Linear Diversity Precoding Design Criterion for Block-Fading Delay Limited MIMO Channel

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***Abstract*— A novel approach utilizing the virtual multiple-access like capacity region with symmetry condition on rates is used to develop the design criterion of linear precoding (inner code) for the block-fading delay limited MIMO channel. The overall coding process is split into two parts—outer and inner code. The outer coders are supposed to work block-wise independently and the only joint processing is assumed inherently in the inner code. The motivation is to develop two-level coding process having the same outage capacity performance but significantly lower complexity of the design compared with the information-theoretically optimal joint coding book.**

I. Introduction

The precoding in space-time communications is extensively investigated today. But almost all attempts aim at achieving the higher diversity order of constellation under ergodic consideration. We see the main contributions in this field in [1], [2], [3], where diversity precoders are acquired from an exhaustive computer search or rigorous algebraic number theory. Other papers, e.g. [4], examine the precoders assuming known channel state information at the transmitter. A detailed treatment of the fundamental trade-off between diversity and multiplexing gains in MIMO (multiple-input multiple-output) channel is developed in [5].

Contrary to aforementioned approaches, in this paper the case of finite length frame of information transmission in the delay limited, frequency flat, and block-wise fading environment is treated. Moreover, no channel state information and even no channel statistics is assumed to be known at the transmitter. The practical motivation lies in the desire for the increase of the reliability of successful data blocks transmissions over the block fading channels with independent fades. The whole approach is developed strictly from the information-theoretic perspective. The reader interested in analytical investigation of general statistical description of MIMO channel capacity should refer to [6].

The outage capacity is the objective of our optimization. A large number of practical communication systems transmit the information at a given minimum required rate per block in finite number of successive blocks—one frame. The quality of such service is described by the outage probability of the block in the frame. With Rayleigh fading, this often leads to the necessity of system design taking a huge energy margin into an account. This is the deleterious effect which we are going to remove utilizing appropriately designed precoder.

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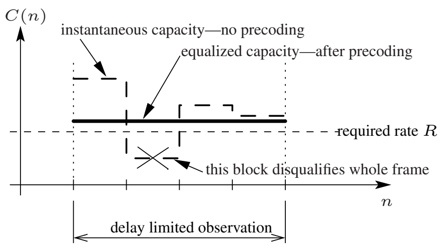

Fig. 1. Desired goal—instantaneous capacity equalization.

The precoder approach can be seen as a two-stage coding with one stage equalizing the channel properties in the frame of the M independently faded blocks. The equalization is understood in a stochastic sense, i.e. no channel information at the transmitter, and the criterion being the stochastic characteristic (capacity region). The precoder design criterion builds on the parallel between the capacity of the block fading MIMO channel and the virtual multiple access capacity region.

There is also an alternative approach directly using one-stage coding but with multiple codebooks [7]. Our linear precoding approach has advantage of simpler processing and common shared codebook at the outer coding stage.

A. *Motivation—problem statement*

The objective of the paper is to develop a design criterion for the precoding (inner code) across the finite set of block fading channel observations such that the virtual channel seen by outputs of the outer coders has uniform achievable rates over all blocks in the frame. Our design criterion is based on frame probability of outage which can be formally defined as

$$\Pr\{\text{frame outage}\} = \Pr\{\bigcup_{n}\{C(n) < R\}\} \tag{1}$$

where $C(n)$ is the instantaneous capacity in the nth block (out of the M blocks in the frame) and R is the required rate. The outage occurs when at least one block in the frame fails to support the required rate. To put it differently, the goal is to equalize or stabilize the instantaneous capacity development within the whole frame as depicted in Fig. 1. There is shown the situation without precoding where the frame probability of

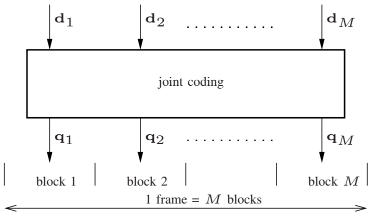

Fig. 2. Optimal joint (one-stage) coder, $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$.

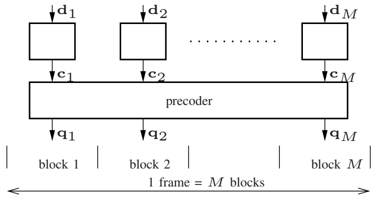

Fig. 3. Two-stage coding processing with independent block-wise outer coders $\mathbf{d}_n \mapsto \mathbf{c}_n$, $\forall n \in \{1, ..., M\}$ and precoding (inner code), $\tilde{\mathbf{c}} \mapsto \tilde{\mathbf{q}}$.

outage is determined by the worst channel realization in the frame. There is high probability that instantaneous capacity per given block does not support the required rate and the outage of the whole frame occurs. Utilizing the appropriate precoder, we can arrive at such virtual channel conditions under which the instantaneous capacity does not vary along the frame so significantly or even at all.

B. Two-stage coder approach

We want to replace the *joint (one-stage)* coding process (Fig. 2) $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$, where $\tilde{\mathbf{d}} = [\mathbf{d}_1^T, \ldots, \mathbf{d}_M^T]^T$ is the raw information data vector and $\tilde{\mathbf{q}} = [\mathbf{q}_1^T, \ldots, \mathbf{q}_M^T]^T$ is the vector of channel coded symbols, by the *two-stage* coding (Fig. 3). In the two-stage approach, the first stage—*block-wise coding* $\mathbf{d}_n \mapsto \mathbf{c}_n$, $\forall n \in \{1, ..., M\}$, is performed by *outer coders* sharing a common codebook and the second stage—*inner code* $\tilde{\mathbf{c}} \mapsto \tilde{\mathbf{q}}$, $\tilde{\mathbf{c}} = [\mathbf{c}_1^T, \ldots, \mathbf{c}_M^T]^T$. The vector $\tilde{\mathbf{c}}$ is the vector of intermediate encoded symbols forming the input to the virtual channel. The inner code is assumed to be a *linear precoding*.

The joint (one-stage) coding is the optimal one. The one-stage code effectively dissolves multiple random instances of the channel in the frame by using long codeword spanning the whole frame. In the two-stage approach, the linear precoding creates a new virtual codebook viewed at the level of $\tilde{\mathbf{q}}$ symbols from the symbols $\mathbf{c}_n$ coded by common codebook from $\mathbf{d}_n$. This linearly composed virtual codebook can be view also as a case of multiple simultaneous codes (refer to [7] with explicit multiple codebook approach).

The design goal for the precoder is to provide the uniform achievable rate per one block (i.e. simultaneously for each individual block in the frame) in two-stage case that should be as close as possible to the one obtained if direct one-stage coding spanning the whole frame was used. The phrase "as close as possible" is understood in probabilistic sense since there is no channel state information at the transmitter and the channel observation is non-ergodic.

C. Outline of the solution

The solution relies on a novel *virtual multiple access capacity region* approach. For the clarity of the further explanation, the detailed outline follows.

- The channel model assumes M independent blocks in the frame. No form of channel state information is assumed on the transmitter side.
- Channel coder is decomposed into outer and inner code.
- The linear precoding (inner code) with unitary matrix is allowed to mix the output among arbitrary blocks and transmitter antennas in the whole frame.
- Codewords of the outer code are required to be independent over the blocks.
- On the level of outer code output, the system can be seen as a virtual multiple access system with outer codes in the individual blocks as separate sources of information.
- The capacity region for such virtual multiple access like system is derived.
- The symmetry conditions enforcing uniform achievable rate for outer code across the blocks are introduced.
- Precoder design criterion guarantees that the pair of precoder plus outer block independent code does not perform worse than the direct coder with codeword spanning the whole frame.

II. MIMO System model

The MIMO system is equipped with N_T antennas at the transmitter and N_R at the receiver. At nth block, the channel input-output equation is

$$\mathbf{x}_n = \mathbf{G}_n \mathbf{q}_n + \mathbf{w}_n \tag{2}$$

where $\mathbf{q}_n = [q_{n,1}, ..., q_{n,N_T}]^T$ are channel symbols (codewords), $\mathbf{x}_n = [x_{n,1}, ..., x_{n,N_R}]^T$ is the received signal and $\mathbf{w}_n = [w_{n,1}, ..., w_{n,N_R}]^T$ is the Gaussian noise. We consider IID (Independent and Identically Distributed) channel $(N_R \times N_T)$ matrix $\mathbf{G}_n$ entries with zero mean and unity variance, and M independently faded channel matrices observed in one frame.

The outer coder codewords must be long enough to justify Shannon sense capacity evaluation per one block. If this holds (at least approximately) we can consider the "instantaneous" capacity per one block. For the mathematical simplicity of the channel capacity evaluation, we *equivalently* denote codewords as having the length one—the stochastic properties are constant within each particular block (block fading) and thus it will provide the same result as for the true long block codewords.

The precoder operates on the frame of M intermediate codewords. The precoded channel symbols are stacked into the $(MN_T \times 1)$ vector

$$\tilde{\mathbf{q}} = \tilde{\mathbf{F}} \tilde{\mathbf{c}} \tag{3}$$

where $\tilde{\mathbf{c}} = [\mathbf{c}_1^T, ..., \mathbf{c}_M^T]^T$, $\mathbf{c}_n = [c_{n,1}, ..., c_{n,N_T}]^T$ are intermediate codewords, $\tilde{\mathbf{q}} = [\mathbf{q}_1^T, ..., \mathbf{q}_M^T]^T$ are channel symbols, and

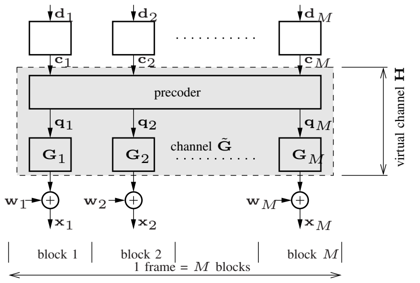

Fig. 4. Virtual channel model.

$\tilde{\mathbf{F}}$ is $(MN_T \times MN_T)$ unitary precoding matrix. The channel input-output relation for the whole frame can be expressed in the form

$$\tilde{\mathbf{x}} = \tilde{\mathbf{G}}\tilde{\mathbf{q}} + \tilde{\mathbf{w}} = \tilde{\mathbf{H}}\tilde{\mathbf{c}} + \tilde{\mathbf{w}} \tag{4}$$

where $\tilde{\mathbf{H}} = \tilde{\mathbf{G}}\tilde{\mathbf{F}}$ is the *virtual channel* (Fig. 4) and stacked channel matrix $\tilde{\mathbf{G}}$ is block-diagonal with the dimension $(MN_R \times MN_T)$

$$\tilde{\mathbf{G}} = \mathrm{diag}(\mathbf{G}_1, \mathbf{G}_2, \ldots, \mathbf{G}_M). \tag{5}$$

The received vector $\tilde{\mathbf{x}}$ has the dimension $(MN_R \times 1)$. The IID Gaussian additive noise $\tilde{\mathbf{w}} = [\mathbf{w}_1^T, \ldots, \mathbf{w}_M^T]^T$ with variance σ_w^2 per dimension is a $(MN_R \times 1)$ vector.

A symbol $\tilde{\mathbf{G}}_{[n]}$, $n = 1, ..., M$, will denote the oblong $(MN_R \times N_T)$ sub-matrix composed from $(N_T(n-1)+1)$th to $(N_T n)$th column of the original matrix $\tilde{\mathbf{G}}$, i.e. for the whole channel matrix it holds $\tilde{\mathbf{G}} = [\tilde{\mathbf{G}}_{[1]}, ..., \tilde{\mathbf{G}}_{[M]}]$. We can rewrite (4) into the form

$$\tilde{\mathbf{x}} = \sum_{n=1}^{M} \tilde{\mathbf{H}}_{[n]}\mathbf{c}_n + \tilde{\mathbf{w}}. \tag{6}$$

This expression has a formal form of the *multiple-access input output relation* with outer coder codewords $\mathbf{c}_n$ as a separate sources of information.

The following notation will be used in the paper. Let $\mathcal{S}(k)$ denote the set of the k-tuples formed as all combinations of k indices from the set $\mathcal{M} = \{1, 2, ..., M\}$, $k \in \mathcal{M}$. Particular k-tuple with its individual elements (indices) will be denoted $\mathbf{s} = \{s_1, \ldots, s_k\} \in \mathcal{S}(k)$. Using this, we denote $\tilde{\mathbf{G}}_{[\mathbf{s}]} = [\tilde{\mathbf{G}}_{[s_1]}, ..., \tilde{\mathbf{G}}_{[s_k]}]$, and similarly for $\tilde{\mathbf{H}}$, $\tilde{\mathbf{F}}$, and other variables.

III. Capacity region—virtual multiple-access approach

This section provides general capacity region definitions of the precoded block-fading channel introduced in Sec. II.

A. Virtual multiple-access like model

Now, we turn back to the channel model (6). The information flow takes a *formal form of multiple-access channel* (Fig. 5) with individual symbols $\mathbf{c}_n$ as separate sources of information. The sub-channel contribution between $\tilde{\mathbf{c}}_{[\mathbf{s}]} = [\mathbf{c}_{s_1}^T, ..., \mathbf{c}_{s_k}^T]^T$, $\forall \mathbf{s} \in \mathcal{S}(k)$ and $\tilde{\mathbf{x}}$ is modeled as

$$\tilde{\mathbf{x}}_\Delta(\tilde{\mathbf{c}}_{[\mathbf{s}]}) = \tilde{\mathbf{H}}_{[\mathbf{s}]}\tilde{\mathbf{c}}_{[\mathbf{s}]} + \tilde{\mathbf{w}}. \tag{7}$$

B. Capacity region

The capacity region is completely specified by the set of inequalities (see [8])

$$\sum_{i=1}^{k} R_{s_i} < \tilde{C}_{\mathbf{s}}, \ \ \forall k \in \mathcal{M}, \mathbf{s} \in \mathcal{S}(k) \tag{8}$$

where the capacity between independent codewords $\tilde{\mathbf{c}}_{[\mathbf{s}]} = [\mathbf{c}_{s_1}^T, ..., \mathbf{c}_{s_k}^T]^T$, $\forall k \in \mathcal{M}, \mathbf{s} \in \mathcal{S}(k)$, and the received $\tilde{\mathbf{x}}$ is given as

$$\tilde{C}_{\mathbf{s}} = \log_2 \det \left(\mathbf{I}_{kN_T} + \frac{\Gamma}{N_T} \sum_{i=1}^{k} \tilde{\mathbf{\Lambda}}_{s_i} \right) \tag{9}$$

where $\Gamma = \mathrm{E}[\|\mathbf{c}_n\|^2]/\sigma_w^2$ is signal to noise ratio and $\mathbf{I}_{kN_T}$ is $(kN_T \times kN_T)$ identity matrix. The $(kN_T \times kN_T)$ matrix $\tilde{\mathbf{\Lambda}}_{s_i}$ is

$$\tilde{\mathbf{\Lambda}}_{s_i} = \tilde{\mathbf{H}}_{[s_i]}^H \tilde{\mathbf{H}}_{[s_i]} = \tilde{\mathbf{F}}_{[s_i]}^H \tilde{\mathbf{G}}^H \tilde{\mathbf{G}} \tilde{\mathbf{F}}_{[s_i]}. \tag{10}$$

The rate R_{s_i} is directly tied to the s_ith virtual user (block in the frame) when multiple-access approach is used.

C. Rate symmetry condition

The essential *design goal* being pursued in the paper lies in the demand of the strictly *equal virtual channel rate conditions* for each separate original codeword $\mathbf{c}_n$ for given arbitrary particular realization of the channel matrix $\tilde{\mathbf{G}}$. This means that all M virtual users must be able to communicate at the same rate with arbitrary low probability of error, i.e. their achievable rates are assumed being equal. We seek whether the point $[R, \ldots, R]$ (i.e. $R_n = R, \forall n \in \{1, \ldots, M\}$) lies within the capacity region. This will be called a *rate symmetry condition*. The rate symmetry condition implies (from (8))

$$R_n = R < \frac{1}{k}\tilde{C}_{\mathbf{s}}, \ \ \forall n \in \mathcal{M}, k \in \mathcal{M}, \mathbf{s} \in \mathcal{S}(k). \tag{11}$$

Furthermore, the maximum *common* achievable rate per block is defined as

$$R_{\max} = \min_{k \in \mathcal{M}, \mathbf{s} \in \mathcal{S}(k)} \frac{1}{k}\tilde{C}_{\mathbf{s}}. \tag{12}$$

The rate $R_{\max}$ is a random entity and thus we define the *probability of outage* as

$$P_{\mathrm{out}}(R) = \Pr\{R_{\max} < R\}. \tag{13}$$

IV. Precoder design criterion

A. One-stage optimal coder

The capacity obtained from standard single user approach (see [9]) if we allow the overall one-stage coder $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$ to span its codewords $\tilde{\mathbf{q}}$ over the *whole* M blocks with arbitrary (including none) unitary precoding is

$$\tilde{C}_M^{\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}} = \log_2 \det \left(\mathbf{I}_{MN_T} + \frac{\Gamma}{N_T} \tilde{\mathbf{\Lambda}}_M \right) \tag{14}$$

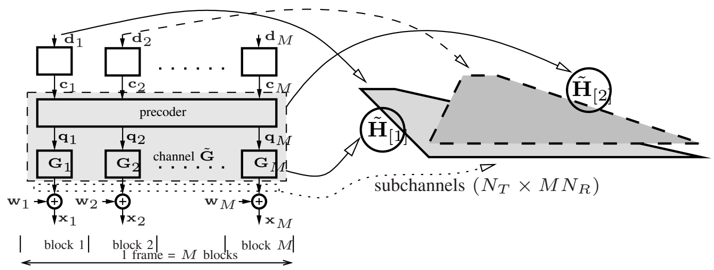

Fig. 5. The formal form of multiple-access channel—the mixture originated from the contributions of independent outer coders.

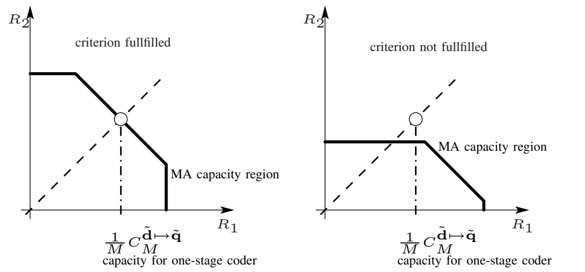

Fig. 6. A graphical representation of the design criterion for $M = 2$.

where

$$\tilde{\mathbf{\Lambda}}_M = \tilde{\mathbf{H}}^H\tilde{\mathbf{H}} = \tilde{\mathbf{F}}^H\tilde{\mathbf{G}}^H\tilde{\mathbf{G}}\tilde{\mathbf{F}}. \tag{15}$$

The channel usage corresponds to the length of the whole frame. No coder can outperform this one. However, our design goal is to split the overall coding process $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$ into two separate parts—precoding $\tilde{\mathbf{c}} \mapsto \tilde{\mathbf{q}}$ which equalizes the individual blocks in such a way that the outer coder $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{c}}$ can assume equal virtual channel rate conditions over individual blocks. Its codewords span only over one block $\mathbf{d}_n \mapsto \mathbf{c}_n$. Each block have independent codewords.

B. Diversity design criterion for two-stage coder

In order to achieve the same outage capacity as for the direct coder $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$, the precoder must guarantee that the *dominant* term in (12) will be *equal* to the *one-stage coder* performance $\frac{1}{M}\tilde{C}_M^{\tilde{\mathbf{d}}\mapsto\tilde{\mathbf{q}}}$. This makes the virtual channel $\tilde{\mathbf{H}}$ to be such that guarantees the equal rate conditions for all $\mathbf{c}_n$ with the capacity per block not being worse than it would be obtained for direct $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$ coder. This can be graphically interpreted (Fig. 6) as a condition that the straight line $R_1 = \cdots = R_M$ in the code rate space $\{R_1, \ldots, R_M\}$ intersection with the hyperplane defined by $R_1 + \cdots + R_M = \tilde{C}_M^{\tilde{\mathbf{d}}\mapsto\tilde{\mathbf{q}}}$ *must not* be outside the capacity region (8) for any given channel state. If this did not hold it would mean that the equally distributed achievable rates $R_n = R$ are lower than they could have been if we had allowed direct M block $\tilde{\mathbf{d}} \mapsto \tilde{\mathbf{q}}$ coder.

One can view the precoding process also as an operation that equalizes the virtual channel $\tilde{\mathbf{H}}$ capacities over individual eigenmodes or the operation that morphs the shape of the capacity region to enlarge its size along the line $R_1 = \cdots = R_M$. This is done inherently by unifying the codeword energy and eigenvalues in individual eigenmodes. If the precoder was not present the performance of independent outer coders for each $\mathbf{d}_n \mapsto \mathbf{c}_n$ would be given by the weakest real channel state $\mathbf{G}_n$ since this block disqualifies the whole frame.

The formal form of the *precoder design criterion* is

$$\frac{1}{M}\tilde{C}_M^{\tilde{\mathbf{d}}\mapsto\tilde{\mathbf{q}}} = \min_{k\in\mathcal{M},\mathbf{s}\in\mathcal{S}(k)} \frac{1}{k}\tilde{C}_{\mathbf{s}}. \tag{16}$$

This condition should hold for any given realization of the channel $\tilde{\mathbf{G}}$ with the probability close to 1.

V. Simulations—Selected Precoders

Our goal in this section is to verify whether selected precoders fulfill the design criterion. In MIMO, there can be used basically two ways of precoding—(1) joint spatial-temporal, and (2) temporal only. Simulations numerically compute the *cumulative distribution function* of the maximum achievable rate per block in the frame or equivalently the probability of outage (13). Via simulation, we have discovered that all tested precoders fail to satisfy the criterion (16) in a strict sense. Although not being optimal, they still provide an improvement in the performance. We want to find out how close to the optimum we can get using such precoders.

A. Spatial-temporal precoding

The first option lies in spreading the symbols $\mathbf{c}_n$ in both time and space. The spatial-temporal precoding is represented by unitary matrix having all non-zero entries. We have chosen the real *Hadamard precoder* consisting of orthogonal columns with elements ± 1 scaled to meet the power constraint. The second evaluated precoder (called *complex field precoder* (CFD)) was obtained using tools of algebraic number theory, [10], [11]. The latter has complex entries with mutually equal magnitudes and orthogonal columns.

B. Temporal only precoding

This precoding spreads the symbols only in time and separately for each particular antenna. That can be ensured by the precoder given as a Kronecker product (*KP precoders*) of the baseline precoder and the identity matrix, i.e. $\tilde{\mathbf{F}} = \mathbf{F}_M \otimes \mathbf{I}_{N_T}$, where $(M \times M)$ matrix $\mathbf{F}_M$ can be given as in subsection V-A. We have observed that such precoding has almost identical outage performance as the previous full spatial-temporal precoding. From that follows that the *most*

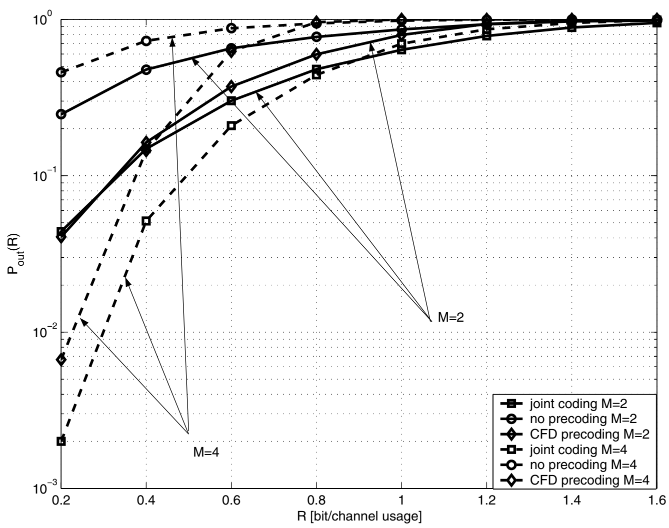

Fig. 7. Outage capacity, SISO case, $M = 2, 4$, $\Gamma = 1$.

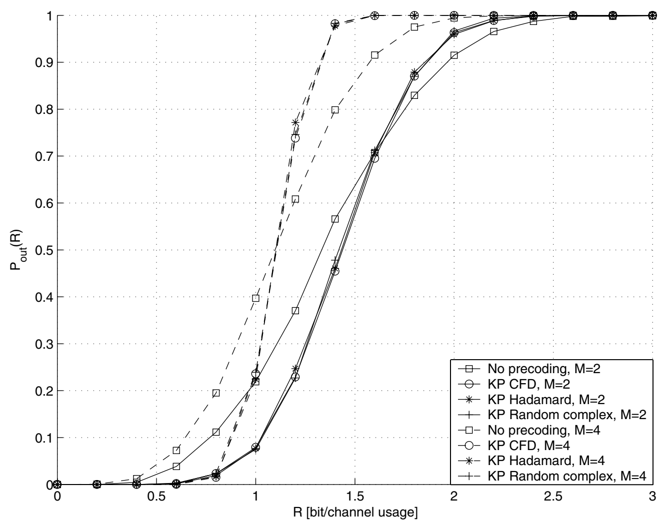

Fig. 8. Outage capacity, Kronecker product precoding, MIMO, $M = 2, 4$, $N_T = N_R = 2$, $\Gamma = 1$.

of achievable *diversity improvement* is provided by precoding along the *time* domain. The MIMO systems have already large degree of *inherent* spatial diversity (especially for large N_T, N_R).

C. Simulation results

Fig. 7 illustrates the beneficial influence of unitary precoding in SISO case $M = 2, 4$ in comparison with no precoding case and the optimal joint (one-stage) coding case. The logarithmic scale of probability of outage is used. With the increase of the precoding order, the frame probability of outage decreases at least for lower desired rate. That was our goal since the region of our main interest encompasses rates achievable with very low probability of outage. Fig. 8 (linear outage probability scale) shows that the influence of the displayed precoders is very beneficial also for MIMO since we are able to provide significantly lower probability of outage for desired rate.

VI. Conclusions

The novel virtual multiple-access like channel approach allowing the outage capacity investigation in precoded block fading delay limited MIMO transmission was presented. The specific design criterion of the precoder (inner code) is developed, ensuring equal outage capacity for each codeword at the outer coder output. The performance of the two-stage approach (outer code with independent codewords and inner precoder) equal to the joint one-stage coding (codeword spanning the whole frame) is desired. The selected precoders, however suboptimal regarding the design criterion, are evaluated. They show that the precoding offers significantly lower probability of outage for given desired rate with very simple processing complexity compared with the direct joint codebook design. The temporal only precoding (KP precoders) is shown to be essential since the additional improvement in the transmission reliability using the precoding in both, space and time, is negligible in MIMO channel.

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