Outage Probability Comparison of CP-OFDM and TDS-OFDM for Broadcast Channels

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Abstract— **The time domain synchronous orthogonal frequency division multiplex (TDS-OFDM) modulation scheme has recently been proposed in a new digital television (DTV) system. The European DTV standard DVB-T, on the other hand, uses the cyclic prefix (CP) OFDM modulation. In this paper, TDS-OFDM and CP-OFDM for nonergodic broadcast inter-symbol-interference channels are compared from an information theoretical perspective. The outage probability expressions are derived for both schemes and evaluated numerically under various system configurations and channel distributions.**

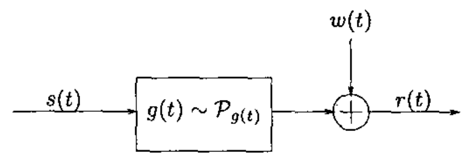

Fig. 1. Channel Model

I. Introduction

The cyclic prefix orthogonal frequency division multiplex (CP-OFDM) technique is well known for its ability to deal with inter-symbol-interference (ISI) in high rate wireless communications. The Digital Video Broadcasting for Terrestrial Television (DVB-T) [1], as the European digital television standard, uses CP-OFDM as its modulation scheme. Although CP-OFDM is easy to demodulate, it carries a non-negligible price. In particular, cyclic prefixes (CP) longer than the channel duration must be sent as the guard intervals between consecutive inverse discrete Fourier transformed (IDFT) blocks, causing a loss in the channel utilization. Furthermore, DVB-T sends a large amount of training symbols (more than 10% of the data symbols) in order to facilitate the channel estimation, causing further losses in the channel throughput.

Recently, a new digital television system, Terrestrial Digital Multimedia/Television Broadcasting (DMB-T) [2], has been proposed for broadcasting in terrestrial environments. A new modulation scheme called time domain synchronous OFDM (TDS-OFDM) is used in DMB-T. The transmitter of TDS-OFDM processes data with IDFT in the same way as CP-OFDM. However, instead of CPs, TDS-OFDM inserts pseudo-noise (PN) sequences as the guard intervals, which also serve as the training symbols. The combination of the guard intervals and the training symbols can reduce transmission overhead and thus provide a better performance.

After removing the PN sequences at the receiver, TDS-OFDM is equivalent to the zero-padded OFDM (ZP-OFDM) [3]. Muquet et al. [3] proposed a low complexity equalizer for ZP-OFDM. They compared ZP-OFDM and CP-OFDM and numerically demonstrated that ZP-OFDM with the equalizer outperformed CP-OFDM in terms of bit error rates, both in coded and uncoded cases.

In this paper, the two modulation schemes, namely CP-OFDM and TDS-OFDM, for nonergodic broadcast ISI channels are compared from an information theoretical perspective. The outage probability [4] is the performance metric used in the comparison. The paper is organized as follows. Section II describes the continuous nonergodic channel model, based on which the comparison is performed. CP-OFDM and TDS-OFDM modulation schemes are presented in detail in Section III and IV respectively, where the outage probability expressions for both schemes are developed as well. In Section V, the outage probabilities are numerically evaluated under various system configurations and channel distributions. The paper is concluded in Section VI.

For clarification the notations used in this paper follow usual conventions. Boldface lowercase symbols represent vectors. Boldface uppercase symbols represent matrices. $(\cdot)^T$ and $(\cdot)^\dagger$ are the transpose and conjugate transpose of $(\cdot)$ respectively. The indexes of the elements of matrices and vectors start from 0.

II. Nonergodic Broadcast Channel

In the terrestrial broadcasting system considered here, the locations of all the receivers are fixed. Since the transmitter is sending the same information to all the receivers, the broadcast channel reduces to a single user compound channel as shown in Fig. 1. The channel $g(t)$ to a user is random and time invariant. Assume it has a finite impulse response duration and is governed by the probability density function $\mathcal{P}_{g(t)}(\cdot)$. Neither the receiver nor the transmitter knows the channel. The channel output to each user is corrupted by the additive zero mean complex white Gaussian noise $w(t)$ with power spectral density (p.s.d.) 1.

The Shannon capacity is not applicable to the nonergodic channel $g(t)$ because there is always a non-zero probability that some channel realizations cannot support the attempted transmission rate no matter how low it is. Similar to the approaches of [4] and [5], the *outage probability* is the performance metric for this nonergodic channel.

The outage probability at a rate R is defined as the probability that a user cannot receive data reliably at this rate with a single code. The outage probability expressions are derived for the CP-OFDM and TDS-OFDM baseband modulation schemes in the following two sections, respectively.

III. CP-OFDM Baseband Modulation Scheme

Let $\mathbf{F}^\dagger$ be an N-point (N is even) IDFT matrix with unit norm, i.e., $\mathbf{F}^\dagger$ is of size $N \times N$ and the element of $\mathbf{F}^\dagger$ at the k^{th} row and the l^{th} column is

$$[\mathbf{F}^\dagger]_{kl} = \frac{1}{\sqrt{N}} \exp \frac{j2\pi kl}{N}. \quad (1)$$

CP-OFDM with the IDFT matrix $\mathbf{F}^\dagger$ partitions the channel into N subcarriers, indexed as $\{-(N/2-1), \ldots, -1, 0, 1, \ldots, N/2\}$, where subcarrier 0 corresponds to the DC frequency in the baseband system. Let $\mathbf{x} = [x_0, x_1, \cdots, x_{N-1}]^T$ be the IDFT input vector, then the i^{th} subcarrier ($-(N/2-1) \leq i \leq N/2$) corresponds to the input $x_{(i \bmod N)}$, where *mod* is the modulo operation. The mapping from the indexes of the subcarriers to those of the IDFT input positions is one-to-one and onto. Assume only subcarriers $\{-M, \ldots, M\}$ are used for the data and training transmission, i.e., $x_i = 0$ for $M+1 \leq i \leq N-M-1$. Let

$$\mathcal{Q} = \{0, \ldots, M\} \cup \{N-M, \ldots, N-1\} \quad (2)$$

be the set of the transmission input positions. Suppose in each IDFT block, there are N_s subcarriers for data and N_t for training, $N_s + N_t = 2M+1$. The set of the data input positions in IDFT block k is denoted as $\mathcal{P}(k) = \{p_0(k), p_1(k), \ldots, p_{N_s-1}(k)\}$, and thus $\mathcal{Q} \setminus \mathcal{P}(k)$ is the set for training.

Let

$$\mathbf{s}(k) = [s_0(k), s_1(k), \cdots, s_{N_s-1}(k)]^T, \quad (3)$$

$$\mathbf{t}(k) = [t_0(k), t_1(k), \cdots, t_{N_t-1}(k)]^T \quad (4)$$

be the vectors of the unknown data symbols and the known training symbols in the k^{th} IDFT block respectively. Assume equal power in the data and training, i.e.,

$$\frac{1}{N_s} \mathrm{E}\{\mathrm{tr}\,[\mathbf{s}(k)\mathbf{s}(k)^\dagger]\} = \frac{1}{N_t} \mathrm{tr}\,[\mathbf{t}(k)\mathbf{t}(k)^\dagger] = \rho. \quad (5)$$

Fig. 2 shows the CP-OFDM scheme with training. Let $\mathbf{0}_{(N-N_s-N_t)\times 1}$ be the vector of the stuffing symbols. The mapping matrix $\mathbf{\Pi}(k)$ defined by $\mathcal{Q}$ and $\mathcal{P}(k)$ maps $\mathbf{s}(k)$, $\mathbf{t}(k)$ and $\mathbf{0}$ to the corresponding subcarriers. To generate the continuous-time complex baseband waveform $s(t)$ sent to the channel (Fig. 1), the following processes are performed after the IDFT $\mathbf{F}^\dagger$: adding the cyclic prefixes of length N_{cp} to the IDFT blocks, parallel to serial (P/S) conversion, discrete to analog (D/A) conversion with sampling period T, and lowpass filtering with

$$p(t) = \frac{1}{\sqrt{T}} sinc(t/T). \quad (6)$$

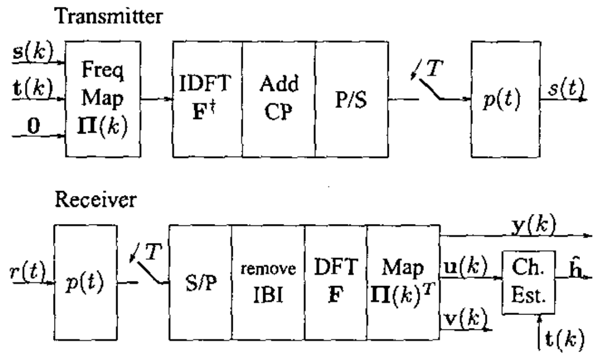

Fig. 2. CP-OFDM Transmitter and Receiver

The p.s.d. of $s(t)$ within frequency $(-\frac{N_s+N_t}{2NT}, \frac{N_s+N_t}{2NT})$ is nearly flat, having an average value of ρ. Outside $(-\frac{N_s+N_t}{2NT}, \frac{N_s+N_t}{2NT})$, the p.s.d. is nearly zero [1]. Therefore, the bandwidth of $s(t)$ $f_B = \frac{N_s+N_t}{NT}$.

The receiver consists of a lowpass filter $p(t)$, an analog to discrete (A/D) converter with sampling period T, a serial to parallel (S/P) converter, an inter-block-interference (IBI) remover, the unit norm DFT $\mathbf{F}$ and the reverse mapping $\mathbf{\Pi}(k)^T$.

The composite channel is defined as $h(t) = p(t) * g(t) * p(t)$ where $*$ denotes a linear convolution. Because $g(t)$ has a finite duration, and $p(t)$ goes to zero fast when $|t|$ goes to infinite (6), $h(t)$ is assumed to have a finite duration $(-n_0 T, n_1 T)$ where n_0 and n_1 are positive integers. The discrete channel impulse response is

$$h(n) = h((n - n_0)T) \quad \text{for } 0 \leq n \leq L = n_0 + n_1, \quad (7)$$

where $L \leq N_{cp}$ (assume there is no IBI).

The input output relation of the CP-OFDM scheme, therefore, is

$$\begin{pmatrix} \mathbf{y}(k) \\ \mathbf{u}(k) \\ \mathbf{v}(k) \end{pmatrix} = \mathbf{\Pi}(k)^T \mathbf{D} \mathbf{\Pi}(k) \begin{pmatrix} \mathbf{s}(k) \\ \mathbf{t}(k) \\ \mathbf{0} \end{pmatrix} + \mathbf{w}(k) \quad (8)$$

where

$$\mathbf{D} = \begin{pmatrix} d_0 & & & \\ & d_1 & & \\ & & \ddots & \\ & & & d_{N-1} \end{pmatrix} \quad (9)$$

$$\mathbf{d} \triangleq [d_0, d_1, \cdots, d_{N-1}]^T = \sqrt{N}\mathbf{F} \begin{pmatrix} \mathbf{h} \\ \mathbf{0}_{(N-L-1)\times 1} \end{pmatrix}, \quad (10)$$

$\mathbf{y}(k)$, $\mathbf{u}(k)$ and $\mathbf{v}(k)$ are the outputs corresponding to $\mathbf{s}(k)$, $\mathbf{t}(k)$ and $\mathbf{0}$ respectively, and $\mathbf{w}(k)$ is the noise term with complex Gaussian distribution $\mathcal{CN}(0, \mathbf{I}_N)$. The outputs $\{\mathbf{u}(0), \mathbf{u}(1), \ldots, \mathbf{u}(k)\}$ are used to estimate the channel. Because in the broadcast scenario a channel realization for one receiver is time invariant as assumed, infinite training outputs are used for the channel estimation. Suppose the channel estimator is consistent, i.e., the channel estimate $\hat{\mathbf{h}}$ converges almost surely to $\mathbf{h}$ as k goes to infinity. Following the arguments in [5], the channel can be assumed known at the receiver in order to analyze the outage probability problem.

Based on the above assumptions the input output relation further reduces to

$$\mathbf{y}(k) = \mathbf{D}(k)\mathbf{s}(k) + \mathbf{w}'(k) \tag{11}$$

where $\mathbf{D}(k)$ is a diagonal matrix with the diagonal entries $d_i, i \in \mathcal{P}(k)$,and $\mathbf{w}'(k)$ has the distribution $\mathcal{CN}(0, \mathbf{I}_{N_s})$.

The following assumptions are made about the data placement pattern $\mathcal{P}(k)$:

- The period of $\mathcal{P}(k)$ is K, i.e., $\mathcal{P}(k) = \mathcal{P}(k+K)$.
- During one period, every transmission subcarrier is used both for data and training, and has the same data-to-training ratio. More precisely, let the indication function $\mathrm{I}(x, \mathcal{X}) = 1$ if $x \in \mathcal{X}$,0 otherwise. Then $\forall i \in \mathcal{Q}$

$$\sum_{k=0}^{K-1} \mathrm{I}(i, \mathcal{P}(k)) = \frac{KN_s}{N_s + N_t}. \tag{12}$$

In Fig. 3, the alternating placement pattern similar to DVB-T [1] is an example which satisfies the above assumptions.

Gathering K successive blocks into one superblock yields

$$\bar{\mathbf{y}}(i) = \bar{\mathbf{D}}\bar{\mathbf{s}}(i) + \bar{\mathbf{w}}(i) \tag{13}$$

where

$$\bar{\mathbf{D}} = \begin{pmatrix} \mathbf{D}(0) & & & \\ & \mathbf{D}(1) & & \\ & & \ddots & \\ & & & \mathbf{D}(K-1) \end{pmatrix}, \tag{14}$$

$\bar{\mathbf{y}}(i)$, $\bar{\mathbf{s}}(i)$ and $\bar{\mathbf{w}}(i)$ are the vectors concatenating $\{\mathbf{y}(k)\}$, $\{\mathbf{s}(k)\}$ and $\{\mathbf{w}(k)\}$ respectively, $iK \leq k < (i+1)K$.

For the channel distribution $\mathcal{P}_{g(t)}(\cdot)$, the outage probability at an attempted transmission rate R is

$$P_{out}(\rho, R) = \mathcal{P}(\Psi(\rho, g(t)) < R) \tag{15}$$

where

$$\begin{aligned} & \Psi(\rho, g(t)) \\ = \ & \frac{1}{KT(N + N_{cp})} \log_2 \det (\mathbf{I} + \rho\bar{\mathbf{D}}^\dagger\bar{\mathbf{D}}) & (16) \\ = \ & \frac{1}{KT(N + N_{cp})} \sum_{k=0}^{K-1} \sum_{i \in \mathcal{P}(k)} \log_2(1 + \rho|d_i|^2) & (17) \\ = \ & \frac{N_s}{T(N + N_{cp})(N_s + N_t)} \sum_{i \in \mathcal{Q}} \log_2(1 + \rho|d_i|^2). & (18) \end{aligned}$$

Equation (15) and (16) are derived from [4]. The unit of $\Psi(\rho, g(t))$ and R is bits per second (b/s). Equation (17) is obtained from the fact that $\bar{\mathbf{D}}$ is diagonal. Equation (18) follows by switching the order of the two summations in (17), and by the assumption (12).

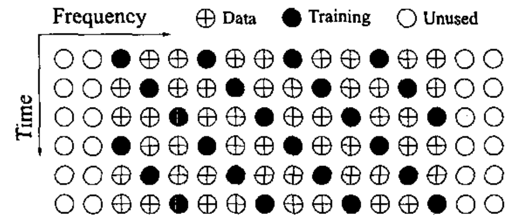

Fig. 3. An example of data placement patterns.

In CP-OFDM, the insertions of the cyclic prefixes and training symbols both introduce transmission overhead. To measure the overhead, the *symbol efficiency* γ is defined as the power ratio of the effective data transmission over the overall transmission. For CP-OFDM,

$$\gamma_{cp} = \frac{N_s}{N_s + N_t} \frac{N}{N + N_{cp}}. \tag{19}$$

IV. TDS-OFDM Baseband Modulation Scheme

TDS-OFDM utilizes PN sequences, instead of CPs, as the guard intervals between successive IDFT blocks. Besides acting as the guard intervals the PN sequences also serve for the purposes of training and synchronization etc.

Fig. 4 shows the transmitter and receiver structure of the TDS-OFDM baseband modulation scheme. Since there are many common components in TDS-OFDM and CP-OFDM, many symbols and components in the previous section are reused. The transmitter consists of an N-point IDFT $\mathbf{F}^\dagger(1)$, a P/S, a D/A with sampling rate $1/T$ and a low pass filter $p(t)$ (6). Let $\mathbf{s}(k)$ of length N_s ($N_s = N$ in this case) and $\mathbf{t}(k)$ of length N_t be the data and training vectors respectively, (3) and (4). The P/S converts the vector $[\mathbf{t}(k); \mathbf{F}^\dagger\mathbf{s}(k)]$ into a serial form. Assume equal power in the data and training transmission with average power ρ (5). The bandwidth of the baseband waveform $s(t)$ $f_B = 1/T$ and the p.s.d. of $s(t)$withi n the passband is ρ.

The discrete channel $\mathbf{h}$ is obtained in the same way as in the previous section (7). Again a finite channel duration is assumed in this case; $h(n) = 0$ for $n < 0$ or $n > L$, and $L \leq N_t$ (no IBI).

The received waveform $r(t)$ is converted into the discrete version $r(n)$. A channel estimator takes $r(n)$ as the input to estimate the channel. The consistency of the channel estimator is assumed, thus the channel can be treated as known at the receiver for the outage probability problem, as in CP-OFDM. The influence of the training is removed by subtracting $h(n) * t(n)$ from $r(n)$, where $t(n)$ is the serial version of $\mathbf{t}(k)$.

The input output relation of TDS-OFDM, therefore, is

$$\mathbf{y}(k) = \mathbf{H}\mathbf{F}^\dagger\mathbf{s}(k) + \mathbf{w}(k) \tag{20}$$

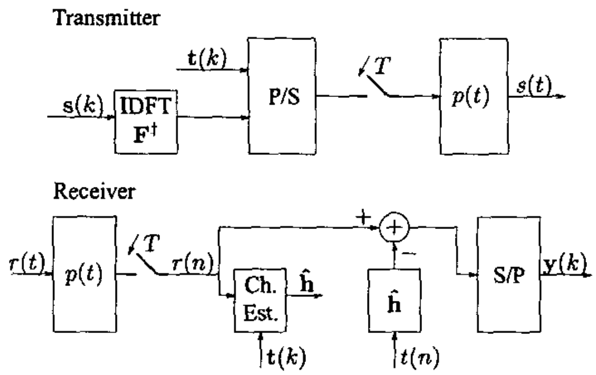

Fig. 4. TDS-OFDM transmitter and receiver

where

$$\mathbf{H} = \begin{pmatrix} h_0 & 0 & \cdots & 0 \\ \vdots & h_0 & \ddots & \vdots \\ h_L & \vdots & \ddots & 0 \\ 0 & h_L & & h_0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & h_L \end{pmatrix} \tag{21}$$

and $\mathbf{w}(k)$ has the distribution $\mathcal{CN}(0, \mathbf{I})$.

The outage probability $P_{out}(\rho, R)$ of TDS-OFDM [5],in a form similar to (15) and (16) in CP-OFDM, is

$$P_{out}(\rho, R) = \mathcal{P}(\Psi(\rho, g(t)) < R) \tag{22}$$

where

$$\Psi(\rho, g(t)) = \frac{1}{T(N_s + N_t)} \log_2 \det(\mathbf{I} + \rho \mathbf{H}^\dagger \mathbf{H}). \tag{23}$$

The symbol efficiency γ of TDS-OFDM is affected only by the insertion of the guard intervals, hence,

$$\gamma_{tds} = \frac{N_s}{N_s + N_t}. \tag{24}$$

V. Simulations

The outage probabilities (15) and (22) depend on the channel distribution $\mathcal{P}_{g(t)}(\cdot)$. To evaluate the outage probabilities, the channel profiles are specified as follows.

In the DVB-T standard, the channel ensemble *Fixed Reception* F_1 is specified for testing the system performance [1] in the fixed reception that corresponds to the nonergodic channels. In this section a modified version of F_1 is defined as

$$g(t) = \frac{r_0 \delta(t) + \sum_{i=1}^{N} r_i \delta(t - \tau_i)}{\sqrt{\sum_{i=0}^{N} \rho_i^2}} \tag{25}$$

where $N = 20$, $\{r_i\}_{i=0}^{N}$ are independent zero-mean complex Gaussians with variances $\{\rho_i^2\}_{i=0}^{N}$ respectively, $\{\rho_i\}_{i=1}^{N}$ and $\{\tau_i\}_{i=1}^{N}$ are specified in [1], and ρ_0 is controlled by a Ricean factor K_r,

$$\rho_0^2 = K_r \sum_{i=1}^{N} \rho_i^2. \tag{26}$$

Suppose that the receiver population consists of several groups, each of which has the same number of receivers, and that the channel distribution within one group is governed by (25) with the Ricean factor associated with the group.

Shown in table I, the configuration parameters of CP-OFDM are those of DVB-T [1], where the length of the cyclic prefixes N_{cp} is chosen from $\{N/4, N/8, N/16, N/32\}$. The first set of configuration parameters of TDS-OFDM comes from DMB-T [2], shown in table I as well.

TABLE I

Configuration Parameters of CP-OFDM and TDS-OFDM

	CP-1 (DVB 8K)	CP-2 (DVB 2K)	TDS-1 (DMB)	TDS-2 (cfg 2)	TDS-3 (cfg 3)
N_s	6116	1529	3780	1705	837
N_t	701	176	378	213	213
N	8192	2048	3780	1705	837
T(μs)	0.1094	0.1094	0.1323	0.1314	0.1314
f_B(MHz)	7.61	7.61	7.56	7.61	7.61

TABLE II

Symbol Efficiencies of CP-OFDM and TDS-OFDM

	CP-1 or 2 (DVB 8K or 2K)			
N_{cp}/N	1/4	1/8	1/16	1/32
γ_{cp}	0.718	0.797	0.844	0.870

	TDS-1 (DMB)	TDS-2 (cfg 2)	TDS-3 (cfg 3)	
γ_{tds}	0.909	0.889	0.797	

The outage probabilities (15) and (22) are evaluated by the Monte Carlo method. Fig. 5 shows the outage probability curves when $\rho = 30$dB and there is only one receiver group with $K_r = 10$dB. Fig. 6 is the plot when $\rho = 30$dB and there are 3 receiver groups with $K_r = \{5, 10, 20\}$dB respectively. In the figure legend, *DVB-T 2K 1/4* stands for CP-OFDM in the configuration of the DVB-T 2K system ($N = 2048$) with $N_{cp}/N = 1/4$; *DMB-T* stands for TDS-OFDM in the DMB-T system configuration. Since DVB-T 8K has almost the same curves as DVB-T 2K, the outage probability curves for DVB-T 8K are omitted. Fig. 5 and 6 show that TDS-OFDM in the configuration of DMB-T has a better performance than CP-OFDM in the configuration of DVB-T in terms of outage probabilities.

In an attempt to understand better why DMB-T has a better coverage performance, the outage probabilities are plotted for two additional configuration parameter sets under the two channel profiles above. The second set of the TDS-OFDM configuration parameters (cfg 2) is chosen such that TDS-OFDM has the same IDFT block duration, the same guard interval du-

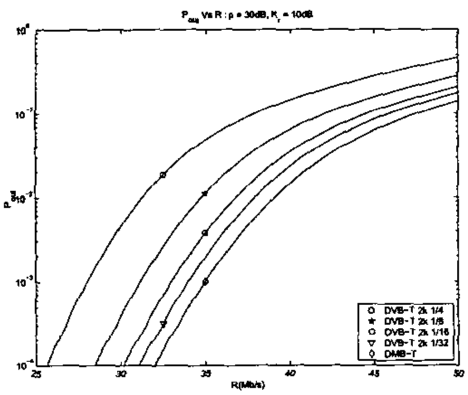

Fig. 5. Outage probability curves for $\rho = 30$dB and $K_r = 10$dB.

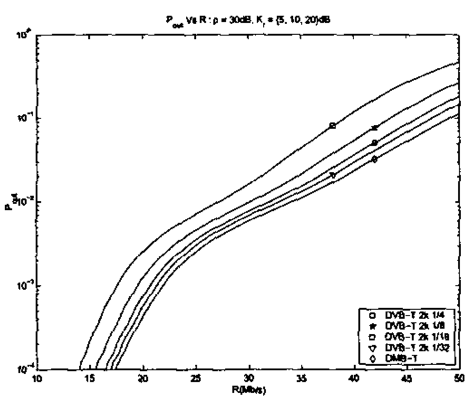

Fig. 6. Outage probability curves for $\rho = 30$dB and $K_r = \{5, 10, 20\}$dB.

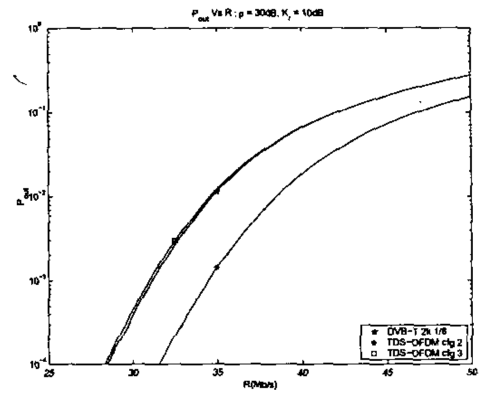

Fig. 7. Additional $\mathcal{P}_{out}$ curves for $\rho = 30$dB and $K_r = 10$dB.

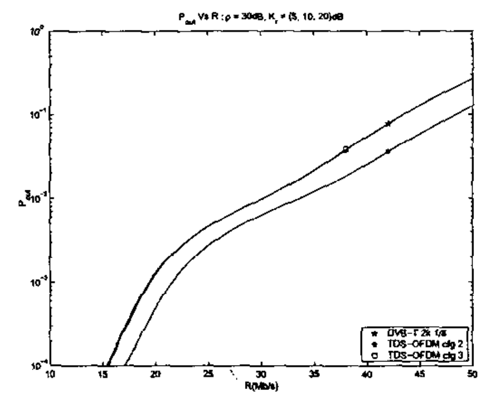

Fig. 8. Additional $\mathcal{P}_{out}$ curves for $\rho = 30$dB and $K_r = \{5, 10, 20\}$dB.

ration and the same bandwidth as *DVB-T 2K 1/8*. In the third set (cfg 3), TDS-OFDM has the same symbol efficiency, guard interval duration and bandwidth as *DVB-T 2K 1/8* (Table I, II). As shown in Fig. 7 and 8, TDS-OFDM in cfg 2 outperforms *DVB-T 2K 1/8* while TDS-OFDM in cfg 3 has almost the same curves as *DVB-T 2K 1/8*.

VI. Conclusions

In this paper,the outage probabilities of the CP-OFDM and TDS-OFDM modulation schemes are derived and evaluated for nonergodic broadcast channels. First, CP-OFDM and TDS-OFDM are configured with the parameters in the DVB-T and DMB-T systems respectively. The simulations show that TDS-OFDM has a lower outage probability than CP-OFDM at any rate in these configurations. Next, TDS-OFDM is configured to have the same symbol efficiency as CP-OFDM. TDS-OFDM has nearly the same outage probability curves as CP-OFDM in this case, which indicates that the symbol efficiency is a major factor affecting the outage probability. Because TDS-OFDM combines the guard intervals and the training symbols, it has a higher symbol efficiency thus a better coverage when TDS-OFDM and CP-OFDM have the same guard interval and IDFT block durations.

Acknowledgment

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