Performance Analysis of the Node Cooperative ARQ Scheme for Wireless Ad-Hoc Networks

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Abstract— **In wireless channels, the bursty nature of block errors render immediate packet retransmissions at the link level ineffective. Cooperative communication is a promising technique to combat the negative impacts of channel fading by providing diverse channels between peers in wireless ad-hoc networks. In this paper, an analytical model is proposed for the throughput of the Node Cooperative Automatic Repeat reQuest scheme for the wireless ad-hoc networks. It is based on a two-state Markov model for block errors in the wireless fading channels. Simulation results are given to demonstrate effectiveness of the analytical model.**

I. Introduction

Wireless ad hoc networks have attracted intensive research attention in recent years. However, wireless channels are characterized by limited bandwidth, high bit error rate, time-varying and location dependent channel condition, etc. Automatic Repeat reQuest schemes (ARQ) are *de facto* parts of wireless link layer protocols to avoid expensive retransmissions of erroneous packets by the transport layer. Due to the inherent characteristics of fading process in the wireless channels, the frame errors appear in bursts rather than randomly. The conventional ARQ schemes which have been designed for the channels with random errors can be less efficient in the wireless ad hoc networks,where an error burst can span over several consecutive data frames, i.e., frame errors are correlated.

Recently, space diversity techniques in the form of multiple transmit and receive antennas have been proposed to improve the quality of communication over wireless fading channels. Multiple Input and Multiple Output (MIMO) communication systems [2] and the corresponding channel coding techniques, such as space time coding [3], have been proposed to implement space diversity in the next generation wireless networks. However, implementation of multiple antennas on small mobile devices is quite difficult due to the device size and cost constraints. An alternative form of space diversity can be achieved in a multi-user environment by allowing nodes to cooperate [3]. In cooperative communications, each node not only transmits and receives data for its own applications, but also provides an alternative path between pairs of other communicating nodes. In other words, each node acts as a relay node to facilitate better communications between the other pairs of nodes at the link level. The theoretical and implementation aspects of cooperative communications in the physical layer have been areas of active interest among researchers [4],[5]. However, to the best of our knowledge, cooperative techniques on the upper layers of communication protocols still needs to be explored.

In [1], a simple and efficient ARQ scheme, namely Node Cooperative Stop and Wait (NCSW) scheme, has been proposed for wireless ad-hoc networks. Preliminary simulation results have indicated the significant throughput gain of the NCSW scheme over conventional non-cooperative retransmission scheme. In this paper, we further investigate the NCSW scheme by developing an analytical model to compute the throughput. The analytical model is based on a two-state Markov chain for block errors in the wireless fading channels. The effectiveness of the model is verified by simulation results.

The rest of this paper is organized as follows. In Section II, the considered ad hoc network is specified. The analytical model of NCSW scheme is developed in Section III. Simulation results are given in Section IV, followed by the concluding remarks in Section V.

II. System Model

We consider an ad-hoc wireless network shown in Fig. 1. For the sake of generality, a group of autonomous nodes without any central control are assumed. Although there is no fixed infrastructure in this system model, it can generalize the other types of wireless networks with infrastructures, such as cellular networks and wireless LANs. In fact, a single node in this model can be viewed as a mobile device, a base station or an access point. A cooperation group is a subset of nodes that can reach one another with a single hop. In other words, nodes in a cooperation group are in the radio coverage area of one another. Those groups may be set up during connection stage or link level handshaking (e.g., RTS/CTS in the IEEE 802.11). Each node may join several cooperation groups depending on its position, capability, and willingness to cooperate. As shown in Fig. 2, each of those cooperation groups can be modelled as a single hop wireless network. At any instant of time, one sender node captures the shared media to send a burst of frames to its intended destination node. During that time period the neighbor nodes in the group keep listening to the shared channel and assist the sender and the receiver nodes if error happens.

A two-state Markov process, as shown in Fig. 3, is assumed to adequately describe the process of frame success/failure [9] over the wireless channels. The channel is deemed to be

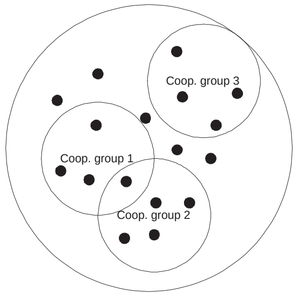

Fig. 1. An ad-hoc wireless network model

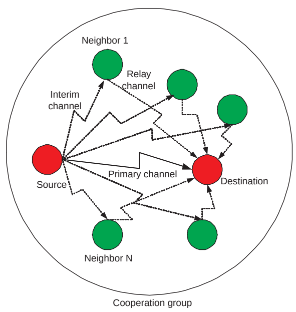

Fig. 2. A single cooperation group

in state G (good) if the fading envelope is above a certain threshold γ during the frame transmission time. Otherwise, it is in state B (bad). The received frame can be decoded correctly by the receiver in the former case, while it can not be decoded properly by the receiver and needs to be retransmitted by the sender in the latter case. Let T_f be the duration of a single frame. It has been shown in [10] that for a Rayleigh fading channel, the parameters of the two-state Markov process in Fig. 3 can be given as

$$r = \frac{Q(\theta, \rho\theta) - Q(\rho\theta, \theta)}{e^{\gamma} - 1}, \quad q = \frac{1 - e^{-\gamma}}{e^{-\gamma}} r, \tag{1}$$

where $Q(\cdot, \cdot)$ is the Marcum Q function, $\theta = \sqrt{\frac{2\gamma}{1-\rho^2}}$, $\rho = J_0(2\pi f_m T_f)$, and $J_0(\cdot)$ is the zero order Bessel function of the first kind.

Since the long range wireless networks, such as satellite networks, are not considered in this paper, the propagation delays among communicating nodes are relatively small (e.g., wireless LANs and cellular networks). In those systems, a Stop and Wait (SW) retransmission scheme is more efficient than the Go Back N (GBN) and Selective Repeat (SR) schemes. In the SW scheme, the sender node does not transmit the next frame until the correct reception of the previous frame is confirmed by an explicit or implicit ACK. We assume that the reverse channel, used for ACK and NAK (Negative Acknowledgment), is error-free. Thus the ACK/NAK frames can be received immediately and correctly by all nodes in a cooperation group.

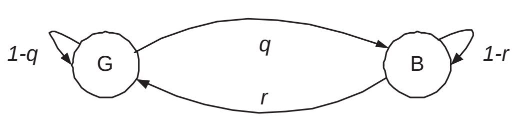

Fig. 3. Markovian model for frame success/failure process over wireless fading channels

III. Node Cooperative SW Scheme

Depending on the relative velocity of communicating nodes, the duration of channel fadings can be as long as the transmission time of several frames. For example, in a slow fading channel with a node speed of 5 Km/h and carrier frequency $f_c = 2400MHz$, the average fading duration is about 60 ms. For a typical frame length of 5 ms, the conventional SW scheme has to retransmit the erroneous frame for an average of 12 times. The situation will be worse in high-rate systems with a shorter frame duration.

With the proposed NCSW ARQ scheme, the neighbor nodes in a cooperation group (Fig. 2) monitor the ongoing communication between a sender and a receiver node, decode the received frame, and store a copy of the most recently received frame. If the receiver can not decode a frame correctly, it will send a NAK to the sender. Consequently, the sender will respond by retransmitting the frame. In the SW scheme, the neighbor nodes are oblivious to the retransmissions. However, in the NCSW scheme, when the neighbor nodes in a cooperation group receive a NAK, they will transmit the requested frame to the receiver concurrently with the retransmission trials by the sender. When a frame is acknowledged by the receiver, all the nodes in the cooperation group drop their corresponding copy of the acknowledged frame. Obviously, a neighbor node in the cooperation group can retransmit only if it has already received a correct copy of the requested frame. Even if a neighbor node has a correct copy, however, cooperation is optional. This guarantees the backward compatibility of the NCSW protocol with the conventional SW protocol.

If some coding and decoding schemes, such as Distributed Space Time Coding [8], are implemented in the physical layer, the chance of successful retransmission will be increased significantly due the existence of independent and diverse paths between a sender and a receiver node. Therefore, the probability of successful frame retransmission for NCSW can be approximated to be the probability of the event that at least one node in the cooperation group can successfully deliver the frame to the receiver.

A. Protocol Analysis Model

In this subsection we develop the analytical model by describing the frame success/failure process for the proposed NCSW ARQ scheme. A proper model for this process is the key component for analyzing throughput. We accomplish the task in three steps: 1) we derive a cooperation model for a sender and receiver pair with a single neighbor node; 2) we model the impacts of all neighbor nodes in the cooperation group as an equivalent *super neighbor node*; 3) we combine

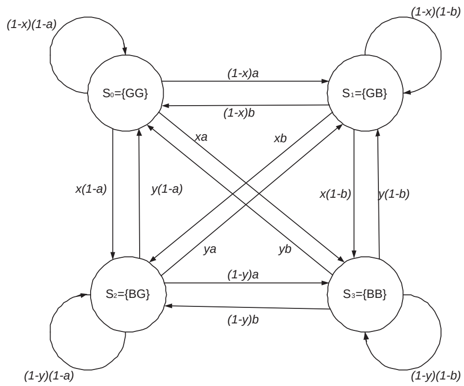

Fig. 4. State transition model for $\{I^{(i)}(k-1), R^{(i)}(k)\}$

the above two models to obtain the model for the frame success/failure process for a sender/receiver pair and an arbitrary number of neighbor nodes.

In step 1, we consider a sender/receiver pair with a single neighbor node. We use a two-state Markov model, as described in Sect. II, to specify the success or failure of frame transmission over wireless fading channels. We use three distinct two-state Markov processes to model the *primary channel* between the sender and the receiver nodes, the *interim channel* between the sender and the neighboring node, and the *relay channel* from the neighbor node to the destination node, as shown in Fig. 2. The corresponding transition probability matrices are denoted by

$$\begin{bmatrix} 1-q & q \\ r & 1-r \end{bmatrix}, \quad \begin{bmatrix} 1-x & x \\ y & 1-y \end{bmatrix}, \quad \text{and} \quad \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$

for the primary, interim, and relay channels, respectively. At the discrete time instant k (the transmission time of the k^{th} frame), the neighbor node is in bad state (B) if it is not able to cooperate in the retransmission of the k^{th} frame. Otherwise, the neighbor node is considered to be in good state (G). Assuming that the neighbor node is always willing to cooperate, B state happens if the previously erroneous frame has not been correctly received by the neighbor node or the relay channel is in bad state at the time instant k. Therefore, the status of the neighbor node i at the time instant k can be formulated as follows:

$$N^{(i)}(k) = \begin{cases} G, & \text{if } I^{(i)}(k-1) = G \text{ and } R^{(i)}(k) = G \\ B, & \text{otherwise,} \end{cases} \tag{2}$$

where $I^{(i)}(k)$ and $R^{(i)}(k)$ denote the states of the i^{th} interim and relay channels, respectively, corresponding to node i.

A four state Markov model, as shown in Fig. 4, can be used to model the transition process between different states of $\{I^{(i)}(k-1), R^{(i)}(k)\}$. Let π_i denote the probability of being in state S_i in Fig. 4. The following set of linear equations can be solved to obtain π_i for $0 \le i \le 3$;

$$\mathbf{\Pi} \cdot \mathbf{A} = \mathbf{\Pi} \tag{3}$$
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1,$$

where $\mathbf{\Pi} = [\pi_0 \pi_1 \pi_2 \pi_3]$ and $\mathbf{A}$ is the transition probability matrix for the model in Fig. 4. From (2), $N^{(i)}(k)$ can be modelled by another two-state Markov process with parameters (u_1, v_1) which are defined as

$$\begin{aligned} u_1 &\triangleq \mathrm{P}\{N^{(i)}(k) = B | N^{(i)}(k-1) = G\} \\ v_1 &\triangleq \mathrm{P}\{N^{(i)}(k) = G | N^{(i)}(k-1) = B\}. \end{aligned} \tag{4}$$

¿From the Markov model shown in Fig. 4, it can be seen that

$$u_1 = 1 - (1-x)(1-a). \tag{5}$$

Then, using the solution to (3), we can obtain v_1 as

$$\begin{aligned} v_1 &= \frac{\mathrm{P}\{N^{(i)}(k-1) = B | N^{(i)}(k) = G\}\mathrm{P}\{N^{(i)}(k) = G\}}{\mathrm{P}\{N^{(i)}(k-1) = B\}} \\ &= \frac{u_1 \pi_0}{1-\pi_0}. \end{aligned} \tag{6}$$

The Markov model specified by (5) and (6) is for only one neighbor node. As the second step, we propose an iterative approach to model a cooperation group with multiple neighbor nodes as a single cooperative super neighbor node. Let $M \ge 2$ be the total number of neighbor nodes. In the first iteration, we combine neighbor nodes 1 and 2 into one equivalent node. Then the resulting equivalent node is combined with node 3, and so on, until all the M neighbor nodes are combined together to form a single super neighbor node.

Denote by $N^{(1)}(k)$ and $N^{(2)}(k)$ the states of nodes 1 and 2, respectively, at time instant k. Since the retransmission will succeed if at least one of the neighbor nodes or the sender node succeeds to correctly deliver the frame to the receiver node, the combined cooperation model of node 1 and 2, denoted by $N^{(1,2)}(k)$, can be represented by

$$N^{(1,2)}(k) = \begin{cases} G, & \text{if } N^{(1)}(k) = G \text{ or } N^{(2)}(k) = G \\ B, & \text{otherwise.} \end{cases} \tag{7}$$

The discrete random process $N^{(1,2)}(k)$ in (7) can also be modelled by a two-state Markov process with its parameters defined by

$$\begin{aligned} u^{(1,2)} &\triangleq \mathrm{P}\{N^{(1,2)}(k) = B | N^{(1,2)}(k-1) = G\} \\ v^{(1,2)} &\triangleq \mathrm{P}\{N^{(1,2)}(k) = G | N^{(1,2)}(k-1) = B\}. \end{aligned} \tag{8}$$

Let (u_1, v_1) and (u_2, v_2) be the corresponding Markov parameters of $N^{(1)}(k)$ and $N^{(2)}(k)$, respectively. The status of $\{N^{(1)}(k), N^{(2)}(k)\}$ can be described by another four-state Markov process as shown in Fig. 5. ¿From this Markov model and (8), it can be easily seen that $v^{(1,2)} = 1-(1-v_1)(1-v_2)$. Then, we can obtain $u^{(1,2)}$ as

$$\begin{aligned} u^{(1,2)} &= \frac{\mathrm{P}\{N^{(1,2)}(k-1) = G | N^{(1,2)}(k) = B\}}{\mathrm{P}\{N^{(1,2)}(k-1) = G\}} \times \\ &\quad \mathrm{P}\{N^{(1,2)}(k) = B\} = \frac{v^{(1,2)}\pi_3}{1-\pi_3}, \end{aligned} \tag{9}$$

where $\begin{bmatrix} \pi_0 & \pi_1 & \pi_2 & \pi_3 \end{bmatrix}$ is obtained by solving the equation set (3) for the Markov model in Fig. 5.

In the next iteration, the two-state Markov model specified by $(u^{(1,2)}, v^{(1,2)})$ is combined with the two-state Markov

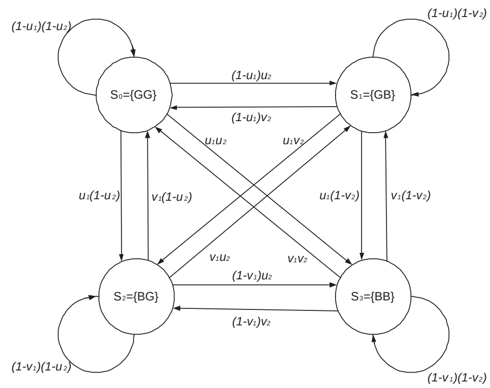

Fig. 5. State transition model for $\{N^{(1)}(k), N^{(2)}(k)\}$

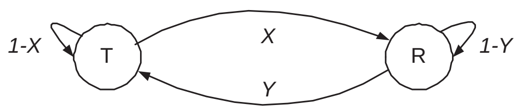

Fig. 6. Markovian model for the NCSW ARQ

model of neighbor node 3, which is specified by (u_3, v_3) in the same way as described above. In the final iteration, we will obtain the two-state model for the super neighbor node, which specifies the impacts of all the neighbor nodes. We denote by $N^{(1,\ldots,M)}(k)$ the status of the super neighbor node at time instant k, which can be represented by a two-state Markov model with the following parameters:

$$\begin{aligned} u &\triangleq \mathrm{P}\{N^{(1,\ldots,M)}(k) = B | N^{(1,\ldots,M)}(k-1) = G\} \\ v &\triangleq \mathrm{P}\{N^{(1,\ldots,M)}(k) = G | N^{(1,\ldots,M)}(k-1) = B\}, \end{aligned}$$

which can be obtained by the above iterative procedure.

The final step is to combine the cooperation of a single super neighbor node with the sender node to completely model the NCSW protocol. Let $O(k)$ denote the state of the NCSW protocol at time instant k. $O(k)$ is either in Transmission (T) state or Retransmission (R) state, according to the two-state Markov model shown in Fig. 6. The parameters of this Markov model are defined as

$$\begin{aligned} X &\triangleq \mathrm{P}\{O(k) = R | O(k-1) = T\} \\ Y &\triangleq \mathrm{P}\{O(k) = T | O(k-1) = R\}. \end{aligned} \tag{10}$$

In state T, the sender transmits a new frame; however, in state R, all nodes in the cooperation group retransmit the previously failed frame. Let $P_c(k)$ represent the state of the primary channel at time instant k; $O(k)$ will transit between T and R states according to the logic given by Table I, with the corresponding transition probability matrix $\mathbf{B}(8 \times 8)$ given by (11).

TABLE I
STATE TRANSITION LOGIC FOR $O(k)$

$\{O(k-1), P_c(k), N^{(1,\ldots,M)}(k)\}$	$O(k)$
S_0:{T,G,G}	T
S_1:{T,G,B}	T
S_2:{T,B,G}	R
S_3:{T,B,B}	R
S_4:{R,G,G}	T
S_5:{R,G,B}	T
S_6:{R,B,G}	T
S_7:{R,B,B}	R

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \bar{r}\bar{v} & \bar{r}v & r\bar{v} & rv \\ 0 & 0 & 0 & 0 & \bar{r}u & \bar{r}\bar{u} & ru & r\bar{u} \\ q\bar{v} & qv & \bar{q}\bar{v} & \bar{q}v & 0 & 0 & 0 & 0 \\ qu & q\bar{v} & \bar{q}u & \bar{q}\bar{u} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{r}\bar{v} & \bar{r}v & r\bar{v} & rv \\ \bar{r}u & \bar{r}\bar{u} & ru & r\bar{u} & 0 & 0 & 0 & 0 \\ q\bar{v} & qv & \bar{q}\bar{v} & \bar{q}v & 0 & 0 & 0 & 0 \\ qu & q\bar{u} & \bar{q}u & \bar{q}\bar{u} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

where $\bar{\alpha} \triangleq (1-\alpha), \alpha \in \{u, v, r, q\}$. Let $\mathbf{P} = [p_{S_0}, \ldots, p_{S_7}]$ denote the steady state probability vector, where p_{S_i} is the steady state probability of being in state S_i in Table I. This vector can be obtained by solving a set of linear equations given by

$$\begin{aligned} \mathbf{P} \cdot \mathbf{B} &= \mathbf{P} \\ p_{S_0} + \ldots + p_{S_7} &= 1. \end{aligned} \tag{12}$$

Having p_{S_i}, for $i = 0, \ldots, 7$, and Table I, the parameters of the two-state Markov model for the NCSW protocol, can be obtained by

$$\begin{aligned} X &= \frac{p_{S_2} + p_{S_3}}{p_{S_0} + p_{S_1} + p_{S_2} + p_{S_3}} \\ Y &= \frac{p_{S_4} + p_{S_5} + p_{S_6}}{p_{S_4} + p_{S_5} + p_{S_6} + p_{S_7}}. \end{aligned} \tag{13}$$

Equation (13) completes the frame success/failure process modelling as a two-state Markov model. Given this model, the throughput can be obtained by

$$\eta_{NCSW} = \frac{Y}{Y+X}. \tag{14}$$

IV. SIMULATION RESULTS

We simulate a single hop ad-hoc network with one pair of sender-receiver nodes and varying number of neighbor nodes, as shown in Fig. 2. The channels among the nodes are generated by the Rayleigh fading model. The impacts of path loss and shadowing are not considered due to their very slow variations compared with the activities of link layer. The carrier frequency is 2400 MHz, relative speed of the mobile nodes is 5 Km/h. The quality of the channels are represented in terms of the ratio of the fading margin over the mean value of the fading envelope, as $L = \frac{\sqrt{\gamma}}{\mathrm{E}[|\zeta(t)|]}$. When the value of fading envelope is below the fading margin, the transmitted frame can not be decoded properly. The mean value of a fading channel is

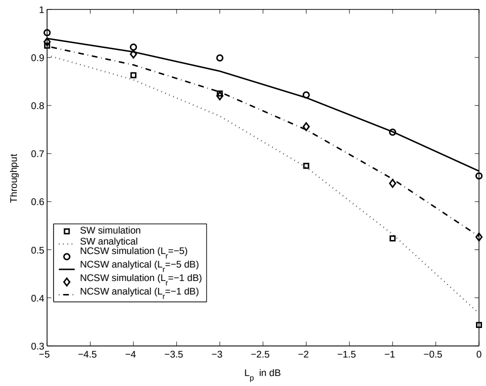

Fig. 7. Throughput vs. the fading margin of the primary channel

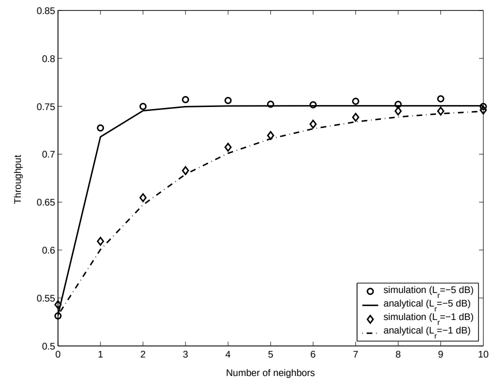

Fig. 8. Throughput vs. the number of the neighbor nodes ($L_p = -1dB$)

normalized to a unit; then, $L = \sqrt{\gamma}$. As the value of the fading margin increases, we will encounter poorer channel qualities and more frame errors. The data frame duration is assumed to be 5 ms, which is a reasonable value for many wireless data networks. Perfect ACK/NAK information on the feedback channels is assumed to be available for the whole cooperation group right after a frame transmission. The numerical results are obtained from Monte-Carlo simulations.

To observe the impact of a small number of cooperative nodes on the system throughput, the SW and the NCSW schemes with only 2 neighbor nodes are simulated. The throughput of both schemes are obtained by simulations and the analytical models. As shown in Fig. 7, the results are plotted against variations of the quality of the primary channel, as denoted by L_p. Quality of all the interim and the relay channels are assumed to be identical, and denoted by L_r. To demonstrate the impact of variations in quality of the interim/relay channels, throughput of the NCSW protocol is plotted for two different values of L_r, namely $-5dB$ and $-1dB$. As it can be seen, with cooperation of only 2 neighbor nodes, throughput of the NCSW scheme can be improved up to 30%, depending on the quality of the interim/relay channels. The simulations results also demonstrate the accuracy of the proposed analytical model for the system throughput.

To evaluate the effect of the number of neighbor nodes on the protocol throughput, the fading margin of the primary channel is set to $L_p = -1dB$. The simulations are performed for two different fading margins for the relay/interim channels with $L_r = -5dB$ and $L_r = -1dB$. As shown in Fig. 8, when the number of the cooperative nodes is increased, the system throughput approaches to some saturation level depending on the quality of the primary and the interim/relay channels. If the qualities of the interim/relay channels are good, having even one or two neighbor nodes can significantly improve the system performance; however, when the qualities of the interim/relay channels are poor, more neighbor nodes are required to achieve the same level of performance gain. Saturation in the system throughput was also expected. In fact, regardless of the number of neighbor nodes or their channel conditions, individual frame errors can not be prevented. However, the cooperation of neighbor nodes can reduce the duration of error bursts.

V. Conclusions

We have proposed an analytical model for the throughput of the Node Cooperative Stop and Wait (NCSW) ARQ scheme for wireless ad hoc networks. Simulation results have demonstrated the accuracy of the proposed model and the performance gain of the NCSW scheme. It is concluded that when the channel condition between the sender and the receiver nodes is poor, node cooperation can significantly improve the throughput performance.

Acknowledgement

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