Correlation Mitigation Schemes for Index-Modulated Fluid Antenna Systems

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***Abstract*—In this paper, we investigate the use of index-modulated (IM) transmissions within the framework of fluid antenna (FA) systems, where an FA port is activated during each transmission interval. The adoption of this approach is motivated by the common physical characteristic exhibited by both FAs and IM transmissions, which entails the use of a single radio-frequency (RF) chain. From this perspective, we derive a closed-form expression for the bit error rate (BER) of IM-FA systems in the presence of spatial correlation. Our results demonstrate that IM-FAs outperform conventional IM systems. Since the FA ports are relatively close to each other, we focus on correlation mitigation techniques to improve performance. Specifically, we first investigate two port selection strategies, namely the maximum norm-based and the Euclidean distance-based selection schemes, assuming full channel knowledge. Then, we introduce the concept of spatial set partition coding for IM-FAs to spatially separate the FA ports. Numerical results demonstrate that the performance of IM-FAs is further improved in the case of high correlation whenever we apply the proposed correlation mitigation strategies.**

***Index Terms*—Fluid antenna systems, index modulation, spatial correlation, transmit port selection, set partition coding.**

I. Introduction

Over the past decades, the use of multiple antenna technologies has attracted considerable research interest. In fact, multiple-input multiple-output (MIMO) systems are considered one of the most important wireless communication technologies for the achievement of remarkable diversity and multiplexing gains [1]. Nonetheless, current MIMO technologies are prone to some physical deployment constraints, including the inter-antenna separation by at least half a wavelength. To overcome this limitation, the novel idea of fluid antenna (FA) systems was presented in [2]. This technology represents any software-controllable radiating fluidic structure that is able to alter its shape and position to reconfigure several parameters such as the operating frequency, polarization, and radiation patterns [3].

Recently, researchers have focused on studying the performance of FAs for single and multi-user environments from a communication theory perspective. Specifically, the authors in [2] show that the outage probability of FA systems decreases as the number of ports increases, and can outperform maximal ratio combining for a large number of ports. This work is extended in [4] to a multi-user scenario in which the performance of FAs is studied in the presence of multi-user interference. The aforementioned works adopt a generalized correlation model by treating the first FA port as a reference to capture the strong spatial correlation over the different FA ports. Although this model constitutes a fundamental initial measure towards acquiring knowledge of the FA technology, it may not accurately capture the correlation between the FA ports. In light of this, the authors in [5] demonstrate that the performance improvements achieved by FAs are restricted when a more detailed spatial correlation model is employed. Additionally, the constraints imposed by these limitations are examined in [6], and the results obtained illustrate that the performance of FA systems is primarily governed by the available physical space.

Moreover, due to the evolution of wireless communications, there is a need to further enhance the systems' spectral and energy efficiencies. To address these challenges, the novel concept of index-modulated (IM) transmissions has emerged as a viable solution [7]. In IM systems, only a fraction of indexed resource entities, e.g., subcarriers, antennas, or time slots are activated for data transmission, while the others are kept unused, hence additional information bits are implicitly conveyed by the index usage. This makes IM-aided systems less complex while consuming substantially less energy [8]. More recently, transmit antenna selection has been extensively studied for IM systems, where several algorithms were proposed to enhance the system outage probability, bit error rate (BER), or capacity [9]. In an effort to reduce the spatial correlation among antennas, the authors in [10] proposed a trellis-coded spatial modulation scheme for spatially correlated MIMO IM systems in which the antenna index bits were processed by a convolutional encoder. Nonetheless, this method was confined to a predetermined code structure for a limited number of transmitting antennas.

Typically, IM transmission in the spatial domain requires one RF chain per active transmit antenna. However, activating a single antenna during transmission allows for the utilization of only one RF chain for the entire system [7]. This approach emulates the functioning of FAs, since the fluid element is confined to a specific location within a dielectric holder [3]. By leveraging this shared physical property between FAs and IM transmissions, the contribution of this paper is to exploit the use of IM transmission techniques in the context of FAs and study spatial correlation mitigation strategies to enhance their performance. Initially, we provide an analytical framework

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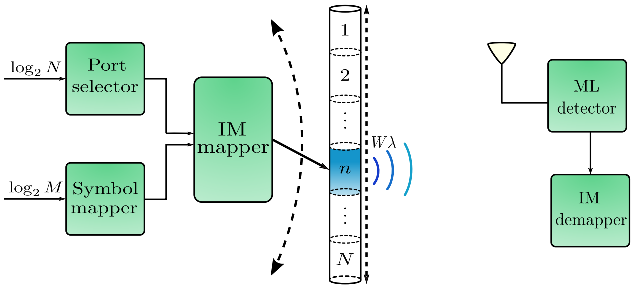

Fig. 1: Transceiver structure for the proposed IM-FA system.

to compute the average BER for IM-FA systems in arbitrarily correlated channels and show an advantage for FAs over conventional IM systems. Then, with the aim of mitigating the impact of correlation, we implement two port selection schemes to choose the least-correlated ports. Moreover, we propose a generalized set partition coding (SPC) scheme for the space domain to spatially separate the FA ports. While the analysis presented in [10] showcases the benefits of utilizing IM transmissions in the context of SPC, it does not delve into the scalability of the system to handle a large number of transmitting indices, a remarkable characteristic of FAs. Hence, this technique facilitates the reduction of correlation between ports, especially when the number of ports is large, and can thus provide large performance gains. Indeed, our theoretical and simulation results reveal that with the employment of correlation mitigation schemes, we can further enhance the performance of IM-FAs.

II. System Model

We consider a point-to-point FA-based IM transmission system between a single FA transmitter and a conventional single-antenna receiver, as depicted in Fig. 1. Specifically, a conductive fluid element is located within a uniform linear tube consisting of N evenly distributed locations (also known as ports) along a linear dimension of $W\lambda$, where λ is the transmission wavelength and W is a scaling constant representing the size of the FA normalized by λ [2]. Moreover, the FA is equipped with a single RF chain and thus a single port is activated for transmission, based on the output of the IM system which is explained in the following discussion. It is assumed that the conductive fluid element can switch locations instantly among the ports, e.g., with the assistance of a mechanical pump [3].

A. Channel Model

The signal is transmitted over a $1 \times N$ wireless channel $\mathbf{h}$. We consider a flat Rayleigh block fading communication channel, i.e., the channel coefficients remain constant during one timeslot, but change independently between different timeslots. Since the ports located within the FA structure are arbitrarily close to each other [2], the channels are considered to be correlated. Typically, we express the correlated Rayleigh fading channel $\mathbf{h}$

$$\mathbf{h} = \widetilde{\mathbf{h}}\, \mathbf{R}^{\frac{1}{2}}, \tag{1}$$

where the N entries of $\widetilde{\mathbf{h}}$ are independent and identically distributed (i.i.d) random variables, each following a complex circular Gaussian random variable with zero mean and variance σ_h^2, and $\mathbf{R}$ is the $N \times N$ transmitter spatial correlation matrix.

B. Index-Based Modulation for FAs

At the FA transmitter, the incoming random bits are split into two streams; one is mapped onto the spatial constellation diagram of size N responsible for the port selection, and the other is mapped onto the signal constellation diagram of size M responsible for the signal modulation. Hence $\eta = \log_2(NM)$ bits are sent at each transmission time. More specifically, these $\log_2(NM)$ bits are mapped onto a constellation vector $\mathbf{x}$ of size N, i.e., $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$. In conventional IM systems, a single antenna is selected during transmission. In the context of FA-based IM systems, at each transmission time, the radiating fluid element is positioned to the port which is selected by the IM mapper based on the incoming bit stream[1]. Therefore, only one element in $\mathbf{x}$ is non-zero, which is at the position of the active port. For instance, given that the FA port is used as an additional resource to transmit information, the active port constitutes a mapping strategy, and the output vector is

$$\mathbf{x} \triangleq \begin{bmatrix} 0 & \cdots & 0 & \underbrace{s_m}_{n\text{-th position}} & 0 & \cdots & 0 \end{bmatrix}^T, \tag{2}$$

where $n = 1, 2, \ldots, N$ represents the index of the activated port, and s_m is the m-th information-bearing symbol from the M-ary constellation at the n-th position. Moreover, we consider an M-ary quadrature amplitude modulation (QAM) constellation design in which $s_m \in \mathcal{S}$, where $\mathcal{S}$ denotes the QAM alphabet set.

C. Maximum-Likelihood Receiver

The received signal experiences additive white Gaussian noise (AWGN) with component w following a circularly symmetric Gaussian distribution with zero mean and variance σ_w^2. We assume a complex baseband signal representation and symbol-by-symbol detection in which the sampled signal at the receiver when the signal is transmitted from the n-th FA port is

$$y = h_n s_m + w, \tag{3}$$

where h_n, $n = 1, 2, \ldots, N$, is the channel between the n-th FA port and the receiver. The receiver's objective is to jointly detect the modulated symbol as well as the active FA port index. Thus, the receiver follows the optimal maximum-likelihood (ML) decision rule[2], which is given by

$$[\hat{n}, \hat{m}] = \arg\min_{n,m} |y - h_n s_m|^2, \tag{4}$$

where $\hat{n}$ and $\hat{m}$ represent the indices of the estimated FA port and the symbol, respectively, and $|\cdot|^2$ is the absolute value squared. In the following discussion, we develop an accurate analytical framework to derive closed-form expressions for the BER of the proposed IM-FA system.

[1]While FAs can be perceived to bear similarities to conventional IM systems, in which a single antenna element is activated, FAs can support a much larger number of ports, and is not limited to a fixed physical structure [3].

[2]The use of the ML detector is validated due to the fact that the channel inputs are equally likely [8].

III. Error Rate Analysis of IM-FA systems

In this section, we consider the analysis of IM-FAs in terms of BER. Due to the specific signal structure of IM systems described in Section II, the transmit vector $\mathbf{x}$ is correctly recovered if both the port and the transmitted symbol are correctly detected. The operation of finding the active port is equivalent to solving an N-hypothesis testing problem at the receiver, hence the analysis involves the computation of multidimensional integrals. Therefore, it is very common in the index modulation literature to compute the average BER by exploiting union-bound methods [7].

Proposition 1: *The conditional pairwise error probability (PEP) that the transmitted vector $\mathbf{x}$ is received as another vector $\hat{\mathbf{x}}$ for IM-FAs is given by*

$$P(\mathbf{x} \to \hat{\mathbf{x}} \mid \mathbf{h}) = Q\left(\sqrt{\frac{|\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2}{4\sigma_w^2}}\right), \tag{5}$$

where $Q(\cdot)$ is the Gaussian Q-function.

Proof. See Appendix A. □

The next step is to obtain the average probability of error. We can rewrite the error probability in (5) as

$$P(\mathbf{x} \to \hat{\mathbf{x}} \mid \mathbf{h}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{|\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2}{8\sigma_w^2 \sin^2\theta}\right)\mathrm{d}\theta, \tag{6}$$

where we used the following alternative (Craig's) representation of the Gaussian Q-function [11, Sec. 4.1, Eq. (4.2)]

$$Q(z) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{z^2}{2\sin^2\theta}\right)\mathrm{d}\theta. \tag{7}$$

Then, by taking the expectation of (7), we obtain

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \mathcal{M}_\Psi\left(-\frac{\bar{\gamma}}{2\sin^2\theta}\right)\mathrm{d}\theta, \tag{8}$$

where $\mathcal{M}_\Psi(\cdot)$ is the moment-generating function (MGF) of the random variable $\Psi = |\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2$ and $\bar{\gamma} \triangleq 1/(4\sigma_w^2)$. In the special case where the channel coefficients are i.i.d, the MGF is decomposed into a product of marginal MGFs. However, in the presence of spatial correlation, this is not possible. In the following proposition, we provide a closed-form expression for the PEP in (8).

Proposition 2: *The PEP of IM-FAs computed in a closed-form expression as*

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = \frac{1}{2}\left(1 - \sqrt{\frac{1}{1+\frac{4\sigma_w^2}{\mu}}}\right), \tag{9}$$

where μ is the eigenvalue of the matrix $\boldsymbol{\delta}\boldsymbol{\delta}^H\mathbf{R}$, and $\boldsymbol{\delta} \triangleq (\mathbf{x}-\hat{\mathbf{x}})$.

Proof. See Appendix B. □

As discussed at the beginning of this section, we aim to provide a bound on the average BER. Using the result from Proposition 2, the average BER of IM-FAs is obtained as

$$\begin{aligned}\overline{P}_{\mathrm{e}} &\leq \frac{1}{\eta 2^\eta}\sum_{\mathbf{x}}\sum_{\hat{\mathbf{x}}} d(\mathbf{x},\hat{\mathbf{x}})P(\mathbf{x}\to\hat{\mathbf{x}}) \\ &= \frac{1}{2\eta 2^\eta}\sum_{\mathbf{x}}\sum_{\hat{\mathbf{x}}} d(\mathbf{x},\hat{\mathbf{x}})\left(1-\sqrt{\frac{1}{1+\frac{4\sigma_w^2}{\mu}}}\right),\end{aligned} \tag{10}$$

where $d(\mathbf{x},\hat{\mathbf{x}})$ is the Hamming distance, i.e., the number of bits in error between the vectors $\mathbf{x}$ and $\hat{\mathbf{x}}$ and $\eta = \log_2(NM)$. It is important to note that the method employed to calculate the BER possesses a high degree of generality in the sense that any correlation model can be adopted, since the expression in (10) depends solely on the eigenvalue of $\boldsymbol{\delta}\boldsymbol{\delta}^H\mathbf{R}$. Hence, the entries of $\mathbf{R}$ are free to follow any arbitrary model.

IV. Spatial Correlation Mitigation Schemes

In this section, we aim to enhance the performance of IM-FA systems in the presence of channel correlation. We first investigate two transmit port selection policies to select the least spatially-correlated ports, and then analyze the performance of IM-FAs by employing coded modulation in the spatial domain.

A. Transmit Port Selection

The basic idea of applying transmit port selection is to overcome the detrimental effects of spatial correlation to improve the system's average BER. In the context of IM-FAs, we aim to select the k best ports according to two different selection criteria which are explained next[3]. We should stress that k represents a system parameter that is decided before transmission. Moreover, since we are selecting a subset k out of N available ports, the system is essentially converted from an N-port to a k-port IM system. Therefore, to keep the same spectral efficiency, performing port selection comes at the expense of increasing the modulation order. In other words, once k is chosen, the modulation order becomes $M' = 2^{\eta-\log_2(k)}$.

1) Maximum Norm-Based Selection: In this scheme, the ports corresponding to the elements in the channel vector with a larger Euclidean norm are favored [12]. Let $\mathcal{S}_k$ represent the set of enumerations of all possible $\binom{N}{k}$ combinations of selecting k out of N FA ports. More specifically, the selection of the k transmit ports is associated with the largest channel norms out of the N ports. Evidently, k must be chosen in a manner to preserve the port selection for IM systems since the number of ports is confined to a power of two. Hence, the selection is

$$\mathbf{h}_{\hat{k}} = \arg\max_{k\in\mathcal{S}_k} \|\mathbf{h}_k\|^2, \tag{11}$$

where $\mathbf{h}_{\hat{k}}$ is the selected vector of size k, and $\|\cdot\|_{\mathrm{F}}^2$ is the Frobenius norm. Subsequently, the receiver performs the ML detection on the reduced-size symbol vector $\mathbf{x}$.

[3]We assume that the transmitter obtains channel information at each coherence time by sequentially sending training symbols and getting feedback from the receiver. The channel estimation for IM-FAs is left for future consideration.

2) Euclidean Distance-Based Selection: To further improve the average BER, we notice from (5) that the PEP is a monotone decreasing function of $d_{min} \triangleq \min_{\mathbf{x} \neq \hat{\mathbf{x}}} |\mathbf{h}(\mathbf{x} - \hat{\mathbf{x}})|^{\mathbf{2}}$. Therefore, we need to maximize d_{min} to minimize the PEP. Out of the $\binom{N}{k}$ possibilities, the optimal port subset that maximizes the minimum Euclidean distance among all transmit vectors is [9]

$$s^* = \arg \max_{k \in \mathcal{S}_k} \left\{ \min_{\mathbf{x} \neq \hat{\mathbf{x}}} |\mathbf{h}_k(\mathbf{x} - \hat{\mathbf{x}})|^2 \right\}, \tag{12}$$

where $\mathbf{h}_k$ is the channel vector of size k. The optimal solution is obtained by searching over all possible channel vectors. We show in Section V that compared to the norm-based selection, this scheme achieves lower error rates at the expense of higher search complexity, especially when the number of FA ports is large, i.e. for high data rates.

B. Spatial Set Partition Coding

In this subsection, we investigate the use of coded modulation to enhance the system's performance. Since the aim is to minimize the spatial correlation between FA ports, we apply the concept of SPC first introduced in [13], on the spatial domain. In other words, we partition the set of transmitting ports into subsets such that the spacing between ports within a specific subset is maximized. For instance, if we consider a 16-port FA in which each port index is represented by 4 bits, the set partition that maximizes the spacing between ports is translated into separating consecutive bit sequences by a large Euclidean distance. In other words, port indices 0000 (first port) and 1000 (ninth port) have a large Euclidean distance, i.e., two port indices that differ only at the most significant bit indicates that the ports are least correlated.

In this paper, we design a rate $3/4$ encoder for a 16-port FA, as shown in Fig. 2. For the sake of brevity, the case of having a general SPC scheme for an arbitrary number of FA ports is left for future consideration. The working principle of IM-FAs with spatial SPC is as follows. The incoming bit stream is split into two parts similar to the uncoded case in Section II. To ensure that the FA ports are selected with maximum spatial separation, the port index bits need to be processed by a convolutional encoder with spectral efficiency $\log_2(N/2)$. Then, the new spectral efficiency becomes $\eta_c = \log_2(MN/2) = \log_2(MN) - 1 = \eta - 1$. Therefore, the coded FA system requires a higher modulation order to achieve the same spectral efficiency as the uncoded system. More specifically, we have $\log_2(16/2) = 3$ bits that enter the encoder at each transmission time.

From Fig. 2, we notice that the uncoded bits $a_u^{(1)}$ and $a_u^{(2)}$ go through a finite state machine (FSM) consisting of three delay blocks. The output of the FSM selects a subset rather than a port index, and the uncoded bit $a_u^{(3)}$ is used to select the actual port index within the subset. This means that the encoder is designed to maximize the minimum interset distance. By following this procedure, we ensure that at each transmission time, the FA ports are separated in a way to minimize correlation. The encoder design is based on tabulated parameters taken from [14, Table 3.3, Sec. 3.4]. Note that other designs that provide a better

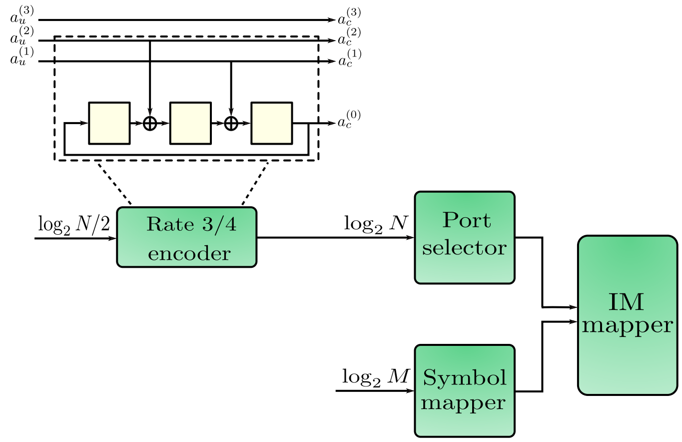

Fig. 2: Set partition coding for IM-FAs for $N = 16$ FA ports.

performance exist. However, the analysis becomes intractable as the number of states in the FSM increases.

In general, the BER analysis of SPC-IM-FAs can be carried out using the same method as that of convolutional codes, whereby the encoder transfer function is obtained under the assumption of a memoryless binary symmetric channel (BSC). The BSC assumption holds whenever the bits are interleaved under perfect or infinite interleaving depth. Hence, the average BER of SPC-IM-FAs is upper bounded by [15, Sec. 8.2.2]

$$\overline{P}_\mathrm{e}^\mathrm{c} \leq T(D), \tag{13}$$

where $T(D)$ is the encoder transfer function, D is given by

$$D = \sqrt{4\overline{P}_\mathrm{e}(1 - \overline{P}_\mathrm{e})}, \tag{14}$$

and $\overline{P}_\mathrm{e}$ is the IM-FA system average BER derived in (10). The parameter D represents the BSC model with crossover probability $\overline{P}_\mathrm{e}$. To obtain the transfer function for the encoder under consideration, the first step involves deriving the state diagram from the encoder in Fig. 2, which is subsequently utilized to obtain the state equations. The transfer function $T(D)$ is then determined by solving the aforementioned state equations and is given in closed-form by (15) on top of the next page. Note that other performance bounds can be considered which would yield similar observations. Nonetheless, the suitability of this approach is attributed to the expression of the transfer function, as well as the utilization of a high code rate. Thus, by plugging (15) into (13), we obtain a performance bound for SPC-IM-FAs.

V. Numerical Results

In this section, we provide extensive Monte-Carlo simulations to validate our analysis and quantify the performance of IM-FAs. Unless stated otherwise, we assume a fixed operating frequency of 5 GHz ($\lambda = 6$ cm) and $W = 1$. For the sake of presentation, the correlation matrix $\mathbf{R}$ is written as

$$\mathbf{R} = \begin{bmatrix} R_{1,1} & \cdots & R_{1,N} \\ \vdots & \ddots & \vdots \\ R_{N,1} & \cdots & R_{N,N} \end{bmatrix}, \tag{16}$$

where the (i, j)-th entry of $\mathbf{R}$, which represents the spatial correlation between ports i and j, is given by [6]

$$T(D) = \frac{D^2+D^3-2D^5+4D^7+4D^8+6D^9+3D^{10}+5D^{11}-18D^{12}-26D^{13}+32D^{15}+21D^{16}-19D^{17}-20D^{18}+3D^{19}+8D^{20}-D^{22}}{1-D^2-5D^3-D^4+D^5+2D^6-4D^7-5D^8+3D^9+7D^{10}+12D^{11}+2D^{12}-11D^{13}-9D^{14}+5D^{15}+7D^{16}-2D^{18}}. \tag{15}$$

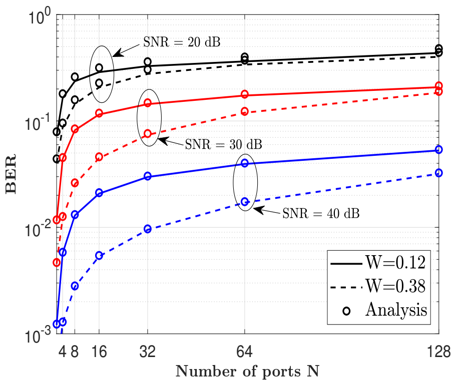

Fig. 3: Impact of the spatial correlation and the number of FA ports on the BER.

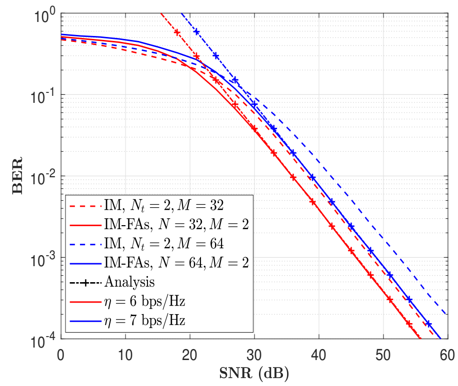

Fig. 4: Performance comparison between IM and IM-FA systems for different η.

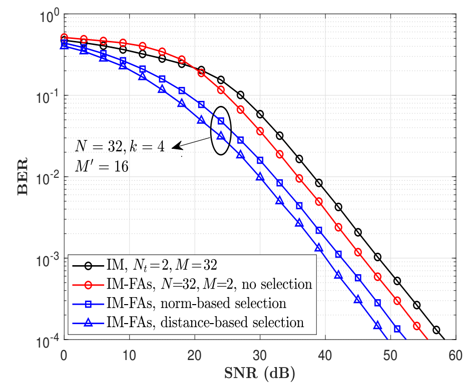

Fig. 5: Performance of IM-FAs with transmit port selection, for $\eta = 6$ bps/Hz.

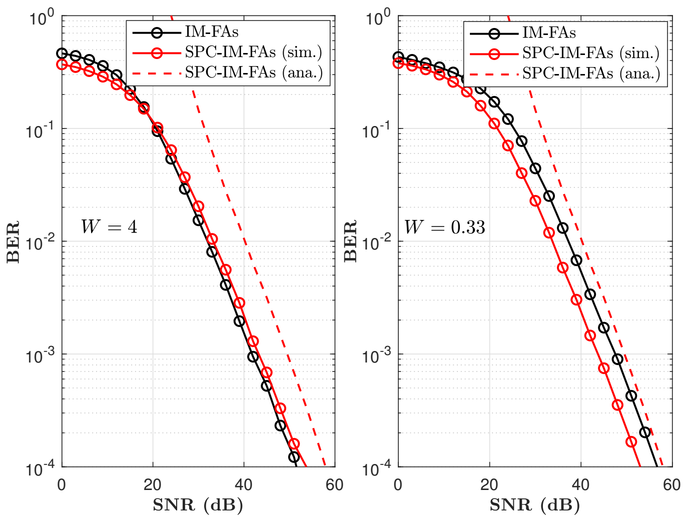

Fig. 6: Performance of SPC-IM-FAs for $N = 16$, $\eta = 5$ bps/Hz.

$$R_{i,j} = J_0\left(2\pi \frac{(i-j)}{N-1} W\right), \tag{17}$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind.

The performance in terms of uncoded BER of the FA system for an increasing number of ports is shown in Fig. 3. Specifically, we plot the results for two different lengths of the FA, i.e., $W = \{0.12, 0.38\}$. We first notice that for all signal-to-noise ratio (SNR) values, the BER performance gets worse as the number of FA ports increases. This is an expected result since the correlation between ports increases as well as the likelihood of erroneous port detection increases with an increasing number of FA ports. We then observe that for a lower number of FA ports, i.e., $N < 64$, the BER increases significantly. Beyond this value, the performance does not degrade as much, meaning that it is sufficient to have a moderate number of FA ports since the BER is not largely affected. Last, we remark that the analytical results match with the simulations at high SNR, which validates the derived bound.

In Fig. 4, we provide a comparison between the IM-FA system and the conventional IM system with N_t transmit antennas. To have a fair comparison, we choose a predefined spectral efficiency η as well as the physical length of the FA. For instance, for $W = 1$, the length of the FA is fixed at 6 cm. Under this limitation, the traditional IM system can have at most $N_t = 2$ transmit antennas if we want to keep them separated by at least half a wavelength. In addition, it should be noted that conventional IM transmissions are susceptible to antenna mutual coupling, resulting in performance degradation when the correlation is low. This inherent effect imposes an additional constraint when compared to FA-based IM transmissions. Having this in mind, for $\eta = 7$ bps/Hz, the traditional IM system requires a modulation order of $M = 64$. On the other hand, IM-FAs can have at most 64 ports with a minimum modulation order of 2. We observe that the FA-based IM system outperforms the N_t-antenna IM system after a certain SNR value (around 24 dB). More specifically, at a BER of 10^{-3}, we observe a gain of approximately 4 dB. This result is also validated for the case where $\eta = 6$ bps/Hz. Finally, we observe that the analytical results match closely with the simulation results, which validates our theoretical framework.

In Fig. 5, we compare the uncoded BER performance of IM-FAs with transmit port selection for the case where the FA has 32 ports. For the port selection, namely the norm-based and the distance-based schemes, we select the $k = 4$ best ports and increase the modulation order from $M = 2$ to $M' = 16$ to keep the same spectral efficiency as the system with no selection. As expected, we observe a significant gain for both port selection schemes over the scheme without selection, with 6 dB and 3 dB for the distance-based and norm-based schemes at a BER of 10^{-3}, respectively. The reason for this gain is that decreasing k means having less but more powerful combinations of channel coefficients despite using a higher modulation order. Evidently, this gain in performance comes at the expense of full channel knowledge at the transmitter.

Finally, we present the performance of the SPC for IM-FAs in Fig. 6 for the specific case where the FA is equipped with 16 ports. Furthermore, we consider two scenarios for the correlation; the first one with a large physical length, i.e., $W = 4$, and the second with $W = 0.33$ to generate a high correlation. We notice that for the first scenario, the use of SPC for the FA ports has no advantage and performs worse than IM-FAs with no SPC. The reason for this behavior is that the uncoded FA-based system uses a lower modulation order as compared to its coded counterpart, hence the expected coding gain is not visible whenever the spatial correlation is low. On the other hand, the second scenario shows the advantage of employing SPC for the FA ports in the presence of high correlation, as it is apparent that we have a significant performance improvement in terms of coding gain (around 4 dB) at a BER of 10^{-4}. The gain improvement is mainly attributed to the encoding structure and the set partitioning of the FA ports. In other words, as the spatial separation between ports increases, the effect of spatial correlation is reduced. On a final note, we observe that the derived theoretical upper bound for SPC-IM-FAs (dashed red curves) follows the same trend as the simulation results, thereby validating our analysis.

VI. Conclusion

In this work, we studied the concept of IM-FA systems, in which an FA transmitter's port indices are used to convey additional information bits. Initially, we derived a closed-form expression for the average BER of IM-FAs. Given that the FA operates in a small physical space, we investigated various correlation mitigation strategies. Our results demonstrated that by using transmit port selection, we can enhance the performance of IM-FAs and outperform IM systems with traditional antennas. Moreover, by applying spatial set partition coding, we observed a coding gain improvement of 4 dB compared to the case without correlation mitigation. This work aimed to introduce IM transmissions for FAs and highlight their influence in next-generation communication systems.

Appendix A
Proof of Proposition 1

Based on the ML detection rule in (4), the conditional PEP is calculated as

$$\begin{aligned} P(\mathbf{x}\to\hat{\mathbf{x}}\mid\mathbf{h}) &= \Pr\big(|y-\mathbf{h}\mathbf{x}|^2 > |y-\mathbf{h}\hat{\mathbf{x}}|^2 \mid \mathbf{h}\big) \\ &= \Pr\big(|w|^2 > |\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})+w|^2 \mid \mathbf{h}\big) \\ &= \Pr\left(|w||\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})| < -\frac{1}{2}|\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2 \;\middle|\; \mathbf{h}\right). \end{aligned} \tag{18}$$

Then, by conditioning on $\mathbf{h}$, we obtain a Gaussian-distributed random variable with zero mean and variance $\sigma_w^2|\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2$. Hence, $P(\mathbf{x}\to\hat{\mathbf{x}}\mid\mathbf{h})$ is reformulated as

$$P(\mathbf{x}\to\hat{\mathbf{x}}\mid\mathbf{h}) = Q\left(\frac{|\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2}{\sqrt{4\sigma_w^2|\mathbf{h}(\mathbf{x}-\hat{\mathbf{x}})|^2}}\right), \tag{19}$$

which is derived as given in (5).

Appendix B
Proof of Proposition 2

By expressing the MGF of the random variable Ψ in terms of the correlation matrix, we obtain [16]

$$\mathcal{M}_\Psi(s) = \prod_{n=1}^{N}\frac{1}{1-s\mu_n}, \tag{20}$$

where μ_n is the n-th eigenvalue of $\boldsymbol{\delta}\boldsymbol{\delta}^H\mathbf{R}$. Note that the analysis, in this case, is simplified due to having a single antenna receiver. Significantly, we notice the matrix $\boldsymbol{\delta}\boldsymbol{\delta}^H\mathbf{R}$ has always rank one due to the term $\boldsymbol{\delta}\boldsymbol{\delta}^H$, which is a consequence of employing IM transmission. Thus, we obtain a single non-zero eigenvalue, denoted by μ, which can be calculated as

$$\mu = \sum_{n=1}^{N}\mu_n = \mathrm{tr}\big(\boldsymbol{\delta}\boldsymbol{\delta}^H\mathbf{R}\big), \tag{21}$$

where $\mathrm{tr}(\cdot)$ is the trace operator. Hence, the MGF is given by

$$\mathcal{M}_\Psi(s) = \frac{1}{1-s\mu}. \tag{22}$$

Finally, by replacing the above expression for the MGF in (8), we obtain a closed-form solution for the PEP of IM-FAs in (9).

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