Linear Closed-Form Precoding of MIMO Multiplexing Systems in the Presence of Transmit Correlation and Ricean Channel

Jabran Akhtar
Department of Informatics
University of Oslo
P. O. Box 1080, Blindern
N-0316 Oslo, Norway
Email: jabrana@ifi.uio.no

David Gesbert
Mobile Communications Department
Eurécom Institute
2229 Route des Crêtes, BP 193
F-06904 Sophia Antipolis Cédex, France
Email: david.gesbert@eurecom.fr

Are Hjørungnes
UniK - University Graduate Center
University of Oslo
P. O. Box 70
N-2027 Kjeller, Norway
Email: arehj@unik.no

Abstract— **This paper presents a closed-form linear precoder for a MIMO spatial multiplexing (SM) system in the presence of transmit correlation and a Ricean component. Existing SM (V-BLAST and similar schemes), based upon channel matrix inversion, rely on the linear independence of antenna channel responses for stream separation and suffer considerably from high levels of fading correlation and/or dominating ill-conditioned line-of-sight channel components. We propose a simple algorithm that adjusts the transmitted constellation through power weighting and phase shifts that can be interpreted in some extreme case as a higher order constellation design scheme. We obtain a rate-preserving MIMO multiplexing scheme that can operate smoothly at any degree of transmit correlation and any type of LOS channel component.**

I. Introduction

Multiple input and multiple output (MIMO) systems, employing several transmit and receive antennas at both ends, are capable of providing a large increase in capacity compared to traditional single antenna systems [1], [2]. However, this increase in capacity is dependent upon the fact that the channels from a transmitter to a receiver follow independent paths. The capacity of MIMO systems can be shown to degrade if there are for example severe correlations present at the transmitter and/or receiver side [3], [4].

Similarly, [5] demonstrated that line of sight (LOS) components, while having a positive effect on the outage behavior of the channel, are also capable of reducing the capacity of MIMO systems. The matrix representing the LOS component of wireless MIMO channels is typically extremely ill-conditioned [4] and thus does not lend itself to a matrix inversion. In fact, with either strong transmit correlations or a high Ricean factor, the capacity behavior of the MIMO channel will ultimately become similar to that of a SIMO/MISO, with a possible additional array gain depending on the partial channel knowledge at the transmitter. If the LOS channel is very dominating, then the capacity falls back to that of a SISO system with additional array gain at the receiver.

Although the negative impact of correlation and the Rice component on average capacity behavior of MIMO systems is significant, the effect it has on the BER behavior of actual SM schemes [1] is much more dramatic. That is because conventional SM schemes rely explicitly (such as in linear MIMO detectors) or implicitly (such as in maximum likelihood (ML) MIMO detectors) on linear separability of the input's spatial signatures to detect the data, unless a form of joint encoding is applied across the streams to differentiate them. For example, current schemes like SM (e.g. V-BLAST) literally break down in the presence of correlation levels close to one or high Ricean factors. As a result, these algorithms simply fail to adapt themselves and extract the non-zero capacity that is present in highly correlated or strongly Ricean channels.

Designing appropriate transmission techniques that can adjust to various kinds of channel and terrain scenarios is therefore an important and practical issue for the successful deployment of MIMO systems. Additionally, the transmit correlation and LOS components can be assumed to remain static over a longer period of time and parameters for these statistics can be fed back to the transmitter on a regular interval. The transmitter can utilize this to transmit information in a more robust manner in the presence of ill-conditioned channels.

Precoding for correlated/Ricean MIMO channels has been considered among other in the case of space-time block codes (including but not limited to [6]). In the case of SM schemes however, the effect of propagation-related ill-conditioning is much more dramatic because the transmitter design no longer guarantees channel orthogonality.

Although precoding for such correlated SM scenarios have previously also been considered [7], [8], [9] the focus has mainly been on transmit correlation and quite often on capacity issues rather than on designing robust practical algorithms. To minimize the BER in the presence of transmit correlation and LOS channel a transmit precoding scheme based on per-antenna phase shifting was proposed in [10] to improve the system performance. The main downside of this approach is that a numerical search is required to find the optimal phases.

A closed-form solution for transmit correlation only, avoiding the need of any numerical optimization was presented in [11], [12]. The precoder is found in a closed-form as the solution to a linear equation parametrized as a function of the transmit correlation coefficient. Unfortunately this work did not deal with the highly practical case of Ricean channels. In this article, we extend these results and jointly tackle the problem of transmit correlation and LOS channel component.

The idea builds on the following principle: When going from MIMO to a SIMO or SISO system, SM can be modified into the form of "constellation multiplexing" (CM) [13] in order to preserve the rate of transmission in a way that is transparent to spatial properties of the channel. The idea of CM is that a higher-order constellation can be designed from the superposition of several low-order constellations with proper phase and power adjustment of each constellation. Of

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course, one antenna is enough to transmit constellation-multiplexed data. In contrast to SM transmission, the substreams in CM schemes are differentiated through power scaling rather than through spatial signatures. By combining constellation- and spatial-multiplexing in a proper way, one obtains an algorithm which can operate smoothly at all levels of correlation and Ricean factors.

II. Signal and channel models

We consider a Ricean MIMO system consisting of N transmit antennas and M ($\geq N$) receive antennas with correlations present at the transmitter only. In this situation, the channel can be described by

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}} + \sqrt{\frac{K}{K+1}}\mathbf{H}_{\text{los}}. \tag{1}$$

The $M \times N$ channel matrix $\mathbf{H}_0$ consists of complex Gaussian zero mean unit-variance independent and identically distributed (iid) elements while $\mathbf{R}_t$ is the $N \times N$ transmit correlation matrix. $\mathbf{H}_{\text{los}}$, also of dimensions $M \times N$, is the LOS channel matrix, possibly being ill-conditioned, and K defines the Ricean factor. The choice of $K = 0$ leads to a standard Rayleigh fading channel.

The baseband equivalent of the N-dimensional signal vector observed at the receiver can be expressed as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{2}$$

where $\mathbf{n}$ is the M-dimensional noise vector whose entries are iid complex Gaussian with zero mean and variance σ_n^2. We consider the spatial multiplexing of independent symbols $s_1, ..., s_N$ and limit ourselves to *diagonal* precoding of these symbols in the form of:

$$\mathbf{s} = [\sqrt{P}_1 s_1 \ \sqrt{P}_2 e^{j\phi_2} s_2 \ ... \ \sqrt{P}_N e^{j\phi_N} s_N]^T. \tag{3}$$

$P_1, ..., P_N$ represent power levels allocated respectively to input symbols $s_1, ..., s_N$, and are selected to satisfy $\sum_{i=1}^{N} P_i = 1$. $\phi_2, ..., \phi_N$ correspond to phase shifts on each transmit antenna. Notice that the first symbol does not undergo a phase change and can be regarded as a reference point for all other phase components. We therefore define $\phi_1 = 0$. Standard SM assigns equal weights $P_i = \frac{1}{N}$ and $\phi_i = 0$ for $1 \leq i \leq N$.

The symbols are all expected to be selected from the same modulation with $E\{|s_i|^2\} = 1$. The minimum distance between two symbols for the given modulation is denoted by $d_{\min}$ while $d_{\max}(\geq d_{\min})$ is the minimum distance between two constellation points with highest amplitude. In the 4-QAM case, $d_{\min} = d_{\max}$ as all symbols are transmitted with equal power. Throughout the paper, $\mathbf{H}_{:,i}$ points to the i'th column of the matrix $\mathbf{H}$ similarly $\mathbf{H}_{i,:}$ denotes the i'th row. $E\{\}$ is the expectation operator while $*$ refers to complex transpose of a vector/matrix.

III. Receiver Structure

With the aim to find a simple closed-form expression for the precoding weights, we assume a particular receiver structure based on maximum ratio combining (MRC) which allows to derive the expressions in a compact fashion. No optimality of this decoding method is assumed as the main goal is to find a closed-form solution to the precoder weights only.

The principle behind the decoding structure is to successively estimate symbols in an iterative fashion, similar to V-BLAST [1], where the matrix inversion procedure is replaced with a MRC. The intuition for utilizing an MRC iterative detection principle as a means to derive the precoder weights is that i) it offers more robustness against an ill-conditioned channel than a straight matrix inversion based detector and ii) it permits with relative ease, computation of the interference factors as a function of the transmit correlation and LOS channel. Through simulations, we show that the precoder offers good performance gains in the case of other decoding methods such as ML decoding.

For the sake of exposition, in the next section, we start by describing the optimization procedure for the 2×2 case. The derivation is later extended to the case of arbitrary number of transmitter and receiver antennas.

IV. Precoder Optimization for 2×2 MIMO system

Writing out in full, the channel matrix in Equation (1) in the 2×2 situation:

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0 \begin{bmatrix} \alpha & \beta e^{j\psi} \\ \beta e^{-j\psi} & \alpha \end{bmatrix} + \sqrt{\frac{K}{K+1}} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}, \tag{4}$$

where by construction $\alpha^2 + \beta^2 = 1$, and $\rho = 2\alpha\beta$ is the modulus of the antenna correlation coefficient, and channel coefficients $h_{i,j}$ describe the components of the LOS matrix.

Without loss of generality in the decoding procedure, we assume $P_1 \geq P_2$ and as the first part of the decoding, the receiver implements a MRC with the first row of $\mathbf{H}^*$:

$$z_1 = (\mathbf{H}^*)_{1,:}\mathbf{y} = \tau_1\sqrt{P}_1 s_1 + \tau_2\sqrt{P}_2 e^{j\phi_2} s_2 + (\mathbf{H}^*)_{1,:}\mathbf{n}. \tag{5}$$

τ_1 denotes the gain for s_1 as a result of the MRC while τ_2 represents the effects of the interference. An estimate for s_1 can be obtained directly from (5), or alternatively from:

$$\frac{1}{\tau_1}z_1 = \sqrt{P}_1 s_1 + \frac{\tau_2}{\tau_1}\sqrt{P}_2 e^{j\phi_2} s_2 + \frac{1}{\tau_1}(\mathbf{H}^*)_{1,:}\mathbf{n} \tag{6}$$

where

$$\tau_1 = (\mathbf{H}^*)_{1,:}\mathbf{H}_{:,1} = ||\mathbf{H}_{:,1}||^2, \quad \tau_2 = (\mathbf{H}^*)_{1,:}\mathbf{H}_{:,2} \tag{7}$$

and $\frac{\tau_2}{\tau_1}$ is the channel related interference factor.

Equations (5) and (6) show that symbol s_2 will be superposed upon s_1 as a function of the channel matrix, which, on average, in turn is composed of the transmit correlation, the K factor and the LOS channel matrix. Notice that, this superposition effectively reduces the minimum distance for detection of symbol s_1.

As long as the interference's magnitude is small enough not to "move" the symbol s_1 out of its decision boundary, i.e.,

$$|\frac{\tau_2}{\tau_1}|\sqrt{P}_2 d_{\max} \leq \frac{1}{2}\sqrt{P_1}d_{\min} \tag{8}$$

a symbol decision can be made on z_1 to obtain an estimate for s_1. For the sake of deriving the precoder, we assume no error propagation and that (8) holds, hence after obtaining s_1, the symbol is subtracted from $\mathbf{y}$,

$$\hat{\mathbf{y}} = \mathbf{y} - \mathbf{H}_{:,1}\sqrt{P}_1 s_1. \tag{9}$$

The robustness of the precoder with respect to the assumption made in (8) is demonstrated in the simulations. An estimate for the second symbol can now be obtained through a second MRC:

$$z_2 = (\mathbf{H}^*)_{2,:}\hat{\mathbf{y}} = \tau_3\sqrt{P}_2 e^{j\phi_2} s_2 + (\mathbf{H}^*)_{2,:}\mathbf{n}, \tag{10}$$

where $\tau_3 = (\mathbf{H}^*)_{2,:}\mathbf{H}_{:,2} = ||\mathbf{H}_{:,2}||^2$.

A. Average Channel Behavior

We wish to design the precoder exclusively based upon knowledge of long-term parameters $h_{i,j}, K, \rho$ and ψ with no dependence on the short-term varying parameter $\mathbf{H}_0$.

The performance of detection of s_1 depends on the instantaneous minimum distance in z_1, however, for the optimization of the weights P_1, P_2 and phase to be independent of $\mathbf{H}_0$, we base ourselves upon an "average" channel behavior. To this end, we introduce the following quantity modified from (5):

$$\hat{z}_1 = E\{\tau_1\}\sqrt{P}_1 s_1 + E\{\tau_2\}\sqrt{P}_2 e^{j\phi_2} s_2. \tag{11}$$

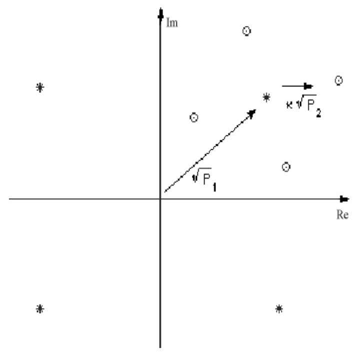

Fig. 1. Illustration of superposed 4-QAM constellations ($\kappa = \frac{\tau_2}{\tau_1}$).

A rather straightforward calculation can then be used to show that (see section V)

$$E\{\tau_1\} = \frac{1}{K+1}\big(2 + K(h_{1,1}^* h_{1,1} + h_{2,1}^* h_{2,1})\big) \tag{12}$$

and

$$E\{\tau_2\} = \frac{1}{K+1}\big(2\rho e^{j\psi} + K(h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2})\big). \tag{13}$$

Clearly, with $\rho = 0$ and $K = 0$ we find $E\{\tau_1\} = 2$ and $E\{\tau_2\} = 0$. This is what would be expected in an ideal situation, as the MRC would only return an array gain factor of 2 with no interference.

B. Evaluation of Minimum Distances

Next, we evaluate the minimum distances, which dictate the error performance of the symbols, under the average channel behavior by considering the absolute average value of each individual gain factor in $E\{\tau_1\}$ and $E\{\tau_2\}$, denoted for the latter by $\hat{E}\{\tau_2\}$ while all factors in $E\{\tau_1\}$ are already non-negative. For s_1, the minimum distance is found from (11):

$$\delta_1 = E\{\tau_1\}\sqrt{P}_1 d_{\min} - \hat{E}\{\tau_2\}\sqrt{P}_2 d_{\max}, \tag{14}$$

where

$$\hat{E}\{\tau_2\} = \frac{1}{K+1}(2\rho + K|h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}|). \tag{15}$$

After a post detection / cancelling of s_1 the average gain for s_2 can be described by

$$\hat{z}_2 = E\{\tau_3\}\sqrt{P}_2 e^{j\phi_2} s_2, \tag{16}$$

where one can show that (section V)

$$E\{\tau_3\} = \frac{1}{K+1}\big(2 + K(h_{1,2}^* h_{1,2} + h_{2,2}^* h_{2,2})\big). \tag{17}$$

This leads to:

$$\delta_2 = E\{\tau_3\}\sqrt{P}_2 d_{\min}. \tag{18}$$

C. Precoding Coefficients with the BER Balancing Criterion (BBC)

1) Phase Optimization: If the average gain coming from $E\{\tau_2\}$ is non-zero, then by selecting the phase ϕ_2 accordingly the distance from the decision boundary can be maximized for s_1. For an arbitrary QAM modulation, this is done by selecting ϕ_2 at the emitter such that

$$\phi_2 = -\angle\, E\{\tau_2\}. \tag{19}$$

This aligns up the symbols in a coherent fashion.

2) Power Optimization: Assuming the noise entries of $\mathbf{H}_{1,:}^*\mathbf{n}$ and $\mathbf{H}_{2,:}^*\mathbf{n}$ follow the same distribution, all components in $\mathbf{H}^*$ also have an identical statistical structure. Thus, the noise factors have identical variance when averaged over $\mathbf{H}_0$. Therefore, we can equate the average probability of error for s_1 and s_2 simply by equating the minimum distances, for the average value of the gains:

$$E\{\tau_1\}\sqrt{P}_1 d_{\min} - \hat{E}\{\tau_2\}\sqrt{P}_2 d_{\max} = E\{\tau_3\}\sqrt{P}_2 d_{\min}, \tag{20}$$

under constraint

$$P_1 + P_2 = 1. \tag{21}$$

For clarity we re-write equation (20) as

$$\mu_1\sqrt{P}_1 - \mu_2\sqrt{P}_2 = \mu_3\sqrt{P}_2, \tag{22}$$

where we have defined $\mu_1 = E\{\tau_1\}d_{\min}$, $\mu_2 = \hat{E}\{\tau_2\}d_{\max}$ and $\mu_3 = E\{\tau_3\}d_{\min}$. The weights for this 2×2 system can easily be computed as functions of μ to be:

$$P_1 = \frac{(\mu_2+\mu_3)^2}{\mu_1^2 + (\mu_2+\mu_3)^2},\ P_2 = \frac{\mu_1^2}{\mu_1^2 + (\mu_2+\mu_3)^2}. \tag{23}$$

D. Interpretations

Observe that $h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2}$ in the expression for $\hat{E}\{\tau_2\}$, directly measures the ill-conditioning of $\mathbf{H}_{\text{los}}$. This is intuitively appealing because one expects the precoder to depend on whether the LOS component is easily invertible or not.

Special cases:

- **No LOS:** With a small K, the expressions give more attention to the effects of transmit correlation. For instance, $K = 0$ gives $\phi_2 = -\psi$ while $\mu_1 = 2, \mu_2 = 2\rho$ and $\mu_3 = 2$ which coincides with the results of [11] given for 4-QAM:
$$P_1 = \frac{(1+\rho)^2}{1+(1+\rho)^2},\ P_2 = \frac{1}{1+(1+\rho)^2}. \tag{24}$$
- **Strong LOS:** With $K \to \infty$ and a strongly ill-conditioned $\mathbf{H}_{\text{los}}$ we find, $\mu_1 \approx \mu_2 \approx \mu_3$, giving $P_1 = 0.8$ and $P_2 = 0.2$ (4-QAM). Interestingly, this corresponds to the power allocation for a regular 2D constellation. For instance a 16-QAM constellation can be seen as the superposition of two 4-QAM constellations with respective powers 0.8 and 0.2 (see Figure 1). Hence SM is here replaced by CM. If the LOS component is better conditioned, the scheme performs a mixture of spatial and constellation-multiplexing.

V. Optimization for an arbitrary MIMO system

For a general MIMO setup, the MRC precoder may easily be extended as following. We first assume that the power weights satisfy

$$P_1 \geq P_2 \geq \ldots \geq P_N. \tag{25}$$

Thus, in an iterative detection procedure, s_1 would become the first symbol to be decoded, followed by s_2 etc. in a chronological order.

To derive the appropriate values of $P_1, \ldots, P_N$ and phases, the average gain and interference factors need to be calculated. The average gain coming from cross-interference of the LOS channel and the remaining channel $\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}}$ is assumed to be zero, i.e., $E\{(\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}})^*\mathbf{H}_{\text{los}}\} = E\{\mathbf{H}_{\text{los}}^*(\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}})\} = \mathbf{0}$.

We therefore obtain

$$E\{\mathbf{H}^*\mathbf{H}\} = \frac{1}{K+1}E\{(\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}})^*(\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}})\} + \frac{K}{K+1}\mathbf{H}_{\text{los}}^*\mathbf{H}_{\text{los}}$$

$$= \frac{M}{K+1}\mathbf{R}_t + \frac{K}{K+1}\mathbf{H}_{\text{los}}^*\mathbf{H}_{\text{los}}. \tag{27}$$

Element k, l $(1 \leq k, l \leq N)$ can then be written out explicitly as:

$$E\{\mathbf{H}^*\mathbf{H}\}_{k,l} = \frac{1}{K+1}\left(M\rho_{k,l} + K\sum_{i=1}^{M} h_{i,k}^* h_{i,l}\right). \quad (28)$$

Taking into account the absolute gain coming from both factors, we define

$$\gamma_{k,l} = \frac{1}{K+1}\left(M|\rho_{k,l}| + K\left|\sum_{i=1}^{M} h_{i,l}^* h_{i,k}\right|\right). \quad (29)$$

As previously, channel coefficients $h_{i,j}$ represent elements of the LOS matrix while $\rho_{k,l}$ describes the coefficients of the correlation matrix $\mathbf{R}_t$, where $\rho_{k,k} = 1$.

Assuming an iterative MRC receiver, the average minimum distance for s_1 becomes

$$\delta_1 = \gamma_{1,1}\sqrt{P_1}d_{\min} - \gamma_{1,2}\sqrt{P_2}d_{\max} - \ \ldots\ - \gamma_{1,N}\sqrt{P_N}d_{\max}. \quad (30)$$

After a symbol estimation/subtraction, the minimum distance for s_2 can be found:

$$\delta_2 = \gamma_{2,2}\sqrt{P_2}d_{\min} - \gamma_{2,3}\sqrt{P_3}d_{\max} - \ \ldots\ - \gamma_{2,N}\sqrt{P_N}d_{\max}. \quad (31)$$

By repeating this N times, we obtain expressions for N minimum distances,

$$\delta_N = \gamma_{N,N}\sqrt{P_N}d_{\min}. \quad (32)$$

1) Phase Optimization: To cancel out the phase-shifts introduced for s_1, the most significant symbol, we set

$$\phi_i = -\angle\, E\{\mathbf{H}^*\mathbf{H}\}_{1,i} = -\angle(M\rho_{1,i} + K\sum_{k=1}^{M} h_{k,1}^* h_{k,i}), \quad (33)$$

for $i = 2, ..., N$.

If the transmitter and receivers are positioned far from each other, and the arrays are placed broadside to each other, which is a practical situation in many applications, then the channel model can be approximated as [5]

$$\mathbf{H} = \sqrt{\frac{1}{K+1}}\mathbf{H}_0\mathbf{R}_t^{\frac{1}{2}} + \sqrt{\frac{K}{K+1}}e^{j\theta}\mathbf{1}_{M\times N}. \quad (34)$$

If a constant phase shift, or an exponential correlation model, is applied for the transmit correlation matrix then the solution provided by (33) will hold for all symbols [12] as long as K is small. For larger values of K, $\mathbf{H}_{\text{los}}$ will dominate and the single phase shift it introduces can be canceled out for all symbols by selecting $\phi_i = -\theta$, $i = 2, ..., N$.

Nevertheless, for this particular MRC receiver, the exact phase rotation is also dependent upon $\mathbf{H}_0$, which the transmitter is unaware of, and therefore the selection of (33) will in practice only have minor effect. In contrast, the decoder of [11] eliminates $\mathbf{H}_0$ before further processing and the phase change thus plays a more important role. The essential information destined to differentiate the signals is though determined by the choice of power weights:

2) Weight Optimization: To guarantee all symbols an equal error rate, it is sufficient that values for $\sqrt{P_1}, \sqrt{P_2}, ..., \sqrt{P_N}$ are selected so that on average the minimum symbol distance observed for each symbol is identical:

$$\delta_1 = \delta_N, \delta_2 = \delta_N, ..., \delta_{N-1} = \delta_N. \quad (35)$$

Based on (35) the following linear system can then be set up as part of the problem to find the appropriate power levels:

$$\mathbf{\Delta p} = \mathbf{0} \quad (36)$$

where $\mathbf{\Delta} =$

$$\begin{bmatrix} \gamma_{1,1}\check{d} & -\gamma_{1,2}\hat{d} & -\gamma_{1,3}\hat{d} & \ldots & -\gamma_{N,N}\check{d} - \gamma_{1,N}\hat{d} \\ 0 & \gamma_{2,2}\check{d} & -\gamma_{2,3}\hat{d} & \ldots & -\gamma_{N,N}\check{d} - \gamma_{2,N}\hat{d} \\ & & \ldots & & \\ 0 & 0 & 0 & \gamma_{N-1,N-1}\check{d} & -\gamma_{N,N}\check{d} - \gamma_{N-1,N}\hat{d} \end{bmatrix}, \quad (37)$$

$$\mathbf{p} = [\sqrt{P}_1\ \sqrt{P}_2\ \ldots\ \sqrt{P}_N]^T \quad (38)$$

and $\mathbf{0}$ is a vector with N zero elements. To obtain a compact notation we have used the following: $\check{d} = d_{\min}$ and $\hat{d} = d_{\max}$.

The upper triangular system (37) only contains $N-1$ equations for N unknowns, however, any solution must also satisfy $\sum_{i=1}^{N} P_i = 1$. Therefore $\mathbf{p}$ can be found as the only unit-norm all-positive vector in the null space of $\mathbf{\Delta}$. For a proof we refer to [12], [13].

Observe that with $K = 0$, $\gamma_{k,l} = M\rho_{k,l}$ and the matrix becomes scale identical to the one presented in [12].

Extreme LOS cases:

- If $K = 0$ and $\rho_{k,l} = 0$, we find $\gamma_{k,l} = 0$, $(1 \leq k, l \leq N, k \neq l)$ and from (37) one can easily see that this gives $P_i = \frac{1}{N}$, i.e., equal power distribution across all streams.
- On the other hand, with a strong K factor and high level of ill-conditionality (e.g. (34)) we can assume all $\gamma_{k,l}$ to be of roughly equal value, giving arise to the following (scale corrected) matrix assuming $d_{\min} = d_{\max}$:

$$\mathbf{\Delta} = \begin{bmatrix} 1 & -1 & -1 & \ldots & -2 \\ 0 & 1 & -1 & \ldots & -2 \\ & & \ldots & & \\ 0 & 0 & & 1 & -2 \end{bmatrix}. \quad (39)$$

This linear system can easily be solved through backsubstitution and under the energy constrain one arrives to:

$$P_i = \frac{3 \cdot 4^N}{4^i(4^N - 1)}, \quad i = 1, ..., N. \quad (40)$$

The energy for this setup decreases by one quarter from symbol s_i to s_{i+1}. The final form of the received signal $\hat{z}_1$, will conclusively simply correspond to a standard 4^N-QAM modulation.

VI. Simulations

This section demonstrates the effectiveness of the precoder through Monte Carlo simulations. The simulations are performed for a 2×2 MIMO system employing 4-QAM modulation. We use the following receiver structures and compare the results with and without precoding:

- An MRC SIC (successive interference cancelling) decoder, as has been described in the text.
- MMSE SIC decoder. The receiver is similar to the one above but rather implements a MMSE matrix inversion to estimate the symbols in each iteration.
- ML, a full exhaustive maximum likelihood search is carried out.

In Figure 2, the simulation results are shown for the MRC decoder assuming $K = 1$, $K = 10$ under the channel model of (34) and no transmit correlation. The use of MRC introduces residual symbol interference showing up as a flooring effect, however, the precoder nevertheless manages to bring in a noticeable improvement.

The second simulation plot, Figure 3 displays the use of MMSE SIC receiver structure with/without precoding under the same channel conditions as previously. In Figure 4, the same simulations have been extended with transmit correlation being set at $\rho = 0.8$. Even at low K-factors having a precoder clearly becomes beneficial.

Finally, Figure 5 uses ML as the decoder, the K-factors being 10 and 15 and with no transmit correlation assumption. A high K factor with precoding makes the slope of the curve steeper as the fading is virtually non-existing. Further simulation results for transmit correlation cases may be found in [11], [12].

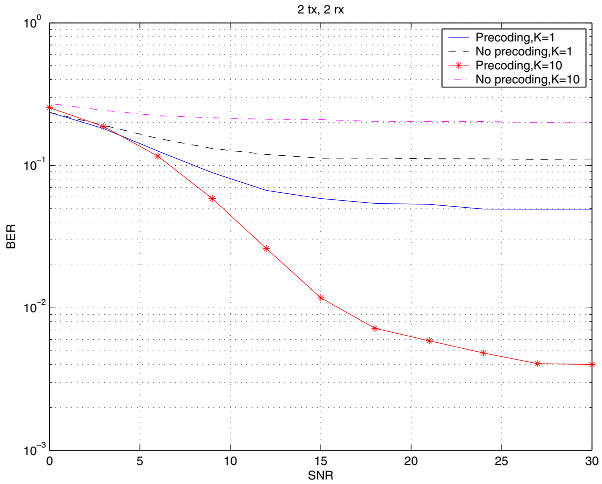

Fig. 2. $K = 1, K = 10$, MRC with/without precoding

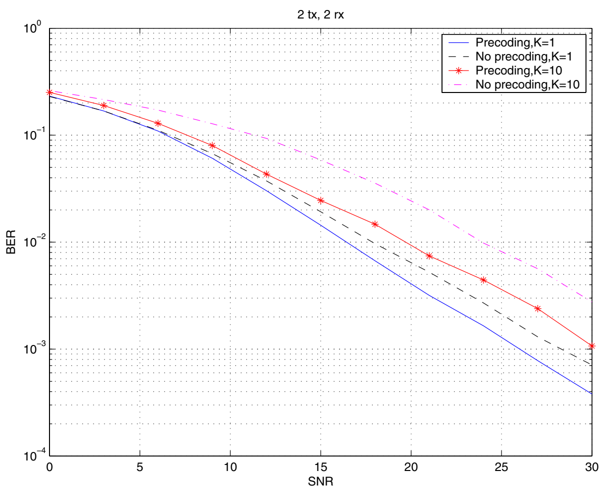

Fig. 3. $K = 1$ and $K = 10$, MMSE SIC with/without precoding

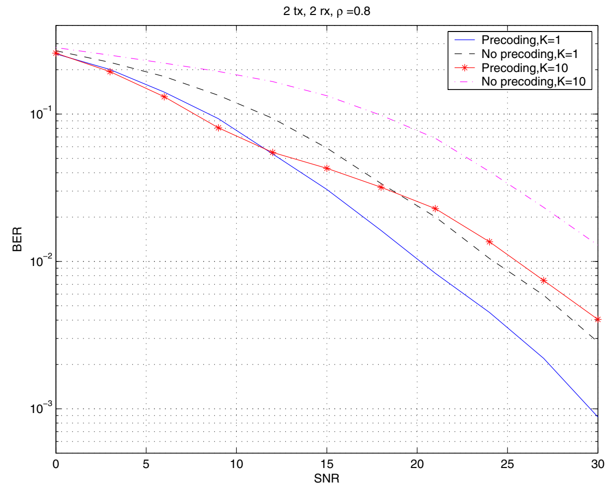

Fig. 4. $K = 1$ and $K = 10$, $\rho = 0.8$, MMSE SIC with/without precoding

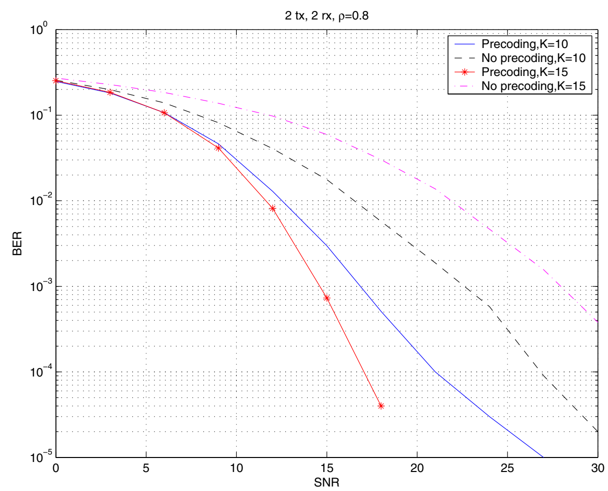

Fig. 5. $K = 10$ and $K = 15$, ML with/without precoding

VII. Conclusions

In this article, we proposed a simple closed-form power weighting approach making use of the average channel knowledge to adapt the transmitted constellation. The derivation assumes a particular decoder structure, however, the weights may be applied on a wider range of receivers. This offers a way to preserve a constant data rate for any correlation level and for well- or ill-behaved LOS components.

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