Universal space-time codes from two-dimensional trellis codes

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Abstract— **In the absence of accurate channel probability distribution information or in broadcast scenarios, code design for consistent channel-by-channel performance, rather than average performance, may be desirable. Root and Varaiya's compound channel theorem promises the existence of universal codes that operate with a consistent proximity to channel mutual information on *any* instance of the compound linear vector Gaussian channel that is similar to the capacity gap of an AWGN-specific code with similar complexity on the AWGN channel. This study presents single-dimensional trellis codes such that when multiplexed over two, three and four transmit antennas, provide universal performance over the compound linear vector Gaussian channel. As a result of their channel-by-channel consistency, the universal trellis codes presented here deliver comparable or in some cases superior frame-error-rate and bit-error-rate performance under quasistatic Rayleigh fading to trellis codes of similar complexity that are designed specifically for the quasistatic Rayleigh fading scenario.**

I. Introduction

The use of multiple antennas at both transmitter and receiver is crucial in order to harvest the capacity of rich propagation environments. For example, when the path gains between transmit and receive antenna pairs are independent Gaussian random variables, Foschini [1] and Telatar [2] showed that capacity increases linearly with the number of transmit-receive antenna pairs. As is common in the current literature, we refer to a channel resulting from the use of multiple antennas as a space-time channel.

An instance of signal transmission over a space-time channel with N_t transmit antennas and N_r receive antennas is often modeled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{H}$ is an $N_r \times N_t$ complex matrix of path gains, $\mathbf{x}$ is the complex input vector, $\mathbf{y}$ is the complex output vector and $\mathbf{w}$ is the complex additive white Gaussian noise vector with variance N_0 per dimension. For vector channels of the form (1), signal design criteria for average error probability performance were established in [3] and [4] for the case when the path gains are characterized by complex Gaussian random variables (Rayleigh fading). Since then, the design of coded space-time diversity schemes based on the average performance criterion for quasistatic Rayleigh fading has been an active research area ([5], [6], [7], [8], [9], [10], [11]).

The work was supported by the National Science Foundation through Grant CCR-0209110 and STMicroelectronics and the state of California through MICRO grant 03-031. This work was supported by Conexant, Skyworks, and the state of California through MICRO grant 03-092.

While good average error performance is desirable, it does not guarantee consistently good "channel-by-channel" performance (i.e. universal performance). In fact, most of the code designs proposed for the average performance cannot maintain coded performance under certain rank-deficient channels. Such channels have been of interest due to their deleterious effect in certain propagation environments ([12], [13], [14]). In constrast, good universal performance implies good average performance irrespective of the quasistatic distribution.

Root and Varaiya's compound channel coding theorem [15] indicates that a *single* code with rate R bits/symbol can achieve reliable transmission of information over any linear Gaussian channel $\mathbf{H}$ that induces more than R bits/symbol of mutual information (MI), i.e. over any channel with $\text{MI}(\mathbf{H}, E_s) > R$ where

$$\text{MI}\left(\mathbf{H}, E_s\right) = \log_2 \det\left(\mathbf{I} + \frac{E_s}{N_0}\mathbf{H}\mathbf{H}^\dagger\right) \tag{2}$$

is the mutual information $I(\mathbf{x};\mathbf{y})$ (in bits per symbol) and E_s is the transmit energy per antenna per symbol[1]. The implication of this result is that good error performance over a set of channels does not have to come at the expense of significant performance degradation over others.

A universal code delivers similar error performance over all channels with the same mutual information. Consider a code that transmits R bits per symbol over N transmit antennas. Let E_s^* denote the transmit power per antenna that a specific code requires to achieve the target bit-error-rate (BER) on the (static) channel $\mathbf{H}$. The excess mutual information requirement of that specific on this channel is the difference between the channel mutual information $\text{MI}(\mathbf{H}, E_s^*)$ and the rate R:

$$\text{EMI}(\mathbf{H}) = \text{MI}(\mathbf{H}, E_s^*) - R \quad \text{bits.} \tag{3}$$

Let $\mathcal{H} = \{\mathbf{H} : \text{MI}(\mathbf{H}) \geq R\}$ denote the set of channels that comprise the compound $N \times N$ channel with capacity R bits per symbol. The goal of universal R bits per symbol code design is to minimize, over the $N \times N$ compound channel with capacity R, the worst-case excess mutual information $\sup_{\mathbf{H}\in\mathcal{H}} \text{EMI}(\mathbf{H}) - R$ at fixed target bit-error-rate (or frame-error-rate), latency, and decoding complexity. This paper extends the ideas in [16] to propose universal space-time trellis codes formed by straightforward multiplexing

[1]Assuming the input vector has a complex Gaussian distribution with covariance matrix $E_s\mathbf{I}_{N_t}$.

of single-dimensional linear trellis-code over two, three and four transmit antennas. The multiplexed trellis codes have simpler maximum-likelihood decoding than general vector-labeled trellis codes.

Section II summarizes our results on the worst-case minimum-distance of a space-time code under linear transformations with equal mutual information and derives a simple approximate criterion for universal behavior. This section also formulates the encoder rate, constellation size and trellis complexity requirements for universal space-time trellis codes formed by multiplexing a single-dimensional trellis code.

Section III presents linear trellis codes found by exhaustive searches over their respective encoder classes, such that when multiplexed over two, three and four transmit antennas provide universal performance with maximum-likelihood decoding. The performance variation of universal codes as compared to other existing space-time codes over different channel instances is illustrated via extensive simulations. Our discussion ends with simulation results showing that the average error performance of the proposed universal codes over quasistatic Rayleigh fading is comparable, and in some cases superior to existing space-time codes with similar decoding complexity designed specifically for the average error probability performance. Section IV delivers the conclusions.

II. Design guidelines for universal space-time trellis codes

Let $\mathbf{X}$ and $\hat{\mathbf{X}}$ be two different codewords of a space-time code $\mathcal{C}$ for N_t transmit antennas, and let $\mathbf{E} = \hat{\mathbf{X}} - \mathbf{X}$ denote the codeword difference matrix. Under maximum-likelihood (ML) decoding, the probability that the decoder mistakes $\hat{\mathbf{X}}$ for $\mathbf{X}$ conditioned on the perfect knowledge of the channel matrix $\mathbf{H}$ at the receiver is given by $P(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}) = Q(\sqrt{d^2(\mathbf{E}|\mathbf{H})/2N_0})$ where $d^2(\mathbf{E}|\mathbf{H}) = ||\mathbf{HE}||^2 = \mathrm{trace}(\mathbf{HEE}^\dagger\mathbf{H}^\dagger)$ is the squared Euclidean norm of the codeword difference matrix $\mathbf{E}$ when transformed by the channel $\mathbf{H}$, and $Q(\cdot)$ is the standard Gaussian tail integral function. For a fixed channel $\mathbf{H}$, the minimum of the squared-distances $d^2(\mathbf{E}|\mathbf{H})$ over all $\mathbf{E}$ will be called the $d^2_{\min}(\mathbf{H})$ of the code. Universal codes should have good $d^2_{\min}(\mathbf{H})$ for all instances $\mathbf{H}$ of the compound channel.

The smallest value of $d^2_{\min}(\mathbf{H})$ over the compound channel is a function of the eigenvalues of the codeword difference matrices. For a given codeword difference matrix $\mathbf{E}$, $\boldsymbol{\zeta} = (\zeta_1, ..., \zeta_{N_t})$ will be the vector of eigenvalues of $\mathbf{EE}^\dagger$ with the ordering $\zeta_1 \geq \zeta_2 \geq ... \geq \zeta_{N_t}$, and we will write $\mathbf{E} \cong \boldsymbol{\zeta}$. The eigenvalues of $\mathbf{HH}^\dagger$, where $\mathbf{H}$ is an $N_r \times N_t$ channel gain matrix, will be denoted by λ_i, $i = 1, ..., N_r$ with the ordering $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_{N_r}$, and we will write $\mathbf{H} \cong \boldsymbol{\lambda}$ where $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_{N_r})$. Throughout the paper, we will assume that $N_r \leq N_t$.

A. Worst-case distance over the compound channel

Consider the $N_r \times N_t$ compound channel with capacity R bits per symbol, $\mathcal{H} = \left\{\mathbf{H} \in \mathbb{C}^{N_r \times N_t} : \mathrm{MI}(\mathbf{H}) \geq R\right\}$. We are interested in the minimum of the squared Euclidean norm, $d^2(\mathbf{E}|\mathbf{H})$, of a codeword difference matrix $\mathbf{E} \cong \boldsymbol{\zeta}$ as transformed by all instances $\mathbf{H}$ of the compound channel $\mathcal{H}$. Subject to the MI constraint $\mathrm{MI}(\mathbf{H}) = \sum_{i=1}^{N_r} \log_2(1 + \lambda_i) \geq R$, the worst case channel has its eigenvalues along the weakest eigenvectors of the error event, with magnitudes determined by a waterpouring solution using the geometric mean of the error event eigenvalues. This exact solution, however, does not yield a simple criterion for universality except for $N_r = 1$. An approximate criterion for universality over the $N \times N$ compound channel is obtained by bounding the worst-case distance over the compound channel. The following lemma summarizes this result. We have omitted the proof for brevity.

Lemma 1: The worst-case minimum distance of a space-time code over the compound $N \times N$ channel $\mathcal{H}(N, N, R)$ of capacity R bits per symbol is bounded as

$$\left(2^{\frac{R}{N}} - 1\right)\Delta_E^* \geq \inf_{\mathbf{H}\in\mathcal{H}(N,N,R)} d^2_{\min}(\mathbf{H}) \geq \min_{\mathbf{E}} \left(2^{R\frac{\zeta_N}{\Delta_E(\mathbf{E})}} - 1\right)\Delta_E(\mathbf{E}) \qquad (4)$$

where $\Delta_E^* = \min_{\mathbf{E}} \mathrm{trace}(\mathbf{EE}^\dagger)$ is the minimum squared Euclidean-distance of the code. The first inequality of Lemma 1 implies that a universal code should have good minimum Euclidean-distance. The second inequality of the lemma leads us to choose, among good minimum Euclidean-distance codes, those codes with high minimum eigenvalue $\zeta^*_{N_t} = \min_{\mathbf{E}} \zeta_{N_t}(\mathbf{EE})$. Ultimately, the universal performance of a code over the compound channel will be measured by its excess mutual information requirement. Nevertheless, this criterion provides us with a basic rule to prune the search for universal space-time trellis codes.

B. Encoder rate and constellation size requirements for universal space-time codes

Consider a rate-k/n convolutional encoder with memory ν. For R bits per symbol transmission over N_t transmit antennas using a 2^mPSK/QAM constellation, we use this encoder $l = R/k$ times (assume for simplicity that k divides R evenly). Let $(b_0, ..., b_{n-1})$, ..., $(b_{(l-1)n}, ..., b_{ln-1})$ denote the ln codeword bits that the binary encoder would produce for lk input bits in succession. If $ln = N_t m$, then we map $(b_{(i-1)m}, ..., b_{im-1})$ onto the 2^mPSK/QAM constellation for the ith transmit antenna, $i = 1, ..., N_t$. If $ln > N_t m$, we puncture $ln - N_t m$ out of ln bits and group the remaining $N_t m$ bits similarly, keeping the index order. Universal performance over the compound channel dictates the following design rules for multiplexed single-dimensional trellis codes.

- Binary encoder rate and constellation size: When the channel has only one nonzero column, the encoder rate effectively rises to kN_t/n, therefore $n > kN_t$ is required. Also, the constellation should be large enough to host R bits redundantly, i.e. $m > R$.
- Trellis complexity: The effective code length (duration of the shortest error event) of a k-bits/symbol linear trellis code is $\mathrm{ECL} = \lfloor \nu/k \rfloor + 1$ where ν is the memory of the encoder [17]. In general, for r-levels of transmit diversity,

TABLE I

Linear trellis codes for multiplexing onto N_t transmit antennas to produce R bits per symbol space-time codes. A rate-$1/n$ binary convolutional encoder outputs nR code bits which are mapped on $N_t \times 2^m$PSK/QAM. Codes #1-#10 are universal codes found via exhaustive searches. Codes #11-#16 are trellis codes with good-AWGN performance and/or good periodic-erasures performance. 16QAM, QPSK: Gray labeling, 8PSK: Gray labeling 0,2,3,1,5,7,6,4 in octal going around the circle.

#	N_t	R	$1/n$, ν	g_i	Const.	$\zeta^*_{\Delta_{H,\min}}$	Δ^*_P	Δ^*_E	Mapping
1	2	1	1/4, 4	03 22 36 04	QPSK	$\zeta^*_2 = 5.53$	80	20	$(b_0, b_1), (b_2, b_3)$
2	2	2	1/3, 4	31 05 35	8PSK	$\zeta^*_2 = 0.67$	16.32	12.6	$(b_0, b_1, b_2), (b_3, b_4, b_5)$
3	2	2	1/3, 5	71 31 61	8PSK	$\zeta^*_2 = 0.89$	≤ 25.02	13.4	$(b_0, b_1, b_2), (b_3, b_4, b_5)$
4	2	2	1/3, 6	161 041 171	8PSK	$\zeta^*_2 = 1.05$	17.7	17.2	$(b_0, b_1, b_2), (b_3, b_4, b_5)$
5	2	3	1/3, 3	06 16 13	16QAM	$\zeta^*_2 = 0.4$	1.6	4	$(b_0, b_1, b_2, b_3), (b_4, b_6, b_7, b_8)$
6	3	1	1/6, 3	15 03 07 05 04 11	QPSK	$\zeta^*_3 = 2.44$	248	26	$(b_0, b_1), (b_2, b_3), (b_4, b_5)$
7	3	2	1/3, 5	62 55 47	QPSK	$\zeta^*_3 = 0.24$	32	22	$(b_0, b_1), (b_2, b_3), (b_4, b_5)$
8	3	3	1/2, 5	75 62	QPSK	$\zeta^*_2 = 2.0$	15.5	12.0	$(b_0, b_1), (b_2, b_3), (b_4, b_5)$
9	4	2	1/4, 4	33 23 26 06	QPSK	$\zeta^*_2 = 4.60$	78.6	24	$(b_0, b_1), (b_2, b_3), (b_4, b_5), (b_6, b_7)$
10	4	2	1/4, 5	75 71 67 53	QPSK	$\zeta^*_3 = 0.29$	49.1	36	$(b_0, b_1), (b_2, b_3), (b_4, b_5), (b_6, b_7)$
Maximal-Δ^*_E codes and good periodic-erasures codes									
11	2	1	1/4, 4	25 27 33 37	QPSK	$\zeta^*_2 = 5.53$	24	24	$(b_0, b_1), (b_2, b_3)$
12	2	2	1/3, 6	173 062 115	8PSK	$\zeta^*_2 = 0.56$	32.3	17.6	$(b_0, b_1, b_2), (b_3, b_4, b_5)$
13	3	3	1/4, 6	117 155 145 137	16QAM	$\zeta^*_2 = 0.28$	5.03	14.4	$(b_0, b_1, b_2, b_3), (b_4, b_5, b_6, b_7)$
14	3	3	1/2, 5	65 57	QPSK	$\zeta^*_2 \leq 1.43$	19.7	16	$(b_0, b_1), (b_2, b_3), (b_4, b_5)$
15	4	2	1/4, 4	25 27 33 37	QPSK	$\zeta^*_2 = 2.62$	74.14	30	$(b_0, b_1), (b_2, b_3), (b_4, b_5), (b_6, b_7)$
16	4	3	1/4, 6	135 147 135 163	8PSK	$\zeta^*_2 = 0.98$	45.01	24	$(b_0, b_1, b_2), (b_3, b_4, b_5), (b_6, b_7, b_8)$

the necessary (but not sufficient) trellis complexity obeys $\lfloor \nu/k \rfloor \geq N_t(r-1)$. Although not universal over the $N_t \times N_t$ rank-unconstrained compound channel, a diversity-r code with good ζ^*_r can provide universal performance over all channels that establish at least $N_t - r + 1$ equally strong spatial eigenmodes. Moreover, missing levels of transmit diversity can be restored by an outer code.

III. Universal space-time codes from standard trellis codes

For our exhaustive code searches we have used the stack-based algorithm [18] following a pruning step that uses a small set of test channels [16] to find the best worst-case eigenvalues. Besides the worst-case eigenvalues, the minimum squared Euclidean-distance, Δ^*_E, the diversity order, $\Delta_{H,\min}$, and the minimum product-distance, Δ^*_P, are important parameters that determine the error probability performance of the code over different channel scenarios. The worst-case ith eigenvalue over all $\mathbf{E}\mathbf{E}^\dagger$ is denoted by $\zeta^*_i = \min_{\mathbf{E}} \zeta_i(\mathbf{E}\mathbf{E}^\dagger)$. The presented codes are found by exhaustive searches over their class of encoders to maximize the worst-case minimum-eigenvalue ζ^*_r under a transmit diversity constraint $\Delta_{H,\min} = r$ while sacrificing no more than twenty percent of the maximum squared Euclidean-distance achievable within the same class, when possible.

A. *Universal codes for $N_t = 2, 3, 4$ transmit antennas*

Table 1 lists rate-1/3 + 8PSK trellis-coded modulations for for 16, 32 and 64 states (codes #2, #3, #4, respectively) such that when multiplexed over two transmit antennas, deliver universal performance over the 2×2 compound channel. Figure 1 displays the simulated bit-error-rate performances of code #4 as well as two other transmit-diversity schemes over the 2×2 compound channel as a function of excess mutual information. At BER=10^{-5} code #4 requires no more than 0.88 bits of excess MI per transmit antenna over singular channels and requires 0.93 bits of excess MI on unitary channels. The rate-1/2 64-state maximal-free-distance convolutional code [19] with QPSK modulation requires 0.84 bits of excess MI on the AWGN channel at BER=10^{-5}. At BER=10^{-5}, code #4 handles *every* instance of the 2×2 compound channel within 0.09 bits of excess MI per transmit antenna of the best rate-1/2 + QPSK convolutional code of similar complexity on the AWGN channel. The performance of the code over singular 2×2 channels *is* the performance of the code over 2×1 channels. The 2 bits/symbol 64-state $\mathbb{Z}_4$-linear 4PSK-TCM of [6], designed to deliver good average error-probability under quasistatic Rayleigh fading channels, requires a worst-case excess MI of 1.10 bits per transmit antenna to achieve BER $= 10^{-5}$. The uncoded QPSK transmission on the AWGN channel has excess MI of 1.13 bits at BER $= 10^{-5}$.

Another scheme that delivers 2 bits/symbol over two transmit antennas consists of a good AWGN-TCM followed by Alamouti repetition [5]. The 64-state rate-2/3 Ungerboeck TCM [20] achieves BER $= 10^{-5}$ at SNR = 8.8 dB on the AWGN channel. On the compound 2×2 channel, the excess MI requirement of this scheme is a linear function of the sum of the channel eigenvalues [16]. On singular channels, this concatenated scheme requires only 0.55 bits of excess MI per antenna at BER $= 10^{-5}$, whereas on unitary channels the excess MI is requirement is 1.26 bits per transmit antenna. Among the three codes examined, code #4

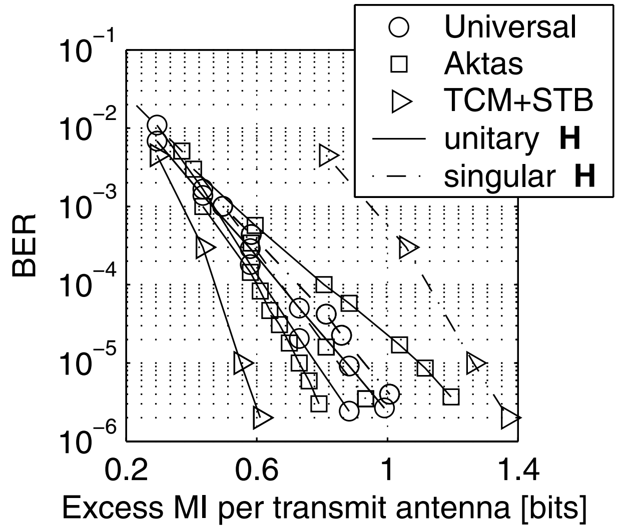

Fig. 1. Channel-by-channel performance of the universal 64-state rate-1/3 + 8PSK code (#4) over the 2×2 compound channel. Best- and worst-case (for BER $= 10^{-5}$) singular channels are identified via extensive simulation. For comparison, the compound channel performance of the 64-state 4PSK code of Aktas *et al.* [6] and the compound channel performance of the 64-state rate-2/3 + 8PSK Ungerboeck-TCM + Alamouti STB signaling is provided. Each frame consists of 127 data symbols and 3 symbols for trellis termination.

has the most consistent channel-by-channel performance.

For $R = 2$ bits/symbol transmission over $N_t = 3$ transmit antennas, our search originally focused on rate-1/5 convolutional encoders. The Δ_E^*-constrained optimal-ζ_3^* search over 32-state encoders and all puncturing patterns resulted in a system that can simply be represented as two rate-1/3 convolutional encoder outputs producing six bits that label three QPSK points (code #7).

For the target bit error rate of BER$=10^{-5}$, the excess MI requirement of proposed trellis codes as well as several other transmit-diversity schemes for two and three transmit antennas over the compound channel is displayed in Table II.

Now consider the MI penalty incurred by using an orthogonal space-time block (STB) scheme for transmit diversity. Figure 2 shows the worst-case mutual information loss over the compound channel of two orthogonal STB schemes as a function of the channel mutual information. For two transmit antennas, Alamouti's full-rate [5] and for three transmit antennas the rate-3/4 scheme of [10] experience heavy MI penalty for channels that support high rates. Universal trellis codes have superior worst-case compound channel performance as compared to orthogonal block schemes.

For $R = 2$ bits per symbol over $N_t = 4$ transmit antennas, code #10 ($\zeta_3^* = 0.28$, $\Delta_P^* = 49.1$, $\Delta_E^* = 36$) is the maximal-ζ_3^* as well as the maximal-Δ_E^* among 32-state rate-1/4 encoders mapping two QPSK points for each information bit. The $R = 2$ bits per symbol 32-state vector-labeled 4PSK trellis-code of [8], which was proposed for good average Rayleigh fading performance has $\zeta_3^* = 0.14$ and $\Delta_P^* = 34.6$, $\Delta_E^* = 36$.

TABLE II

EXCESS MUTUAL INFORMATION (EMI) REQUIREMENT OF $R = 2$ BITS PER SYMBOL SCHEMES FOR $L_t = 2$ AND $L_t = 3$ AS A FUNCTION OF CHANNEL EIGENVALUE SKEW. TARGET BER IS 10^{-5} ON 127-DATA-SYMBOL FRAMES WITH TRELLIS TERMINATION.

$L_t = 2$, $R = 2$ bits per symbol schemes

$\lambda_2/\lambda_1 = 0$	$\lambda_2/\lambda_1 = 0.5$	$\lambda_2/\lambda_1 = 1$
64-state rate-2/3 8PSK TCM + STB		
EMI $= 0.55$	EMI $= 1.20$	EMI $= 1.26$
64-state universal rate-1/3 + 8PSK code (#4)		
$0.80 \leq$ EMI ≤ 0.93	$0.82 \leq$ EMI ≤ 0.89	EMI $= 0.88$
64-state 4PSK code of Aktas *et al.* [6]		
$0.73 \leq$ EMI ≤ 1.10	$0.80 \leq$ EMI ≤ 0.93	EMI $= 0.86$
64-state universal rate-2/6 + $2 \times$ 8PSK code of [16]		
$0.67 \leq$ EMI ≤ 0.95	$0.75 \leq$ EMI ≤ 0.95	EMI $= 0.95$
Uncoded QPSK		
		EMI $= 1.13$

$L_t = 3$, $R = 2$ bits per symbol schemes

$\lambda_3 = 0, \lambda_2 = 0$	$\lambda_3 = \lambda_2 = \lambda_1$
32-state 4PSK code of Aktas *et al.* [6]	
$0.62 \leq$ EMI ≤ 0.90	EMI $= 0.81$
32-state universal 4PSK code (#7)	
$0.67 \leq$ EMI ≤ 0.85	EMI $= 0.83$
(31,21) binary BCH code + 32-state code (#13)	
$0.83 \leq$ EMI ≤ 0.98	EMI $= 0.94$

B. Universal trellis codes under quasistatic Rayleigh fading

Universal space-time trellis codes deliver good average error performance under quasistatic Rayleigh fading as long as the quasistatic period is longer than several traceback depths of the codes. Figure 3 compares the frame-error-rate (FER) and the bit-error rate (BER) performances of code #4 and the 64-state code of [6] over 2×1 and 2×2 quasistatic Rayleigh fading with quasistatic duration of 130 channel symbols, three of which are used for trellis termination. The universal code has slightly better FER and BER over the SNR range displayed. The universal code performs 1.7 dB away from outage capacity. Over 3×1 and 3×3 quasistatic Rayleigh fading channels, code #7 and the 32-state code of [6] have similar BER and FER performances over a wide range of SNRs, with code #7 performing slightly worse in FER for the 3×3 scenario. Similarly, code #10 has approximately the same FER and BER performance with the 32-state code of [8] over 4×1, 4×2 and 4×4 quasistatic Rayleigh fading channels, and slightly better BER performance over 4×1 quasistatic Rayleigh fading.

It is interesting to note that for 2×2 Rayleigh fading, the probability that the eigenvalues λ_1, λ_2 of $\mathbf{H}\mathbf{H}^\dagger$ are more than 10 dB apart is 0.45. For 3×3 Rayleigh fading, this probability is 0.83. Universal code design which takes into account the performance over singular channels (through high ζ_N) results in good codes for the average Rayleigh fading performance.

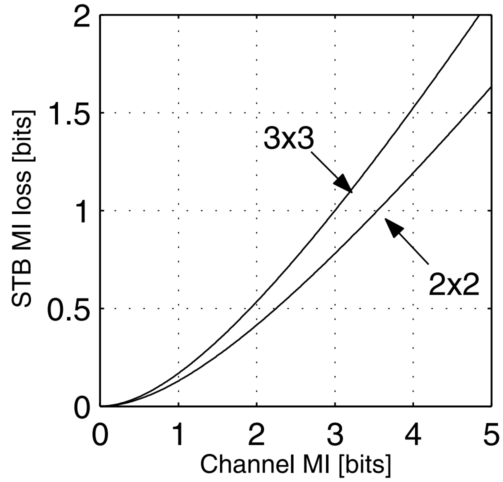

Fig. 2. Worst-case mutual information loss of orthogonal space-time block codes over the compound channel as a function of channel mutual information. Alamouti repetition [5] for two antennas and tha rate-3/4 block scheme of [10].

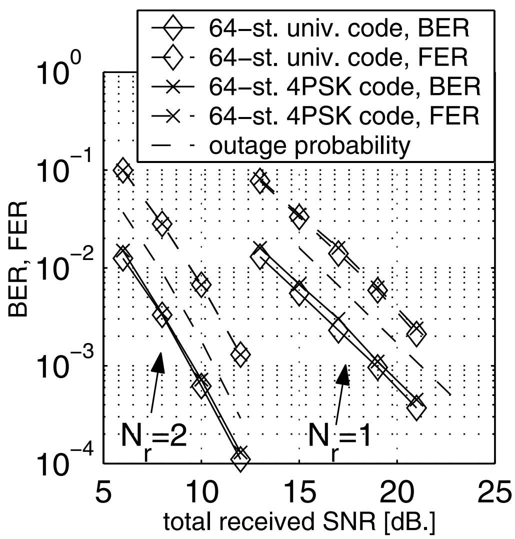

Fig. 3. Bit-error-rate and frame-error-rate performance of the 64-state universal TCM (code #4) as compared to 64-state code of Aktas et. al, over the quasistatic Rayleigh fading channel, $N_t = 2$. Each frame consists of 127 data symbols and three symbols for trellis termination. Maximum-likelihood decoding on the entire frame.

IV. Conclusions

Wireless communication with multiple transmit antennas exposes the transmitted signals to a rich variety of channels. When accurate statistical characterization of the path gains is not possible or when the code is used in a broadcast scenario, universal code design which aims to deliver consistently good error probability performance on any instance of the channel may be desirable. For consistent channel-by-channel performance across the rank-unconstrained $N \times N$ compound channel with maximum-likelihood decoding, a universal space-time code should have good minimum Euclidean-distance and a good smallest minimum-eigenvalue over all codeword differences. Perhaps more importantly, universal codes should obey binary encoder rate, effective code length and constellation size requirements. The proposed universal codes for two, three and four transmit antennas have similar or superior average frame-error-rate and bit-error-rate performances over quasistatic Rayleigh fading channels as compared to trellis codes of similar complexity designed specifically for the quasistatic scenario. Rayleigh fading with independent path gains has high occurrences of singular and almost-singular channels taken into account by universal code design.

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