Scaling Laws for Overlaid Wireless Networks: A Cognitive Radio Network vs. a Primary Network

Changchuan Yin, Long Gao, and Shuguang Cui

***Abstract*— We study the scaling laws for the throughputs and delays of two coexisting wireless networks that operate in the same geographic region. The primary network consists of Poisson distributed legacy users of density n, and the secondary network consists of Poisson distributed cognitive users of density m, with $m > n$. The primary users have a higher priority to access the spectrum without particular considerations for the secondary users, while the secondary users have to act conservatively in order to limit the interference to the primary users. With a practical assumption that the secondary users only know the locations of the primary transmitters (not the primary receivers), we first show that both networks can achieve the same throughput scaling law as what Gupta and Kumar [1] established for a stand-alone wireless network if proper transmission schemes are deployed, where a certain throughput is achievable for each individual secondary user (i.e., zero outage) with high probability. By using a fluid model, we also show that both networks can achieve the same delay-throughput tradeoff as the optimal one established by El Gamal *et al.* [2] for a stand-alone wireless network.**

***Index Terms*— Ad hoc networks, overlaid wireless networks, throughput, delay, cognitive radio networks.**

I. Introduction

Initiated by the seminal work of Gupta and Kumar [1], the throughput scaling law for large-scale wireless networks has become an active research topic [3]-[14]. Scaling laws provide a fundamental way to measure the achievable throughput of a wireless network. Considering n nodes that are randomly distributed in a unit area and grouped independently into one-to-one source-destination (S-D) pairs, Gupta and Kumar [1] showed that typical time-slotted multi-hop architectures with a common transmission range and adjacent-neighbor communication can achieve a sum throughput that scales as $\Theta\left(\sqrt{n/\log n}\right)$. They also showed that an alternative arbitrary network structure with optimally chosen traffic patterns, node locations, and transmission ranges can achieve a sum throughput of order $\Theta\left(\sqrt{n}\right)$. Thus, they suggested that a factor of $\sqrt{\log n}$ is the price to pay for the randomness of the node locations. In [3], with percolation theory, Franceschetti *et al.* showed that the $\Theta\left(\sqrt{n}\right)$ sum throughput scaling is achievable even for randomly deployed networks under certain special conditions. In [4], Grossglauser and Tse showed that by allowing the nodes to move independently and uniformly, a constant throughput scaling $\Theta(1)$ per S-D pair can be achieved. Later, Diggavi *et al.* showed that a constant throughput per S-D pair is achievable even with a one-dimensional mobility model [5]. In these approaches, the network area is fixed and the throughput scales with the node density n. We call this kind of network as *dense network*. On the other hand, based on the *extended network* model where the density of nodes is fixed and the network area increases with n, the information-theoretic scaling laws of transport capacity were studied for different values of the pathloss exponent α in [8]-[14]. In particular, Özgür *et al.* [14] proposed a hierarchical cooperation scheme to achieve a sum throughput that scales as $n^{2-\alpha/2}$ for $2 \leq \alpha < 3$, i.e., asymptotically linear for $\alpha = 2$.

In wireless networks, another key performance metric is delay, which incurs the interesting problems regarding the interactions between throughput and delay. The issues of delay-throughput tradeoff for static and mobile wireless networks have been addressed in [2], [15]-[21]. In [2], El Gamal *et al*. established the optimal delay-throughput tradeoff for static and mobile wireless networks. For static networks, they showed that the optimal delay-throughput tradeoff is given by $D(n) = \Theta\left(n\lambda(n)\right)$, where $\lambda(n)$ and $D(n)$ are the throughput and delay per S-D pair, respectively. Using a random-walk mobility model, they showed that a much higher delay of $\Theta\left(n \log n\right)$ is associated with the higher throughput of $\Theta(1)$ for mobile networks. The delay-throughput tradeoffs in mobile wireless networks have been investigated under many other mobility models, which include the i.i.d. model [15], [17], [18], the hybrid random walk model [20], and the Brownian motion model [19]. For the hierarchical cooperation scheme in a static wireless network, Özgür and Lévêque [21] showed that a significantly larger delay was introduced compared with the traditional multi-hop scheme, and the delay-throughput tradeoff is $D(n) = \Theta\left(n\left(\log n\right)^2\lambda(n)\right)$ for $\lambda(n)$ between $\Theta\left(1/(\sqrt{n}\log n)\right)$ and $\Theta\left(1/\log n\right)$.

All the aforementioned results focus on the throughput scaling laws or the delay-throughput tradeoffs for a single wireless network. In recent years, the ever-growing demand for frequency resource from wireless communication industries imposes more stress over the already-crowded radio spectrum. However, a recent report by the Federal Communications Commission (FCC) Spectrum Policy Task Force indicated that over 90 percent of the licensed spectrum remains idle at a given time and location [22]. This motivated the regulation bodies to consider the possibility of permitting secondary networks to coexist with licensed primary networks, which is the main driving force behind the cognitive radio technology [23]. In a secondary network, the cognitive users opportunistically access the spectrum licensed to primary users according to the spectrum sensing result [24], where the primary users have

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a higher priority and the secondary users need to prevent any harmful interference to the primary users [25], [26]. In this overlaid regime, the throughput scaling law and the delay-throughput tradeoff for both the primary and secondary networks are interesting and challenging problems. Some preliminary work along this line appeared recently. In [27], [28], Vu *et al.* considered the throughput scaling law for a single-hop cognitive radio network, where a linear scaling law is obtained for the secondary network with an outage constraint for the primary network. In [29], Jeon *et al*. considered a multi-hop cognitive network on top of a primary network and assumed that the secondary nodes know the location of each primary node regardless of whether it is a transmitter (TX) or a receiver (RX). With an elegant transmission scheme, they showed that by defining a preservation region around each primary node, both networks can achieve the same throughput scaling law as a stand-alone wireless network, while the secondary network may suffer from a finite outage probability.

In a practical cognitive network, it is hard for the secondary users to know the locations of primary receiving nodes since they may keep passive all the time. A reasonable assumption is that the secondary network knows the locations of the primary TXs. Based on this assumption, in this paper we define a preservation region just around each primary TX and propose corresponding transmission schemes for the two networks. We show that when the secondary network has a higher density as requested in [29], both networks can achieve the same throughput scaling law as a stand-alone wireless network, with zero outage for the secondary users with high probability. Considering a fluid model, we also show that both networks can achieve the same delay-throughput tradeoff as the optimal one established for a stand-alone static wireless network in [2]. In our approach, the primary network deploys a time-slotted multi-hop transmission scheme similar to that in [1] and does not need to cooperate with the secondary network. Note that, as mentioned in [29], if both the primary network and the secondary network are willing to cooperate and do time-sharing, both of them could easily achieve the same throughput scaling law as a stand-alone wireless network.

The rest of the paper is organized as follows. The system model, definitions, and main results are described in Section II. The proposed protocols for the primary and secondary networks are discussed in Section III. The delay and throughput scaling laws for the primary network are established in Section IV. The delay and throughput scaling laws for the secondary network are derived in Section V. Finally, Section VI summarizes our conclusions.

II. System Model, Definitions, and Main Results

In this section, we first describe the system model and assumptions about the primary and secondary networks, and then define the throughput and delay. We use $p(E)$ to represent the probability of event E and claim that an event E_n occurs with high probability (*w.h.p.*) if $p(E_n) \to 1$ as $n \to \infty$. We use the following order notations throughout this paper. Given non-negative functions $f(n)$ and $g(n)$:

1) $f(n) = O(g(n))$ means that there exists a positive constant c_1 and an integer m_1 such that $f(n) \leq c_1 g(n)$ for all $n \geq m_1$.
2) $f(n) = \Omega(g(n))$ means that there exists a positive constant c_2 and an integer m_2 such that $f(n) \geq c_2 g(n)$ for all $n \geq m_2$. Namely, $g(n) = O(f(n))$.
3) $f(n) = \Theta(g(n))$ means that both $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ hold for all $n \geq \max(m_1, m_2)$.

A. Network Model

Consider the scenario where a network of primary nodes and a network of secondary nodes coexist over a unit square. The primary nodes are distributed according to a Poisson point process (P. P. P.) of density n and randomly grouped into one-to-one source-destination (S-D) pairs. The distribution of the secondary nodes is following a P. P. P. of density m. The secondary nodes are also randomly grouped into one-to-one S-D pairs. As the model in [29], we assume that the density of the secondary network is higher than that of the primary network, i.e.,

$$m = n^{\beta}, \tag{1}$$

with $\beta > 1$.

For the wireless channel, we only consider the large-scale pathloss and ignore the effects of shadowing and small-scale multipath fading. As such, the normalized channel power gain $g(r)$ is given as

$$g(r) = \frac{A}{r^{\alpha}}, \tag{2}$$

where A is a system-dependent constant, r is the distance between the TX and the corresponding RX, and $\alpha > 2$ denotes the pathloss exponent. In the following discussion, we normalize A to be unity for simplicity.

The primary network and the secondary network share the same spectrum, time, and space, while the former one is the licensed user of the spectrum and thus has a higher priority to access the spectrum. The secondary network opportunistically access the spectrum while keeping its interference to the primary network at an "acceptable level". In this paper, the "acceptable level" means that the presence of the secondary network does not degrade the throughput scaling law of the primary network.

We assume that the secondary network only knows the locations of the primary TXs and has no knowledge about the locations of the primary RXs. This is the essential difference between our model and the model in [29], where the authors assumed that the secondary network knows the locations of all the primary nodes. Some other aspects of our model are defined in a similar way to that in [29], as we will discuss later.

B. Transmission Rate and Throughput

The ambient noise is assumed as additive white Gaussian noise (AWGN) with an average power N_0. During each transmission, we assume that each TX-RX pair deploys a capacity-achieving scheme, and the channel bandwidth is normalized to be unity for simplicity. Thus the data rate of the k-th primary TX-RX pair is given by

$$R_p(k) = \log\left(1 + \frac{P_p(k) g\left(\|X_{p,\text{tx}}(k) - X_{p,\text{rx}}(k)\|\right)}{N_0 + I_p(k) + I_{sp}(k)}\right), \quad (3)$$

where $\|\cdot\|$ stands for the norm operation, $P_p(k)$ is the transmit power of the k-th primary TX-RX pair, $X_{p,\text{tx}}(k)$ and $X_{p,\text{rx}}(k)$ are the TX and RX locations of the k-th primary TX-RX pair, respectively, $I_p(k)$ is the sum interference from all other primary TXs to the RX of the k-th primary TX-RX pair, and $I_{sp}(k)$ is the sum interference from all the secondary TXs to the RX of the k-th primary TX-RX pair. Specifically, $I_p(k)$ can be written as

$$I_p(k) = \sum_{i=1, i\neq k}^{Q_p} P_p(k) g\left(\| X_{p,\text{tx}}(i) - X_{p,\text{rx}}(k) \|\right), \quad (4)$$

where Q_p is the number of active primary TX-RX pairs, and $I_{sp}(k)$ is given by

$$I_{sp}(k) = \sum_{i=1}^{Q_s} P_s(i) g\left(\| X_{s,\text{tx}}(i) - X_{p,\text{rx}}(k) \|\right), \quad (5)$$

where Q_s is the number of active secondary TX-RX pairs, $P_s(i)$ is the transmit power of the i-th secondary TX-RX pair, and $X_{s,\text{tx}}(i)$ is the TX location of the i-th secondary TX-RX pair. Likewise, the data rate of the l-th secondary TX-RX pair is given by

$$R_s(l) = \log\left(1 + \frac{P_s(l) g\left(\|X_{s,\text{tx}}(l) - X_{s,\text{rx}}(l)\|\right)}{N_0 + I_s(l) + I_{ps}(l)}\right), \quad (6)$$

where $X_{s,\text{rx}}(l)$ is the RX location of the l-th secondary TX-RX pair, $I_s(l)$ is the sum interference from all other secondary TXs to the RX of the l-th secondary TX-RX pair, and $I_{ps}(l)$ is the sum interference from all primary TXs to the RX of the l-th secondary TX-RX pair. Specifically, $I_s(l)$ is given by

$$I_s(l) = \sum_{i=1, i\neq l}^{Q_s} P_s(i) g\left(\| X_{s,\text{tx}}(i) - X_{s,\text{rx}}(l) \|\right), \quad (7)$$

and $I_{ps}(l)$ is given by

$$I_{ps}(l) = \sum_{i=1}^{Q_p} P_p(i) g\left(\| X_{p,\text{tx}}(i) - X_{s,\text{rx}}(l) \|\right). \quad (8)$$

Now we give the definitions of throughput per S-D pair and sum throughput.

Definition 1: The *throughput per S-D pair* $\lambda(n_t)$ is defined as the average data rate that each source node can transmit to its chosen destination $w.h.p.$ in a multi-hop fashion with a particular scheduling scheme, where n_t is the number of nodes in the network. We have

$$p\left(\min_{1\leq i\leq n_t/2} \liminf_{t\to\infty} \frac{1}{t} M_i(t) \geq \lambda(n_t)\right) \to 1, \quad (9)$$

as $n_t \to \infty$, where $M_i(t)$ is the number of bits that S-D pair i transmitted in t time slots.

Definition 2: The *sum throughput* $T(n_t)$ is defined as the product between the throughput per S-D pair $\lambda(n_t)$ and the number of S-D pairs in the network, i.e.,

$$T(n_t) = \frac{n_t}{2} \lambda(n_t). \quad (10)$$

According to the network model defined in Section II.A, the number of nodes in the primary network (or in the secondary network) is a random variable. However, we will show in Lemma 1 and Lemma 3 at Section III that the number of nodes in the primary network (or in the secondary network) will be bounded by functions of the node density *w.h.p.*. As such, in the following discussion, we use $\lambda_p(n)$ and $\lambda_s(m)$ to denote the throughputs per S-D pair for the primary network and the secondary network, respectively. We use $T_p(n)$ and $T_s(m)$ to denote the sum throughputs for the primary network and the secondary network, respectively.

C. Fluid Model and Delay

As in [2], we use a fluid model to study the delay-throughput tradeoffs for the primary and secondary networks. In this model, we divide each time slot into multiple packet slots, and the size of the data packets can be scaled down to arbitrarily small with the increase of the node density n (or m) in the networks.

Definition 3: The *delay* $D(n_t)$ *of a packet* is defined as the average time that it takes to reach the destination node after the departure from the source node.

Let $D_i(j)$ denote the delay of packet j for S-D pair i. The sample mean of delay over all packets transmitted for S-D pair i is defined as

$$D_i = \limsup_{k\to\infty} \frac{1}{k} \sum_{j=1}^{k} D_i(j), \quad (11)$$

and the average delay over all S-D pairs is given by

$$\overline{D(n_t)} = \frac{2}{n_t} \sum_{i=1}^{n_t/2} D_i.$$

The average delay over all realizations of the network is

$$D(n_t) = E\left[\overline{D(n_t)}\right] = \frac{2}{n_t} \sum_{i=1}^{n_t/2} E\left[D_i\right]. \quad (12)$$

As what we did over the notations of throughput, in the following discussion, we use $D_p(n)$ and $D_s(m)$ to denote the packet delays for the primary network and the secondary network, respectively.

D. Main Results

The main results of this paper are as follows.

1) We propose a coexistence scheme for two overlaid ad hoc wireless networks: a primary network vs. a secondary network. These two networks operate in the same geographic region and share the same spectrum. The primary network has a higher priority to access the spectrum and has no special considerations over the

presence of the secondary network, while the secondary network operates opportunistically to access the spectrum in order to limit the interference to the primary network. We assume that the primary network uses a typical time-slotted adjacent-neighbor transmission protocol (similar to that in [1]) and the secondary network has a higher density and only knows the locations of the primary TXs. By a properly designed secondary protocol, we show that each secondary source node has a finite opportunity to transmit its packets to the chosen destination *w.h.p.*, i.e., no outage compared with the result in [29].

2) For the primary network, we show that the throughput per S-D pair is $\lambda_p(n) = \Theta(\sqrt{\frac{1}{n \log n}})$ *w.h.p.* and the sum throughput is $T_p(n) = \Theta(\sqrt{\frac{n}{\log n}})$ *w.h.p.*. These results are the same as those in a stand-alone ad hoc wireless network considered in [1]. Following the fluid model [2], we give the delay-throughput tradeoff for the primary network as $D_p(n) = \Theta(n\lambda_p(n))$ for $\lambda_p(n) = O(\frac{1}{\sqrt{n \log n}})$, which is the optimal delay-throughput tradeoff for a stand-alone wireless ad hoc network established in [2].
3) For the secondary network, we prove that the throughput per S-D pair is $\lambda_s(m) = \Theta(\sqrt{\frac{1}{m \log m}})$ *w.h.p.* and the sum throughput is $T_s(m) = \Theta(\sqrt{\frac{m}{\log m}})$ *w.h.p.*. Although due to the presence of the preservation regions, the secondary packets seemingly experience larger delays compared with that of the primary network, we show that the delay-throughput tradeoff for the secondary network is the same as that in the primary network, i.e., $D_s(m) = \Theta(m\lambda_s(m))$ for $\lambda_s(m) = O(\frac{1}{\sqrt{m \log m}})$.

III. Network Protocols

In our proposed scheme, the primary network deploys a modified time-slotted multi-hop transmission scheme over that in [1], [2], [29]. The secondary network adapts its protocol according to the primary transmission scheme. We first describe the primary protocol, then introduce the secondary protocol, and finally give a lemma to show that with our proposed protocols the secondary users can communicate without outage *w.h.p.*. Similarly as in [29], we claim that an outage event occurs when a node has zero opportunity to communicate. The outage probability is defined as the fraction of nodes that have zero opportunity to communicate.

A. Primary Network Protocol

- We divide the unit square into small-square primary cells. The area of each primary cell is $a_p = \frac{k_1 \log n}{n}$, with $k_1 \geq 1$.
- We group the primary cells into primary clusters, and each cluster has $K_p^2 = 25$ primary cells. We split the transmission time into time division multiple access (TDMA) frames, where each frame has 25 time slots that correspond to the number of cells in each primary cluster with each slot of length t_p. In each time slot, one cell in each primary cluster is chosen to be active. The cells in each primary cluster take turns to be active in a round-robin fashion. All primary clusters follow the same 25-TDMA transmission pattern, as shown in Fig. 1.

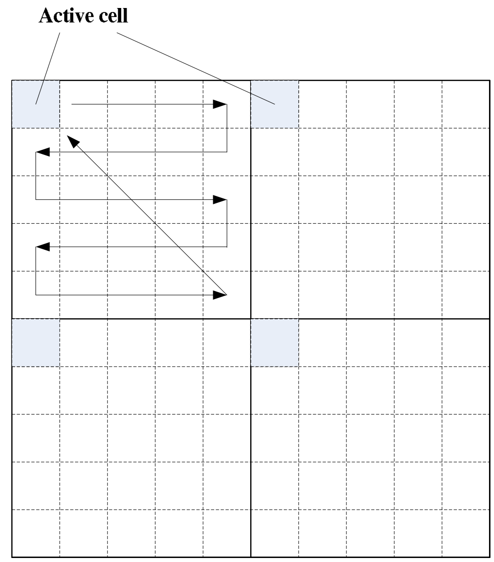

Fig. 1. A four-cluster example with 25 cells per cluster. The cells in each cluster take turns to be active along the arrowed line over time.

- We define the data path along which the packets traverse as the horizontal line and then the vertical line connecting a source and its corresponding destination, as shown in Fig. 2. One node within a primary cell is defined as a designated relay node, which is responsible for relaying the packets of all the data paths passing through the cell. The packets will be forwarded from cell to cell by the relay nodes first along the horizontal data path (HDP), then along the vertical data path (VDP). Nodes in a particular cell take turns to serve as the designated relay node.
- When a primary cell is active, it transmits a single packet for each of the data paths passing through the cell. The transmission is also deployed in a TDMA fashion. The TDMA frame structure for the primary network is shown in Fig. 3, where one packet slot is assigned to one S-D data path that passes through or originates from a particular primary cell. As such, the number of packet slots is determined by the total number of data paths in the cell, which is based on the so-called fluid model [2]. The specific packet transmission procedure is as follows:
  – The designated relay node first transmits a single packet for each of the S-D paths passing through the cell; and then each of the source nodes within the cell takes turns to transmit a single packet.
  – The receiving node must be located in one of the neighboring primary cells along the predefined data path, unless it is a destination node, which may be located in the same cell. If the next-hop of the packet is the final destination, it will be directly delivered to the destination node; otherwise, the packet will be

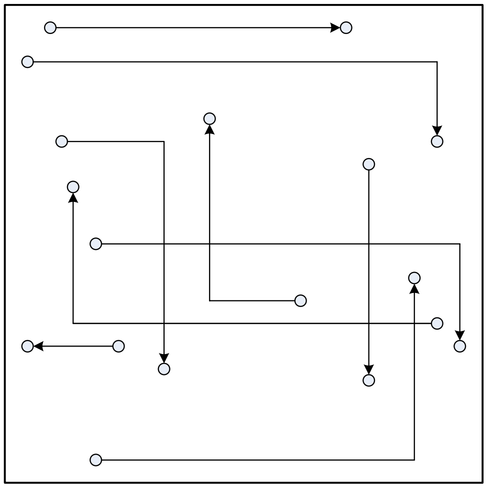

Fig. 2. Examples of HDPs and VDPs for the primary S-D pairs.

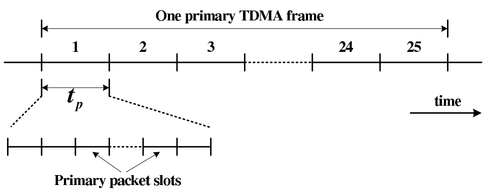

Fig. 3. Structure of the primary TDMA frame, where t_p is the time-slot duration of the primary TDMA scheme.

transmitted to a designated relay node.

  – The designated relay node in each primary cell maintains a buffer to temporarily store the packets received from its neighboring cells, and each packet will be transmitted to the next hop in the next active time slot of the cell.

- At each packet slot, the TX node transmits with power of $P_0 a_p^{\frac{\alpha}{2}}$, where P_0 is a constant.

The primary protocol in this paper is similar to that in [2] but with different data paths and TDMA transmission patterns. As a result, we have the following two lemmas.

Lemma 1: Let n_{pt} denote the number of total primary nodes in the unit square; then we have $\frac{n}{2} < n_{pt} < en$ *w.h.p.*.

Proof: Since n_{pt} is a Poisson random variable with parameter $\mu = n$, using the Chernoff bound in Lemma 11 (see Appendix), we have

$$\begin{aligned} p\left(n_{pt} \leq \frac{n}{2}\right) &\leq \frac{e^{-n}(en)^{\frac{n}{2}}}{\left(\frac{n}{2}\right)^{\frac{n}{2}}} \\ &= \left(\frac{2}{e}\right)^{\frac{n}{2}} \to 0 \end{aligned} \tag{13}$$

as $n \to \infty$, and

$$\begin{aligned} p\left(n_{pt} \geq en\right) &\leq \frac{e^{-n}(en)^{en}}{(en)^{en}} \\ &= e^{-n} \to 0 \end{aligned} \tag{14}$$

as $n \to \infty$. Combining (13) and (14) via the union bound, we obtain

$$p\left(n_{pt} \leq \frac{n}{2} \text{ or } n_{pt} \geq en\right) \leq p\left(n_{pt} \leq \frac{n}{2}\right) + p\left(n_{pt} \geq en\right) \to 0$$

as $n \to \infty$. Hence

$$p\left(\frac{n}{2} < n_{pt} < en\right) = 1 - p\left(n_{pt} \leq \frac{n}{2} \text{ or } n_{pt} \geq en\right) \to 1$$

as $n \to \infty$, which completes the proof. ■

Lemma 2: For $k_1 \geq 1$, each primary cell contains at least one but no more than $k_1 e \log n$ primary nodes *w.h.p.*.

Proof: Let n_p denote the number of primary nodes in a particular primary cell; then n_p is a Poisson random variable with parameter $\mu = n a_p = k_1 \log n$. The probability of $n_p = 0$ is given by

$$p\left(n_p = 0\right) = \left.\frac{e^{-k_1 \log n}(k_1 \log n)^k}{k!}\right|_{k=0} = \frac{1}{n^{k_1}}. \tag{15}$$

By the union bound, the probability that at least one primary cell having no nodes is upper-bounded by the total number of cells multiplied by $p\left(n_p = 0\right)$, which is

$$\begin{aligned} & p\left(\text{At least one primary cell has no nodes}\right) \\ & \leq \frac{1}{a_p} p\left(n_p = 0\right) = \frac{1}{k_1 n^{k_1 - 1} \log n} \to 0 \end{aligned}$$

as $n \to \infty$ for $k_1 \geq 1$.

Now consider the upper bound of n_p. By the Chernoff bound in Lemma 11 (see Appendix), we have

$$p\left(n_p \geq k_1 e \log n\right) \leq \frac{e^{-k_1 \log n}\left(e k_1 \log n\right)^{k_1 e \log n}}{\left(k_1 e \log n\right)^{k_1 e \log n}} = n^{-k_1}.$$

As long as $k_1 \geq 1$, by the union bound, we have

$$\begin{aligned} & p\left(\text{At least one primary cell has more than } k_1 e \log n \text{ nodes}\right) \\ & \leq \frac{1}{a_p} n^{-k_1} = \frac{1}{k_1 n^{k_1 - 1} \log n} \to 0 \end{aligned}$$

as $n \to \infty$. This completes the proof. ■

B. Secondary Network Protocol

- We divide the unit area into square secondary cells with size $a_s = \frac{k_2 \log m}{m}$, with $k_2 \geq 1$.
- We group the secondary cells into secondary clusters. Each secondary cluster has $K_s^2 = 25$ cells. Similar to the primary network protocol, the secondary network also follows a 25-TDMA pattern to communicate. We let the duration of each secondary TDMA frame equal to that of one primary time slot. The relationship between the primary TDMA frame and the secondary TDMA frame is shown in Fig. 4, where each secondary time slot is further divided into packet slots.
- To limit the interference from the secondary nodes to the primary nodes, we define a preservation region as a square containing M^2 secondary cells around a particular primary cell in which an active primary TX (not the RX) is located, where M is an integer and the value will be defined later. No secondary nodes in the preservation regions are allowed to transmit.
- The designated relay nodes and data paths for the secondary network are defined in the same way as those for the primary network. As shown in Fig. 5, when a particular secondary cell outside the preservation region

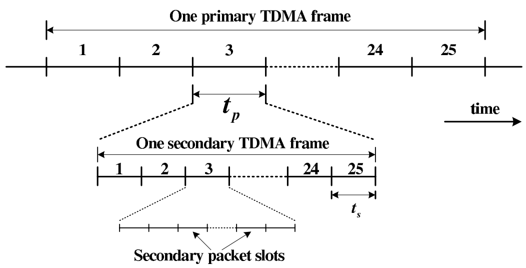

Fig. 4. Structure of the secondary TDMA frame and its relationship with the primary TDMA frame, where t_s is the time-slot duration for the secondary TDMA scheme.

is active, its designated relay node transmits a single packet for each of the data paths passing through the cell, and each of the secondary source nodes within the cell takes turns to transmit a single packet. The packet is transmitted to the next-hop relay node or the destination node in neighboring secondary cells along the HDP or VDP path. Note that if the RX node is the destination node, it may be located in the same cell, as we discussed for the primary protocol.

- When a secondary cell falls into a preservation region[1], its designated relay node buffers the packets that it receives; it waits until the preservation region is cleared and the cell is active to deliver the packets to the next hop.
- At each packet slot, the active secondary TX node transmits with power of $P_1 a_s^{\frac{\alpha}{2}}$, where P_1 is a constant.

Similarly as in the primary network case, we have the following two lemmas for the secondary network.

Lemma 3: Let n_{st} denote the total number of secondary nodes in the unit square; then we have $\frac{m}{2} < n_{st} < em$ *w.h.p.*.

Proof: The proof is similar to that of Lemma 1. ■

Lemma 4: For $k_2 \geq 1$, each secondary cell contains at least one but no more than $k_2 e \log m$ secondary nodes *w.h.p.*.

Proof: The proof is similar to that of Lemma 2. ■

Regarding the cluster size, note that the value of K_s is not necessarily the same as that of K_p. Here we choose $K_s = K_p$ for simplicity. Without loss of generality, we also choose $k_1 = k_2$ in the following discussion.

Now, let us discuss how to choose the value of M, i.e., the size for the preservation region. Considering the fact that the primary TX may only transmit to a node in its adjacent cells or within the same cell, the preservation region should accommodate at least 9 primary cells to protect the potential primary RX. Since the primary RX may be located close to the outer boundary of the 9-cell region, we should add another layer of protective secondary cells. As such, any active secondary TXs outside the preservation region are at least certain-distance-away from the potential primary RX. Therefore, we define the side length of the preservation square region as

$$M\sqrt{a_s} \geq 3\sqrt{a_p} + 2\epsilon_p, \tag{16}$$

[1]Note that the secondary nodes located in the preservation regions can still receive packets from TXs outside the preservation regions, although they are not permitted to transmit packets.

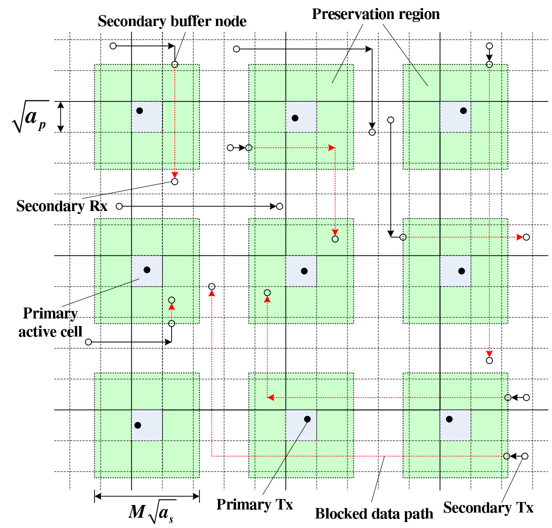

Fig. 5. Preservation region and examples of secondary data paths.

where $\epsilon_p > 0$ defines the width of the protective secondary strip around the 9 primary cells in the preservation region. There is a tradeoff in choosing the value of ϵ_p. If we choose a larger ϵ_p, the interference from the secondary network to the primary network will be less. However, the opportunity for the secondary network to access the spectrum will also be less since the unpreserved area in the unit square will be reduced. In the following discussion, we set $\epsilon_p = \sqrt{a_s}$ for simplicity. Accordingly, the minimum value of M can be set as

$$\begin{aligned} M &= \lfloor \frac{3\sqrt{a_p} + 2\sqrt{a_s}}{\sqrt{a_s}} \rfloor \\ &= \lfloor 3\sqrt{\frac{a_p}{a_s}} \rfloor + 2 \\ &\approx 3\sqrt{\frac{n^{\beta-1}}{\beta}}, \end{aligned} \tag{17}$$

where $\lfloor \cdot \rfloor$ denotes the flooring operation. In the last equation of (17), we applied $a_p = \frac{k_1 \log n}{n}$, $a_s = \frac{k_2 \log m}{m}$, $k_1 = k_2$, and (1), assuming that n is large enough. In the following discussion, "n is large" or "n is large enough" means that, for a fixed β, n is chosen to satisfy $a_s \ll a_p$. For example, when $k_1 = k_2$, $\beta = 2$, $n = 1000$, we have $m = 1000000$ and $\frac{a_p}{a_s} = \frac{n^{\beta-1}}{\beta} = 500$.

Note that the preservation region defined here is larger than that in [29] due to the fact that we only know the locations of primary TXs. If a secondary node falls inside a preservation region, it will be silenced. If not, it may become active and has an opportunity to transmit its packets. Accordingly, we call the unpreserved region as the "active region". Since the locations of preservation regions change periodically according to the active time slots in the primary TDMA frame, from the point view of a specific secondary node, it is periodically located in the active region. We define the following terminology to measure the fraction of time in which a secondary cell is located in the active region.

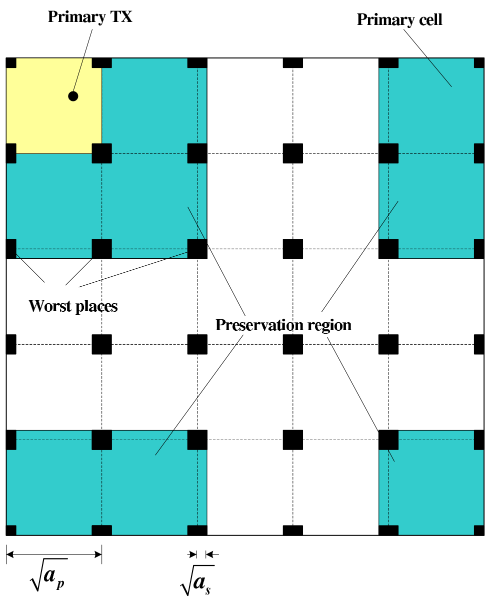

Fig. 6. Preservation regions and worst places in one primary cluster.

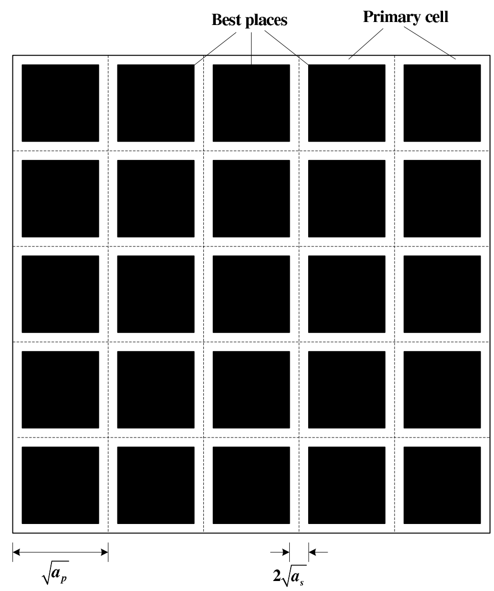

Fig. 7. The best places in one primary cluster.

Definition 4: The *opportunistic factor* of a secondary cell is defined as the fraction of time in which it is located in the active region.

We use the following lemma to show that, with the protocols defined previously, each individual secondary source node has a finite opportunity to transmit its packets to the chosen destination *w.h.p.*.

Lemma 5: With the proposed transmission protocol, we have the following results:

1) The opportunistic factor for a secondary cell is $\frac{9}{25} \leq \eta \leq \frac{16}{25}$, for n is large enough.
2) Each individual secondary node has a finite opportunity to transmit its packets to the chosen destination, i.e., zero outage, *w.h.p.*.

Proof: Consider one primary cluster of 25 primary cells as shown in Fig. 6, where the preservation regions are illustrated as the shaded area when the upper-left primary cell is active in this and neighboring clusters. The primary cells will take turns to be active over time (see Fig. 1) and the locations of the preservation regions will change accordingly. We can easily verify that any point in the cluster has a finite opportunity to be in the active region when n is large. However, during each period of a primary TDMA frame, the fractions of time for different secondary nodes to be in the active region are not the same. The worst places are the squares with side length of $2\sqrt{a_s}$ around the vertices of each primary cell, as shown by those deeply-shaded small squares in Fig. 6. The opportunistic factor of the secondary cells in these squares is $\frac{9}{25}$. The best places are the squares with side length of $\sqrt{a_p} - 2\sqrt{a_s}$ inside each primary cell, as shown by the deeply-shaded squares in Fig. 7. The opportunistic factor of the secondary cells in these squares are $\frac{16}{25}$. When the secondary cell lies in other places, the opportunistic factor is between $\frac{9}{25}$ and $\frac{16}{25}$.

The condition that a secondary node is located in the active region is not sufficient to ensure that it can transmit packets to the destination along the predefined data path. Recall that the secondary network also deploys a TDMA scheme with adjacent-neighbor transmission. The sufficient condition to ensure that each individual secondary node has a finite chance to transmit packets is that the secondary cell in which the node is located will be assigned with at least one active secondary TDMA slot within each secondary frame, whenever the cell is in the active region. Since in each primary time slot, we have one complete secondary TDMA frame in our protocol, the above sufficient condition is indeed satisfied.

Based on the above discussions, during each period of a primary TDMA frame, each secondary cell has a finite opportunity to be located in the active region with an opportunistic factor of $\frac{9}{25} \leq \eta \leq \frac{16}{25}$, and each of them is assigned with a secondary TDMA slot. According to the secondary protocol, when a secondary cell is active, each packet buffered in this cell will be assigned with a packet slot *w.h.p.* to be transmitted, since the total number of data paths that pass through or originate from each secondary cell is upper-bounded *w.h.p.* (see Lemma 10 in Section V). Thus, the packets from any secondary source node have a finite opportunity to be transmitted along the predefined data path to the chosen destination *w.h.p.*. This completes the proof for the zero outage property. ∎

There is a significant difference between our result here and that in [29]. The authors in [29] defined preservation regions of 9 secondary cells around each primary node, and the positions of the preservation regions are fixed. If the secondary nodes are located in the preservation regions, they will never be active. Therefore, the secondary network in [29] usually suffers from a non-zero outage probability, even though the outage probability is upper-bounded *w.h.p.*. In our case, each secondary node has a finite opportunity to be active such that we have zero outage *w.h.p.*.

IV. Delay and Throughput Analysis for the Primary Network

In this section, we discuss the delay and throughput scaling laws as well as the delay-throughput tradeoff for the primary network. The main results are given in three theorems. We first present the delay and throughput scaling laws, then establish the delay-throughput tradeoff for the primary network.

A. Delay Analysis for the Primary Network

The packet delay for the primary network is given by the following theorem.

Theorem 1: According to the primary network protocol in Section III, the packet delay is given by

$$D_p(n) = \Theta\left(\frac{1}{\sqrt{a_p(n)}}\right), \quad w.h.p.. \tag{18}$$

Proof: We first derive the average number of hops for each packet to traverse along the primary S-D data path, then use the fact that the time for each primary packet to spend at each hop is a constant, $25t_p$, as shown in Fig. 3, and finally calculate the average delay for each primary S-D pair .

Since each primary hop spans a distance of $\Theta\left(\sqrt{a_p(n)}\right)$ *w.h.p.*, the number of hops for a primary packet along the S-D data path i is $\Theta\left(\frac{d_p(i)}{\sqrt{a_p(i)}}\right)$ *w.h.p.*, where $d_p(i)$ is the length of the primary S-D data path i. Hence, the number of hops traversed by a primary packet, averaged over all S-D pairs, is $\Theta\left(\frac{2}{n_{pt}}\sum_{i=1}^{n_{pt}/2}\frac{d_p(i)}{\sqrt{a_p(n)}}\right)$ *w.h.p.*.

The data path length $d_p(i)$ is a random variable, with a maximum value of 2. According to the law of large numbers, as $n_{pt} \to \infty$, the average distance between primary S-D pairs is

$$\frac{2}{n_{pt}}\sum_{i=1}^{n_{pt}/2} d_p(i) = \Theta(1).$$

Therefore, the average number of hops for a primary packet to traverse is $\Theta\left(\frac{1}{\sqrt{a_p(n)}}\right)$ *w.h.p.*. Since we use a fluid model such that the packet size of the primary network scales proportionally to the throughput $\lambda_p(n)$, each packet arrived at a primary cell will be transmitted in the next active time slot of the cell. As such, the maximum time spent at each primary hop for a particular packet is $25t_p$. Hence, the average delay for each primary packet is given by

$$D_p(n) = \Theta\left(\frac{25t_p}{\sqrt{a_p(n)}}\right) = \Theta\left(\frac{1}{\sqrt{a_p(n)}}\right), \quad w.h.p, \tag{19}$$

which completes the proof. ■

The above proof follows the same logic as the proof of Theorem 4 in [2]. The two differences are that we use HDPs and VDPs as the packet routing paths instead of the direct S-D links and we use a different TDMA transmission pattern.

B. Throughput Analysis for the Primary Network

For the primary network, the throughput per S-D pair and the sum throughput scaling laws are given in the following theorem.

Theorem 2: With the primary protocol defined in Section III, the primary network can achieve the following throughput per S-D pair and sum throughput *w.h.p.*:

$$\lambda_p(n) = \Theta\left(\sqrt{\frac{1}{n\log n}}\right) \tag{20}$$

and

$$T_p(n) = \Theta\left(\sqrt{\frac{n}{\log n}}\right). \tag{21}$$

Before we give the proof of the above theorem, we first give two lemmas, then use these lemmas to prove the theorem. The main logical flows in the proofs of these lemmas and the theorem are motivated by that in [29] and [15].

Lemma 6: With the primary protocol defined in Section III, each TX node in a primary cell can support a constant data rate of K_1, where $K_1 > 0$ is independent of n.

Proof: In a given primary packet slot, suppose we have Q_p active primary cells and Q_s active secondary cells. The data rate supported for a TX node in the i-th active primary cell can be calculated as follows:

$$R_p(i) = \frac{1}{25}\log\left(1 + \frac{P_p(i)g(\|X_{p,\text{tx}}(i) - X_{p,\text{rx}}(i)\|)}{N_0 + I_p(i) + I_{sp}(i)}\right), \tag{22}$$

where $\frac{1}{25}$ denotes the rate loss due to the 25-TDMA transmission in the primary network. Note that since there is only one active primary link initiated in each primary cell at a given time, we index the active link initiated in the i-th active primary cell as the i-th active primary link in the whole network. In Fig. 8, we show the primary interference sources to the primary RX of the i-th active primary link, where the shaded cells represent the active primary cells based on the 25-TDMA protocol. From the figure, we see that we have 8 primary interferers with a distance of at least $3\sqrt{a_p}$, 16 primary interferers with a distance of at least $7\sqrt{a_p}$, and so on. Thus, $I_p(i)$ is upper-bounded as

$$\begin{aligned} I_p(i) &= \sum_{k=1,k\neq i}^{Q_p} P_p(k)g(\|X_{p,\text{tx}}(k) - X_{p,\text{rx}}(i)\|) \\ &< P_0\sum_{t=1}^{\infty} 8t(4t-1)^{-\alpha} \\ &= I_p < \infty, \end{aligned} \tag{23}$$

where we used the relationship that $P_p(k) = P_0 a_p^{\frac{\alpha}{2}}$ for all k's and the fact that the series $\sum_{t=1}^{\infty} 8t(4t-1)^{-\alpha}$ converges to a constant for $\alpha > 2$ (see Lemma 12 in Appendix). Due to the preservation regions, a minimum distance $\sqrt{a_s}$ can be guaranteed from all secondary active TXs to any active primary RXs. Thus, $I_{sp}(i)$ is upper-bounded as

$$\begin{aligned} I_{sp}(i) &= \sum_{k=1}^{Q_s} P_s(k) g(\|X_{s,\text{tx}}(k) - X_{p,\text{rx}}(k)\|) \\ &\quad + P_1 a_s^{\frac{\alpha}{2}} (\sqrt{a_s})^{-\alpha} \\ &< P_1 \sum_{t=1}^{\infty} 8t(4t-1)^{-\alpha} + P_1 \\ &= I_{sp} < \infty, \end{aligned} \tag{24}$$

where we used the fact that $P_s(k) = P_1 a_s^{\frac{\alpha}{2}}$ for all k's and Lemma 12. Therefore, we have

$$R_p(i) > \frac{1}{25} \log \left(1 + \frac{P_0(\sqrt{5})^{-\alpha}}{N_0 + I_p + I_{sp}} \right) = K_1 > 0, \tag{25}$$

where the relationship that $\|X_{p,\text{tx}}(i) - X_{p,\text{rx}}(i)\| \leq \sqrt{5a_p}$ is used (see Fig. 8). This completes the proof. ■

Lemma 7: For $a_p(n) = k_1 \log n/n$, the number of primary S-D paths (including both HDPs and VDPs) that pass through or originate from each primary cell is $O\left(n\sqrt{a_p(n)}\right)$ *w.h.p.*.

Proof: See the proof of Lemma 3 in [29] or the proof of Lemma 2 in [15]. ■

Now we give the proof for Theorem 2.

Proof: Consider the proof of the per-node throughput in (20). According to the definitions in Section II, we need to show that there are deterministic constants $c_2 > 0$ and $c_1 < +\infty$ to satisfy

$$\lim_{n\to\infty} p\left(\frac{c_2}{\sqrt{n \log n}} \leq \lambda_p(n) \leq \frac{c_1}{\sqrt{n \log n}} \right) = 1. \tag{26}$$

A loose upper bound of the per-node throughput for the primary network is achieved when the secondary network is absent. Gupta and Kumar [1] have already showed that such an upper bound given in (26) exists. We then only need to consider the proof for the lower bound.

Since a given TX node in each primary cell can support a constant data rate of K_1 (see Lemma 6), each primary S-D pair can achieve a data rate of at least K_1 divided by the maximum number of data paths that pass through and originate from the primary cell. From Lemma 7, we know that the number of data paths that pass through or originate from each primary cell is $O\left(n\sqrt{a_p(n)}\right)$ *w.h.p.*. Therefore, the throughput per S-D pair $\lambda_p(n)$ is lower-bounded by $\Omega\left(\frac{1}{n\sqrt{a_p(n)}}\right)$ *w.h.p.*, i.e., the lower bound is $\Omega\left(\frac{1}{\sqrt{n \log n}}\right)$ $w.h.p.$.

From Lemma 1, the number of primary S-D pairs is lower-bounded by $\frac{n}{4}$ *w.h.p.*. Thus, the sum throughput $S_p(n)$ is lower-bounded by $\frac{n}{4}\lambda_p(n)$ *w.h.p.*, i.e., the lower bound is $\Omega\left(\sqrt{\frac{n}{\log n}}\right)$ *w.h.p.*. The upper bound of $S_p(n)$ is already established in [1]. This completes the proof. ■

From the proof of Theorem 2, the throughput per S-D pair for the primary network can be written as

$$\lambda_p(n) = \Theta\left(\frac{1}{n\sqrt{a_p(n)}}\right), \quad w.h.p.. \tag{27}$$

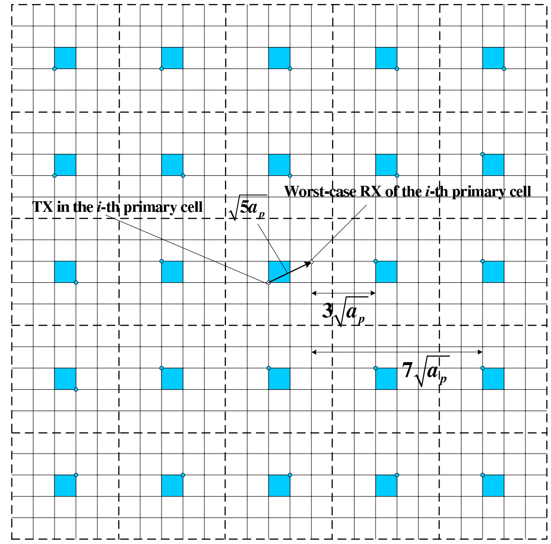

Fig. 8. Interference from the concurrent primary transmissions to the worst-case primary RX of the transmission from the i-th primary cell.

C. Delay-throughput Tradeoff for the Primary Network

Combining the results in (18) and (27), the delay-throughput tradeoff for the primary network is given by the following theorem.

Theorem 3: With the primary protocol defined in Section III, the delay-throughput tradeoff is

$$D_p(n) = \Theta(n\lambda_p(n)), \text{ for } \lambda_p(n) = O\left(\frac{1}{\sqrt{n \log n}}\right). \tag{28}$$

V. Delay and Throughput Analysis for the Secondary Network

The difference between the primary and the secondary transmission schemes arises from the presence of the preservation regions. When their paths are blocked by the preservation regions, the secondary relay nodes buffer the packets and wait until the next hop is available. Due to the presence of the preservation region, the secondary packets will experience a larger delay compared with the primary packets. However, the average packet delay per hop for each secondary S-D data path is still a constant as we discussed later. Thus, we can show that the throughput scaling law and the delay-throughput tradeoff for the secondary network are the same as those in the primary network. In the following discussion, we first analyze the average packet delay, then discuss the throughput scaling law, and finally describe the delay-throughput tradeoff.

A. Delay Analysis for the Secondary Network

The average packet delay for the secondary network is given by the following theorem.

Theorem 4: According to the proposed secondary network protocol in Section III, the packet delay is given by

$$D_s(m) = \Theta\left(\frac{1}{\sqrt{a_s(m)}}\right), \quad w.h.p.. \tag{29}$$

Before giving the proof of Theorem 4, we present the following lemma.

Lemma 8: The average packet delay for each secondary hop is $\Theta(1)$.

Proof: Let $D^j_{s,h}(i)$ denote the packet delay for the secondary network over hop j and S-D pair i. As shown in Fig. 4, if there are no preservation regions, each secondary cell has one active time slot in each primary time slot. In another word, each secondary packet will experience a worst-case delay of t_p at each hop, i.e., $D^j_{s,h}(i) = t_p$. When we have the preservation regions, according to Lemma 5, $D^j_{s,h}(i)$ is a bounded random variable. It depends on the location of the active TX from which the secondary packet departs. As shown in Fig. 4 and Fig. 6, when the active TX is located in the worst places as shown in Fig. 6, $D^j_{s,h}(i)$ is $\frac{1}{\eta_{\min}}t_p$, where $\eta_{\min} = \frac{9}{25}$ is the minimum value of the opportunistic factor η. Similarly, when the active TX is located in the best places as shown in Fig. 7, $D^j_{s,h}(i)$ is $\frac{1}{\eta_{\max}}t_p$, where $\eta_{max} = \frac{16}{25}$ is the maximum value of the opportunistic factor η. Hence, the ensemble average of $D^j_{s,h}(i)$ will be a constant c_0, where $\frac{1}{\eta_{\max}}t_p < c_0 < \frac{1}{\eta_{\min}}t_p$, i.e., $E\left[D^j_{s,h}(i)\right] = \Theta(1)$. This completes the proof. ■

Now, let us prove Theorem 4.

Proof: Since each secondary hop covers a distance of $\Theta\left(\sqrt{a_s(m)}\right)$ *w.h.p.*, and similarly as in the proof of Theorem 2, the average length of each secondary S-D data path is $\Theta(1)$, the average number of hops for each secondary packet is $\Theta\left(\frac{1}{\sqrt{a_s(m)}}\right)$ *w.h.p.*. From Lemma 8, the average packet delay for each secondary hop is $\Theta(1)$. Therefore, the average packet delay for the secondary network is

$$D_s(m) = \Theta\left(\frac{1}{\sqrt{a_s(m)}}\right)$$

w.h.p., which completes the proof. ■

B. Throughput Analysis for the Secondary Network

For the secondary network, the throughput scaling law is given by the following theorem.

Theorem 5: With the secondary protocol defined in Section III, the secondary network can achieve the following throughput per-node and sum throughput *w.h.p.*:

$$\lambda_s(m) = \Theta\left(\sqrt{\frac{1}{m\log m}}\right) \tag{30}$$

and

$$T_s(m) = \Theta\left(\sqrt{\frac{m}{\log m}}\right). \tag{31}$$

Similarly as in the primary network case, we first present two lemmas, then use these lemmas to prove Theorem 5.

Lemma 9: With the proposed secondary protocol, each TX node in a secondary cell can support a data rate of K_2, where $K_2 > 0$ is independent of m.

Proof: Due to the presence of the preservation regions, a minimum distance of $1.5\sqrt{a_p}$ from all primary TXs to a specific active secondary RX can be guaranteed. At a given secondary packet slot and at the i-th secondary link (i.e., the active transmission initiated in the i-th secondary cell), the interference from all active primary TXs is upper-bounded as

$$\begin{aligned} I_{ps}(i) &< P_0 a_p^{\frac{\alpha}{2}} \sum_{t=1}^{\infty} 8t((3t-1)\sqrt{a_p})^{-\alpha} \\ &\quad + P_0 a_p^{\frac{\alpha}{2}} \left(1.5\sqrt{a_p}\right)^{-\alpha} \\ &< P_0 \sum_{t=1}^{\infty} 8t(3t-1)^{-\alpha} + P_0(1.5)^{-\alpha} \\ &= I_{ps} < \infty, \end{aligned} \tag{32}$$

where we applied the same technique as in the proof of Lemma 6 to obtain the upper bound. Likewise, $I_s(i)$ is upper-bounded by $I_s = P_1 \sum_{t=1}^{\infty} 8t(4t-1)^{-\alpha}$, which converges to a constant as shown in Lemma 12 (see the Appendix). Considering the effects of the preservation region, the lower bound of the data rate that is supported in each secondary cell can be written as

$$R_s(i) > \frac{1}{25}\eta_{\min} \log\left(1 + \frac{P_0(\sqrt{5})^{-\alpha}}{N_0 + I_{ps} + I_s}\right) = K_2 > 0, \tag{33}$$

where $\eta_{\min} = \frac{9}{25}$ represents the penalty due to the presence of the preservation region. Thus, we can guarantee a constant data rate $K_2 > 0$ for a given TX node in each secondary cell, which completes the proof. ■

Lemma 10: For $a_s(m) = k_2 \log m/m$, the number of secondary S-D paths (including both HDPs and VDPs) that pass through or originate from each secondary cell is $O\left(m\sqrt{a_s(m)}\right)$ *w.h.p.*.

Proof: The proof of Lemma 10 follows the same logic as that in the proof of Lemma 7. ■

Now, let us prove Theorem 5.

Proof: The proof of Theorem 5 is similar to the proof of Theorem 2. ■

Similarly as in Theorem 2, the throughput per S-D pair of the secondary network can be written as

$$\lambda_s(m) = \Theta\left(\frac{1}{m\sqrt{a_s(m)}}\right), \quad w.h.p.. \tag{34}$$

C. Delay-throughput Tradeoff for the Secondary Network

Combining the results in (29) and (34), the delay-throughput tradeoff for the secondary network is given by the following theorem.

Theorem 6: With the secondary protocol defined in Section III, the delay-throughput tradeoff for the secondary network is

$$D_s(m) = \Theta\left(m\lambda_s(m)\right), \text{ for } \lambda_s(m) = O\left(\frac{1}{\sqrt{m\log m}}\right). \tag{35}$$

VI. Conclusion

In this paper, we studied the coexistence of two wireless networks with different priorities, where the primary network has a higher priority to access the spectrum, and the secondary network opportunistically explore the spectrum. When the secondary network has a higher density, with our proposed protocols, both of these networks can achieve the throughput scaling law promised by Gupta and Kumar in [1]. Comparing with the recent result in [29], we only assumed the knowledge about the primary TX locations and there is no outage penalty for the secondary nodes. By using a fluid model, we also showed that both networks can achieve the same delay-throughput tradeoff as the optimal one established for a stand-alone wireless network in [2].

APPENDIX

In the appendix, we first recall the useful Chernoff bound for a Poisson random variable from [30]; then we give a lemma to show that the infinite series sums in Lemma 6 and Lemma 9 converge to a constant.

Lemma 11: (Theorem 5.4 in [30]) Let X be a Poisson random variable with parameter μ.

1) If $x > \mu$, then

$$p\left(X \geq x\right) \leq \frac{e^{-\mu}(e\mu)^x}{x^x};$$

2) If $x < \mu$, then

$$p\left(X \leq x\right) \leq \frac{e^{-\mu}(e\mu)^x}{x^x}.$$

Lemma 12: The sum $\sum_{t=1}^{\infty} at(bt-1)^{-\alpha}$ converges to a constant, where $\alpha > 2$, a and b are positive integers.

Proof:

$$\begin{aligned}\sum_{t=1}^{\infty}\frac{at}{(bt-1)^{\alpha}} &= \frac{a}{b^{\alpha}}\sum_{t=1}^{\infty}\frac{t}{(t-\frac{1}{b})^{\alpha}} \\ &= \frac{a}{b^{\alpha}}\sum_{t=1}^{\infty}\frac{1}{(t-\frac{1}{b})^{\alpha-1}} \\ &\quad +\frac{a}{b^{\alpha+1}}\sum_{t=1}^{\infty}\frac{1}{(t-\frac{1}{b})^{\alpha}}. \qquad (36)\end{aligned}$$

Applying the following inequality

$$\sum_{t=1}^{\infty}\frac{1}{(t-\frac{1}{b})^{\alpha}} \leq \frac{1}{(1-\frac{1}{b})^{\alpha}} + \int_{1}^{\infty}\frac{1}{(t-\frac{1}{b})^{\alpha}}\,dt$$

to (36), we obtain

$$\begin{aligned}\sum_{t=1}^{\infty}\frac{at}{(bt-1)^{\alpha}} &\leq \frac{a}{b^{\alpha}}\left(\frac{1}{(1-\frac{1}{b})^{\alpha-1}} + \int_{1}^{\infty}\frac{1}{(t-\frac{1}{b})^{\alpha-1}}\,dt\right) \\ &\quad +\frac{a}{b^{\alpha+1}}\left(\frac{1}{(1-\frac{1}{b})^{\alpha}} + \int_{1}^{\infty}\frac{1}{(t-\frac{1}{b})^{\alpha}}\,dt\right) \\ &= \frac{a}{b^{\alpha}\left(1-\frac{1}{b}\right)^{\alpha-1}} + \frac{a\left(1-\frac{1}{b}\right)^{-\alpha+2}}{b^{\alpha}(\alpha-2)} \\ &\quad +\frac{a}{b^{\alpha+1}\left(1-\frac{1}{b}\right)^{\alpha}} + \frac{a\left(1-\frac{1}{b}\right)^{-\alpha+1}}{b^{\alpha+1}(\alpha-1)},\end{aligned}$$

where the last equation is a constant when $\alpha > 2$. This completes the proof. ∎

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