

UC Irvine

UC Irvine Electronic Theses and Dissertations

Title

Age of Information in Multiple Sensing

Permalink

<https://escholarship.org/uc/item/0636c28g>

Author

Javani, Alireza

Publication Date

2021

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA,
IRVINE

Age of Information in Multiple Sensing

THESIS

submitted in partial satisfaction of the requirements
for the degree of

MASTER OF SCIENCE

in Electrical Engineering & Computer Science

by

Alireza Javani

Thesis Committee:
Professor Zhiying Wang, Chair
Professor Hamid Jafarkhani
Professor Ender Ayanoglu

2021

DEDICATION

I dedicate this thesis to my lovely father whom I didn't see for a long time. Undeniably, I would not be the place that I am now if it was not because of your sincere support. I will miss you forever.

Contents

	Page
LIST OF FIGURES	iv
LIST OF TABLES	v
ACKNOWLEDGMENTS	vi
ABSTRACT OF THE THESIS	vii
1 Introduction	1
1.1 Background	3
2 AoI in Homogeneous Networks	8
2.1 Single Source and Multiple Servers	8
2.2 Multiple Sources and Multiple Servers	14
3 AoI in Heterogeneous Networks	21
3.1 Heterogeneous Networks with a Single Source	21
3.2 Conclusion	26
Bibliography	28

List of Figures

	Page
1.1 The n -server monitoring network with S_1, S_2, \dots, S_n being the servers and I_1, I_2, \dots, I_m being the independent information sources, sending the updates from the sources to the monitor.	5
1.2 AoI for a network with n servers.	5
2.1 SHS for a single source and n homogeneous servers.	9
2.2 AoI versus the number of servers, for fixed total arrival rate. For each server, the service rate $\mu = 1$ and the total arrival rate $n\lambda$ is shown in the x-axis. . .	13
2.3 Comparison of LCFS, FCFS, and LCFS with preemption in waiting (LCFS-W). The number of servers is $n = 4$ and $\mu = 1$ for each server.	14
3.1 Optimal value of λ_1 as a function of μ_1 . $\lambda_1 + \lambda_2 = \lambda, \mu_1 + \mu_2 = 100$	24

List of Tables

	Page
2.1 Table of transitions for a single source and n homogeneous servers.	10
3.1 Table of transitions for $n = 2$ heterogeneous servers.	22

ACKNOWLEDGMENTS

First and foremost, I would like to thank my advisor Prof. Zhiying Wang for her consistent guidance from the beginning of this project up to its last moments. Her kind advice, and technical instructions showed me the proper direction to succeed in this project. My sincere gratitude goes to her because she devotedly and patiently cooperated in our publication.

I would also like to thank Dr. Marwen Zorgui for his time and helps during this project which have been undoubtedly effective.

My appreciation also extends to committee members, Prof. Jafarkhani and Prof. Ayanoglu not only because of their valuable time reviewing but also because I learned many concepts in their courses which indirectly contributes to this project.

At the end, I should give credit to all referenced publishers who inspired me for doing this project and pursuing this line of research.

ABSTRACT OF THE THESIS

Age of Information in Multiple Sensing

By

Alireza Javani

Master of Science in Electrical Engineering & Computer Science

University of California, Irvine, 2021

Professor Zhiying Wang, Chair

Having timely and fresh knowledge about the current state of information sources is critical in a variety of applications. In particular, a status update may arrive at the destination much later than its generation time due to processing and communication delays. The freshness of the status update at the destination is captured by the notion of age of information. In this study, we first analyze a network with a single source, n servers, and the monitor (destination). The servers independently sense the source of information and send the status update to the monitor. We then extend our result to multiple independent sources of information in the presence of n servers. We assume that updates arrive at the servers according to Poisson random processes. Each server sends its update to the monitor through a direct link, which is modeled as a queue. The service time to transmit an update is considered to be an exponential random variable. We examine both homogeneous and heterogeneous service and arrival rates for the single-source case, and only homogeneous arrival and service rates for the multiple sources case. We derive a closed-form expression for the average age of information under a last-come-first-serve (LCFS) queue for a single source and arbitrary n homogeneous servers. For $n = 2, 3$, we derive the explicit average age of information for arbitrary sources and homogeneous servers, and for a single source and heterogeneous servers. For $n = 2$ we find the optimal arrival rates given fixed sum arrival rate and service rates.

Chapter 1

Introduction

Widespread sensor network applications such as health monitoring using wireless sensors [1] and the Internet of things (IoT) [2, 3], as well as applications like stock market trading and vehicular networks [4], require sending several status updates to their designated recipients (called monitors). Outdated information in the monitoring facility may lead to undesired situations. As a result, having the data at the monitor as fresh as possible is crucial.

In order to quantify the freshness of the received status update, the age of information (AoI) metric was introduced in [5]. For an update received by the monitor, AoI is defined as the time elapsed since the generation of the update. AoI captures the timeliness of status updates, which is different from other standard communication metrics like delay and throughput. It is affected by the inter-arrival time of updates and the delay that is caused by queuing during update processing and transmission.

The AoI has been applied to different network models as a performance metric for various communication systems that timeliness of data is critical, e.g., trust-aware resource allocation schemes [6, 7], source nodes powered by energy harvesting [8–12], wireless erasure networks and coding [13–17], scheduling in networks [18–22], and unmanned aerial vehicle

(UAV)-assisted communication systems [23–26].

In [5], authors considered the single-source single-server and first-come-first-serve (FCFS) queue model and determined the arrival rate that minimizes AoI. Different cases of multiple-source single-server under FCFS and last-come-first-serve (LCFS) were considered in [27] and the region of feasible age was derived. In [28,29], the system is modeled as a source that submits status updates to a network of parallel and serial servers, respectively, for delivery to a monitor and AoI is evaluated. The parallel-server network is also studied in [30] when the number of servers is 2 or infinite, and the average AoI for FCFS queue model was derived.

Authors in [31] formulated a discrete-time decision problem in order to find a scheduling policy for minimizing the expected weighted sum of AoI. A multi-source multi-hop setting in broadcast wireless networks was investigated in [32] and a fundamental lower bound on the average AoI was derived. Different scheduling policies with throughput constraints were considered in [33] to minimize AoI. Another age-related metric of peak AoI was introduced in [34], which corresponds to the age of information at the monitor right before the receipt of the next update. The average peak AoI minimization in IoT networks and wireless systems was considered in [35,36]. The problem of minimizing the average age in energy harvesting sources by manipulating the update generation process was studied in [37,38]. Maximizing energy efficiency of wireless sensor networks that include constraints on AoI is investigated in [39].

We consider AoI in a multiple-server network. We assume that a number of shared sources are sensed and then the data is transmitted to the monitor by n independent servers. For example, the sources of information can be some shared environmental parameters, and independently operated sensors in the surrounding area obtain such information. For another example, the source of information can be the prices of several stocks which is transmitted to the user by multiple independent service providers. Throughout this thesis, a sensor or a service provider is called a server, since it is responsible to serve this update to the monitor.

We assume that status updates arrive at the servers independently according to Poisson random processes, and the server is modeled as a queue whose service time for an update is exponentially distributed. We assume information sources are independent and are sensed by n independent servers.

In this thesis we study the average age of information as in [5]. We mainly consider LCFS with preemption in service (in short, LCFS) queue model, namely, upon the arrival of a new update, the server immediately starts to serve it and drops any old update being served. We derive a closed-form formula of the average AoI for LCFS and a single source. For multiple sources, AoI formula is derived for arbitrary number of sources and $n = 2, 3$ servers. In addition, the heterogeneous network with a single source is considered. To obtain the AoI, we use the stochastic hybrid system (SHS) analysis similar to [27, 28].

This thesis is organized as follows. Section 1.1 formally introduces the system model of interest, and provides preliminaries on SHS. In subsection 2.1, we derive the average age of information formula by applying SHS method to our model when we have a single information source and the network is homogeneous. In subsection 2.2 we derive AoI for arbitrary number of information sources when $n = 2, 3$. In section 3.1, we investigate the heterogeneous network when we have a single source and $n = 2, 3$ and find the optimal arrival rate at each server when $n = 2$. At the end, the conclusion follows in section 3.2.

1.1 Background

Notation: we use boldface for vectors, and normal font with a subscript for its elements. For example, for a vector \mathbf{x} , the j -th element is denoted by x_j . For non-negative integers a and $b \geq a$, we define $[a : b] \triangleq \{a, \dots, b\}$, $[a] \triangleq [1 : a]$. If $a > b$, $[a : b] = \emptyset$.

In this section, we first present our network model, and then briefly review the stochastic

hybrid system analysis from [27]. The network consists of m information sources that are sensed by n independent servers as illustrated in Figure 1.1. Updates after going through separate links are aggregated at the monitor side. The interest of this thesis is the average AoI at the monitor. Server j collects updates of source i following a Poisson random process with rate $\lambda_j^{(i)}$ and the service time is an exponential random variable with average $\frac{1}{\mu_j}$, independent of all other servers, $j \in [n], i \in [m]$. A network is called homogeneous if $\lambda_j^{(i)} = \lambda^{(i)}, \mu_j = \mu$, for all $j \in [n], i \in [m]$, otherwise, it is heterogeneous. In case of a single source in a homogeneous network, we denote $\lambda^{(1)}$ simply by λ .

Consider a particular source. Suppose the freshest update at the monitor at time t is generated at time $u(t)$, the *age of information* at the monitor (in short, AoI) is defined as $\Delta(t) = t - u(t)$, which is the time elapsed since the generation of the last received update. From the definition, it is clear that AoI linearly increases at a unit rate with respect to t , except some reset jumps to a lower value at points when the monitor receives a fresher update from the source. The age of information of our network is shown in Figure 1.2. Let t_1, t_2, \dots, t_N be the generation time of all updates at all servers in increasing order. The black dashed lines show the age of every update. Let T_1, T_2, \dots, T_N be the receipt time of all updates. The red solid lines show AoI.

We note a key difference between the model in this work and most previous models. Updates come from different servers, therefore they might be out of order at the monitor and thus a new arrived update might not have any effect on AoI because a fresher update is already delivered. As an example, from the 6 updates shown in Figure 1.2, *useful* updates that change AoI are updates 1, 3, 4 and 6, while the rest are disregarded as their information when arrived at the monitor is obsolete. Thus among all the received updates for AoI analyses, we only need to consider the *useful* ones that lead to a change in AoI.

The average AoI is the limit of the average age over time $\Delta \triangleq \lim_{T \rightarrow \infty} \int_0^T \Delta(t) / T$, and for a stationary ergodic system, it is also the limit of the average age over the ensemble

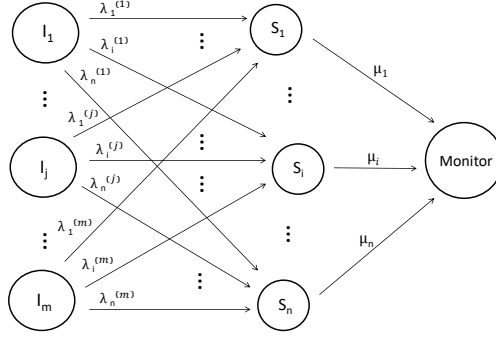


Figure 1.1: The n -server monitoring network with S_1, S_2, \dots, S_n being the servers and I_1, I_2, \dots, I_m being the independent information sources, sending the updates from the sources to the monitor.

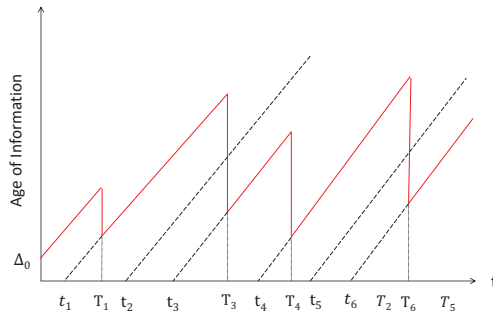


Figure 1.2: AoI for a network with n servers.

$$\Delta = \lim_{t \rightarrow \infty} \mathbb{E}[\Delta(t)].$$

We view our system as a stochastic hybrid system (SHS) and apply a method first introduced in [27] in order to calculate AoI. We can thus obtain the average AoI under LCFS with preemption in service, or in short, LCFS.

In SHS, the state is composed of a discrete state and a continuous state. The discrete state $q(t) \in \mathcal{Q}$, for a discrete set \mathcal{Q} , is a continuous-time discrete Markov chain (e.g., to represent the number of idle servers in the network), and the continuous-time continuous state $\mathbf{x}(t) = [x_0(t), x_1(t), \dots, x_n(t)] \in \mathbb{R}^{n+1}$ is the stochastic process for AoI. We use $x_0(t)$ to represent the age at the monitor, and $x_j(t)$ for the age at the j -th server, $j = 1, 2, \dots, n$. Graphically, we represent each state $q \in \mathcal{Q}$ by a node. For the discrete Markov chain $q(t)$, transitions happen from one state to another through directed transition edge l , and the time spent before the transition occurs is exponentially distributed with rate $\lambda(l)$. Note that it is possible to transit from the same state to itself. The transition occurs when an update arrives at a server, or an update is received at the monitor. Thus the transition rate is the update arrival rate or the service rate $\lambda(l) \in [\lambda_1^{(1)}, \dots, \lambda_n^{(n)}, \mu_1, \dots, \mu_n]$. Denoted by L'_q and L_q the sets of incoming and outgoing transitions of state q , respectively. When transition l occurs, we write that the discrete state transits from q_l to q'_l . For instance, if we have 2 states and considering the transition l from state 1 to state 2, we have $q_l = 1$ and $q'_l = 2$ which shows that state 2 is an outgoing transition for state 1 and state 1 is an incoming transition for state 2. For a transition, we denote that the continuous state changes from \mathbf{x} to \mathbf{x}' . In our problem, this transition is linear in the vector space of \mathbb{R}^{n+1} , i.e., $\mathbf{x}' = \mathbf{x}A_l$, for some real matrix A_l of size $(n+1) \times (n+1)$. Note that when we have no transition, the age grows at a unit rate for the monitor and relevant servers, and is kept unchanged for irrelevant servers. Hence, within the discrete state q , $\mathbf{x}(t)$ evolves as a piece-wise linear function in time, namely, $\frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{b}_q$, for some $\mathbf{b}_q \in \{0, 1\}^{n+1}$. In other words, the age grows at a unit rate for the monitor and relevant servers; and the age is kept unchanged for

irrelevant servers. For our purpose, we consider the discrete state probability

$$\pi_{\hat{q}}(t) \triangleq \mathbb{E}[\delta_{\hat{q},q(t)}] = P[q(t) = \hat{q}], \quad (1.1)$$

and the correlation between the continuous state $\mathbf{x}(t)$ and the discrete state $q(t)$:

$$\mathbf{v}_{\hat{q}} = [v_{\hat{q}0}(t), \dots, v_{\hat{q}n}(t)] \triangleq \mathbb{E}[\mathbf{x}(t)\delta_{\hat{q},q(t)}]. \quad (1.2)$$

Here $\delta_{\cdot,\cdot}$ denotes the Kronecker delta function. When the discrete state $q(t)$ is ergodic, $\pi_q(t)$ converges uniquely to the stationary probability π_q , for all $q \in \mathcal{Q}$. We can find these stationary probabilities from the following set of equations knowing that $\sum_{q \in \mathcal{Q}} \pi_q = 1$,

$$\pi_q \sum_{l \in L_q} \lambda(l) = \sum_{l \in L'_q} \lambda(l) \pi_{q_l}, \quad q \in \mathcal{Q}$$

A key lemma we use to develop AoI for our LCFS queue model is the following from [27], which was derived from the general SHS results in [40].

Lemma 1. [27] *If the discrete-state Markov chain $q(t)$ is ergodic with stationary distribution π and we can find a non-negative solution of $\{\mathbf{v}_q, q \in \mathcal{Q}\}$ such that*

$$\mathbf{v}_q \sum_{l \in L_q} \lambda(l) = \mathbf{b}_q \pi_q + \sum_{l \in L'_q} \lambda(l) \mathbf{v}_{q_l} A_l, \quad q \in \mathcal{Q}, \quad (1.3)$$

then the average age of information is given by

$$\Delta = \sum_{q \in \mathcal{Q}} v_{q0}. \quad (1.4)$$

Chapter 2

AoI in Homogeneous Networks

2.1 Single Source and Multiple Servers

In this section, we derive AoI with the LCFS queue for the single-source n -server homogeneous network with arrival rate λ and service rate μ at all servers. Note that to compute the average AoI, Lemma 1 requires solving $|\mathcal{Q}|(n+1)$ linear equations of $\{\mathbf{v}_q, q \in \mathcal{Q}\}$. To obtain explicit solutions for these equations, the complexity grows with the number of discrete states. Since the discrete state typically represents the number of idle servers in the system for homogeneous servers, $|\mathcal{Q}|$ should be $n+1$. In what follows, we introduce a method inspired by [28] to reduce the number of discrete states and efficiently describe the transitions.

We define our continuous state \mathbf{x} at a time as follows: the first element x_0 is AoI at the monitor, x_1 is the freshest update among all updates in the servers, and x_2 is the second freshest update in the servers, etc. With this definition, we have $x_1 \leq x_2 \leq \dots \leq x_n$, for any time. Note that the index i of x_i does not represent a physical server index, but the i -th smallest age of information among the n servers. The physical server index for x_i changes

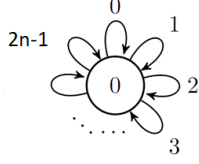


Figure 2.1: SHS for a single source and n homogeneous servers.

with each transition. We say that the server corresponding to x_i is the i -th *virtual* server.

A transition l is triggered by the arrival of an update at a server, or the delivery of an update to the monitor. Recall that we use \mathbf{x} and \mathbf{x}' to denote AoI continuous state vector right before and after the transition l .

When one update arrives at the monitor and the server for that update becomes idle, we put a *fake update* to the server using the method introduced in [28]. Thus the number of discrete states is reduced to one, indicating that all servers are virtually busy. We denote this state by $q = 0$. In particular, we put the current update that is in the monitor to an idle server until the next update reaches this server. This assumption does not affect our final calculation for AoI, because even if the fake update is delivered, AoI at the monitor does not change.

When an update is delivered to the monitor from the k -th virtual server, the server becomes idle and as previously stated, receives the fake update. The age at the monitor becomes $x'_0 = x_k$, and the age at the k -th virtual server becomes $x'_k = x'_0 = x_k$. In this scenario, consider the update at the j -th virtual server, for $j > k$. Its delivery to the monitor does not affect AoI since it is older than the current update of the monitor, i.e., $x_j \geq x_k = x'_0$. Hence, we can adopt a *fake preemption* where the update for the j -th virtual server, for all $k \leq j \leq n$, is preempted and replaced with the fake current update at the monitor. Physically, these updates are not preempted and as a benefit, the servers do not need to cooperate and can work in a distributed manner.

By utilizing virtual servers, fake update, and fake preemption, we reduce SHS to a single

l	$\lambda(l)$	$\mathbf{x}' = \mathbf{x}A_l$
0	λ	$[x_0, 0, x_2, x_3, x_4, \dots, x_n]$
1	λ	$[x_0, 0, x_1, x_3, x_4, \dots, x_n]$
2	λ	$[x_0, 0, x_1, x_2, x_4, \dots, x_n]$
\vdots	\vdots	
$n-1$	λ	$[x_0, 0, x_1, x_2, x_3, \dots, x_{n-1}]$
n	μ	$[x_1, x_1, x_1, x_1, \dots, x_1]$
$n+1$	μ	$[x_2, x_1, x_2, x_2, \dots, x_2]$
$n+2$	μ	$[x_3, x_1, x_2, x_3, \dots, x_3]$
\vdots	\vdots	
$2n-1$	μ	$[x_n, x_1, x_2, x_3, \dots, x_n]$

Table 2.1: Table of transitions for a single source and n homogeneous servers.

discrete state with linear transition A_l . In Figure 2.1, we illustrate our SHS with discrete state space of $Q = \{0\}$. The stationary distribution π_0 is trivial and $\pi_0 = 1$. We set $\mathbf{b}_q = [1, \dots, 1]$ which indicates that the age at the monitor and the age of each update in the system grows at a unit rate. The transitions are labeled $l \in [0 : 2n - 1]$, and for each transition l we list the transition rate and the transition mapping in Table 2.1. For simplicity, we drop the index $q = 0$ in the vector \mathbf{v}_0 , and write it as $\mathbf{v} = [v_0, v_1, \dots, v_n]$. Because we have one state, $\mathbf{x}A_l$ and $\mathbf{v}A_l$ are in correspondence. Next, we describe the transitions in Table 2.1.

Case I. $l \in [0 : n - 1]$: When a fresh update arrives at virtual server $l + 1$, the age at the monitor remains the same and x_{l+1} becomes zero. This server has the smallest age, so we take this zero and reassign it to the first virtual server, namely, $x'_1 = 0$. Accordingly, virtual server $l + 1$ becomes virtual server 1, and virtual server 1 becomes virtual server 2, ..., virtual server l becomes virtual server $l + 1$. The transition rate is the arrival rate of the update, λ .

Case II. $l \in [n : 2n - 1]$: When an update is received at the monitor from virtual server $l + 1 - n$, the age at the monitor changes to x_{l+1-n} and this server becomes idle. Using fake updates and fake preemption we assign $x'_j = x_{l+1-n}$, for all $l + 1 - n \leq j \leq n$. The transition rate is the service rate of a server, μ .

Below we state our main theorem on the average AoI for the single-source n -server network.

THEOREM 2.1. *The average age of information at the monitor for homogeneous single-source n -server network where each server has a LCFS queue is:*

$$\Delta = \frac{1}{\mu} \left[\frac{1}{n\rho} \sum_{j=1}^{n-1} \prod_{i=1}^j \frac{\rho(n-i+1)}{i+(n-i)\rho} + \frac{1}{n\rho} + \frac{1}{n^2} \prod_{i=1}^{n-1} \frac{\rho(n-i+1)}{i+(n-i)\rho} \right], \quad (2.1)$$

where $\rho = \frac{\lambda}{\mu}$.

Proof. Recall that \mathbf{v} denotes the vector \mathbf{v}_0 for the single state $q = 0$. By Lemma 1 and the fact that there is only one state, we need to calculate the vector \mathbf{v} as a solution to (1.3), and the 0-th coordinate v_0 is AoI at the monitor. As mentioned before, $\mathbf{v}A_l$ is in correspondence with $\mathbf{x}A_l$, so we have:

$$\begin{aligned} (n\lambda + n\mu)\mathbf{v} = & [1, 1, 1, 1, 1, 1, 1, \dots, 1] \\ & + \lambda[v_0, 0, v_2, v_3, v_4, \dots, v_n] \\ & + \lambda[v_0, 0, v_1, v_3, v_4, \dots, v_n] \\ & + \lambda[v_0, 0, v_1, v_2, v_4, \dots, v_n] \\ & \quad \vdots \quad \quad \quad \vdots \\ & + \lambda[v_0, 0, v_1, v_2, v_3, \dots, v_{n-1}] \\ & + \mu[v_1, v_1, v_1, v_1, v_1, \dots, v_1] \\ & + \mu[v_2, v_1, v_2, v_2, v_2, \dots, v_2] \\ & + \mu[v_3, v_1, v_2, v_3, v_3, \dots, v_3] \\ & \quad \vdots \quad \quad \quad \vdots \\ & + \mu[v_n, v_1, v_2, v_3, \dots, v_{n-1}, v_n]. \end{aligned} \quad (2.2)$$

From the 0th coordinate of (2.2), we have $(n\lambda + n\mu)v_0 = 1 + n\lambda v_0 + \mu \sum_{j=1}^n v_j$, implying

$$v_0 = \frac{1}{n\mu} + \frac{\sum_{j=1}^n v_j}{n}. \quad (2.3)$$

From the 1st coordinate of (2.2), it follows that $v_1 = \frac{1}{n\lambda}$. Then, to calculate v_0 , we have to calculate v_i for $i \in [2 : n]$. From the i -th coordinate of (2.2),

$$((n-i+1)\lambda + (i-1)\mu)v_i = 1 + \mu \sum_{j=1}^{i-1} v_j + \lambda(n-i+1)v_{i-1}. \quad (2.4)$$

For $i \in [2 : n-1]$, from (2.4), we obtain

$$(i\mu + (n-i)\lambda)(v_{i+1} - v_i) = \lambda(n-i+1)(v_i - v_{i-1}).$$

Hence, $w_{i+1} \triangleq v_{i+1} - v_i = \frac{\lambda(n-i+1)}{(i\mu + (n-i)\lambda)}w_i$. Setting $i = 2$ in (2.4), we have

$$((n-1)\lambda + \mu)v_2 = 1 + \mu v_1 + \lambda(n-1)v_1. \quad (2.5)$$

Simplifying (2.5), we obtain $w_2 = v_2 - v_1 = \frac{1}{(n-1)\lambda + \mu}$. Therefore, we write

$$w_j = \frac{1}{n\lambda} \prod_{i=1}^{j-1} \frac{\lambda(n-i+1)}{i\mu + (n-i)\lambda}, 2 \leq j \leq n. \quad (2.6)$$

Finally, setting $i = n$ in (2.4),

$$(\lambda + (n-1)\mu)v_n = 1 + \mu \sum_{j=1}^{n-1} v_j + \lambda v_{n-1},$$

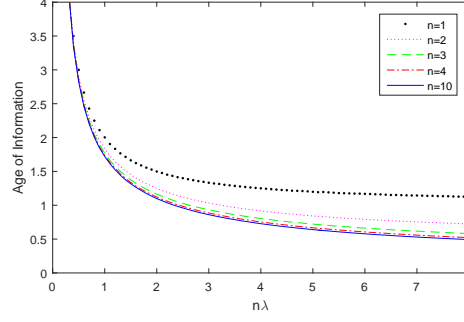


Figure 2.2: AoI versus the number of servers, for fixed total arrival rate. For each server, the service rate $\mu = 1$ and the total arrival rate $n\lambda$ is shown in the x-axis.

implying $\mu \sum_{i=1}^n v_i = \mu \sum_{j=1}^{n-1} v_j + \mu v_n = (\lambda + (n-1)\mu)v_n + \mu v_n - 1 - \lambda v_{n-1}$. Hence,

$$\frac{1}{n} \sum_{i=1}^n v_i = \frac{\lambda}{n\mu} w_n + v_n - \frac{1}{n\mu}. \quad (2.7)$$

Combining (2.3) and (2.7), we obtain the average AoI as

$$\Delta = v_0 = v_n + \frac{\lambda}{n\mu} w_n = \sum_{j=2}^n w_j + \frac{1}{n\lambda} + \frac{\lambda}{n\mu} w_n,$$

which is simplified to (2.1) using (2.6). □

Figure 2.2 shows AoI when the total arrival rate $n\lambda$ is fixed and $n = 1, 2, 3, 4, 10$. We observe that for up to 4 servers, a significant decrease in AoI occurs with the increase of n . However, increasing the number of servers beyond 4 provides only a negligible decrease in AoI. In Figure 2.3, LCFS (with preemption in service), LCFS with preemption in waiting, and FCFS queue models are compared numerically. As can be seen from the figure, LCFS outperforms the other two queue models, which coincides with the intuition that exponential service time is memoryless and older updates in service should be preempted. Moreover, we observe that the optimal arrival rate for FCFS queue is approximately 0.5 for all $n \leq 50$.

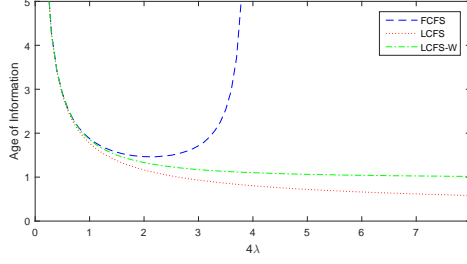


Figure 2.3: Comparison of LCFS, FCFS, and LCFS with preemption in waiting (LCFS-W). The number of servers is $n = 4$ and $\mu = 1$ for each server.

2.2 Multiple Sources and Multiple Servers

In this subsection, we present AoI calculation with the LCFS queue for the m -source n -server homogeneous network. The arrival rate of source i at any server is $\lambda_j^{(i)} = \lambda^{(i)}$, for all $i \in [m], j \in [n]$. The arrival rate of the sources other than source i is $\bar{\lambda}^{(i)} \triangleq \sum_{i' \neq i} \lambda^{(i')}, i \in [m]$. The service rate at any server is μ . Let Δ_i denote the average AoI at the monitor for source $i \in [m]$. Without loss of generality, we calculate Δ_1 for Source 1 under LCFS.

The continuous state \mathbf{x} represents the age for Source 1, and similar to the single-source case, it is defined as follows: x_0 is AoI of source 1 at the monitor, x_i is the age of the i -th freshest update among all updates of source 1 in the servers. Therefore $x_1 \leq x_2 \leq \dots \leq x_n$, for any time. Using fake updates and fake preemption as explained in Section 2.1, we obtain an SHS with a single discrete state and $3n$ transitions described below:

Case I. $l \in [0 : n - 1]$: A fresh update arrives at virtual server l from source 1. This update is the freshest update, so $x'_1 = 0$. Now, the previous freshest update becomes the second freshest update, that is $x'_2 = x_1$, and so on. Then $\mathbf{x}' = [x_0, 0, x_1, \dots, x_l, x_{l+2}, \dots, x_n]$. The transition rate is $\lambda^{(1)}$.

Case II. $l \in [n : 2n - 1]$: A fresh update arrives at virtual server $l' \triangleq l + 1 - n$ from source $i \neq 1$. The age at the monitor does not change, namely, $x'_0 = x_0$. The l' -th freshest update is preempted. Moreover, if the virtual server l' does complete service, it does not reduce the

age of the source of interest. Thus, the l' -th virtual server becomes the n -th virtual server with age x_0 . Therefore, we have $\mathbf{x}' = [x_0, x_1, \dots, x_{l'-1}, x_{l'+1}, \dots, x_n, x_0]$. The transition rate is $\overline{\lambda^{(1)}}$.

Case III. $l \in [2n : 3n - 1]$: the update of source 1 in virtual server $h \triangleq l + 1 - 2n$ is delivered. The age x_0 is reset to x_h and the virtual server h becomes idle. Using fake update and fake preemption, we reset $x'_l = x_h, h \leq j \leq n$. The transition rate is μ .

Dropping the index $q = 0$ and denoting $\mathbf{v}_0 = \mathbf{v} = [v_0, v_1, \dots, v_n]$, the system of equations for the model is

$$\begin{aligned}
n\mu v_0 &= 1 + \mu \sum_{i=1}^n v_i, \\
v_1(\overline{\lambda^{(1)}} + n\lambda^{(1)}) &= 1 + \overline{\lambda^{(1)}}v_2, \\
n(\lambda + \mu)v_i &= 1 + (i-1)\lambda^{(1)}v_i + (n-i+1)\lambda^{(1)}v_{i-1} \\
&\quad + i\overline{\lambda^{(1)}}v_{i+1} + (n-i)\overline{\lambda^{(1)}}v_i \\
&\quad + \mu \sum_{j=1}^{i-1} v_j + (n-i+1)\mu v_i, \quad 2 \leq i \leq n,
\end{aligned} \tag{2.8}$$

where $v_{n+1} \triangleq v_0$ and $\lambda = \overline{\lambda^{(1)}} + \lambda^{(1)} = \sum_{i=1}^n \lambda^{(i)}$.

The theorems below state the average AoI for $n = 2, 3$ servers, and determine the optimal arrival rate given the sum arrival rate.

THEOREM 2.2. *For m information sources and $n = 2$ homogeneous servers, the average AoI at the monitor for source i , $1 \leq i \leq m$, is*

$$\Delta_i = \frac{1}{2(\lambda + \mu)} + \frac{\lambda + \mu}{2\mu\lambda^{(i)}}. \tag{2.9}$$

Proof. From (2.8), we write

$$\begin{aligned}
n(\lambda + \mu)[v_0, v_1, v_2] &= [1, 1, 1] \\
&+ \lambda^{(1)}[v_0, 0, v_2] \\
&+ \lambda^{(1)}[v_0, 0, v_1] \\
&+ \overline{\lambda^{(1)}}[v_0, v_2, v_0] \\
&+ \overline{\lambda^{(1)}}[v_0, v_1, v_0] \\
&+ \mu[v_1, v_1, v_1] \\
&+ \mu[v_2, v_1, v_2]
\end{aligned}$$

From the 0-th coordinate, we have

$$\begin{aligned}
n(\lambda + \mu)v_0 &= 1 + n\lambda v_0 + \mu(v_1 + v_2) \\
n\mu v_0 &= 1 + \mu(v_1 + v_2) \\
v_0 &= \frac{1}{n\mu} + \frac{v_1 + v_2}{n}
\end{aligned}$$

From the 1-st coordinate, we have

$$\begin{aligned}
2(\lambda + \mu)v_1 &= 1 + \overline{\lambda^{(1)}}(v_1 + v_2) + 2\mu v_1 \\
2\lambda v_1 &= 1 + \overline{\lambda^{(1)}}v_1 + \overline{\lambda^{(1)}}v_2 \\
(\lambda + \lambda^{(1)})v_1 &= 1 + \lambda_2 v_2 \\
v_1 &= \frac{1}{\lambda + \lambda^{(1)}} + \frac{\overline{\lambda^{(1)}}v_2}{\lambda + \lambda^{(1)}}
\end{aligned}$$

From the 2-nd coordinate, we have

$$2(\lambda + \mu)v_2 = 1 + \lambda^{(1)}(v_1 + v_2) + 2\overline{\lambda^{(1)}}v_0 + \mu(v_1 + v_2)$$

$$2(\lambda + \mu)v_2 = 1 + (\lambda^{(1)} + \mu)v_1 + (\lambda^{(1)} + \mu)v_2 + 2\overline{\lambda^{(1)}}v_0$$

$$(\lambda + \overline{\lambda^{(1)}} + \mu)v_2 = 1 + (\lambda^{(1)} + \mu)v_1 + n\overline{\lambda^{(1)}}v_0$$

$$v_2 = \frac{1}{\lambda + \overline{\lambda^{(1)}} + \mu} + \frac{(\lambda^{(1)} + \mu)v_1}{\lambda + \overline{\lambda^{(1)}} + \mu} + \frac{n\overline{\lambda^{(1)}}v_0}{\lambda + \overline{\lambda^{(1)}} + \mu}$$

Solving these equations followed by algebraic simplifications results in (2.9). □

THEOREM 2.3. *For homogeneous m sources and $n = 3$ servers,*

$$\Delta_i = \frac{1}{3\mu} \frac{(5\rho^{(1)} + 2(\rho + 1)^2)(\rho + 1)}{2\rho^3 + 5\rho^{(1)}\rho + 2\rho^{(1)}}, \quad 1 \leq i \leq m,$$

where $\rho = \frac{\lambda}{\mu}$ and $\rho^{(i)} = \frac{\lambda^{(i)}}{\mu}$.

Proof.

$$\begin{aligned} n(\lambda + \mu)[v_0, v_1, v_2, v_3] = & [1, 1, 1, 1] \\ & + \lambda^{(1)}[v_0, 0, v_2, v_3] \\ & + \lambda^{(1)}[v_0, 0, v_1, v_3] \\ & + \lambda^{(1)}[v_0, 0, v_1, v_2] \\ & + \overline{\lambda^{(1)}}[v_0, v_2, v_3, v_0] \\ & + \overline{\lambda^{(1)}}[v_0, v_1, v_3, v_0] \\ & + \overline{\lambda^{(1)}}[v_0, v_1, v_2, v_0] \\ & + \mu[v_1, v_1, v_1, v_1] \\ & + \mu[v_2, v_1, v_2, v_2] \\ & + \mu[v_3, v_1, v_2, v_3]. \end{aligned}$$

At the 0-th coordinate

$$n(\lambda + \mu)v_0 = 1 + n\lambda v_0 + \mu(v_1 + v_2 + v_3)$$

$$n\mu v_0 = 1 + \mu(v_1 + v_2 + v_3)$$

$$v_0 = \frac{1}{n\mu} + \frac{v_1 + v_2 + v_3}{3}$$

At the 1-st coordinate

$$3(\lambda + \mu)v_1 = 1 + \overline{\lambda^{(1)}}v_2 + 2\overline{\lambda^{(1)}}v_1 + 3\mu v_1$$

$$3\lambda v_1 = 1 + \overline{\lambda^{(1)}}v_2 + 2\overline{\lambda^{(1)}}v_1$$

$$(\lambda + 2\lambda^{(1)})v_1 = 1 + \overline{\lambda^{(1)}}v_2$$

$$v_1 = \frac{1}{\lambda + 2\lambda^{(1)}} + \frac{\overline{\lambda^{(1)}}}{\lambda + 2\lambda^{(1)}}v_2$$

At the 2-nd coordinate

$$3(\lambda + \mu)v_2 = 1 + \lambda^{(1)}v_2 + 2\lambda^{(1)}v_1 + 2\overline{\lambda^{(1)}}v_3 + \overline{\lambda^{(1)}}v_2 + \mu v_1 + 2\mu v_2$$

$$(3\lambda + 3\mu - \lambda^{(1)} - \overline{\lambda^{(1)}} - 2\mu)v_2 = 1 + (2\lambda^{(1)} + \mu)v_1 + 2\overline{\lambda^{(1)}}v_3$$

$$(2\lambda + \mu)v_2 = 1 + (2\lambda^{(1)} + \mu)v_1 + 2\overline{\lambda^{(1)}}v_3$$

At the 3-rd coordinate

$$(3\lambda + 3\mu)v_3 = 1 + 2\lambda^{(1)}v_3 + \lambda^{(1)}v_2 + 3\overline{\lambda^{(1)}}v_0 + \mu(v_1 + v_2 + v_3)$$

$$(3\lambda + 3\mu - 2\lambda^{(1)} - \mu)v_3 = 1 + \mu v_1 + (\lambda^{(1)} + \mu)v_2$$

$$(\lambda + 2\overline{\lambda^{(1)}} + 2\mu)v_3 = 1 + \mu v_1 + (\lambda^{(1)} + \mu)v_2$$

Then, we have

$$v_0 = \frac{(\lambda + \mu)(2\lambda^2 + 4\lambda\mu + 2\mu^2 + 5\lambda^{(1)}\mu)}{6\lambda^3\mu + 15\lambda^{(1)}\lambda\mu^2 + 6\lambda^{(1)}\mu^3} \quad (2.10)$$

The age is

$$\begin{aligned} v_0 &= \frac{(\lambda + \mu)(2(\lambda + \mu)^2 + 5\lambda^{(1)}\mu)}{3\mu(2\lambda^3 + 5\lambda\lambda^{(1)}\mu + 2\lambda^{(1)}\mu^2)} \\ &= \frac{(\lambda + \mu)(2(\lambda + \mu)^2 + 5\lambda^{(1)}\mu)}{3\mu(2\lambda^{(1)}(\lambda + \mu)^2 + 2\lambda^2\lambda^{(1)} + \lambda\lambda^{(1)}\mu)} \\ &= \frac{1}{3\mu} \frac{(5\rho^{(1)} + 2(\rho + 1)^2)(\rho + 1)}{2\rho^3 + 5\rho^{(1)}\rho + 2\rho^{(1)}} \end{aligned}$$

□

THEOREM 2.4. *Consider homogenous m sources and 2 servers. The optimal arrival rate $\lambda^{(i)*}$ minimizing the weighted sum of AoIs, i.e., $w_1\Delta_1 + w_2\Delta_2 + \dots + w_n\Delta_n$ for $w_i \geq 0$, subject to the constraint $\lambda^{(1)} + \lambda^{(2)} + \dots + \lambda^{(m)} = \lambda$, is given by*

$$\lambda^{(i)*} = \frac{\lambda\sqrt{w_i}}{\sum_{i=1}^m \sqrt{w_i}}, i \in [m].$$

Proof. The objective function that we are trying to minimize is convex (it is obvious from the second derivative matrix) and therefore we just have to put the derivative with respect to each $\lambda^{(i)}$ equal to zero.

$$\frac{\partial}{\partial \lambda^{(i)}} (w_1\Delta_1 + w_2\Delta_2 + \dots + w_n\Delta_n + a(\sum_{i=1}^n \lambda^{(i)} - \lambda)) = 0, \quad (2.11)$$

for $i \in [m]$. Simplifying (2.11) results in:

$$\frac{w_1}{(\lambda^{(1)})^2} = \frac{w_2}{(\lambda^{(2)})^2} = \dots = \frac{w_n}{(\lambda^{(n)})^2} = a. \quad (2.12)$$

Knowing the fact that $\lambda^{(1)} + \lambda^{(2)} + \dots + \lambda^{(m)} = \lambda$, we obtain the result in theorem 2.4. \square

Chapter 3

AoI in Heterogeneous Networks

3.1 Heterogeneous Networks with a Single Source

In this section, we consider a single source and assume that the arrival and service rates of the servers are arbitrary. We denote by $\lambda_j^{(1)} \triangleq \lambda_j$ the arrival rate of the single source at server j , and μ_j the service rate of server $j \in [n]$. For this setting, we can no longer use the same technique used in the homogeneous case to reduce the state space and derive AoI. In particular, we need to keep track of the age of updates at the physical servers as well as their ordering, resulting in $n!$ number of states. In the following, we illustrate the steps for deriving AoI in the case of $n = 2$ servers.

THEOREM 3.1. *Consider $m = 1$ source and $n = 2$ heterogeneous servers. The average AoI is given by*

$$\Delta = \frac{1}{\mu_1 + \mu_2} + \frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\mu_1 + \mu_2} \frac{1}{\lambda_1 + \lambda_2} \left(\frac{\mu_1 \lambda_2}{\lambda_1 + \mu_2} + \frac{\mu_2 \lambda_1}{\lambda_2 + \mu_1} \right). \quad (3.1)$$

Proof. We define state 1 as the state that server 1 contains a fresher update compared to

l	$\lambda(l)$	Transition	$\mathbf{x}' = \mathbf{x}A_l$	$v_{ql}A_l$
1	λ_1	$1 \rightarrow 1$	$[x_0, 0, x_2]$	$[v_{10}, 0, v_{12}]$
2	λ_1	$2 \rightarrow 1$	$[x_0, 0, x_2]$	$[v_{20}, 0, v_{22}]$
3	λ_2	$1 \rightarrow 2$	$[x_0, x_1, 0]$	$[v_{10}, v_{11}, 0]$
4	λ_2	$2 \rightarrow 2$	$[x_0, x_1, 0]$	$[v_{20}, v_{21}, 0]$
5	μ_1	$1 \rightarrow 1$	$[x_1, x_1, x_1]$	$[v_{11}, v_{11}, v_{11}]$
6	μ_1	$2 \rightarrow 2$	$[x_1, x_1, x_2]$	$[v_{21}, v_{21}, v_{22}]$
7	μ_2	$1 \rightarrow 1$	$[x_2, x_1, x_2]$	$[v_{12}, v_{11}, v_{12}]$
8	μ_2	$2 \rightarrow 2$	$[x_2, x_2, x_2]$	$[v_{22}, v_{22}, v_{22}]$

Table 3.1: Table of transitions for $n = 2$ heterogeneous servers.

server 2 and state 2 as the state that server 2 has the fresher update. Upon arrival of an update at each server or receipt of an update at the monitor, we observe some self-transition and intra-state transitions. Transitions rate and mappings are illustrated in Table 3.1.

Steady states probabilities are found knowing that $\pi_1 + \pi_2 = 1$ and $\pi_1\lambda_2 = \pi_2\lambda_1$. Therefore, we will have $\pi = [\frac{\lambda_1}{\lambda_1+\lambda_2}, \frac{\lambda_2}{\lambda_1+\lambda_2}]$.

$$\begin{aligned}
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)\mathbf{v}_1 &= b_1\pi_1 + \lambda_1(v_{10}, 0, v_{12}) + \lambda_1(v_{20}, 0, v_{22}) \\
&\quad + \mu_1(v_{11}, v_{11}, v_{11}) + \mu_2(v_{12}, v_{11}, v_{12}) \quad (3.2)
\end{aligned}$$

$$\begin{aligned}
(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)\mathbf{v}_2 &= b_2\pi_2 + \lambda_2(v_{10}, v_{11}, 0) + \lambda_2(v_{20}, v_{21}, 0) \\
&\quad + \mu_1(v_{21}, v_{21}, v_{22}) + \mu_2(v_{22}, v_{22}, v_{22}) \quad (3.3)
\end{aligned}$$

Where $\mathbf{v}_1 = (v_{10}, v_{11}, v_{12})$ and $\mathbf{v}_2 = (v_{20}, v_{21}, v_{22})$. Therefore, we have six equations and six unknowns here. We can easily see that $v_{11} = \frac{\pi_1}{\lambda_1+\lambda_2}$ and $v_{22} = \frac{\pi_2}{\lambda_1+\lambda_2}$.

$$\begin{aligned}
v_{12} &= \frac{\pi_1}{\lambda_2 + \mu_1} + \frac{\lambda_1\pi_2}{(\lambda_1 + \lambda_2)(\lambda_2 + \mu_1)} + \frac{\mu_1\pi_1}{(\lambda_1 + \lambda_2)(\lambda_2 + \mu_1)} \\
&= \pi_1\left(\frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_2 + \mu_1}\right)
\end{aligned}$$

$$\begin{aligned}
v_{21} &= \frac{\pi_2}{\lambda_1 + \mu_2} + \frac{\lambda_2 \pi_1}{(\lambda_1 + \lambda_2)(\lambda_1 + \mu_2)} + \frac{\mu_2 \pi_2}{(\lambda_1 + \lambda_2)(\lambda_1 + \mu_2)} \\
&= \pi_2 \left(\frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\lambda_1 + \mu_2} \right)
\end{aligned}$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)v_{10} = \pi_1 + \lambda_1 v_{10} + \lambda_1 v_{20} + \mu_1 v_{11} + \mu_2 v_{12}$$

$$(\lambda_1 + \lambda_2 + \mu_1 + \mu_2)v_{20} = \pi_2 + \lambda_2 v_{10} + \lambda_2 v_{20} + \mu_1 v_{21} + \mu_2 v_{22}$$

We add this 2 equations together and simplify it. Age of Information at the monitor is equal to $v_{10} + v_{20}$ which is:

$$\begin{aligned}
AoI &= \frac{1}{\mu_1 + \mu_2} + \frac{\mu_1(v_{11} + v_{21}) + \mu_2(v_{12} + v_{22})}{\mu_1 + \mu_2} = \\
&= \frac{1}{\mu_1 + \mu_2} + \frac{1}{\lambda_1 + \lambda_2} + \frac{1}{\mu_1 + \mu_2} \frac{1}{\lambda_1 + \lambda_2} \left(\frac{\mu_1 \lambda_2}{\lambda_1 + \mu_2} + \frac{\mu_2 \lambda_1}{\lambda_2 + \mu_1} \right)
\end{aligned}$$

□

For $n = 2$ servers, we also find the optimal arrival rates.

THEOREM 3.2. *For $m = 1$ and $n = 2$ heterogeneous servers, given μ_1, μ_2 and fixed $\lambda_1 + \lambda_2 = \lambda$, the optimal λ_1^* satisfies*

- *if $\mu_1 < \mu_2$ and $\mu_2^2 - \frac{\mu_1(\lambda + \mu_1)(\lambda + \mu_2)}{\mu_2} < 0$:*

$$\lambda_1^* = \frac{-(\mu_2 + c(\lambda + \mu_1)) + \sqrt{\mu_1(\lambda + \mu_2)(2 + \frac{\mu_2}{\lambda + \mu_1} + \frac{\lambda + \mu_1}{\mu_2})}}{1 - \frac{\mu_1(\lambda + \mu_2)}{\mu_2(\lambda + \mu_1)}},$$

- *if $\mu_1 < \mu_2$ and $\mu_2^2 \geq \frac{\mu_1(\lambda + \mu_1)(\lambda + \mu_2)}{\mu_2}$: $\lambda_1^* = 0, \lambda_2^* = \lambda$,*
- *if $\mu_1 > \mu_2$ and $\mu_1^2 \geq \frac{\mu_2(\lambda + \mu_1)(\lambda + \mu_2)}{\mu_1}$: $\lambda_1^* = \lambda, \lambda_2^* = 0$,*

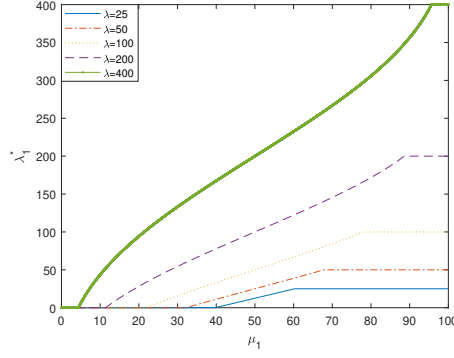


Figure 3.1: Optimal value of λ_1 as a function of μ_1 . $\lambda_1 + \lambda_2 = \lambda$, $\mu_1 + \mu_2 = 100$.

- if $\mu_1 > \mu_2$ and $\mu_1^2 < \frac{\mu_2(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_1}$:

$$\lambda_1^* = \lambda - \frac{-(\mu_1 + \frac{(\lambda+\mu_2)}{c}) + \sqrt{\mu_2(\lambda + \mu_1)(2 + \frac{\mu_1}{\lambda+\mu_2} + \frac{\lambda+\mu_2}{\mu_1})}}{1 - \frac{\mu_2(\lambda+\mu_1)}{\mu_1(\lambda+\mu_2)}},$$

where $c = \frac{\mu_1(\lambda+\mu_2)}{\mu_2(\lambda+\mu_1)}$.

Proof. In order to find the optimal values of λ_1 and λ_2 for a given values of μ_1, μ_2, λ where $\lambda_1 + \lambda_2 = \lambda$, we set the derivative of the following equation with respect to λ_1, λ_2 and a to zero.

$$\frac{1}{\mu_1 + \mu_2} + \frac{\mu_1(v_{11} + v_{21}) + \mu_2(v_{12} + v_{22})}{\mu_1 + \mu_2} - a(\lambda_1 + \lambda_2 - \lambda)$$

$$\begin{aligned} \frac{\partial AoI}{\partial \lambda_1} &= \frac{-1}{(\lambda_1 + \lambda_2)^2} - \frac{\mu_1 \lambda_2 (2\lambda_1 + \lambda_2 + \mu_2)}{(\lambda_1 + \lambda_2)^2 (\lambda_1 + \mu_2)^2} \\ &\quad + \frac{(\lambda_2 + \mu_1)(\mu_2 \lambda_2)}{(\lambda_1 + \lambda_2)^2 (\lambda_2 + \mu_1)^2} - a = 0 \end{aligned}$$

$$\begin{aligned}\frac{\partial AoI}{\partial \lambda_2} = & \frac{-1}{(\lambda_1 + \lambda_2)^2} - \frac{\mu_2 \lambda_1 (2\lambda_2 + \lambda_1 + \mu_1)}{(\lambda_1 + \lambda_2)^2 (\lambda_2 + \mu_1)^2} \\ & + \frac{(\lambda_1 + \mu_2)(\mu_1 \lambda_1)}{(\lambda_1 + \lambda_2)^2 (\lambda_1 + \mu_2)^2} - a = 0\end{aligned}$$

Also, we know that $\lambda_1 + \lambda_2 = \lambda$. With some algebraic simplification we reach to this 2nd order polynomial in order to find the optimal value of λ_1 and consequently λ_2 .

$$\lambda_1^2(1 - c) + 2\lambda_1(\mu_2 + c(\lambda + \mu_1)) + \mu_2^2 - c(\lambda + \mu_1)^2, \quad (3.4)$$

where $c = \frac{\mu_1(\lambda + \mu_2)}{\mu_2(\lambda + \mu_1)}$.

When $c = 1$ it is equivalent to $\mu_1 = \mu_2$ and the equation 3.4 becomes a first order polynomial which results in $\lambda_1 = \lambda_2 = \frac{\lambda}{2}$. This polynomial has 2 real roots because of its positive discriminant and therefore solving the equation 3.4 gives us 2 possible candidate for our optimization problem. When $\mu_1 < \mu_2$ then $c < 1$. Knowing the fact that for 2 roots of 3.4 we have,

$$\begin{aligned}r_1 + r_2 &= \frac{\mu_2 + \frac{\mu_1(\lambda + \mu_2)}{\mu_2}}{c - 1}, \\ r_1 r_2 &= \frac{\mu_2^2 - \frac{\mu_1(\lambda + \mu_1)(\lambda + \mu_2)}{\mu_2}}{1 - c}.\end{aligned}$$

As a result, when $\mu_1 < \mu_2$ and $\mu_2^2 - \frac{\mu_1(\lambda + \mu_1)(\lambda + \mu_2)}{\mu_2} \geq 0$, the 2 roots are negative and therefore in this regime our optimal values become $\lambda_1 = 0, \lambda_2 = \lambda$. When $\mu_1 < \mu_2$ and

$\mu_2^2 - \frac{\mu_1(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_2} \geq 0$, the positive root is the optimal rate which is equal to:

$$\lambda_1 = \frac{-(\mu_2 + c(\lambda + \mu_1)) + \sqrt{\mu_1(\lambda + \mu_2)(2 + \frac{\mu_2}{\lambda+\mu_1} + \frac{\lambda+\mu_1}{\mu_2})}}{1 - \frac{\mu_1(\lambda+\mu_2)}{\mu_2(\lambda+\mu_1)}}.$$

Similarly by writing the $2 - nd$ order polynomial for λ_2 , we reach to the conclusion that when $\mu_1 > \mu_2$, if $\mu_1^2 \geq \frac{\mu_2(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_1}$ the optimal rates are $\lambda_1 = \lambda, \lambda_2 = 0$. In the regime that $\mu_1 > \mu_2$ and $\mu_1^2 < \frac{\mu_2(\lambda+\mu_1)(\lambda+\mu_2)}{\mu_1}$, the positive root is the optimal rate.

$$\lambda_2 = \frac{-(\mu_1 + \frac{(\lambda+\mu_2)}{c}) + \sqrt{\mu_2(\lambda + \mu_1)(2 + \frac{\mu_1}{\lambda+\mu_2} + \frac{\lambda+\mu_2}{\mu_1})}}{1 - \frac{\mu_2(\lambda+\mu_1)}{\mu_1(\lambda+\mu_2)}}.$$

□

The optimal λ_1^* is illustrated in Figure 3.1. When $\mu_1 = \mu_2$ the optimal rates that minimize AoI are $\lambda_1^* = \lambda_2^* = \frac{\lambda}{2}$. As Figure 3.1 illustrates, for $\mu_1 = \mu_2 = 50$, optimal rates are $\lambda_1^* = \frac{\lambda}{2}$ and in the regimes that one of the service rates is much greater than the other one, AoI minimizes when all the updates are sent to the server with the greater service rate.

3.2 Conclusion

In this thesis, we studied the age of information in the presence of multiple independent servers monitoring several information sources. We derived AoI for the LCFS queue model using SHS analysis when we had a homogeneous network and a single source. We also provided the AoI formula for m sources and $n = 2, 3$ servers in a homogeneous network. For a single-source heterogeneous network, the case of $n = 2$ servers were investigated. Moreover, optimal arrival rates are obtained when the sum arrival rate and the service rates are given. Future directions include deriving explicit formula of AoI for multiple sources in a

homogeneous and heterogeneous sensing networks where the update arrival rate and/or the service rate are different among the servers for any number of sources and servers.

Bibliography

- [1] Ruhul Amin, SK Hafizul Islam, GP Biswas, Muhammad Khurram Khan, and Neeraj Kumar. A robust and anonymous patient monitoring system using wireless medical sensor networks. *Future Generation Computer Systems*, 80:483–495, 2018.
- [2] L Siri Chandana and AJ Raja Sekhar. Weather monitoring using wireless sensor networks based on IOT. 2018.
- [3] George Stamatakis, Nikolaos Pappas, Alexandros Fragkiadakis, and Apostolos Traganiotis. Autonomous maintenance in iot networks via aoi-driven deep reinforcement learning, 2020.
- [4] Rong Du, Cailian Chen, Bo Yang, Ning Lu, Xinping Guan, and Xuemin Shen. Effective urban traffic monitoring by vehicular sensor networks. *IEEE Transactions on Vehicular Technology*, 64(1):273–286, 2015.
- [5] Sanjit Kaul, Roy Yates, and Marco Gruteser. Real-time status: How often should one update? In *INFOCOM, 2012 Proceedings IEEE*. IEEE, 2012.
- [6] N. Torkzaban, C. Papagianni, and J. S. Baras. Trust-aware service chain embedding. In *2019 Sixth International Conference on Software Defined Systems (SDS)*, pages 242–247, 2019.
- [7] N. Torkzaban and J. S. Baras. Trust-aware service function chain embedding: A path-based approach. In *2020 IEEE Conference on Network Function Virtualization and Software Defined Networks (NFV-SDN)*, pages 31–36, 2020.
- [8] N. Pappas, Z. Chen, and M. Hatami. Average aoi of cached status updates for a process monitored by an energy harvesting sensor. In *2020 54th Annual Conference on Information Sciences and Systems (CISS)*, pages 1–5, 2020.
- [9] Navid Nouri, Darya Ardan, and Mahmood Mohassel Feghhi. Age of information-reliability trade-offs in energy harvesting sensor networks, 2020.
- [10] O. M. Sleem, S. Leng, and A. Yener. Age of information minimization in wireless powered stochastic energy harvesting networks. In *2020 54th Annual Conference on Information Sciences and Systems (CISS)*, pages 1–6, 2020.

- [11] Yunquan Dong, Pingyi Fan, and Khaled Ben Letaief. Energy harvesting powered sensing in iot: Timeliness versus distortion, 2019.
- [12] S. Feng and J. Yang. Minimizing age of information for an energy harvesting source with updating failures. In *2018 IEEE International Symposium on Information Theory (ISIT)*, pages 2431–2435, 2018.
- [13] S. C. Bobbili, S. Bhambay, and P. Parag. Variable length differential encoding for real-time status updates. *IEEE Communications Letters*, pages 1–1, 2020.
- [14] Maice Costa, Yalin Sagduyu, Tugba Erpek, and Muriel Médard. Robust improvement of the age of information by adaptive packet coding, 2021.
- [15] Ahmed Arafa, Jing Yang, Sennur Ulukus, and H. Vincent Poor. Online timely status updates with erasures for energy harvesting sensors, 2018.
- [16] Ahmed Arafa, Karim Banawan, Karim G. Seddik, and H. Vincent Poor. Sample, quantize and encode: Timely estimation over noisy channels, 2020.
- [17] A. Javani, M. Zorgui, and Z. Wang. On the age of information in erasure channels with feedback. In *ICC 2020 - 2020 IEEE International Conference on Communications (ICC)*, pages 1–6, 2020.
- [18] A. Maatouk, S. Kriouile, M. Assaad, and A. Ephremides. Asymptotically optimal scheduling policy for minimizing the age of information. In *2020 IEEE International Symposium on Information Theory (ISIT)*, pages 1747–1752, 2020.
- [19] B. Sombabu and S. Moharir. Age-of-information based scheduling for multi-channel systems. *IEEE Transactions on Wireless Communications*, 19(7):4439–4448, 2020.
- [20] Mohamed A. Abd-Elmagid, Harpreet S. Dhillon, and Nikolaos Pappas. Aoi-optimal joint sampling and updating for wireless powered communication systems, 2020.
- [21] C. Kam, S. Kompella, and A. Ephremides. Age of incorrect information for remote estimation of a binary markov source. In *IEEE INFOCOM 2020 - IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, pages 1–6, 2020.
- [22] Howard H. Yang, Ahmed Arafa, Tony Q. S. Quek, and H. Vincent Poor. Optimizing information freshness in wireless networks: A stochastic geometry approach, 2020.
- [23] Anousheh Gholami, Nariman Torkzaban, John S. Baras, and Chrysa Papagianni. Joint mobility-aware uav placement and routing in multi-hop uav relaying systems, 2020.
- [24] G. Ahani, D. Yuan, and Y. Zhao. Age-optimal uav scheduling for data collection with battery recharging. *IEEE Communications Letters*, pages 1–1, 2020.
- [25] H. Hu, K. Xiong, G. Qu, Q. Ni, P. Fan, and K. B. Letaief. Aoi-minimal trajectory planning and data collection in uav-assisted wireless powered iot networks. *IEEE Internet of Things Journal*, 8(2):1211–1223, 2021.

- [26] Aidin Ferdowsi, Mohamed A. Abd-Elmagid, Walid Saad, and Harpreet S. Dhillon. Neural combinatorial deep reinforcement learning for age-optimal joint trajectory and scheduling design in uav-assisted networks, 2020.
- [27] Roy D Yates and Sanjit K Kaul. The age of information: Real-time status updating by multiple sources. *IEEE Transactions on Information Theory*, 2018.
- [28] Roy D Yates. Status updates through networks of parallel servers. In *2018 IEEE International Symposium on Information Theory (ISIT)*, pages 2281–2285. IEEE, 2018.
- [29] Roy D Yates. Age of information in a network of preemptive servers. *arXiv preprint arXiv:1803.07993*, 2018.
- [30] Clement Kam, Sastry Kompella, Gam D Nguyen, and Anthony Ephremides. Effect of message transmission path diversity on status age. *IEEE Transactions on Information Theory*, 62(3):1360–1374, 2016.
- [31] Igor Kadota, Elif Uysal-Biyikoglu, Rahul Singh, and Eytan Modiano. Minimizing the age of information in broadcast wireless networks. In *Communication, Control, and Computing (Allerton), 2016 54th Annual Allerton Conference on*, pages 844–851. IEEE, 2016.
- [32] Shahab Farazi, Andrew G Klein, John A McNeill, and D Richard Brown. On the age of information in multi-source multi-hop wireless status update networks. In *2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, pages 1–5. IEEE, 2018.
- [33] Igor Kadota, Abhishek Sinha, and Eytan Modiano. Optimizing age of information in wireless networks with throughput constraints. In *IEEE INFOCOM 2018-IEEE Conference on Computer Communications*. IEEE, 2018.
- [34] Maice Costa, Marian Codreanu, and Anthony Ephremides. On the age of information in status update systems with packet management. *IEEE Transactions on Information Theory*, 62(4):1897–1910, 2016.
- [35] Mohamed A Abd-Elmagid and Harpreet S Dhillon. Average peak age-of-information minimization in UAV-assisted IoT networks. *IEEE Transactions on Vehicular Technology*, 2018.
- [36] Qing He, Di Yuan, and Anthony Ephremides. On optimal link scheduling with min-max peak age of information in wireless systems. In *Communications (ICC), 2016 IEEE International Conference on*. IEEE, 2016.
- [37] Xianwen Wu, Jing Yang, and Jingxian Wu. Optimal status update for age of information minimization with an energy harvesting source. *IEEE Transactions on Green Communications and Networking*, 2(1):193–204.

- [38] Songtao Feng and Jing Yang. Minimizing age of information for an energy harvesting source with updating failures. In *2018 IEEE International Symposium on Information Theory (ISIT)*, pages 2431–2435. IEEE, 2018.
- [39] Ali Valehi and Abolfazl Razi. Maximizing energy efficiency of cognitive wireless sensor networks with constrained age of information. *IEEE Transactions on Cognitive Communications and Networking*, 3(4):643–654, 2017.
- [40] Joao P Hespanha. Modelling and analysis of stochastic hybrid systems. *IEE Proceedings-Control Theory and Applications*, 153(5):520–535, 2006.