Orthogonal-Like Space-Time Coded CPM with Fast Demodulation for Three and Four Transmit Antennas

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***Abstract*—The Alamouti's orthogonal space-time block codes are for QAM modulations and two transmit antennas. We have recently generalized it for the continuous phase modulation (CPM) by maintaining the orthogonality (for the fast ML decoding/demodulation) and the phase continuity of two signals from two transmit antennas denoted as OST-CPM. In this paper, we design orthogonal-like space-time coded CPM systems for three and four transmit antennas based on the existing orthogonal and quasi-orthogonal space-time codes in the literature. Although the signals from the transmit antennas in the proposed orthogonal-like space-time coded CPM systems are not orthogonal, the fast decoding/demodulation is maintained as in the two transmit antenna case. Simulation results show that the performance of the proposed orthogonal-like space-time coded CPM systems for four transmit antennas is much better than that of the OST-CPM systems for two transmit antennas.**

I. INTRODUCTION

Continuous phase modulation (CPM) systems with single transmit antenna have been widely used in wireless systems due to its spectral efficiency and resistance to wireless channel fading, see for example [10]. Lately, space-time coding for multiple transmit antennas has attracted much attention due to the potential capacity increase, see for example [1] – [8]. Zhang and Fitz [9] recently proposed trellis-coded space-time (TC-ST) coding for continuous phase modulation (CPM) systems. Similar to the trellis-coded space-time coding (TC-ST) for QAM modulations, it may have a high decoding/demodulation complexity.

Most recently, based on the Alamouti's scheme [4], we proposed CPM systems with orthogonal space-time (OST) coding in [13] for two transmit antennas, where the orthogonality and the continuity of the two signal phases from two transmit antennas are maintained. The orthogonality provides us the fast decoding similar to the Alamouti's scheme for QAM modulations. The difficulty of the design comes from the maintaining of both the phase continuity and the orthogonality of the signals from two transmit antennas.

As it is already a difficult task to design high rate OST for more than two transmit antennas for QAM modulations [5], it is even more challenging to keep the continuity of the signal phases. Although there exist OST of rate $3/4$ for 3 and 4 transmit antennas, unfortunately, they can not be directly used in the proposed OST-CPM systems [13]. For example, for 3 and 4 transmit antennas, the orthogonal space-time codes [6], [7], [8]

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & 0 \\ -x_3^* & 0 & x_1^* \\ 0 & -x_3^* & x_2^* \end{bmatrix}, \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & x_3 \\ -x_3^* & 0 & x_1^* & -x_2 \\ 0 & -x_3^* & x_2^* & x_1 \end{bmatrix}, \quad (1)$$

do not suit for CPM systems, since there are some zero values in the code matrices. Notice that for 3 and 4 transmit antennas, there are other orthogonal space-time codes with linear processing of symbols presented in [5], but they are hard to be used in the proposed OST-CPM systems too due to that the linear processing is hard to deal with for the signal phases.

In this paper, for 3 and 4 transmit antennas, we modify the codes in (1) as

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2^* & x_1^* & e^{j\phi_3} \\ -x_3^* & e^{j\phi_2} & x_1^* \\ e^{j\phi_1} & -x_3^* & x_2^* \end{bmatrix}, \begin{bmatrix} x_1 & x_2 & x_3 & e^{j\phi_4} \\ -x_2^* & x_1^* & e^{j\phi_3} & x_3 \\ -x_3^* & e^{j\phi_2} & x_1^* & -x_2 \\ e^{j\phi_1} & -x_3^* & x_2^* & x_1 \end{bmatrix}, \quad (2)$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are some real constants which will be specified later. Notice that, the modified codes in (2) are *not* orthogonal, but their behaviors are similar to those of codes in (1) and the fast maximum-likelihood (ML) decoding is maintained. Then, based on the modified codes in (2), we design CPM systems with fast decoding algorithm. Similar to the OST-CPM for two transmit antennas in [13], the main difficulty is to have the continuity of the signal phase at each transmit antenna.

Moreover, in this paper, we also consider CPM systems with quasi-orthogonal space-time coding for 3 and 4 transmit antennas. The CPM systems with quasi-orthogonal space-time coding still have fast decoding algorithm and may further improve the performance of the above orthogonal design, but the decoding complexity is higher than that of the CPM systems with the above modified orthogonal space-time coding. We design space-time coded CPM systems only for four transmit antennas. We can simply turn off one of the transmit antennas to get the design for three transmit antennas.

II. SYSTEM MODEL

We consider a system with four transmit antennas, $L_t = 4$, and one receive antenna, $L_r = 1$, for simplicity. For the coming information sequence $\mathbf{I} = (I_{1,1}, ...I_{L,1}, ..., I_{1,l}, ..., I_{L,l}, ...)$, each information block $I_{1,l}, ..., I_{L,l}$ of length L is mapped into an information symbol matrix

$$\mathbf{d}_l = \begin{bmatrix} d_{1,1}(l) & d_{1,2}(l) & d_{1,3}(l) & d_{1,4}(l) \\ d_{2,1}(l) & d_{2,2}(l) & d_{2,3}(l) & d_{2,4}(l) \\ d_{3,1}(l) & d_{3,2}(l) & d_{3,3}(l) & d_{3,4}(l) \\ d_{4,1}(l) & d_{4,2}(l) & d_{4,3}(l) & d_{4,4}(l) \end{bmatrix}, \quad (3)$$

where $d_{m,n}(l)$ is the modulation symbol at the m-th transmit antenna and comes from a signal constellation set, for example

$$\Omega \triangleq \{-2M+1, -2M+3, ..., -1, 1, ..., 2M-1\}. \quad (4)$$

During time period $4lT \le t \le 4(l+1)T$, the symbol matrix $\mathbf{d}_l$ is used to generate the following signal matrix

$$S = \begin{bmatrix} s_1(t,4l+1) & s_1(t,4l+2) & s_1(t,4l+3) & s_1(t,4l+4) \\ s_2(t,4l+1) & s_2(t,4l+2) & s_2(t,4l+3) & s_2(t,4l+4) \\ s_3(t,4l+1) & s_3(t,4l+2) & s_3(t,4l+3) & s_3(t,4l+4) \\ s_4(t,4l+1) & s_4(t,4l+2) & s_4(t,4l+3) & s_4(t,4l+4) \end{bmatrix}. \quad (5)$$

The m-th row of the signal matrix S is transmitted by the m-th transmit antenna, and in the time periods $(4l+n-1)T \le t \le (4l+n)T, n = 1, 2, 3, 4$, all signals in the n-th column are transmitted simultaneously, denote this time period as the $(4l+n)$-th time slot.

At time slot $k = 4l+n$, $n = 1, 2, 3, 4$, the received signal $y(t, k)$ can be written as [9], [10]

$$y(t,k) = \sum_{m=1}^{L_t} \alpha_m(t) s_m(t,k) + W(t), \tag{6}$$

where $W(t)$ is the additive noise, $\alpha_m(t)$ is the channel gain from the m-th transmit antenna to the receive antenna, and $s_m(t, k)$ is the transmitted signal from the m-th transmit antenna at time slot k which is specified as

$$s_m(t,k) = \sqrt{1/T} \exp\left\{j2\pi\left[\phi_0 + \Phi_m(t,k)\right]\right\}. \tag{7}$$

The term $\Phi_m(t, k)$ in (7) contains the information symbol $d_{m,n}(l)$. More precisely,

$$\Phi_m(t,k) = \sum_{i=0}^{k} \left\{h_m d_{m,i} q\left(t-(i-1)T\right) + c_{m,i} q_0\left(t-(i-1)T\right)\right\}, \tag{8}$$

where $d_{m,i} = d_{m,n}(l)$ with $i = 4l + n$, h_m is the modulation index of the CPM system, $q(t)$ and $q_0(t)$ are the phase smoothing response functions with

$q_0(t) = q(t) = 0$ for $t \le 0$, $q_0(t) = q(t) = \frac{1}{2}$ for $t > T$, and $c_{m,i} = c_{m,n}(l)$ with $i = 4l + n$ is generated by the following matrix

$$\mathbf{c}_l = \begin{bmatrix} c_{1,1}(l) & c_{1,2}(l) & c_{1,3}(l) & c_{1,4}(l) \\ c_{2,1}(l) & c_{2,2}(l) & c_{2,3}(l) & c_{2,4}(l) \\ c_{3,1}(l) & c_{3,2}(l) & c_{3,3}(l) & c_{3,4}(l) \\ c_{4,1}(l) & c_{4,2}(l) & c_{4,3}(l) & c_{4,4}(l) \end{bmatrix}, \tag{9}$$

which depends on the information symbol matrix $\mathbf{d}_l$ and is used to maintain the phase continuity and the orthogonality of the four signals. The choice of the matrix $\mathbf{c}_l$ is the key of the fast decoding, and will be specified later. If we consider a full response CPM system, let $h_m = h \triangleq \frac{m_0}{p}$, where m_0 and p are relatively prime integers, then the phase of the CPM system, $\Phi_m(t, k)$, can be expressed as [9], [10]

$$\Phi_m(t,k) = \theta_m(k-1) + h d_{m,k} q(t-(k-1)T) + c_{m,k} q_0(t-(k-1)T),$$

in which

$$\theta_m(k-1) = \frac{h}{2} \sum_{i \le k-1} d_{m,i} + \frac{1}{2} \sum_{i \le k-1} c_{m,i} \tag{10}$$

belongs to the set $\Omega_\theta \triangleq \{0, \frac{1}{p}, \frac{2}{p}, ..., \frac{p-1}{p}\}$ with modulo 1. Thus, $\Phi_m(t, k)$ has a trellis structure with states in Ω_θ, and $(\Phi_1(t, k), \Phi_2(t, k), \Phi_3(t, k), \Phi_4(t, k))$ has a trellis structure with states in the product set $\Omega_\theta \times \Omega_\theta \times \Omega_\theta \times \Omega_\theta$. Clearly, the number of states increases exponentially with the number of transmit antennas.

The ML demodulation of the information sequence $\mathbf{I}$ is [9], [10]

$$\hat{\mathbf{I}} = \arg\min_{\mathbf{I}} \left\{ \sum_{k=1}^{K} \int_0^T \left| y(t,k) - \sum_{m=1}^{L_t} \alpha_m(t) s_m(t,k) \right|^2 dt \right\}. \tag{11}$$

If the Viterbi algorithm is used to solve the above ML demodulation, each state in the trellis structure has $(2M)^{L_t}$ coming branches and $(2M)^{L_t}$ leaving branches. The decoding complexity is high if there is no fast decoding algorithm. In the following, we propose some special ST-CPM schemes such that the branches in each state can be separated into several independent sets and the ML demodulation complexity can be reduced.

III. Full Response CPM System with Modified Orthogonal Space-Time Coding

In this section, we present a CPM system based on the modified orthogonal space-time codes (2) for four transmit antennas, i.e., $L_t = 4$.

A. Design of CPM Signals

For an information sequence $\{I_{1,1}, I_{2,1}, I_{3,1}, ..., I_{1,l}, I_{2,l}, I_{3,l}, ...\}$, each information block $I_{1,l}, I_{2,l}, I_{3,l}$ of length 3 is mapped into symbols $e_{1,l}, e_{2,l}, e_{3,l}$, which are chosen from a signal constellation Ω in (4). The matrix $\mathbf{d}_l$ in (3) is generated by $e_{1,l}, e_{2,l}, e_{3,l}$ as follows

$$\begin{bmatrix} d_{1,1}(l) & d_{1,2}(l) & d_{1,3}(l) & d_{1,4}(l) \\ d_{2,1}(l) & d_{2,2}(l) & d_{2,3}(l) & d_{2,4}(l) \\ d_{3,1}(l) & d_{3,2}(l) & d_{3,3}(l) & d_{3,4}(l) \\ d_{4,1}(l) & d_{4,2}(l) & d_{4,3}(l) & d_{4,4}(l) \end{bmatrix} = \begin{bmatrix} e_{1,l} & -e_{2,l} & -e_{3,l} & 0 \\ e_{2,l} & -e_{1,l} & 0 & -e_{3,l} \\ e_{3,l} & 0 & -e_{1,l} & -e_{2,l} \\ 0 & e_{3,l} & e_{2,l} & e_{1,l} \end{bmatrix}.$$

To generate the CPM signal waveforms $s_m(t, k)$ in (7), we also need the matrix $\mathbf{c}_l$ in (9), which is related to the symbol matrix $\mathbf{d}_l$ and is specified as follows:

$$\mathbf{c}_l = \begin{bmatrix} 1-a_{1,l} & 1+a_{2,l}+a_{3,l} & 1+a_{2,l}+a_{3,l} & 1+a_{1,l} \\ -a_{2,l} & a_{1,l}+a_{3,l} & 1+a_{2,l} & a_{1,l}+a_{3,l} \\ 1-a_{3,l} & a_{3,l} & 1+a_{1,l}+a_{2,l} & 1+a_{1,l}+a_{2,l} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

where

$$a_{i,l} = \mathrm{mod}(\frac{e_{i,l} m_0}{p}, 2), \quad i = 1, 2, 3, \tag{13}$$

where $\mathrm{mod}(x, y)$ is the modulo operation of x with base y and $m_0/p = h$ is the modulation index. The reason why modulo 2 rather modulo 1 in the phase component is used is because the smoothing response functions $q(T) = q_0(T) = 1/2$ in (8) and thus, $1/2$ appears in the phase modulation in (10) We can see that the matrix $\mathbf{c}_l$ depends only on $a_{1,l}, a_{2,l}$ and $a_{3,l}$, and all of $a_{i,l}$ have at most $2p_0$ possible values for all possible values of $e_{i,l}$ in Ω, where

$$p_0 = \begin{cases} p & \text{if } p \text{ is odd;} \\ p/2 & \text{if } p \text{ is even,} \end{cases} \tag{14}$$

since all of $e_{1,l}$, $e_{2,l}$ and $e_{3,l}$ are odd numbers, and m_0 and p are relatively prime integers.

At the time period between $4lT$ and $4(l+1)T$, the following signals are sent through the m-th transmit antenna

$$s_m(t, 4l+n) = \sqrt{1/T} \exp\left\{j2\pi\Phi_m(t, 4l+n)\right\}, \tag{15}$$

for $n = 1, 2, 3, 4$. The transmitted signal matrix S in (5) can be written as follows[14],

$$S = EC(x_1, x_2, x_3)F_1, \tag{16}$$

where,

$$F_1 = \mathrm{diag}\left\{1,\ 1,\ e^{j\pi a_{3,l}},\ e^{j\pi(a_{3,l}+a_{2,l})}\right\} F, \tag{17}$$

$$F = \mathrm{diag}\left\{1,\ e^{j2\pi a_{3,l} q_0(\tau)},\ e^{j2\pi a_{2,l} q_0(\tau)},\ e^{j2\pi a_{1,l} q_0(\tau)}\right\}, \tag{18}$$

$$C(x_1, x_2, x_3) \triangleq \begin{bmatrix} x_1 & -x_2^* & x_3^* & -1 \\ x_2 & x_1^* & e^{j2\pi q_0(\tau)} & -x_3^* \\ x_3 & -e^{-j2\pi q_0(\tau)} & -x_1^* & x_2^* \\ 1 & x_3 & x_2 & x_1 \end{bmatrix}, \tag{19}$$

$$x_i = e^{j2\pi\left[e_{i,l} h q(\tau) - a_{i,l} q_0(\tau)\right]}, \quad i = 1, 2, 3, \tag{20}$$

Notice that $C(x_1, x_2, x_3)$ in (19) has the same form of (2). E and F are diagonal matrices. The structure (16) provides the existence of fast decoding algorithm which will be discussed in next section.

B. Fast Demodulation Algorithm

By the trellis structure of the CPM system, the sequence detection in (11) can be implemented using the Viterbi algorithm. From each state of the trellis, there are $(2M)^3$ coming branches and $(2M)^3$ leaving branches. In order to search the survivor paths, the input symbol block $(e_{1,l}, e_{2,l}, e_{3,l})$ and the distance of current state $\theta_m(4l)$ to next state $\theta_m(4(l+1))$ need to be obtained, where the input block $(e_{1,l}, e_{2,l}, e_{3,l})$ causes the state transfer from $\theta_m(4l)$ to $\theta_m(4(l+1))$. Thus, we need to search all the branch metrics at the stage l as follows

$$(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}) = \arg \min_{(e_{1,l},e_{2,l},e_{3,l})\in\Omega\times\Omega\times\Omega} \Bigg\{ \sum_{k=4l+1}^{4(l+1)} \int_{(k-1)T}^{kT} \left| y_n(t,k) - \sum_{m=1}^{4} \alpha_{m,n}(t) s_m(t,k) \right|^2 dt \Bigg\}. \quad (21)$$

We next want to simplify the above branch searching by taking the advantage of the special trellis structure of the proposed CPM system.

Assume that the channel $\alpha_m(t)$ is known and constant during a space-time coding block $[4lT, 4(l+1)T]$. From the orthogonality of $s_m(t,k)$ and the properties of $c_{m,n}(l)$, the equation (21) can be rewritten as the following [14]

$$(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}) = \arg \min_{(e_{1,l},e_{2,l},e_{3,l})\in\Omega\times\Omega\times\Omega} \int_0^T ||Y(l) - AS||_F^2 d\tau, \quad (22)$$

where $||V||_F$ is the Frobenius norm. From [14], we know that

$$\int_0^T ||Y(l) - AS||_F^2 d\tau = f_1(x_1, a_{1,l}, a_{2,l}, a_{3,l}) + f_2(x_2, a_{1,l}, a_{2,l}, a_{3,l}) + f_3(x_3, a_{1,l}, a_{2,l}, a_{3,l}). \quad (23)$$

Recall that all of $a_{i,l}, i = 1,2,3$, have only $2p_0$ possible values, where p_0 is specified in (14). More precisely, since $a_{i,l} = \text{mod}(e_{i,l} m_0/p,\, 2), i = 1,2,3$, every $a_{i,l}$ belongs to the following set G:

$$G \triangleq \begin{cases} \{0, \frac{1}{p_0}, \frac{2}{p_0}, \cdots, \frac{2p_0-1}{p_0}\}, & \text{if } p \text{ is odd;} \\ \frac{1}{p} + \{0, \frac{1}{p_0}, \frac{2}{p_0}, \cdots, \frac{2p_0-1}{p_0}\}, & \text{if } p \text{ is even.} \end{cases} \quad (24)$$

Again, since $a_{i,l} = \text{mod}(e_{i,l} m_0/p,\, 2)$, for a fixed $a_{i,l}$, symbol $e_{i,l}$ has to belong to the following set $\Omega(a_{i,l})$:

$$\Omega(a_{i,l}) \triangleq \{n \in \Omega : \text{mod}(n m_0/p,\, 2) = a_{i,l}\}, \quad (25)$$

where Ω is specified in (6). The number of elements in $\Omega(a_{i,l})$ is approximately $2M/(2p_0)$ that will be used to calculate the demodulation complexity later. Therefore, the branch searching in (21) or (22) can be simplified as

$$(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}) = \arg \min_{(a_{1,l},a_{2,l},a_{3,l})\in G\times G\times G} \Bigg\{ \min_{e_{1,l}\in\Omega(a_{1,l})} f_1(x_1, a_{1,l}, a_{2,l}, a_{3,l}) + \min_{e_{2,l}\in\Omega(a_{2,l})} f_2(x_2, a_{1,l}, a_{2,l}, a_{3,l}) + \min_{e_{3,l}\in\Omega(a_{3,l})} f_3(x_3, a_{1,l}, a_{2,l}, a_{3,l}) \Bigg\}. \quad (26)$$

The decoding complexity of the original branch searching (21) is $(2M)^3/p_0$ from a state to the next state, while the one in (26) is at most $(2p_0)^3 3(2M/(2p_0))/p_0 = 24p_0 M$ as shown in Fig.1. Notice that, p_0 depends only on the CPM modulation index h, not on the signal constellation size $2M$, and p_0 is usually much smaller than $2M$. As an example, when $h = 1/2$ is used, then $p_0 = 1$. In this case, the number of branch searching is at most $24M$ while the original one is $8M^3$.

The similar idea for the parallel path searching reduction has independently appeared in space-time trellis coding for PSK signal by using MTCM [17] [18].

IV. Full Response CPM System with Quasi-Orthogonal Space-Time Coding

It is known that the rate of orthogonal space-time code can not be greater than $3/4$ for more than two transmit antennas [5], [15]. Recently, Jafarkhani [11], Tirkkonen, Boariu and Hottinen [12] proposed quasi-orthogonal space-time code

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}, \quad (27)$$

with rate 1 for four transmit antennas. The code (27) has fast decoding, but does not have full diversity. More recently, Su and Xia [16] proposed a quasi-orthogonal space-time code with full diversity based on (27). The full diversity is achieved by properly choosing the signal constellations. In this section, we want to use the quasi-orthogonal space-time code with full diversity in CPM systems.

A. Design of CPM Signals

A binary information sequence $\{\cdots, I_{1,l}, \cdots, I_{L,l}, \cdots\}$ is mapped to a symbol sequence $\{\cdots,\ e_{1,l}, e_{2,l}, e_{3,l}, e_{4,l},\ \cdots\}$, where $e_{1,l}$ and $e_{2,l}$ are chosen from a signal constellation

$$\Omega = \{-2M+1, ..., -1, 1, ..., 2M-1\}, \quad (28)$$

and $e_{3,l}$ and $e_{4,l}$ are chosen from another signal constellation

$$\tilde{\Omega} = \{-2(\tilde{M}-1), ..., -2, 0, 2, ..., 2\tilde{M}\}, \quad (29)$$

The matrix $\mathbf{d}_l$ in (3) is generated by $e_{1,l}, ..., e_{4,l}$ as follows

$$\mathbf{d}_l = \begin{bmatrix} e_{1,l} & e_{2,l} & e_{3,l} & e_{4,l} \\ -e_{2,l} & -e_{1,l} & -e_{4,l} & -e_{3,l} \\ e_{3,l} & e_{4,l} & e_{1,l} & e_{2,l} \\ -e_{4,l} & -e_{3,l} & -e_{2,l} & -e_{1,l} \end{bmatrix}. \quad (30)$$

Similar to Section 3, to generate the CPM signal waveforms $s_m(t,k)$ in (7), we also need matrix $\mathbf{c}_l$ in (9), which is related to the symbol matrix $\mathbf{d}_l$ and specified as follows:

$$\mathbf{c}_l = \begin{bmatrix} a_{3,l} - a_{1,l} & a_{4,l} - a_{2,l} & a_{1,l} - a_{3,l} & a_{2,l} - a_{4,l} \\ 1 + a_{2,l} + a_{3,l} & 1 + a_{1,l} + a_{4,l} & 1 + a_{1,l} + a_{4,l} & 1 + a_{2,l} + a_{3,l} \\ 0 & 0 & 0 & 0 \\ 1 + a_{3,l} + a_{4,l} & 1 + a_{3,l} + a_{4,l} & 1 + a_{1,l} + a_{2,l} & 1 + a_{1,l} + a_{2,l} \end{bmatrix}, \quad (31)$$

where

$$a_{i,l} = \text{mod}(\frac{e_{i,l} m_0}{p},\, 2), \quad i = 1,2,3,4, \quad (32)$$

where $m_0/p = h$ is the modulation index. Similar to (12)-(13), matrix $\mathbf{c}_l$ depends only on $a_{1,l}, a_{2,l}, a_{3,l}$ and $a_{4,l}$, and all of $a_{i,l}$ have at most $2p_0$ possible values.

At the time period between $4lT$ and $4(l+1)T$, the transmitted signal matrix S can be written as [14]

$$S = EC(x_1, x_2, x_3, x_4)F_1, \tag{33}$$

where

$$E = \text{diag}\left\{e^{j2\pi\theta_1(4l)},\, e^{j2\pi[\theta_2(4l)+q_0(\tau)]},\, e^{j2\pi\theta_3(4l)},\, e^{j2\pi[\theta_4(4l)+q_0(\tau)]}\right\}, \tag{34}$$

$$C(x_1, x_2, x_3, x_4) = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ x_2^* & -x_1^* & x_4^* & -x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ x_4^* & -x_3^* & x_2^* & -x_1^* \end{bmatrix}, \tag{35}$$

and

$$F_1 = \text{diag}\Big\{e^{j2\pi a_{3,l}q_0(\tau)},\, e^{j\pi[2a_{4,l}q_0(\tau)+a_{3,l}]},\, e^{j\pi[2a_{1,l}q_0(\tau)+a_{3,l}+a_{4,l}]},\, e^{j\pi[2a_{2,l}q_0(\tau)+a_{1,l}+a_{3,l}+a_{4,l}]}\Big\}. \tag{36}$$

B. Fast Demodulation Algorithm

Similar to the fast demodulation algorithm developed in Section 3, we assume that the channel state information $\alpha_m(t)$ is known and constant during a space-time coding block $[4lT, 4(l+1)T]$. Let $A = [\alpha_1(t)\ \alpha_2(t)\ \alpha_3(t)\ \alpha_4(t)]$, and $Y(l) = [y(t, 4l+1)\ y(t, 4l+2)\ y(t, 4l+3)\ y(t, 4l+4)]$, then the branch metric at stage l can be calculated as [14]

$$(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}, \hat{e}_{4,l}) = \underset{(e_{1,l},e_{2,l},e_{3,l},e_{4,l})\in\Omega\times\Omega\times\tilde{\Omega}\times\tilde{\Omega}}{\arg\min} \int_0^T ||Y(l) - AS||_F^2 d\tau. \tag{37}$$

In here

$$\int_0^T ||Y(l) - AS||_F^2 d\tau = f_{13}(x_1, x_3, a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}) + f_{24}(x_2, x_4, a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}).$$

Recall that all of $a_{i,l}, i = 1, 2, 3, 4$, have at most $2p_0$ possible values, where p_0 is specified in (14). More precisely, $a_{1,l}$ and $a_{2,l}$ belong to set G as in (24) and $a_{3,l}$ and $a_{4,l}$ belong to the following set $\tilde{G}$

$$\tilde{G} = \left\{0, \frac{1}{p_0}, \frac{2}{p_0}, \cdots, \frac{2p_0-1}{p_0}\right\}, \tag{38}$$

which is different from G in (24) because constellation $\tilde{\Omega}$ in (29) is different from constellation Ω in (28). Since $a_{i,l} = \text{mod}(e_{i,l}m_0/p,\, 2)$, $i = 1, 2, 3, 4,$, if $a_{1,l}$ and $a_{2,l}$ are fixed, then $e_{1,l}$ and $e_{2,l}$ belong to the following sets $\Omega(a_{i,l})$, $i = 1, 2$, respectively:

$$\Omega(a_{i,l}) = \{n \in \Omega : \text{mod}(nm_0/p,\, 2) = a_{i,l}\}, \quad i = 1, 2, \tag{39}$$

where Ω is specified in (28). The number of elements in $\Omega(a_{i,l})$ is approximately $2M/(2p_0)$. If $a_{3,l}$ and $a_{4,l}$ are fixed, then $e_{3,l}$ and $e_{4,l}$ belong to the following sets $\tilde{\Omega}(a_{i,l})$, $i = 3, 4$, respectively,

$$\tilde{\Omega}(a_{i,l}) = \{n \in \tilde{\Omega} : \text{mod}(nm_0/p,\, 2) = a_{i,l}\}, \quad i = 3, 4, \tag{40}$$

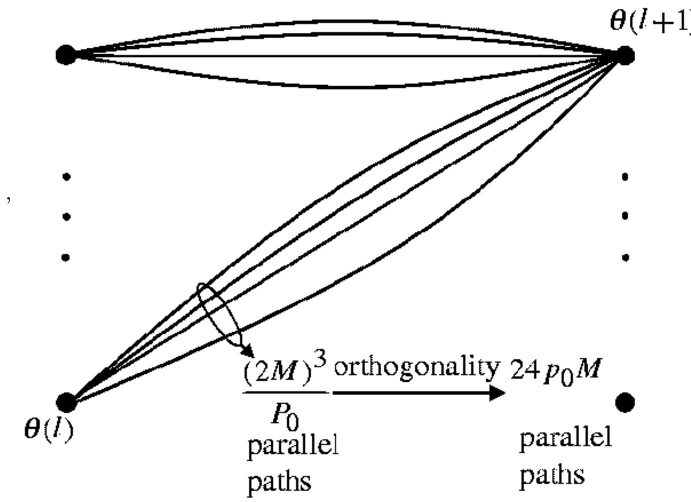


Fig. 1. Parallel paths between two states.

where $\tilde{\Omega}$ is specified in (29). The number of elements in $\tilde{\Omega}(a_{i,l})$ is approximately $2\tilde{M}/(2p_0)$. Therefore, the branch searching (37) can be simplified as

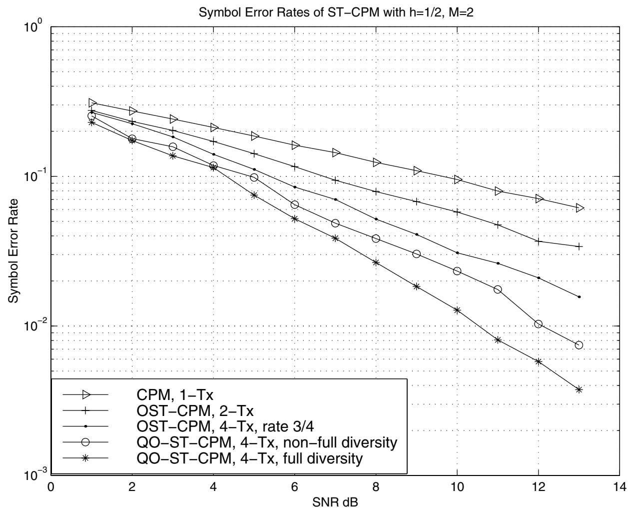
$$(\hat{e}_{1,l}, \hat{e}_{2,l}, \hat{e}_{3,l}, \hat{e}_{4,l}) = \arg \min_{(a_{1,l},a_{2,l},a_{3,l},a_{4,l})\in G\times G\times\tilde{G}\times\tilde{G}} \Bigg\{ \min_{(e_{1,l},e_{3,l})\in\Omega(a_{1,l})\times\tilde{\Omega}(a_{3,l})} f_{13}(x_1, x_3, a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}) + \min_{(e_{2,l},e_{4,l})\in\Omega(a_{2,l})\times\tilde{\Omega}(a_{4,l})} f_{24}(x_2, x_4, a_{1,l}, a_{2,l}, a_{3,l}, a_{4,l}) \Bigg\} \tag{41}$$

The decoding complexity of the above branch searching is $(2p_0)^4(2M + 2\tilde{M})/(2p_0)/p_0 = 16p_0^2(M + \tilde{M})$, while the original one is $(2M)^2(2\tilde{M})^2/p_0 = 16(M\tilde{M})^2/p_0$. Notice that, p_0 depends only on the CPM modulation index h, not on the signal constellation size $2M$ or $2\tilde{M}$, and p_0 is usually much smaller than $2M$ and $2\tilde{M}$.

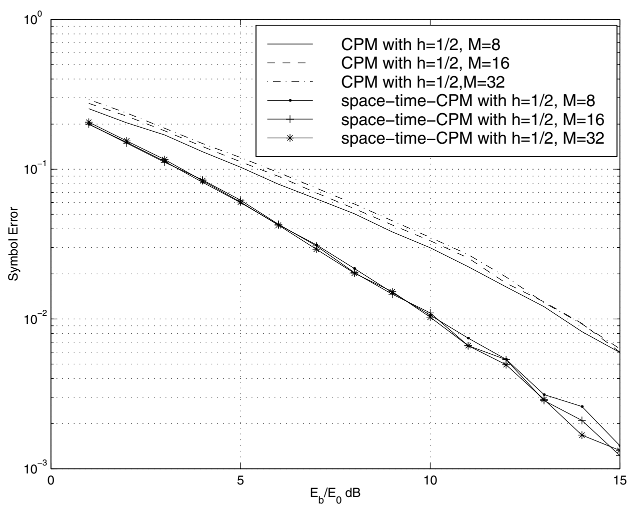
V. Simulation Results

In this section, we compare the performances of the modified orthogonal ST-CPM system for four transmit antennas, the quasi-orthogonal ST-CPM system also for four transmit antennas, and the OST-CPM system [13] for two transmit antennas. One receiver is used in all the simulations. The channel coefficients are zero mean complex Gaussian random variables with variance 1. We assume the channel is *quasi-static*, i.e., the channel coefficients are constant during one block transmission, and change independently from one block to another. In all simulations, we set the full response CPM systems with modulation index $h = 1/2$ and smoothing phase functions $q(t) = \frac{1}{2T}t$, $q_0 = 2\left\{1/2 - \frac{T}{2\pi}sin\left(2\pi\frac{t}{T}\right)\right\}$ if $t \in [0, T]$, $q_0(t) = q(t) = 0$ if $t \leq 0$, and $q_0(t) = q(t) = 1/2$ if $t > T$.

The signal constellation $\Omega = \{-2M+1, ..., -1, 1, 2M-1\}$ is used in the conventional one transmitter CPM system, the OST-CPM system, the modified orthogonal ST-CPM system, and the quasi-orthogonal ST-CPM system *not* with full diversity. For the quasi-orthogonal ST-CPM system with full diversity, signal constellation $\Omega = \{-2M+1, ..., -1, 1, ..., 2M-1\}$ is used for $e_{1,l}$ and



(a) M = 4



(b) M = 8

Fig. 2. Performances of the conventional CPM with 1 Tx antenna (line with $\triangleright$), the OST-CPM with 2 Tx antennas (line with $+$), the modified OST-CPM with 4 Tx antennas (line with $\cdot$), and the quasi-orthogonal ST-CPM with 4 Tx antennas (line with $\circ$ for that *not* with full diversity, and line with $*$ for that with full diversity).

$e_{3,l}$, signal constellation $\tilde{\Omega} = \{-2(M-1), ..., -2, 0, 2, ..., 2M\}$ is used for $e_{2,l}$ and $e_{4,l}$.

From Fig.2 (a) and (b), we can see that the performance of the modified orthogonal ST-CPM system for four transmit antennas is much better than that of the OST-CPM system for two transmit antennas. Furthermore, the quasi-orthogonal ST-CPM systems outperform the modified orthogonal ST-CPM system. However, the decoding complexity of the former is higher than that of the latter.

VI. Conclusion

In this paper, we proposed a modified orthogonal ST-CPM system and a quasi-orthogonal ST-CPM system for three and four transmit antennas, and derived fast ML demodulation algorithms for the proposed systems. Simulation results showed that the performances of the proposed ST-CPM schemes for four transmit antennas are much better than that of the OST-CPM system for two transmit antennas. Although the idea of the fast demodulation algorithm in this paper is similar to the one in [13] for two transmit antennas, the orthogonal-like space-time coded CPM design for 4 transmit antennas is much more difficult than the design for 2 transmit antennas.

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