Cache-Aided Modulation for Heterogeneous Coded Caching over a Gaussian Broadcast Channel

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Abstract—Coded caching is an information theoretic scheme to reduce high peak hours traffic by partially prefetching files in the users local storage during low peak hours. This paper considers heterogeneous decentralized caching systems where users' caches and content library files may have distinct sizes. The server communicates with the users through a Gaussian broadcast channel. The main contribution of this paper is a novel modulation strategy to map the *multicast* messages generated in the coded caching delivery phase to the symbols of a signal constellation, such that users can leverage their cached content to demodulate the desired symbols with higher reliability. For the sake of simplicity, in this paper we focus only on "uncoded" modulation and symbol-by-symbol error probability. However, our scheme in conjunction with multilevel coded modulation can be extended to channel coding over a larger block lengths.

I. INTRODUCTION

Coded caching, originally proposed by Maddah-Ali and Niesen (MAN) in their seminal work [1], leads to an additional coded multicast gain compared to the conventional uncoded caching. In the MAN model, a server has access to a library of N files and is connected to K users through an error-free shared link of unit capacity. Each user is equipped with a cache of size equivalent to M files. The MAN coded caching scheme consists of two phases: placement and delivery. During the placement phase, users partially store files from the library in their cache memories. Of course, placement is agnostic of the future user demands. After the user demands are revealed, the server broadcasts a sequence of multicast messages to the users. Such messages are computed as a function of the user demands, of the library files, and of the user cache content, such that after receiving the multicast messages all users obtain their requested file with zero error probability (or vanishing error probability in the limit of large file size). The placement can be done in a several manners distinguished by two characteristics, coded vs. uncoded and centralized vs. decentralized. Uncoded placement refers to the fact that segments of the library files are stored directly in the caches, and not more general functions thereof. It is known that for the MAN shared link scenario uncoded placement is optimal within a factor of 2 [2]. Therefore, in this paper we consider uniquely uncoded placement. A coded caching system is called centralized [1] if the server assigns the files segments to the users as a function of the number of users in the system. In contrast, in the decentralized case [3], each user individually and independently of the others fills up its cache with bits from the library files without knowing how other users are in the system and which segment have been already cached by the other users. A vast class of decentralized caching placements consists of *random independent caching*, where the set of library bits cached by each user k be a random variable Z_k according to some distribution $p_{Z_k}(\cdot)$ independent of the number of users K, and let the $\{Z_k : k = 1, \ldots, K\}$ be statistically independent.

In a practice, it may be more realistic to consider the case where users and files have distinct sizes (heterogeneous caching systems). In [4], the authors proposed a decentralized coded caching scheme with varying cache sizes by applying zero-padding to subfiles of different length to enable their encoding in a joint multicast message. Coded caching with distinct file sizes with uncoded placement was originally investigated in [5] where the users could request a file multiple times. They proposed a caching scheme for different file sizes by considering random independent caching where the bits of each file are cached independently at random with a probability proportional to the file size. Further improvements on heterogeneous caching could be found in [6]-[8]. A common point of the existing heterogeneous caching schemes is that the delivery phases are based on clique-covering method, which is a direct extension of the MAN delivery.¹

In this paper, we consider the implementation of a heterogeneous decentralized coded caching system over a Gaussian broadcast channel, which is a more realistic model for the actual communication physical layer than of the error-free capacitated shared link. Our main focus is to map the coded packets generated in the caching delivery phase to a signal constellation. ² In heterogeneous caching systems, the subfiles in each multicast message generated by a clique-covering method may have distinct sizes, i.e., there is some inherent redundancy in each multicast message. The idea is to leverage

¹Each transmitted multicast message in the delivery phase is a binary sum of a set of subfiles and useful to a subset of users, where each corresponding user knows all subfiles in the sum except one such that it can decode the remaining subfile.

²This modulation with side information strategy was originally proposed in [9] for index coding, where the authors [9] considered how to modulate the index codes. The relationship between index coding and coded caching was discussed in [1], [10], and the main difference is that the stored content of each user is fixed in the index coding problem while the cache of each user can be designed in the caching problem (such that the "worst" cache configurations can be avoided.)

this redundancy in the modulation/demodulation step, such that the average symbol error rate of users can be reduced. Besides introducing the novel caching-modulation problem, our main contributions are

- We propose a novel modulation/demodulation strategy, where users can leverage their cached content to demodulate.
- We use that the *set partitioning* labelling proposed in [11]–[14] is optimal (i.e., where the minimum distance is maximized) in our modulation with side information strategy.
- We prove that the proposed cache-aided modulation scheme outperforms the conventional modulation scheme with zero padding.

II. SYSTEM MODEL AND PROBLEM SETTING

A. System model

We consider a content delivery system with a server having access to a library of N independent files W = $\{W_1, W_2, \ldots, W_N\}$ with distinct sizes. For each $i \in [N]$, File W_i has $F_i B$ bits where $B = \sum_{j=1}^{N} |W_j|$ is the total library size in bits, and $F_i = \frac{|W_i|}{B}$. The server (e.g., a wireless base station) transmits a signal x(t) to the users which receive $y_k(t) = \sqrt{\gamma_k} x(t) + \nu_k(t)$, where $\nu_k(t)$ is the Additive White Gaussian Noise (AWGN) at the k-th receiver, with unit power, x(t) is also normalized to have unit power, and γ_k denotes the receiver Signal-to-Noise Ratio (SNR). Without loss of generality we shall adopt the standard complex baseband discrete-time model and since we focus on symbolby-symbol demodulation we can omit the discrete time index and simply write $y_k = \sqrt{\gamma_k} x + \nu_k$ for a generic symbol at user k receiver, use X and Y_k to denote the whole transmit and receive sequences over many symbols. Each user k has a cache memory with size M_k bits where $M_k \in [0, B]$. We defined normalized cache sizes as $\mu_k = M_k/B$. The users have different cache sizes, without loss of generality, $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_K.$

The caching system comprises a placement and a delivery phase. In the placement phase users store contents from the library in a decentralized manner without any knowledge about demands. We define ϕ_k as the caching function for user k, which maps the library W to the cache content $Z_k \triangleq \phi_k(W_1, W_2, \ldots, W_N)$ for user k with the content of all caches being denoted by $\mathbf{Z} := (Z_1, Z_2, \ldots, Z_K)$. In the delivery phase, each user requests one file from the library. We denote the file demanded by user k as d_k and demands of all users by $\mathbf{d} := (d_1, d_2, \ldots, d_k)$. Given (\mathbf{d}, \mathbf{Z}) , the server sends the codewords $X \in \mathscr{C}^L$, where \mathscr{C} is a q-dimensional signal constellation (i.e., a discrete set of points in \mathbb{C}^q), and L is the broadcast codeword length in terms of constellation symbols.³ Upon receiving Y_k , user k needs to decode W_{d_k} from Y_k and Z_k .

Given the cache sizes of the users and the file sizes we design a shared link caching scheme to fill the users' caches in the placement phase and to generate broadcast messages P of total size RB bits in the delivery phase. The broadcast messages are designed such that each requested file W_{d_k} can be recovered from (Z_k, P) for each $k \in [K]$. This caching scheme is agnostic of the users' SNR and physical layer modulation. The transmitter then maps the coded bits in P into a sequence of q-dimensional constellation points, by dividing P into labels of m bits each, and using these labels to index the 2^m points of the constellation \mathscr{C} , transmitted sequentially over the Gaussian broadcast channel defined before. The normalized load of the broadcast channel in terms of channel uses per B bits is given by $\frac{R}{m/a}$, where R is the load of the coded caching scheme as defined above and q/m is the spectral efficiency of the underlying physical layer modulation scheme, expressed in bits per complex signal dimension. Such spectral efficiency depends on the physical layer modulation used, which in turns should be optimized with respect to the expected typical receiver SNR.4

The goal of the coded caching delivery scheme is to minimize R while guaranteeing that each user demand can be satisfied, subject to correct decoding of the multicast messages.

The objective of the physical layer modulation is to minimize the average symbol error rate \overline{T} among all users, where

$$\bar{T} = \frac{1}{K} \sum_{k \in [K]} \frac{S_k}{L_k},\tag{1}$$

where S_k represents the number of symbols in X which are useful to user k and decoded wrongly by user k, and L_k represents the number of symbols in X which are useful to user k.

B. Decentralized MAN caching scheme for heterogeneous network

In decentralized coded caching, during the placement phase user k independently fills his memory with $\mu_k F_i B$ bits of file W_i . For each $S \subseteq [K]$ and each $i \in [N]$, $W_{i,S}$ represents the set of bits of W_i which are uniquely cached by users in S. Since B is large enough, by the law of the large number, we have

$$|W_{i,\mathcal{S}}| = F_i B\left(\prod_{j\in\mathcal{S}}\mu_j\right)\left(\prod_{k\notin\mathcal{S}}(1-\mu_k)\right).$$
 (2)

In the delivery phase, for every non-empty subset of users $S \subseteq [K]$, the server transmits the following coded multicast messages

$$P_{\mathcal{S}} = \bigoplus_{k \in \mathcal{S}} W_{d_k, \mathcal{S} \setminus \{k\}},\tag{3}$$

of length $|P_{\mathcal{S}}| = \max_k |W_{d_k,\mathcal{S}\setminus k}|$, where enough zeros are added to the shorter subfiles to make their length to $\max_k |W_{d_k,\mathcal{S}\setminus k}|$ in [4]. In addition, for all distinct demands

³In this paper, for the sake of simplicity, we assume the modulation is uncoded, i.e., we let q = 1 and consider classical QAM/PSK signal constellations.

 $^{^4 {\}rm In}$ general, the value of m/q can be adapted depending on the worst-case user SNR $\min \gamma_k$

 d_k the server must also broadcast directly to all users having requested file d_k the subfiles $W_{d_k,\emptyset}$, which are requested but not cached at any user. Also, notice that the subfiles indexed by S = [K] is cached by everybody; therefore, there is no need to transmit subfiles $W_{d_k,[K]}$ to users. In this paper, for the simplicity of illustration, we use the decentralized MAN caching scheme. We will propose a novel cache-aided modulation scheme, which can be concatenated with any caching scheme based on clique-covering method.

III. CACHED-AIDED MODULATION

The main idea of the proposed modulation scheme is that the different lengths of the subfiles in each multicast message P_S provide some side information to the users with larger cache size to demodulate P_S . We use a toy example to illustrate the idea.

Example 1: Consider a library of two files A = (101001010) of length 9 bits and B = (111001) of length 6 bits. Let K = 2 with cache memory $M_1 = M_2 = 5$ bits. Two users randomly cache $\mu_1 = \mu_2 = \frac{5}{15} = \frac{1}{3}$ of each file, and suppose that the cache realizations is such that the subfile division is $A = (A_{\emptyset}, A_{\{1\}}, A_{\{2\}}, A_{\{1,2\}}) = (101, 001, 010, \emptyset)$ and $B = (B_{\emptyset}, B_{\{1\}}, B_{\{2\}}, B_{\{1,2\}}) = (11, 10, 01, \emptyset)$, i.e., the cache contents are $Z_1 = \{A_{\{1\}}, B_{\{1\}}, A_{\{1,2\}}, B_{\{1,2\}}\} = \{101, 10\}$ and $Z_2 = \{A_{\{2\}}, B_{\{2\}}, A_{\{1,2\}}, B_{\{1,2\}}\} = \{001, 01\}$. Let's assume the demands $d_1 = A$ and $d_2 = B$, and 1 focus on the coded multicast message

$$P_{\{1,2\}} = A_{\{2\}} \oplus B_{\{1\}} = 010 \oplus x10 = 000, \tag{4}$$

where the symbol x denote a "blank" position due to the difference in length of the two subfiles. Suppose that the 8PSK modulation constellation is used with labeling as shown in Fig. 1, such that $P_{\{1,2\}}$ is mapped directly onto the constellation point indexed by the 3-bit label $P_{\{1,2\}}$. User 2 has subfile $A_{\{2\}} = (010)$ in its cache memory and wishes to decode subfiles $B_{\{1\}} = (10)$. Since $|A_{\{2\}}| > |B_{\{1\}}|$, user 2 knows the first bit in the label of the transmitted modulation symbol, which must be equal to the first bit of $A_{\{2\}}$. Hence, it can demodulate the symbol by considering only the subconstellation of points whose first label bit is 0 (the blue points in Fig. 1). On the other hand, user 1 does not know any bit in the symbol label because its known subfile $B_{\{1\}}$ is shorter. Therefore, user 1 must decode the symbol considering the whole constellation. The minimum distance of the 8PSK constellation is $2\sin(\pi/16)$ while the minimum distance of the "blue" subconstellation $2\sin(\pi/4)$. It follows that user 2 has a lower decoding error for the coded multicast message $P_{\{1,2\}}$ than user 1.

In the following, we provide the general description of provide our proposed cache-aided modulation scheme. We define $\ell_S = \max_{k \in S} |W_{d_k,S \setminus k}|$. To transmit requested subfiles, we need to transmit $n_S = \frac{\ell_S}{m}$ constellation symbols.⁵ First we divide each subfile $W_{d_k,S \setminus k}$ into n_S pieces, each of which is

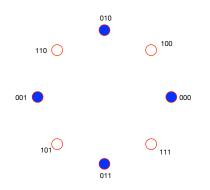


Fig. 1: Set partitioning labelling for 8-PSK

denoted by $W^i_{d_k,S\backslash k}$ where $i\in[n_S]$. ⁶ We generate one coded block

$$P_{\mathcal{S}}^{i} = \bigoplus_{k \in \mathcal{S}} W_{d_{k}, \mathcal{S} \setminus \{k\}}^{i}$$

for all $i \in [n_S]$ and then transmit each block. Notice that each code block has size m, and therefore can be mapped directly onto a modulation point. We define $n_{S,k}$ as the number of useful symbols to user k among the symbols for P_S . Notice that, in the proposed scheme, $n_{S,k} = n_S$ for all k. On the other hand, the difference between the conventional zero padding scheme and our proposed scheme is the way to partition each subfile into blocks. In the delivery phase of the zero padding scheme, we pad enough zero to at the end of $W_{d_k,S\setminus k}$ to make it same length as the longest subfiles $\max_k |W_{d_k,S\setminus k}|$ subfiles and divide it to n_S partitions denoted as $W_{d_k,S\setminus k}^i$, $i \in$ $[n_S]$. Recall that $n_{S,k}$ is the number of useful symbols among n_S symbols for P_S . Notice that, in the zero padding scheme, $n_{S,k}$ might be different for users in S and in general we have $n_{S,k} \leq n_S$.

In Fig. 2, comparison between zero padding scheme and proposed scheme is illustrated through an example. In coded multicast message we would like to transmit subfiles $A_{\{2\}}$ of length 9 bits and subfiles $B_{\{1\}}$ of length 3 bits. Consider m = 3, for our proposed scheme we first divide $A_{\{2\}}$ and $B_{\{1\}}$ into 3 blocks of equal size and then pad with zeros to create blocks of length 3. After this per symbol padding, we encode them together as messages $P_{\{1,2\}}^1$ and $P_{\{1,2\}}^2$ and $P_{\{1,2\}}^3$. In conventional zero padding, we pad with zeros the whole subfile $B_{\{1\}}$ to create a subfile of size 9 bits. Then we divide both subfiles into 3 blocks and encode each block as messages $P_{\{1,2\}}^1$ and $P_{\{1,2\}}^2$ and $P_{\{1,2\}}^3$. In one word, the proposed scheme uses a zero padding on a "symbol level", while the conventional scheme uses a zero padding on a "subfile level".

⁶We also assume that $n_{\mathcal{S}}$ divides $|W_{d_k, \mathcal{S} \setminus k}|$. Because of this assumption, for user k and $\mathcal{S} \in [K]$ we have $|W_{d_k, \mathcal{S} \setminus k}^i| = |W_{d_k, \mathcal{S} \setminus k}^j|$ for $\forall i, j \in [n_{\mathcal{S}}]$.

⁵Since B is large enough, we assume that m divides ℓ_{S} .

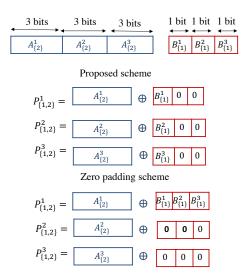


Fig. 2: proposed and zero padding scheme

A. Derivation of the uncoded symbol error rate

The error probability for 2^m -PSK is bounded as follows [15]

$$P_{\rm e,2^m-PSK} \le 2Q\left(\frac{d_{\min}}{\sqrt{2N_0}}\right),$$
 (5)

where d_{\min} is the minimum distance between any two data symbols in a signal constellation and the Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{u^2}{2}} du.$$
 (6)

Since with the proposed binary labeling some users will have some of the most significant bits (leftmost in the label arranged from left to right) known, as seen in the example, we wish to use a binary labeling such that the d_{\min} of the sub constellation indexed by the label with fixed first n most significant bits is maximized for any configuration of the nbits. This is known to be the labeling by set partitioning, wellknown in the coded modulation literature [11]–[14]. For this labeling for the 2^m -PSK modulation we have

$$d_{\min,n} = 2\sin\left(\frac{\pi}{2^{m-n}}\right)d.$$
(7)

A similar reasoning applied to QAM constellations of size 2^m obtained by carving 2^m from the infinite squared grid on the complex plane. In this case we have

$$P_{\rm e,2^m-QAM} \le \left(1 - \left[1 - 2Q\left(\frac{d_{\rm min}}{\sqrt{2N_0}}\right)\right]\right)^2 \le 4Q\left(\frac{d_{\rm min}}{\sqrt{2N_0}}\right)$$
(8)

and the labeling by set partitioning yields the subconstelaltion minimum distance

$$d_{\min,n} = (\sqrt{2})^n d. \tag{9}$$

Fig. 3 shows a typical labeling by set partitioning for the 16-QAM constellation. One can check that for any number of

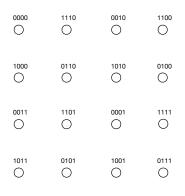


Fig. 3: Set partitioning labelling for 16-QAM

n = 1, 2, 3 known most significant bits the minimum distance of the resulting sub constellation satisfies (9).

The next step is to derive the error probability of decoding subfile $W_{d_k,S\setminus\{k\}}^i$, in coded multicast P_S^i , at receiver user k. In order to do that, we first need to calculate how many bits are known to each receiver. The number of bits are known to user k in the multicast message P_S^i is equal to $\max_{k\in S} |W_{d_k,S\setminus k}^i| - |W_{d_k,S\setminus k}^i|$ since user k has all other subfiles in his cache memory. By using (5) and (7), the error probability of decoding $W_{d_k,S\setminus\{k\}}^i$ with modulation 2^m -PSK is given by

$$P_{e,k,\mathcal{S}} = (10)$$

$$2Q\left(\sqrt{2\gamma_k \sin^2\left(\pi/2^{m-\max_{j\in\mathcal{S}}|W_{d_j,\mathcal{S}\setminus j}^{i_0}|+|W_{d_k,\mathcal{S}\setminus k}^{i_0}|\right)}\right)},$$

where γ_k denotes the SNR at receiver of user k and $i_0 = 1$. Notice that $|W_{d_k,S\setminus k}^i|$ is invariant regarding to index i. In other words, $|W_{d_k,S\setminus k}^i| = |W_{d_k,S\setminus k}^{i_0}|$ where $\forall i \in [n_S]$. After calculating $P_{e,k,S}$ for $\forall i \in [n_{S,k}]$ The total number of useful symbols for user k is given by

$$L_k = \sum_{\mathcal{S}:k\in\mathcal{S}} n_{\mathcal{S},k}.$$
 (11)

The average total number of symbol errors among the symbols useful to user k is give by

$$S_k = \sum_{\mathcal{S}:k\in\mathcal{S}} n_{\mathcal{S},k} P_{e,k,\mathcal{S}}.$$
 (12)

For each user k average symbol error rate is defined as

$$\bar{T}_k = \frac{S_k}{L_k}.$$
(13)

Finally, \overline{T} in (1) is obtained by using above equations.

Lemma 1: The proposed cache-aided modulation achieves lower or equal symbol error rate \overline{T}_k (13) for all user k than conventional zero-padding scheme.

Proof: By considering file sizes are very large, first we derive \overline{T}_k for user k for zero padding scheme. In zero padding scheme, for any S and any user $k \in S$ we have two cases: first

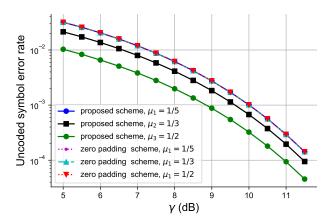


Fig. 4: \overline{T}_k symbol error rate of each user versus SNR for a network with parameters K = 3, $\mu = (1/5, 1/3, 1/2)$ and for each user $\gamma_k = \gamma$.

case if $i \in [n_{\mathcal{S},k}]$ we have $|W^i_{d_k,\mathcal{S}\backslash k}| = m$ and second case if $i \in [n_{\mathcal{S}}] \setminus [n_{\mathcal{S},k}]$ we have $|W^i_{d_k,\mathcal{S}\backslash k}| = 0$. In any useful symbol for user k with indexes in $[n_{\mathcal{S},k}]$ the number of knows bits for user k is zero, i.e. $\max_{j \in \mathcal{S}} |W^{i_0}_{d_j,\mathcal{S}\backslash j}| - |W^{i_0}_{d_k,\mathcal{S}\backslash k}| = 0$. In zero padding scheme, for all user k for all $\mathcal{S} \subseteq [K]$ we have $P_{e,k} = P_{e,k,\mathcal{S}}$, where $P_{e,k} = 2Q\left(\sqrt{2\gamma_k \sin^2(\pi)}\right)$. This implies $\bar{T}^{\text{zp}}_k = P_{e,k}$, where \bar{T}^{zp}_k is denoted the uncoded symbol error rate in (13) for user k. \bar{T}^{p}_k is denoted the symbol error rate for user k for proposed scheme is given by

$$\bar{T}_{k}^{\mathrm{p}} = \frac{\sum_{\mathcal{S}:k\in\mathcal{S}} n_{\mathcal{S},k} P_{e,k,\mathcal{S}}}{\sum_{\mathcal{S}:k\in\mathcal{S}} n_{\mathcal{S},k}}$$
(14a)

$$\leq \max_{\mathcal{S}} P_{e,k,\mathcal{S}} \tag{14b}$$

$$\leq 2Q\left(\sqrt{2\gamma_k \sin^2\left(\pi\right)}\right) \tag{14c}$$

$$=\bar{T}_{k}^{\mathrm{zp}}.$$
 (14d)

For any received signal in any receiver for proposed scheme the number of known bits and d_{\min} areat least as big as the number of known bits for zero padding ones.

IV. SIMULATION RESULTS

In Fig. 4 and Fig. 5, \overline{T}_k for each user k in (13) and \overline{T} in (1) versus SNR are plotted. In Fig. 4 we compare \overline{T}_k in (13) for different users for proposed and zero padding scheme. User 1 has lower cache size among other users, which implies that for this user our scheme does not have any improvement compare to zero padding scheme, the symbol error rate of user 1 with $\mu_1 = 1/5$ for both case are same. User 2 has larger cache size $\mu_2 = 1/3$, symbol error rate of proposed algorithm achieves noticeable gain for user 3 with cache size $\mu_3 = 1/2$ who has biggest cache size among other users.

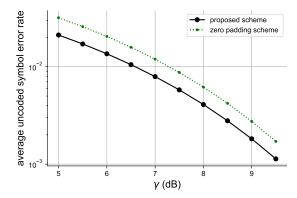


Fig. 5: \overline{T} average symbol error rate versus SNR for a network with parameters K = 3, $\mu = (1/5, 1/3, 1/2)$ and for each user $\gamma_k = \gamma$.

V. CONCLUSION

In this paper, for heterogeneous caching systems, we proposed a novel scheme to map the multicast messages generated by the clique-cover delivery method onto physical layer modulation symbols such that users with larger subfiles can take advantage of the known bits in the constellation labels, effectively restricting their detection problem to a sub-constellation of increased minimum distance. We showed that our scheme achieves better or equal symbol error rate for all users with respect to the conventional zero-padding scheme, which does not use cache-aided side information for the demodulation process to any user. In addition, it can be seen that the best labeling for our scheme is the well-known binary set partitioning labeling, widely used in standard coded modulation techniques. Finally, it is possible to extend the proposed scheme to constellation constructions of longer dimension q using the technique of multilevel coded modulation [16], and replace uncoded error rate with coded block error rate using finite-length coding results [17]. In this paper we have focused on the uncoded case for simplicity and for the sake of space limitation, while the full characterization of the achievable tradeoff between coding rate and block error rate is work in progress.

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