transformation of the channel by the inverse of the SD matrix. The SD scheme is adopted in this paper for our joint Massive-MIMO and PNC.

In this paper, we propose a massive SD matrix according to $\boldsymbol{P}_{\text{sd}} = \begin{bmatrix} \boldsymbol{I}_{\mathrm{Q}} & \boldsymbol{I}_{\mathrm{Q}} \\ \boldsymbol{I}_{\mathrm{Q}} & -\boldsymbol{I}_{\mathrm{Q}} \end{bmatrix}_{\mathrm{L\times L}}$, of which $\boldsymbol{P}_{\text{sd}}$ denotes the massive

SD matrix of dimension $L \times L$, where $L = K \times N$, $Q = \frac{1}{2}L$ and $\boldsymbol{I}_{\mathrm{Q}}$ denoting a $Q \times Q$ square Identity Matrix. $\boldsymbol{P}_{\mathrm{sd}}$ and its inverse, $\boldsymbol{P}_{\mathrm{sd}}^{-1}$, are related as $\boldsymbol{P}_{\mathrm{sd}} = 2\boldsymbol{P}_{\mathrm{sd}}^{-1}$. Thus, a linear transformation of the vector of source symbols from all UEs by the $\boldsymbol{P}_{\mathrm{sd}}$ and an equal transformation of the channel, $\boldsymbol{H}$, with the inverse of $\boldsymbol{P}_{\mathrm{sd}}$ leads to reformulating (3) as

$$\boldsymbol{r} = (\boldsymbol{H}\boldsymbol{P}_{\mathrm{sd}}{}^{-1})(\boldsymbol{P}_{\mathrm{sd}}\boldsymbol{s}) + \boldsymbol{z} \tag{5}$$

$$= \boldsymbol{H}_{\mathrm{sd}}\boldsymbol{s}_{\mathrm{sd}} + \boldsymbol{z} \ , \tag{6}$$

where the transformed channel matrix $\boldsymbol{H}_{\mathrm{sd}} = \boldsymbol{H}\boldsymbol{P}_{\mathrm{sd}}{}^{-1}$, and the transformed vector of source symbols $\boldsymbol{s}_{\mathrm{sd}} = \boldsymbol{P}_{\mathrm{sd}}\boldsymbol{s}$. The transformation creates a correlation of pairs of summation and difference symbols from pairs of antennas from multiple UEs. A vector of the transformed source symbols considering the Massive MIMO system model in Fig.1 is shown as

$$\begin{bmatrix} s_{\mathrm{sd}_{,1}} \\ s_{\mathrm{sd}_{,2}} \\ \vdots \\ s_{\mathrm{sd}_{,\mathrm{Q}}} \\ s_{\mathrm{sd}_{,\mathrm{Q+1}}} \\ \vdots \\ s_{\mathrm{sd}_{,2\mathrm{Q}}} \end{bmatrix}_{\mathrm{L}\times 1} = \begin{bmatrix} s_1 + s_{\mathrm{Q+1}} \\ s_2 + s_{\mathrm{Q+2}} \\ \vdots \\ s_{\mathrm{Q}} + s_{2\mathrm{Q}} \\ s_1 - s_{\mathrm{Q+1}} \\ \vdots \\ s_{\mathrm{Q}} - s_{2\mathrm{Q}} \end{bmatrix}_{\mathrm{L}\times 1} . \tag{7}$$

It can be observed from (7) that $s_{\mathrm{sd}_{,1}}$ and $s_{\mathrm{sd}_{,\mathrm{Q+1}}}$ are correlated because they form respectively, a summation and difference of the source symbols s_1 and $s_{\mathrm{Q+1}}$, and can therefore be paired for network coding. In this example, the source symbols s_1 and $s_{\mathrm{Q+1}}$ can come from antennas from the same UE or different UEs depending on the dimension of the massive SD matrix, hence

$$\begin{bmatrix} s_1 \oplus s_{\mathrm{Q+1}} \\ s_2 \oplus s_{\mathrm{Q+2}} \\ \vdots \\ s_{\mathrm{Q}} \oplus s_{2\mathrm{Q}} \end{bmatrix}_{\mathrm{Q}\times 1} = \begin{bmatrix} f(s_{\mathrm{sd}_{,1}}, s_{\mathrm{sd}_{,\mathrm{Q+1}}}) \\ f(s_{\mathrm{sd}_{,2}}, s_{\mathrm{sd}_{,\mathrm{Q+2}}}) \\ \vdots \\ f(s_{\mathrm{sd}_{,\mathrm{Q}}}, s_{\mathrm{sd}_{,2\mathrm{Q}}}) \end{bmatrix}_{\mathrm{Q}\times 1} . \tag{8}$$

The relation in (8) shows the vectors of SD symbols that are fed to the network coding function, f, which extracts the network-coded symbols. The function $f(x, y)$ maps the pair of inputs x and y to the designated network coded symbols $s_i \oplus s_j$, with i, j, respectively, representing the indices of the inputs, x and y in the SD-transformed vector of input source symbols. Evidently, as $N \gg K$, the pair of the SD symbols are more likely to come from different UEs.

Considering a frequency-flat and slow fading Rayleigh channel, each pair of the correlated SD symbols can be estimated or detected at the BS using linear detectors such as Zero Forcing (ZF) and Minimum Mean Square Error (MMSE). The detection of the SD symbols at the BS can be accomplished, if the channel, $\boldsymbol{H}$ is known. Knowing $\boldsymbol{H}$, the equalization matrix to estimate SD symbols, $\boldsymbol{G}_{\mathrm{sd}}$ is derived from the SD-transformed channel matrix, $\boldsymbol{H}_{\mathrm{sd}}$. The estimated SD transformed input sources symbols, $\widehat{\boldsymbol{s}}_{\mathrm{sd}}$, at the BS is expressed as

$$\widehat{\boldsymbol{s}}_{\mathrm{sd}} = \boldsymbol{G}_{\mathrm{sd}}\boldsymbol{r} \ , \tag{9}$$

TABLE I
PNC Mapping Of I/Q-Component Of QPSK, Based On SD Scheme

$\boldsymbol{s_1}$	$\boldsymbol{s_2}$	$\boldsymbol{s}_{\mathrm{sd}_{,1}} = \boldsymbol{s_1} + \boldsymbol{s_2}$	$\boldsymbol{s}_{\mathrm{sd}_{,2}} = \boldsymbol{s_1} - \boldsymbol{s_2}$	$\boldsymbol{s_1} \oplus \boldsymbol{s_2} = f(\boldsymbol{s}_{\mathrm{sd}_{,1}}, \boldsymbol{s}_{\mathrm{sd}_{,2}})$
1	1	2	0	-1
1	-1	0	2	1
-1	1	0	-2	1
-1	-1	-2	0	-1

where $\boldsymbol{G}_{\mathrm{sd}}$, respectively, for ZF and MMSE, are

$$\boldsymbol{G}_{\mathrm{sd}}{}^{\mathrm{ZF}} = (\boldsymbol{H}_{\mathrm{sd}}{}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{sd}})^{-1}\boldsymbol{H}_{\mathrm{sd}}{}^{\mathrm{H}} \ , \tag{10}$$

$$\boldsymbol{G}_{\mathrm{sd}}{}^{\mathrm{MMSE}} = (\boldsymbol{H}_{\mathrm{sd}}{}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{sd}} + \sigma^2 \mathrm{I})^{-1}\boldsymbol{H}_{\mathrm{sd}}{}^{\mathrm{H}}, \tag{11}$$

and where σ^2 is the noise variance at the receive antennas of the BS, which are assumed to be identical for all antennas, I, the identity Matrix and $(\cdot)^{\mathrm{H}}$, $(\cdot)^{-1}$, respectively, are the conjugate transpose and inverse.

B. PNC Mapping

Table I demonstrates how PNC is achieved for a pair of QPSK modulated symbols based on the SD Matrix transformation. We assume $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ are independent vectors of input sources from 2 antennas, each using QPSK modulation scheme. For the sake of simplicity, we will focus on only the in-phase (I) component constellation of QPSK. However, since quadrature (Q) component of QPSK is similar to the I-component and independently modulated or demodulated, the PNC mapping of the I-component is equally applicable to the Q-component. The SD symbols of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are $\boldsymbol{s}_{\mathrm{sd}_{,1}}$ and $\boldsymbol{s}_{\mathrm{sd}_{,2}}$, while $\boldsymbol{s}_1 \oplus \boldsymbol{s}_2$, is the network coded symbols vector, whose alphabets also conform to the input source's QPSK constellation. The bit-wise XOR operation, $\oplus$, is not applicable at the Physical Layer because it cannot be performed on modulated symbols, and therefore, in this very context, it only denotes network coding operation. The mapping of bits to the I-component of QPSK modulation is according to: $\{0, 1\} \rightarrow \{-1, +1\}$. This is how $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ are derived in Table I. Given that we want to estimate the set of PNC symbols, $\boldsymbol{s}_1 \oplus \boldsymbol{s}_2$, Maximum A Posteriori (MAP) estimator is employed. The goal of the MAP estimator is to determine the PNC symbol that maximizes the Likelihood of $\boldsymbol{s}_1 \oplus \boldsymbol{s}_2$, taking into consideration that each of the expected PNC symbols is equally likely, i.e. $(s_1 \oplus s_2)_{MAP} = \mathrm{argmax}_{s_1 \oplus s_2} P(r_1 r_2 | s_1 \oplus s_2)$. A compact approach is a Log-Likelihood Ratio based on the MAP estimator, which is employed to extract the PNC symbols of $+1$ and -1 from the decoded pair of SD symbols [6]. It has to be noted that the input source symbols do not necessarily have to be precoded with a massive SD matrix before transmission, as the received symbol vector, $\boldsymbol{r}$, does not change irrespective of either following (5) or (6). The same goes for the channel coefficient Matrix. Most importantly, the channel state information, $\boldsymbol{H}$, has to be known at the relay node or BS.

To simplify the explanation here, assuming r_1 and r_2 are the received symbols after $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ are transmitted over an AWGN channel, $\boldsymbol{H}$, and the pair, $\widehat{\boldsymbol{s}}_{\mathrm{sd}_{,1}}$, $\widehat{\boldsymbol{s}}_{\mathrm{sd}_{,2}}$, represent the estimated SD symbols after equalization (see (9) – (11)), then the Likelihood Ratio, LR, of $\boldsymbol{s}_1 \oplus \boldsymbol{s}_2$ is given as

$$LR(s_1 \oplus s_2) = \frac{P(r_1 r_2 | s_1 \oplus s_2 = +1)}{P(r_1 r_2 | s_1 \oplus s_2 = -1)} \ , \tag{12}$$

where $P(r_1 r_2 | s_1 \oplus s_2 = +1)$ is the conditional probability of receiving r_1 and r_2 given that the expected network coded symbol, $s_1 \oplus s_2$, is $+1$, and $P(r_1 r_2 | s_1 \oplus s_2 = -1)$ is the conditional probability of receiving r_1 and r_2 given that the expected network coded symbol, $s_1 \oplus s_2$, is -1.

Using the columns of 3, 4 and 5 in Table I, (12) is further expanded as

$$= \frac{P(r_1 | \widehat{s}_{\mathrm{sd}_{,1}} = 0)\Big(P(r_2 | \widehat{s}_{\mathrm{sd}_{,2}} = -2) + P(r_2 | \widehat{s}_{\mathrm{sd}_{,2}} = +2)\Big)}{P(r_2 | \widehat{s}_{\mathrm{sd}_{,2}} = 0)\Big(P(r_1 | \widehat{s}_{\mathrm{sd}_{,1}} = -2) + P(r_1 | \widehat{s}_{\mathrm{sd}_{,1}} = +2)\Big)}. \tag{13}$$

Based on the Gaussian probability density function, $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)}{2\pi\sigma^2}}$, where x is the received symbol, μ the expected value of the combined input symbols and σ^2, the noise variance, (13) above can further be simplified as follows

$$\begin{aligned}
LR(s_1 \oplus s_2) &= \frac{e^{-\frac{{r_1}^2}{2{\sigma_1}^2}}\left(e^{-\frac{(r_2+2)^2}{2{\sigma_2}^2}} + e^{-\frac{(r_2-2)^2}{2{\sigma_2}^2}}\right)}{e^{-\frac{{r_2}^2}{2{\sigma_2}^2}}\left(e^{-\frac{(r_1+2)^2}{2{\sigma_1}^2}} + e^{-\frac{(r_1-2)^2}{2{\sigma_1}^2}}\right)} \\
&= \frac{e^{-\frac{{r_1}^2}{2{\sigma_1}^2}} e^{-\frac{{r_2}^2+4}{2{\sigma_2}^2}}\left(e^{\frac{2r_2}{{\sigma_2}^2}} + e^{-\frac{2r_2}{{\sigma_2}^2}}\right)}{e^{-\frac{{r_2}^2}{2{\sigma_2}^2}} e^{-\frac{{r_1}^2+4}{2{\sigma_1}^2}}\left(e^{\frac{2r_1}{{\sigma_1}^2}} + e^{-\frac{2r_1}{{\sigma_1}^2}}\right)} \\
&= \frac{e^{-\frac{4}{2{\sigma_2}^2}}\left(e^{\frac{2r_2}{{\sigma_2}^2}} + e^{-\frac{2r_2}{{\sigma_2}^2}}\right)}{e^{-\frac{4}{2{\sigma_1}^2}}\left(e^{\frac{2r_1}{{\sigma_1}^2}} + e^{-\frac{2r_1}{{\sigma_1}^2}}\right)},
\end{aligned}$$

where ${\sigma_1}^2$ and ${\sigma_2}^2$ are respectively, the noise variances at the received antennas for the received symbols r_1 and r_2. The Log-Likelihood Ratio is then derived as

$$\begin{aligned}
LLR(s_1 \oplus s_2) &= \log\Big[LR\big(s_1 \oplus s_2\big)\Big] \\
&= \log\left[e^{\left(\frac{2}{{\sigma_1}^2} - \frac{2}{{\sigma_2}^2}\right)} \frac{\left(e^{\frac{2r_2}{{\sigma_2}^2}} + e^{-\frac{2r_2}{{\sigma_2}^2}}\right)}{\left(e^{\frac{2r_1}{{\sigma_1}^2}} + e^{-\frac{2r_1}{{\sigma_1}^2}}\right)}\right] \\
&= \log\left[\frac{\left(e^{\frac{2r_2-2}{{\sigma_2}^2}} + e^{-\frac{2r_2-2}{{\sigma_2}^2}}\right)}{\left(e^{\frac{2r_1-2}{{\sigma_1}^2}} + e^{-\frac{2r_1-2}{{\sigma_1}^2}}\right)}\right] \\
&= LL_2 - LL_1 \ ,
\end{aligned} \tag{14}$$

where

$$LL_2 = \log\left(e^{\frac{2r_2-2}{{\sigma_2}^2}} + e^{-\frac{2r_2-2}{{\sigma_2}^2}}\right) , \tag{15}$$

$$LL_1 = \log\left(e^{\frac{2r_1-2}{{\sigma_1}^2}} + e^{-\frac{2r_1-2}{{\sigma_1}^2}}\right) , \tag{16}$$

and ${\sigma_i}^2 = \{\boldsymbol{G}_{\mathrm{sd}}^{\mathrm{H}} \boldsymbol{G}_{\mathrm{sd}}\}_{i,i}\sigma^2$ is the noise variance of the i^{th} stream upon estimation of the SD-based received symbols, where in this case, $i = \{1, 2\}$.

The PNC mapping in column 5 of Table I can be reversed as in $\{\{\{0,-2\},\{0,2\}\},\{\{0,-2\},\{0,2\}\}\} \rightarrow \{-1,+1\}$, where the 2 elements in either the set $\{0,-2\}$ or $\{0,2\}$ are, respectively, $\boldsymbol{s}_{\mathrm{sd}_{,1}}$ and $\boldsymbol{s}_{\mathrm{sd}_{,2}}$, and $\{-1,+1\}$ is the set of the mapped I/Q-component symbols of QPSK. This mapping is only valid if bits to I/Q-component symbol mapping of QPSK is also reversed, as in $\{0,1\} \rightarrow \{+1,-1\}$. If this is the case, then the LLR will be as follows:

$$LLR(s_1 \oplus s_2) = LL_1 - LL_2. \tag{17}$$

To reduce the computational complexity, (15) and (16) can be approximated using the log sum of exponential property [11], $\log(e^x + e^y) \approx \max(x,y) + \log(1 + e^{-|x-y|})$, where $\max(x,y)$ is the maximum value of the two variables, x and y. The approximations are, hence, simplified as follows

$$LL_1 \approx \max\left(\frac{2r_1-2}{{\sigma_1}^2}, -\frac{2r_1-2}{{\sigma_1}^2}\right) + \log\left(1 + e^{-\left|\frac{4r_1}{{\sigma_1}^2}\right|}\right) , \tag{18}$$

$$LL_2 \approx \max\left(\frac{2r_2-2}{{\sigma_2}^2}, -\frac{2r_2-2}{{\sigma_2}^2}\right) + \log\left(1 + e^{-\left|\frac{4r_2}{{\sigma_2}^2}\right|}\right). \tag{19}$$

Finally, the PNC symbol for the pair, s_1 and s_2, for the I/Q-component of QPSK can be derived from the approximated LLR using (20) below.

$$\widehat{s_1 \oplus s_2} = \begin{cases} +1, \; LLR \geq 0 \\ -1, \; LLR < 0 \end{cases} . \tag{20}$$

The equation in (20) is for a pair of correlated SD symbols mapped to a PNC symbol. In Multi-user Massive MIMO, this can be generalized for any other pair of SD symbols according to

$$LLR_i(s_i \oplus s_j) = LL_i - LL_j \;, \tag{21}$$

where

$$LL_i \approx \max\left(\frac{2r_i - 2}{{\sigma_i}^2}, -\frac{2r_i - 2}{{\sigma_i}^2}\right) + \log\left(1 + e^{-\left|\frac{4r_i}{{\sigma_i}^2}\right|}\right) , \tag{22}$$

and where $i = 1, \ldots, Q$, $j = Q+1, \ldots, 2Q$, and i is the index of the LLR-based estimated symbols from the pair of correlated SD input source symbols, s_i and s_j.

Finally, for Massive MIMO, the estimated PNC mapped symbols are according to the equation below

$$\widehat{s_i \oplus s_j} = \begin{cases} +1, \; LLR_i \geq 0 \\ -1, \; LLR_i < 0 \end{cases} . \tag{23}$$

III. Simulation and Analysis

In this section, we present Monte-Carlo system simulation results to evaluate the performance of our joint Massive MIMO and Physical Layer Network Coding scheme. The goal of the simulation is to evaluate the Bit-Error-Rate (BER) of the massive SD scheme using linear detectors for the received symbols estimation.

TABLE II
Simulation Parameters

Parameters	***Values***
No. antennas at relay, M	2, 16, 32, 64, 120
No. antennas per UE, K	1, 4, 4, 4, 10
No. UEs, N	2, 4, 8, 16, 12
Channel, H	$i.i.d$ Rayleigh
SNR	0..25
Modulation	QPSK
Packet size per UE	100 QPSK symbols
No. iterations	10^4
Channel Coding	Uncoded
Linear Detectors	ZF, MMSE

The simulation parameters are listed in Table II. The Multi-user Massive MIMO simulation setup is flexible with respect to selecting particularly, the number of antennas at the relay, M, the number of UEs, N, and the number of antennas per UE, K. Therefore, first 3 rows in Table II have to be paired in interpreting the setup. For example, $\{2, 1, 2\}$, a set of values in the first elements of the first 3 rows, corresponds to MU-MIMO setup of $M = 2$, $N = 2$ and $K = 1$

The focus of the simulation was on uplink transmission. For the sake of simplicity, K is the same for all UEs, and each UE transmits 100 QPSK modulated symbols in every timeslot. The uplink transmission is over flat-fading i.i.d Rayleigh channel and at the BS's receive antennas, the channel noise is AWGN. The channel gains are randomly generated, considering SNR values ranging from 0 dB to 25 dB. The transmitted QPSK symbols are uncoded and the number of Monte-Carlo iterations for each simulation is 10K. The channel paths among the UEs are uncorrelated, and so are the individual antenna paths per UE. Linear detectors, such as ZF and MMSE are used.

Fig. 2 compares the error performance of ZF and MMSE against MIMO with and without PNC. Although this is not Massive MIMO, it is presented to compare with Zhang's 2×2 MIMO-PNC [6]. Our simulation result is consistent with that in [6], as 2×2 MIMO-PNC performs better than 2×2 MIMO-NC and MIMO without NC.

Fig. 3 and Fig. 4, respectively, show simulation results of the error performance of the joint 64×64 and 120×120 Multi-user Massive-MIMO systems with PNC, where PNC is achieved with LLR and the massive SD scheme. In these plots, joint Massive MIMO-PNC performs better than the Massive MIMO with NC and one without network coding, but the performance benefit is slightly higher in the lower half of the SNR. As the SNR moves towards the higher end, while M increases, the performance converges, particularly when ZF is used.

The simulation results indicate that PNC is feasible without necessarily decoding each of the input source symbols when the massive SD scheme is adopted in a Massive MIMO system. The benefit of this is increased capacity in uplink and also downlink when the PNC symbols are broadcasted, less SNR requirement using ZF and MMSE to detect SD transformed input source symbols, enhanced energy efficiency as PNC symbols that are broadcasted requires half of the relay antennas. It also shows that in cellular systems, UEs on the edge of the cell, where SNR is low, can be scheduled to transmit with QPSK when performing network coding.

IV. Conclusion

In this paper, we presented an approach that allows PNC to be deployed in Multi-user Massive MIMO system. The PNC Mapping scheme presented uses a massive SD matrix to transform channel coefficient matrix, the wireless channel between multiple users with multiple number of antennas and a relay or Base Station with massive number of antennas, and a vector of input streams from the multiple users, respectively into a massive SD channel coefficient and SD input streams. The SD transformed input streams, inherently, creates pairs of correlated streams that are paired for Network Coding. Using

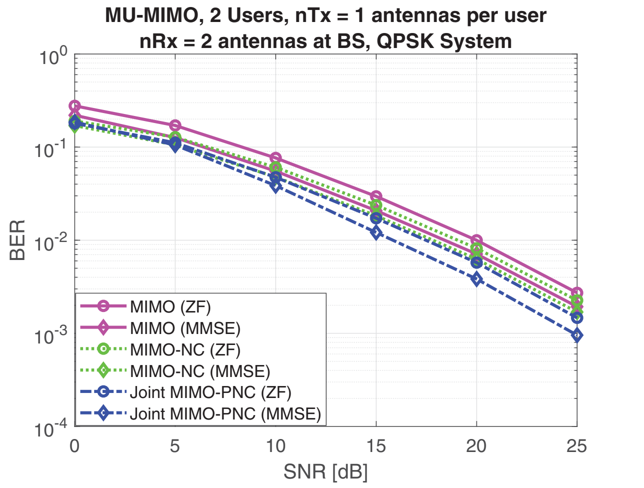

Fig. 2. Simulated BER vs SNR for QPSK (M=2, K=1, N=2) in a 2 × 2 MIMO System

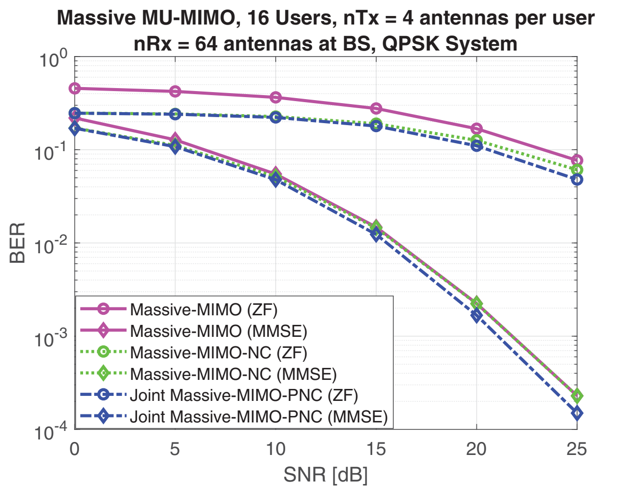

Fig. 3. Simulated BER vs SNR for QPSK (M=64, K=4, N=16) in a 64 × 64 Massive-MIMO System

an equalization matrix based on the massive SD transformed channel coefficient matrix, estimates of pairs of SD input streams can be extracted and after feeding this to a MAP-based Log-Likelihood Ratio, Network Coded streams are generated that can be broadcasted back to the multiple users.

A Monte-Carlo simulation was carried out to evaluate the error performance of the joint Multi-user Massive MIMO and PNC scheme. The modulation scheme adopted was QPSK and over 64 number of antennas at the relay was chosen. The BER against SNR, besides the success in achieving PNC in a Massive MIMO system, showed a better performance for the joint Massive MIMO and PNC scheme over Massive MIMO without PNC, although in the mid SNRs, they seem to converge. The results indicate that Network Coding, in general, is possible in Massive MIMO systems at the physical layer without decoding the individual input streams (PNC), and a

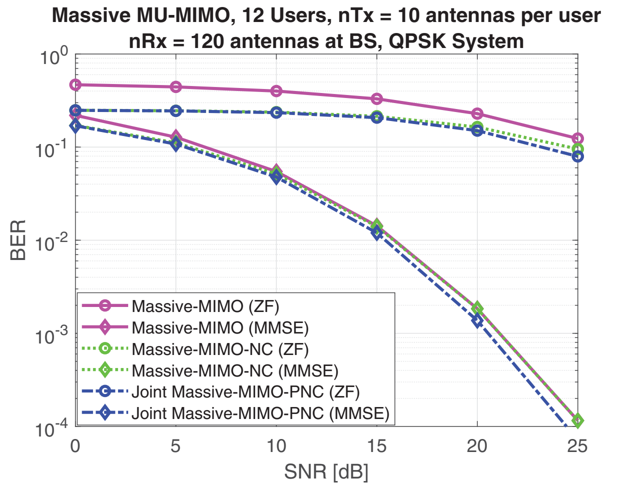

Fig. 4. Simulated BER vs SNR for QPSK (M=120, K=10, N=12) in a 120 × 120 Massive-MIMO System

better performance is achieved with QPSK-based transmission if the massive SD scheme is adopted.

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