Trajectory Optimization for Rotary-Wing UAVs in Wireless Networks with Random Requests

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Abstract—This paper studies the trajectory optimization problem in a scenario where a single rotary-wing UAV acts as a relay of data payloads for downlink transmission requests generated randomly by two ground nodes (GNs) in a wireless network. The goal is to optimize the UAV trajectory in order to minimize the expected average communication delay to serve these random requests. It is shown that the problem can be cast as a semi-Markov decision process (SMDP), and the resulting minimization problem is solved via multi-chain policy iteration. The optimality of a twoscale optimization approach is proved: the optimal trajectory in the communication phase greedily minimizes the communication delay of the current request while moving between the current start position and a target end position (inner optimization); the end positions are selected to minimize the expected average long-term delay in the SMDP (outer optimization). Numerical simulations show that the expected average delay is minimized when the UAV moves towards the geometric center of the GNs during phases in which it is not actively servicing transmission requests, and demonstrate significant improvements over sensible heuristics. Finally, it is revealed that the optimal end positions of communication phases become increasingly independent of the data payload, for large data payload values.

Index Terms—UAV-assisted wireless networks, adaptive trajectory optimization, semi-Markov decision process

I. INTRODUCTION

Recently, much research has gone into UAVs operating in wireless networks [1]–[5]. The drive for this is due to the unique benefits that UAVs acting as flying base stations, mobile relays, etc., provide in enhancing the overall network performance, thanks to their unique advantages over terrestrial counterparts in terms of mobility, maneuverability, and improved line-of-sight (LoS) link probability [1]. However, the design of UAV deployment strategies comes with challenges, namely the determination of optimal positioning or trajectories in the face of constraints imposed on UAV energy consumption, network throughput, and/or delay requirements [1]–[4].

Some research has focused on the trajectory optimization under energy constraints, as in [2] and [3]. In [6], the finegrained structure of LoS conditions is exploited to position UAVs optimally with the goal to maximize throughput. In [4], a model-free Q-learning approach is taken in the trajectory design so as to maximize the transmission sum-rate.

All of these efforts consider situations that are solved in the *offline* case, i.e., the pattern of transmission requests is known in advance, so that the trajectory may be pre-planned accordingly. However, this may be impractical as transmission requests are often random and cannot be determined in advance. In these cases, trajectory design is much more challenging, since it must be continuously adjusted based on the realization of these random processes, and incorporate the uncertainty in the future evolution of the system dynamics. In this paper, we investigate this problem by developing policies that adapt the trajectory based on the random realization of downlink transmission requests generated by two ground nodes (GNs), so as to optimize the average long-term performance.

To further motivate the need for this new formulation, consider the scenario depicted in Fig. 1. In this context, the minimum communication delay to serve GN₁ is achieved by flying towards it to improve the distance-dependent pathloss. With this design, for a sufficiently large data payload, the UAV will terminate the data transmission hovering above GN₁, where channel conditions are most favorable. However, if the UAV is to service a random request generated by GN_2 shortly after terminating the transmission to GN₁, the delay incurred to serve this second request may be large due to the large distance that separates the UAV from GN₂, causing severe pathloss conditions. In other words, under random transmission requests, the greedy delay minimization to serve a certain request may lead the UAV to a position where subsequent random requests cannot be served effectively, yielding poor delay performance in an average long-term sense. This example points to the need to incorporate the random nature of transmission requests in the trajectory design.

To address this question, we consider a scenario in which an UAV is serving two GNs far apart, and receives transmission requests according to a Poisson random process. We formulate the problem as that of designing an *adaptive* trajectory, so as to minimize the average long-term communication delay incurred to serve the requests of both GNs. We prove that the optimal trajectory in the communication phase operates according to a two-scale optimization: in the outer optimization, the UAV selects a target end position, which optimizes the trade-off between minimizing the delay of the current request, and minimizing the expected average long-term delay; then, in the inner optimization, the UAV travels greedily from the current position to the selected end position while communicating, following the trajectory that greedily minimizes the communication delay for the current request, provided in closed form. We utilize a multi-chain policy iteration algorithm to optimize the selection of the end position in the communication phase and the trajectory during the *waiting phase*, in which the UAV is not actively servicing downlink transmission requests. Our numerical results reveal that the UAV should always move towards the geometric center of the two GNs during the waiting phase, and that the optimal trajectory during communication phases becomes increasingly independent of the data payload and only determined by system parameters as the data payload value becomes sufficiently large.

The rest of the paper is organized as follows. In Sec. II, we introduce the system model and state the optimization

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problem; in Sec. III, we formalize the problem as a semi-Markov decision process (SMDP); in Sec. IV, we provide numerical results; lastly, in Sec. V, we conclude the paper with some final remarks.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

Consider the scenario depicted in Fig. 1, where one rotarywing UAV services two ground nodes (GNs) with random downlink¹ transmission requests of L bits. The two ground units GN₁ and GN₂ are located at positions $x_1=-a$ and $x_2=a$ along the x-axis, respectively, both at ground level (height 0). The UAV moves along the line segment connecting the two GNs, at height H from the ground. We let $q(t) \in [-a, a]$ be the UAV's position along the x-axis at time t, and we assume that it is either hovering or moving at speed V, hence $|q'(t)| \in \{0, V\}$, where f' denotes derivative of f over time.

A base station (BS) connected to the rest of the network is the source of downlink traffic to the two GNs. When a downlink request is generated by a certain GN, the BS transmits the data payload to the UAV, which then relays it to the GN using a decode and forward strategy [7]. We assume that the UAV has a high-capacity link to the BS, hence the communication link between the UAV and the GN constitutes the bottleneck of the overall BS-UAV-GN communication. In the rest of the paper, we thus focus on the UAV-GN communication and neglect the delay over the BS-UAV link. We assume that the UAV transmits at fixed power P_c and that the communication intervals experience LoS links with no probabilistic elements. This is motivated by the fact that UAVs in low-altitude platforms generally tend to have a much higher occurrence of LoS links [8]. We model the instantaneous communication rate between the UAV in position q(t) and $GN_r, r \in \{1, 2\}$ in position x_r as

$$R_r(q(t)) = B \log_2\left(1 + \frac{\gamma}{H^2 + (q(t) - x_r)^2}\right), \quad (1)$$

where $H^2 + (q(t) - x_r)^2$ is the squared distance between the UAV and GN_r , B is the channel bandwidth, and γ is the SNR referenced at 1 meter (see [3]).

When the UAV has no active transmission requests, future requests arrive according to a Poisson process with mean $\lambda/2$ requests/second, independently at each GN. Each request requires the transmission of L bits to the corresponding destination. Upon receiving a request from GN_r, the UAV enters the *communication phase*, where it services it by transmitting the L bits to GN_r; any additional requests received during this communication interval are dropped (see also Fig. 1). After the data transmission is completed, the UAV enters the *waiting phase*, where it awaits for new requests (with rate $\lambda/2$ for each GN), and the process is repeated indefinitely. During this periodic process of communication and waiting for new requests, the UAV follows a trajectory, part of our design, with the goal to minimize the average long-term communication delay, as discussed next.



Fig. 1: System model depicting a downlink transmission request from GN₁; the request from GN₂ is dropped during the active communication interval.

B. Problem Formulation

In this work, we consider the unconstrained delay minimization and neglect the propulsion energy consumption from our problem. In fact, it has been shown that a rotary-wing UAV exhibits comparable energy consumption when either moving or hovering [3]; in the special case when the moving and hovering powers are equal (for instance, based on the model in [3], this occurs at speed V=38 m/s), the finite energy in the UAV battery translates into a constraint on the total service time of the UAV, independent of the trajectory followed.

The goal is to define the optimal policy (UAV trajectory) so as to minimize the average communication delay. To this end, let Δ_l be the delay incurred to complete the transmission of the *l*th request serviced by the UAV. Let M_t be the total number of requests served and completed up to time *t*. Then, we define the expected average delay under a given trajectory policy μ (to be defined), starting from q(0) = 0 as²

$$\bar{D}_{\mu} = \lim_{t \to \infty} \mathbb{E}\left[\frac{\sum_{l=0}^{M_t - 1} \Delta_l}{M_t} \middle| q(0) = 0\right].$$
 (2)

We then seek to determine μ^* to minimize \bar{D}_{μ} , i.e.,

$$\mu^* = \arg\min_{\mu} \bar{D}_{\mu}.$$
 (3)

Note that this is a non-trivial optimization problem. While the minimum delay to serve a request, say from GN_1 , is achieved by flying towards GN_1 at maximum speed to improve the link quality, this strategy may not be optimal in an *average* delay sense: if the UAV receives a new request from GN_2 shortly after completing the request to GN_1 , the delay to serve this second request may be large due to the large distance that must be covered by the UAV.

C. Semi-Markov Decision Process (SMDP) formulation

In general, a solution to (3) would involve the optimization of an intractable number of variables over time (i.e., all possible trajectories followed by the UAV at any given time), over a continuous state space (the interval [-a, a]). Therefore,

¹This formulation and the analysis can be directly applied to uplink transmissions as well.

²While in practice the operation time of the UAV is constrained by the amount of energy stored in its battery, and the policy *should* depend on the amount of time left, the asymptotic case $t \to \infty$ is convenient since it gives rise to *stationary policies* (i.e., time-independent); this is a good approximation when the dynamics of the waiting and communication phases occur at much faster time scales than the total travel time, i.e., when M_t in (2) is large for practical values of the travel time t. For perspective, [9] places typical rotary-wing hovering endurance times in the 15-30 minute range.

it is advantageous to approximate the system model through discretization and reformulate (3) as an average-cost SMDP.

We define the state space as $S=\mathcal{I}\times\mathcal{R}$, where $\mathcal{R}=\{0,1,2\}$ denotes the request status, i.e., no active request (0), or a request is received from GN_r ($r \in \{1,2\}$), and

$$\mathcal{I} \triangleq \{-N, N+1, \dots, N-1, N\}$$
(4)

is the set of 2N + 1 indices corresponding to discretized positions $\mathcal{Q} \triangleq \{q_i = \frac{i}{N}a, \forall i \in \mathcal{I}\}$ along the interval $q(t) \in$ [-a, a]. This is a good approximation for sufficiently large N, as $\frac{a}{NV}\lambda \ll 1$, making the expected number of requests received over the travel time between two adjacent discretized positions much smaller than one. It is also useful to further partition the state space into *waiting states*, $\mathcal{S}_{wait} = \mathcal{I} \times \{0\}$, and *communication states*, $\mathcal{S}_{comm} = \mathcal{I} \times \{1, 2\}$.

To define this SMDP, we sample the continuous time interval to define a discrete sequence of states $\{s_n, n \ge 0\} \subseteq S$ with the Markov property. We now define the actions available, the transition probabilities, duration and cost of each state visit.

If the UAV is in state $s_n = (i, 0) \in S_{\text{wait}}$ at time t, i.e., it is in the discretized position q_i and there are no active requests, then the actions available are $a_n \in \{-1, 0, 1\}$, i.e. move right $(a_n=1 \text{ to position } q_{i+1})$, hover $(a_n=0)$, or move left by one discretized position $(a_n=-1 \text{ to } q_{i-1})$. The amount of time required to take this action, i.e., to fly between two adjacent discretized positions, is

$$\Delta_0 \triangleq \frac{a}{NV}.$$
(5)

The new state is then sampled at time $t+\Delta_0$, and is given by $s_{n+1}=(i+a_n, r_{n+1})$. The transition probability from state $s_n=(i,0)$ under action $a_n \in \{-1,0,1\}$ is then given by

$$\mathbb{P}(s_{n+1}=(i+m,0)|s_n=(i,0),a_n=m) = e^{-\lambda\Delta_0},$$
(6)
$$\mathbb{P}(s_{n+1}=(i+m,r)|s_n=(i,0),a_n=m) = \frac{1-e^{-\lambda\Delta_0}}{2}, \forall r \in \{1,2\},$$

depending on whether no request is received during this time interval $(r_{n+1}=0)$, with probability $e^{-\lambda\Delta_0}$, or a request is received from GN_r $(r_{n+1}=r \in \{1,2\})$, with probability $[1-e^{-\lambda\Delta_0}]/2$ for each GN).

Upon reaching state $s_n = (i, r) \in S_{\text{comm}}$ with $r \in \{1, 2\}$ at time t, the UAV has received a request to serve L bits to GN_r . The actions available at this point are all trajectories that start from q_i and allow the UAV to transmit the entire data payload of L bits. Assuming a *move and transmit* strategy (see [3]), the selected trajectory $q(\cdot)$ of duration Δ must satisfy

$$\int_{0}^{\Delta} R_{r}(q(\tau)) d\tau \ge L,$$
(7)

since all bits need to be transmitted during this phase. Under this trajectory, the communication delay is thus Δ . We define the action space in state $(i, r) \in S_{\text{comm}}$ as the set of all feasible trajectories, $\mathcal{T}_r(i) = \bigcup_j \mathcal{T}_r(i \to j)$, where we have defined $\mathcal{T}_r(i \to j)$ as the set of feasible trajectories starting in q_i , ending in q_j , and serving GN_r , i.e.,

$$\mathcal{T}_r(i \to j) = \left\{ q : [0, \Delta] \to [-a, a] : \int_0^\Delta R_r(q(\tau)) d\tau \ge L_r \right\}$$

$$|q'(t)| \le V, \ q(0) = q_i, \ q(\Delta) = q_j, \ \exists \Delta > 0 \bigg\}.$$
 (8)

Upon completing the communication phase, the UAV enters the waiting phase again; the new state is then sampled at time $t+\Delta$ (the amount of time elapsed to complete the selected trajectory), and is given by $s_{n+1}=(j,0)\in S_{\text{wait}}$, corresponding to the position q_j reached at the end of the communication phase. Thus, we have defined the transition probability in the SMDP from state $s_n = (i, r)$ under action $q \in \mathcal{T}_r(i \to j)$ as

$$\mathbb{P}(s_{n+1} = (j,0) | s_n = (i,r), q) = 1, \ \forall q \in \mathcal{T}_r(i \to j).$$
(9)

In other words, the trajectory selection process in the communication phase can be described via a two-scale decision process: 1) given (i, r), i.e., the current position q_i of the UAV and the request received from GN_r , the UAV first selects some $j \in \mathcal{I}$, which defines the target position q_j to be reached at the end of the communication phase; 2) the UAV selects a feasible trajectory q from $\mathcal{T}_r(i \rightarrow j)$, executes the trajectory while communicating to GN_r , and terminates the communication phase in the new position q_j , corresponding to state (j, 0). After this point, the UAV is in the waiting phase again.

With the states and actions defined, we can define a policy μ . Specifically, for states $(i, 0) \in S_{\text{wait}}$, $\mu(i, 0) \in \{-1, 0, 1\}$. Likewise, for states $(i, r) \in S_{\text{comm}}$, $\mu(i, r) = (j, q(\cdot))$, where $j \in \mathcal{I}$ (position reached at the end of the communication phase) and $q(\cdot) \in \mathcal{T}_r(i \to j)$ (feasible trajectory starting in q_i , ending in q_j , to serve GN_r).

The communication delay cost during the waiting phase is zero, i.e. $\Delta_{i,0}(m) = 0$, for all states $(i, 0) \in S_{\text{wait}}$ and actions $m \in \{-1, 0, 1\}$. When the UAV is in a communicating phase, we denote the communication delay incurred in state (i, r)under action $(j, q(\cdot))$ as $\Delta_{i,r}(j, q(\cdot))$. Compactly, we write $\Delta_s(\mu(s))$ to denote the delay incurred in state $s \in S$ under the action $\mu(s)$ dictated by policy μ .

With this notation, and having now defined a *stationary* policy μ , we can rewrite the average delay \bar{D}_{μ} in (2) in the context of the SMDP as

$$\bar{D}_{\mu} = \lim_{K \to \infty} \mathbb{E} \left[\frac{\frac{1}{K} \sum_{n=0}^{K-1} \Delta_{s_n}(\mu(s_n))}{\frac{1}{K} \sum_{n=0}^{K-1} \chi(s_n \in \mathcal{S}_{\text{comm}})} \middle| s_0 = (0,0) \right],$$
(10)

where $\chi(A)$ is the indicator function of the event A. In fact, the numerator in (2) counts the sample average delay incurred in the communication phases up to slot K of the SMDP, whereas the denominator in (2) counts the sample average number of communication slots in the SMDP up to slot K. Now, using Little's Theorem [10], we can rewrite (10) as

$$\bar{D}_{\mu} = \frac{\sum_{s \in \mathcal{S}} \Pi_{\mu}(s) \Delta_{s}(\mu(s))}{\sum_{s \in \mathcal{S}} \Pi_{\mu}(s) \chi(s \in \mathcal{S}_{\text{comm}})} = \frac{\sum_{s \in \mathcal{S}_{\text{comm}}} \Pi_{\mu}(s) \Delta_{s}(\mu(s))}{\sum_{s \in \mathcal{S}_{\text{comm}}} \Pi_{\mu}(s)},$$
(11)

where $\Pi_{\mu}(s)$ is the steady-state probability in the SMDP of the UAV being in state *s* under policy μ , and the second equality holds since $\Delta_s(\mu(s)) = 0$ and $\chi(s \in S_{\text{comm}}) = 0$ for $s \in S_{\text{wait}}$.

III. POLICY OPTIMIZATION AND ANALYSIS

In this section, we tackle the solution to the optimization problem (3), with D_{μ} given by (11). However, (3) cannot be directly solved using dynamic programming techniques, due to the presence of the denominator in (11), which depends on the policy selected μ , hence it affects the optimization. The next lemma demonstrates that the denominator of (11) can be expressed as a positive constant, *independent* from policy μ and only dependent on system parameters. In doing so, the optimization of μ only needs to focus on the minimization of $\sum_{s \in S} \prod_{\mu} (s) \Delta_s(\mu(s))$, so that (3) can be cast as an *aver*age cost per stage problem, solvable with standard dynamic programming techniques.

Lemma 1. Let π_{wait} and π_{comm} be the steady-state probabilities that the UAV is in the waiting and communication phases, $\pi_{\text{comm}} = \sum_{s \in \mathcal{S}_{\text{comm}}} \prod_{\mu}(s)$ and $\pi_{\text{wait}} = 1 - \pi_{\text{comm}}$. We have that

$$\pi_{\text{wait}} = \frac{1}{2 - e^{-\lambda \Delta_0}}, \ \pi_{\text{comm}} = \frac{1 - e^{-\lambda \Delta_0}}{2 - e^{-\lambda \Delta_0}}.$$
 (12)

Proof. Let p_{ww} , p_{wc} , p_{cw} , and p_{cc} be the probabilities of a state request status, $r \in \mathcal{R} = \{0, 1, 2\}$, transitioning in the SMDP as $0 \rightarrow 0, 0 \rightarrow \{1, 2\}, \{1, 2\} \rightarrow 0$, and $\{1, 2\} \rightarrow \{1, 2\}, \{1$ respectively. Then, $p_{ww} = e^{-\lambda \Delta_0}$ (if no request is received, the SMDP remains in the waiting state), $p_{wc} = 1 - p_{ww}$, $p_{cw} = 1$, and $p_{cc} = 0$ (if the SMDP is in the communication state, the next state of the SMDP will be a waiting state, see (9)). Therefore, the steady-state probabilities of being in the waiting and communication states, π_{wait} and π_{comm} , satisfy

$$\pi_{\text{wait}} = p_{ww} \pi_{\text{wait}} + p_{cw} \pi_{\text{comm}} = e^{-\lambda \Delta_0} \pi_{\text{wait}} + \pi_{\text{comm}},$$

$$\pi_{\text{comm}} = p_{wc} \pi_{\text{wait}} + p_{cc} \pi_{\text{comm}} = (1 - e^{-\lambda \Delta_0}) \pi_{\text{wait}},$$

$$\pi_{\text{wait}} + \pi_{\text{comm}} = 1,$$

whose solution is given as in the statement of the lemma.

When we refer to the denominator of (11), it is evident that it is equal to the steady-state probability that the UAV is in a communication state while following policy μ , π_{comm} . However, with the result of Lemma 1, π_{comm} is simply a positive constant determined by system parameters, yielding

$$\bar{D}_{\mu} = \frac{\sum_{s \in \mathcal{S}} \Pi_{\mu}(s) \Delta_s(\mu(s))}{\pi_{\text{comm}}},$$
(13)

which we now aim to minimize with respect to policy μ .

As the problem stands now, the *communication* phase selects an action from $\mathcal{T}_r(i)$, which is a set containing an uncountable number of trajectories. By exploiting the two-scale structure of the problem outlined earlier, we now demonstrate that only a finite set of trajectories from $\mathcal{T}_r(i)$ are eligible to be optimal, for each state $(i, r) \in S_{comm}$, hence making the problem a finite state and action SMDP.

A. Decomposition of Policy μ

Note from (9) that the transition probability from a communication state $s_n = (i, r)$ under action $(j, q(\cdot))$ is only affected by the selection of j and not the particular trajectory $q(\cdot) \in \mathcal{T}_r(i \to j)$ that leads from q_i to q_j during the communication phase. It follows that the steady-state probability Π_{μ}

under $\mu(i, r) = (j, q(\cdot))$ is only affected by the selection of j and not the specific trajectory within $\mathcal{T}_r(i \to j)$.

By establishing this property, we decompose the policy μ into the waiting policy $\theta(i) \in \{-1, 0, 1\}$, which defines the optimal action in state $(i, 0) \in S_{wait}$ of the waiting phase; the end position policy J(i, r), which selects the end position q_i with j=J(i,r) to be reached at the end of the communication phase; and the *trajectory policy* $\rho(i, r, j)$ which, given j=J(i,r), selects a trajectory $q(\cdot)=\rho(i,r,j)$ from $\mathcal{T}_r(i \to j)$. Owing to the independence of Π_{μ} on the trajectory policy ρ , the delay minimization problem can then be rewritten as

$$\bar{D}_{\mu}^{*} = \frac{\min_{\theta, J} \sum_{s \in \mathcal{S}_{\text{comm}}} \Pi_{\theta, J}(s) \min_{\rho(s, J(s)))} \Delta_{s}(J(s), \rho(s, J(s)))}{\pi_{\text{comm}}}$$

Letting

$$\Delta_r^*(i,j) \stackrel{\triangleq}{=} \min_{q(\cdot) \in \mathcal{T}_r(i \to j)} \Delta_{i,r}(j,q), \quad \forall \ (i,r) \in \mathcal{S}_{\text{comm}}, \forall j \in \mathcal{I}, \quad (14)$$

we can finally write

$$\bar{D}^*_{\mu} = \frac{\min_{\theta, J} \sum_{(i,r) \in \mathcal{S}_{\text{comm}}} \Pi_{\theta, J}(i, r) \Delta^*_r(i, J(i, r))}{\pi_{\text{comm}}}.$$
 (15)

Note that $\Delta_r^*(i, j)$ yields the trajectory that greedily minimizes the communication delay when starting from state (i, r), ending in position q_i while serving GN_r . This result proves that, for any communication state (i, r), there exist only 2N + 1 trajectories that are eligible to be optimal, one for each possible ending position $q_i \in \mathcal{Q}$. Hence, the problem is finally reduced to that of finding the optimal waiting policy θ and end position policy J, which can be solved efficiently via dynamic programming (Algorithm 1). In the next section, we provide a closed form expression of $\Delta_r^*(i, j)$.

B. Closed-form Delay Minimizing Trajectory

With the independence of the steady-state probabilities from ρ , we can proceed to solve (14) and then provide the dynamic programming algorithm to solve for θ^* and J^* in (15). By definition of $\mathcal{T}_r(i \rightarrow j)$ in (II-C), we can rewrite $\Delta_r^*(i, j)$ as

$$\Delta_r^*(i,j) = \min_{\Delta,q} \left\{ \Delta \left| \int_0^\Delta R_r(q(\tau)) d\tau \ge L, \right. \\ \left. |q'(\tau)| \le V, q(0) = q_i, q(\Delta) = q_j \right\}.$$
(16)

The minimizer q^* is the trajectory that the UAV should follow when receiving a request from GN_r starting in position q_i and ending in position q_i , selected by the end position policy J.

In defining the optimal trajectory, the following definitions will be useful. Let $\tau_{p_1,p_2} \triangleq \frac{|p_2-p_1|}{V}$ be the time needed to fly at maximum speed from p_1 to $p_2 \in [-a, a]$. Along this straight trajectory, let

$$\ell_{p_1,p_2}^{(r)} \triangleq \int_0^{\tau_{p_1,p_2}} R_r \left(p_1 + \frac{\tau}{\tau_{p_1,p_2}} (p_2 - p_1) \right) d\tau \qquad (17)$$

be the amount of bits transmitted to serve GN_r. Clearly, $\ell_{p_1,p_1}^{(r)} = 0$ ($\tau_{p_1,p_1} = 0$), $\ell_{p_1,p_2}^{(r)} = \ell_{p_2,p_1}^{(r)}$ ($\tau_{p_1,p_2} = \tau_{p_2,p_1}$), and $\ell_{p_1,p_2}^{(1)} = \ell_{-p_1,-p_2}^{(2)}$ ($\tau_{p_1,p_2} = \tau_{-p_1,-p_2}$). The integral $\ell_{p_1,p_2}^{(r)}$ can be determined in closed form and is found in [2], for example. We also define the trajectory $v\{p_1 \rightarrow (p_2, \delta) \rightarrow p_3\}(\tau), \tau \in$

 $[0, \delta + \tau_{p_1, p_2} + \tau_{p_2, p_3}]$, as the one in which the UAV starts at position p_1 , flies at maximum speed to p_2 , hovers at p_2 for δ amount of time, and finally flies at maximum speed from p_2 to p_3 . Mathematically,

$$v\{p_{1} \to (p_{2}, \delta) \to p_{3}\}(\tau)$$

$$=\begin{cases} p_{1} + \frac{\tau}{\tau_{p_{1}, p_{2}}}(p_{2} - p_{1}), & \tau \in [0, \tau_{p_{1}, p_{2}}] \\ p_{2}, & \tau \in [\tau_{p_{1}, p_{2}}, \tau_{p_{1}, p_{2}} + \delta] \\ p_{2} + \frac{\tau - \tau_{p_{1}, p_{2}} - \delta}{\tau_{p_{2}, p_{3}}}(p_{3} - p_{2}), & \tau \in [\tau_{p_{1}, p_{2}} + \delta, \tau_{p_{1}, p_{2}} + \tau_{p_{2}, p_{3}} + \delta]. \end{cases}$$

$$(18)$$

The traffic delivered to GN_r when following this trajectory is $\ell_{p_1,p_2}^{(r)} + \delta R_r(p_2) + \ell_{p_2,p_3}^{(r)}$, with delay $\tau_{p_1,p_2} + \delta + \tau_{p_2,p_3}$. With these definitions, we are now ready to state the main result.

Theorem 1. Let $q^*(\cdot) \in \mathcal{T}_r(i \to j)$ be the trajectory that minimizes the communication delay $\Delta_r^*(i, j)$. If $\ell_{q_i,q_j}^{(r)} \ge L$, then

$$q^*(\cdot) = v\{q_i \to (q_j, 0) \to q_j\}(\cdot), \ \Delta_r^*(i, j) = \tau_{q_i, q_j},$$
 (19)

i.e., the UAV flies at maximum speed from q_i *to* q_j *without interruption; otherwise, if* $\ell_{q_i,x_r}^{(r)} + \ell_{x_r,q_i}^{(r)} \leq L$, then

$$q^*(\cdot) = v\{q_i \to (x_r, \delta^*) \to q_j\}(\cdot), \ \Delta^*_r(i, j) = \tau_{q_i, x_r} + \tau_{x_r, q_j} + \delta^*,$$

where

$$\delta^* = \frac{L - \ell_{q_i, x_r}^{(r)} - \ell_{x_r, q_j}^{(r)}}{R_r(x_r)};$$
(20)

i.e., the UAV flies at maximum speed from q_i to x_r , hovers over x_r for δ^* amount of time, and then flies to q_j ; finally, if $\ell_{q_i,x_r}^{(r)} + \ell_{x_r,q_j}^{(r)} > L$, but $\ell_{q_i,q_j}^{(r)} < L$, then

$$q^*(\cdot) = v\{q_i \to (p^*, 0) \to q_j\}(\cdot), \ \Delta_r^*(i, j) = \tau_{q_i, p^*} + \tau_{p^*, q_j},$$

where p^* is the unique solution in $[x_r, \min\{q_i, q_j\}]$ (if r=1) or $[\max\{q_i, q_j\}, x_r]$ (if r=2) of $\ell_{q_i, p^*}^{(r)} + \ell_{p^*, q_j}^{(r)} = L$; i.e., the UAV flies at maximum speed towards x_r to the farthest point p^* and then back to q_j , with p^* uniquely defined in such a way as to transmit exactly the data payload upon reaching q_j .

Proof. Due to space limitations, we provide an outline of the proof. Assume r=2 (a similar argument applies to r=1 by symmetry). 1) for any trajectory $q(\cdot) \in \mathcal{T}_2(i \to j)$ of duration Δ , one can construct another trajectory $\tilde{q}(\cdot) \in \mathcal{T}_2(i \to j)$ of same duration Δ , and such that $|q(t) - x_r| \ge |\tilde{q}(t) - x_r|$, $\forall t \in [0, \Delta]$; such trajectory is obtained by flying at maximum speed towards GN₂, possibly hovering on top of GN₂ for δ amount of time (if time allows), and then returning to q_j , yielding $\tilde{q}(\cdot)=v\{q_i\to(p^*,\delta^*)\to q_j\}(\cdot)$, for a proper choice of p^* and δ^* such that $\tau_{q_i,p^*}+\tau_{p^*,q_j}+\delta=\Delta$; 2) note that the UAV is always closer to GN₂ under $\tilde{q}(\cdot)$ than it is under $q(\cdot)$, hence it delivers a larger data payload than $q(\cdot)$ while incurring the same delay; therefore, $q(\cdot)$ is suboptimal; 3) $\tilde{q}(\cdot)$ can be further improved by minimizing the delay (by optimizing (p^*, δ^*)), yielding the three cases provided in the statement of the theorem.

C. Multi-chain Policy Iteration (PI) Algorithm

We opt to use a multi-chain PI algorithm to solve (15), as there exist some policies whose induced Markov chain structures are multi-chain. For example, if the *waiting policy* is $\theta(i) = 0$, and the *end position policy* is J(i, r) = i, then the induced Markov chain has 2N+1 recurrent classes (hence multi-chain). To accommodate this structure, the pseudocode that follows is based upon the multi-chain PI methods of [11] and succinctly describes how to solve for μ^* .

In Algorithm 1, we use a vector notation for $\mathbf{\bar{D}}_k$ and \mathbf{h}_k , which denote the average delay and relative value for all states, respectively, following the *k*th policy iterate $\mu_{(k)}$. Likewise, \mathbf{c}_{μ} is the vector notation for the delay cost function under policy μ , supplemented by the optimal minimized communication delays described by (14) and (16), and \mathbf{P}_{μ} is the transition matrix under policy μ .

Algorithm	1	Multi chain	DI to	colve (15)	
AIYOFILIIII		wiulu-chain	$\mathbf{r} \mathbf{u}$	SOIVE (1.)	,

- 1: Initialize k = -1, arbitrary policy $\mu_{(0)}$;
- 2: repeat

3: $k \leftarrow k+1$

- 4: Evaluation: Solve for gain $\bar{\mathbf{D}}_k$ and relative value \mathbf{h}_k under policy $\mu_{(k)}$ by gain-relative value optimality equations [11];
- 5: Improvement: Find $\mu_{(k+1)} \in \arg \min_{\mu} \{ \mathbf{P}_{\mu} \mathbf{\bar{D}}_{k} \};$ choose $\mu_{(k+1)} = \mu_{(k)}$ if $\min_{\mu} \{ \mathbf{P}_{\mu} \mathbf{\bar{D}}_{k} \} = \mathbf{P}_{\mu_{(k)}} \mathbf{\bar{D}}_{k};$
- 6: **if** $\mu_{(k+1)} = \mu_{(k)}$ **then**
- 7: Find $\mu_{(k+1)} \in \arg \min_{\mu} \{ \mathbf{c}_{\mu} + \mathbf{P}_{\mu} \mathbf{h}_{k} \}$; choose $\mu_{(k+1)} = \mu_{(k)}$ if $\min_{\mu} \{ \mathbf{c}_{\mu} + \mathbf{P}_{\mu} \mathbf{h}_{k} \} = \mathbf{c}_{\mu_{(k)}} + \mathbf{P}_{\mu_{(k)}} \mathbf{h}_{k}$;

9: **until** $\mu_{(k+1)} = \mu_{(k)}$; return $\mu^* = \mu_{(k+1)}$.

IV. NUMERICAL RESULTS

We use the following system parameters, unless specified otherwise: number of states 2N+1=101; channel bandwidth B=1MHz; 1-meter reference SNR $\gamma_{dB}=40$ dB; UAV height H=100m; GN locations $x_1=-400$ m, $x_2=400$ m; UAV speed V=20m/s; and request arrival rate $\lambda=0.4$ requests/second.

We vary the data payload L across a range of values and find numerically that, regardless, the optimal policy in the *waiting phase* optimized with Algorithm 1 is

$$\theta^*(i) = \begin{cases} 1, & i \in \{-N, -N+1, ..., -1\} \\ 0, & i = 0 \\ -1, & i \in \{1, 2, ..., N\}. \end{cases}$$
(21)

In other words, in the *waiting phase* it is optimal for the UAV to move towards the geometric center of the two GNs along the line segment connecting the two. Intuitively, the UAV can more readily service a request that is originated equally likely from GN_1 or GN_2 , if it is located in the geometric center when the request arrives, since the UAV is equally distant from both GNs, and can thus serve them equally well.

In Fig. 2, we plot the optimal *end position policy* $J^*(i, 2)$ for different data payload values.³ We note that, for large data payload values L, the optimal end position in the communication phase becomes independent of the initial position i (in this case, $J^*(i, 2) \approx 336$ m, irrespective of i for $L \gg 1$). In fact, for large data payload L, the UAV hovers over the receiver for a significant amount of time during the communication phase

³We omit the figure for states $(i, 1) \in S_{\text{comm}}$, due to the inherent symmetry of the problem. Specifically, if the optimal end point $J^*(i, 2) = j$ is observed, then $J^*(-i, 1) = -j$ is also observed.



Fig. 2: End position in the communication phase as a function of the start position under the optimal policy, when transmitting to GN_2 in position *a*, for different values of the data payload. The small fluctuations are due to the discretization of the state space.



Fig. 3: End position for all states (i, 2) in the communication phase as a function $1/\lambda$, when transmitting to GN₂ in position *a*, for a fixed large data payload L = 15 Mbits (varied across UAV height).

(case $\ell_{q_i,x_r}^{(r)} + \ell_{x_r,q_j}^{(r)} \leq L$ in Theorem 1), hence the final part of the trajectory from x_r to the selected end position q_j becomes irrespective of the actual data payload value. However, $J^*(i, 2)$ does depend on other system parameters, such as the request rate λ and UAV height H, as seen in Fig. 3. Interestingly, as the request rate increases (the inter-arrival request time $1/\lambda$ decreases) the end position is closer to the geometric center (i.e., farther away from the receiver); this is because requests arrive more often, hence it is desirable for the UAV to terminate the communication phase closer to the center, in order to more readily serve future requests.

Next, we illustrate how the optimal expected average delay \bar{D}^*_{μ} , across the same set of data payload values, fares against a heuristic policy which operates as follows: in the <u>waiting phase</u>, hover in the current position; in the <u>data communication phase</u>, greedily minimize the delay by flying at maximum speed towards the receiver until completion. The comparison between the optimal policy μ^* and the heuristic policy is shown for the span of data payload values in Fig. 4. Note that the slope of the line for both the optimal and heuristic policies saturates to $[B \log_2(1+\gamma/H^2)]^{-1}$. In fact, when $L\gg1$, the UAV spends most of the communication time hovering above the receiver (case $\ell_{q_i,x_r}^{(r)} + \ell_{xr,q_j}^{(r)} \leq L$ in Theorem 1), hence $\Delta^*_r(i, j) \approx \frac{L}{R_r(x_r)}$ in (15), yielding

$$\bar{D}^*_{\mu} \approx \frac{\min_{\theta,J} \sum_{(i,r) \in \mathcal{S}_{\text{comm}}} \Pi_{\theta,J}(i,r)L}{\pi_{\text{comm}} B \log_2(1+\gamma/H^2)} = \frac{L}{B \log_2(1+\gamma/H^2)}$$



Fig. 4: Comparison of expected average delay \bar{D}_{μ} vs. data payload L for both optimal and heuristic policy.

Overall, the heuristic scheme performs worse, roughly by 2 seconds for large L. In fact, when hovering during the waiting phase instead of moving towards the center, the UAV incurs a larger delay to serve a request generated by the more distant GN, due to the longer distance that needs to be covered.

V. CONCLUSIONS

In this paper, we studied the trajectory optimization problem of one UAV servicing random downlink transmission requests by two GNs, to minimize the expected communication delay. We formulated the problem as an SMDP, and exploited the structure of the problem to simplify the trajectory design in the communication phase. We showed that the problem exhibits an interesting two-scale structure in the optimal trajectory design, and can be solved efficiently via dynamic programming. Numerical evaluations demonstrate consistent improvements in the delay performance over a sensible heuristic, for a variety of data payload values.

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