K-Best Sphere Detection for the Sphere Packing Modulation Aided SDMA/OFDM Uplink

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***Abstract*—Recently, the turbo-detected Sphere Packing (SP) aided Space-Time Block-Coding (STBC) STBC-SP scheme was demonstrated to provide useful performance improvements over conventionally-modulated orthogonal design based STBC schemes in the context of single-user multiple-input multiple-output (SU-MIMO) systems. Hence, in this treatise we extend its employment to the multi-user MIMO (MU-MIMO) scenario. For the sake of achieving a near Maximum A Posteriori (MAP) performance while imposing a moderate complexity, we specifically design the K-best sphere detection (SD) scheme for supporting the operation of the SP-modulated system, since the conventional SD cannot be directly applied to such a system. Consequently, with the aid of our modified SD, a significant performance gain can be achieved by the SP-modulated system over its conventionally-modulated counterpart in the context of MU-MIMO systems. For example, a performance gain of 3.5 dB can be achieved by the proposed 256-SP over the identical-throughput 16-QAM benchmark system in the scenario of a four-receive-antenna SDMA uplink (UL) system supporting two users both employing Alamouti's $\mathbf{G}_2$ space-time code, given a target BER of 10^{-4}.**

I. INTRODUCTION

The family of multiple-input-multiple-output (MIMO) transmission schemes typically fall into two main categories, i.e. spatial multiplexing and spatial diversity schemes, which aim for maximizing the data rate and for minimizing the Bit Error Ratio (BER), respectively. Research efforts have also been invested in striking a flexible compromise between the achievable rate and the BER [1]. To be more specific, in uplink (UL) multiuser-MIMO (MU-MIMO) scenarios, Spatial Division Multiple Access (SDMA) [2] significantly increases the overall system's throughput in a given bandwidth. In other words, SDMA substantially increases the achievable spectral efficiency by exploiting the unique, user-specific channel impulse responses (CIR) between the transmit and receive antenna pairs. Although traditionally only the BS is equipped with more than one transmit/receive antenna elements, laptop computers and sophisticated multimedia communicators can be equipped with multiple UL transmit antennas in the interest of improving their performance. Orthogonal space-time code (STC) designs have recently attracted considerable research interests, which was inspired by the two-transmit-antenna scenario proposed by Alamouti in [3]. This concept was then further generalised for an arbitrary number of transmit antennas by Tarokh *et al.* in [4], which facilitate linear detection at the receiver, while achieving full transmit diversity.

The concept of combining orthogonal transmit diversity designs with the principle of sphere packing (SP) [5] was introduced by Su *et al.* in [6] in order to maximise the achievable coding advantage, demonstrating that the proposed SP-aided STBC (STBC-SP) scheme, was capable of outperforming the conventional orthogonal design based STBC schemes of [3, 4, 7] in the single-user MIMO (SU-MIMO) scenario. In parallel, sphere detection (SD) techniques [8, 9] designed for conventionally-modulated MIMO systems, have received wide interest in both the research and industrial communities, which are capable of achieving a near-MAP performance at a significantly lower computational complexity compared to the classic MAP detectors.

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Against this background, we design a STBC-SP scheme for a MU-MIMO SDMA/OFDM scenario with the aid of the SD technique. *Our main contribution in this paper is the challenging deisgn of the K-best SD for SP-modulated systems, which extends the employment of STBC-SP schemes to MU-MIMO scenarios, while approaching the MAP performance at a moderate complexity: 1) We improve the STBC performance by jointly designing the space-time signals of the two time slots of an SDMA/OFDM scheme using SP modulation, while existing designs make no attempt to do so owing to its complex detection. 2) We solve this complexity problem by further developing the K-best SD for the detection of SP modulation, because SP offers a substantial SNR reduction at a potentially excessive complexity, which is reduced by the new SD.*

Notation: The bold uppercase variables, such as $\mathbf{X}$ and the lowercase variables $\mathbf{x}$ denote matrices and vectors, respectively. The element in the ith row and jth column of a matrix $\mathbf{X}$ is represented by $\mathbf{X}_{i,j}$, while the ith element of a vector $\mathbf{x}$ is denoted by $\mathbf{x}_i$. Furthermore, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $||\cdot||$ denote the transposition, the conjugate, the Hermitian transposition and the Frobenius norm operations, respectively. Finally, $\mathcal{R}\{\cdot\}$ and $\mathcal{I}\{\cdot\}$ denote the real and imaginary components of a complex number.

II. JOINT ORTHOGONAL SPACE-TIME SIGNAL DESIGN OF TWO ANTENNAS USING SPHERE PACKING

A. Orthogonal Design Using Sphere Packing Modulation

Conventionally, the orthogonal design of STBCs [3, 4, 7] is based on conventional PSK/QAM modulated symbols. Without loss of generality, we take the Alamouti $\mathbf{G}_2$ scheme as a simple example, where the corresponding Space-Time (ST) signaling matrix is constructed as [3]:

$$\mathbf{G}_2(x_1, x_2) = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}, \tag{1}$$

and the rows as well as columns respectively represent the spatial and temporal dimensions. The inputs (x_1, x_2) of the $\mathbf{G}_2$ encoder are chosen independently from the constellation corresponding to a specific modulation scheme, then mapped to two transmit antennas, and finally transmitted during two concecutive symbol periods. Therefore, no efforts are made to jointly design the input symbols (x_1, x_2). However, it was shown by Su *et al.* in [6] that combining the orthogonal design with *sphere packing* (SP) [5] is capable of attaining extra coding gains by maximizing the diversity product[1] of the STBC signals in the presence of temporally correlated fading channels. The diversity product expression for the square-shaped orthogonal STBC matrix $\mathbf{G}_{2^k}$ [7] in the context of time-correlated fading channels is given by [6]:

$$\zeta_{\mathbf{G}_{2^k}} = \frac{1}{2\sqrt{k+1}} \min_{(x_1,\cdots,x_{k+1})\neq(\tilde{x}_1,\cdots,\tilde{x}_{k+1})} (\sum_{i=1}^{k+1} |x_i - \tilde{x}_i|^2)^{1/2}, \tag{2}$$

where x_i and $\tilde{x}_i$ are the elements of two distinct space-time signaling matrices $\mathbf{C}$ and $\tilde{\mathbf{C}}$, respectively. From Eq.(2) we can observe that the diversity product is actually determined by the *Minimum*

[1] The diversity product or coding advantage was defined as the estimated SNR gain over an uncoded system having the same diversity order as the coded system [4]

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Euclidean Distance (MED) among all the possible ST signal vectors $(x_1, x_2, \cdots, x_{k+1})$. Thus, the idea of combining the individual antenna signals into a joint ST design using SP is both straighforward and desirable, since the SP modulated symbols have the best known MED in the $2(k+1)$-dimensional real-valued Euclidean space $\mathbb{R}^{2(k+1)}$ [5]. Hence, the system becomes capable of maximizing the achievable diversity product of STBC codes, which in turn minimizes the transmission error probability.

Again, we consider the $\mathbf{G}_2$ scheme as an example. Let us define the lattice D_4 as a SP having the best MED from all other $(L-1)$ legitimate constellation points in $\mathbb{R}^4$ [5], which may be also defined as a lattice that consists of all legitimate SP constellation points having integer coordinates $[a_1, a_2, a_3, a_4]$. These coordinates uniquely and unambiguously describe the legitimate combinations of the two time-slots' modulated symbols in the $\mathbf{G}_2$ scheme, while obeying the SP constraint of $a_1+a_2+a_3+a_4 = p$, where p is an even integer [10]. Furthermore, each input vector of the $\mathbf{G}_2$ scheme, i.e. (x_1, x_2), in the two-dimensional complex-valued space $\mathbb{C}^2$, can be represented by four real numbers, which as a whole corresponds to the coordinates of a four-dimensional real-valued phasor in the $\mathbb{R}^4$ space represented in the following way:

$$(x_1, x_2) \Longleftrightarrow (\mathcal{R}\{x_1\}, \mathcal{I}\{x_1\}, \mathcal{R}\{x_2\}, \mathcal{I}\{x_2\}). \quad (3)$$

Hence, the joint design of (x_1, x_2) in $\mathbb{C}^2$ is readily transformed into $\mathbb{R}^4$. Consequently, with the aid of the above-mentioned SP scheme, the joint ST signal design of the individual transmit antennas can be achieved by maximizing the coding advantage of $\mathbf{G}_2$ by maximizing the Euclidean distance of the four-tuples [10]:

$$\begin{aligned} (x_{l,1}, x_{l,2}) &= F_4(a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}), \\ &= (a_{l,1} + ja_{l,2}, a_{l,3} + ja_{l,4}). \end{aligned} \quad (4)$$

Upon choosing L legitimate constellation points from the lattice D_4 to construct a set denoted by $\mathcal{A} = \{\vec{a}_l = [a_{l,1}, a_{l,2}, a_{l,3}, a_{l,4}]^T \in \mathbb{R}^4 : 0 \leq l \leq L-1\}$, the L resultant energy-normalised codewords given by:

$$\mathbf{C}_l = \sqrt{\frac{2L}{E_{total}}}\mathbf{G}_2(F_4(\vec{a}_l)), \quad l = 0, 1, \cdots, L-1, \quad (5)$$

where $E_{total} \triangleq \sum_{l=1}^{L}(|a_{l,1}|^2 + |a_{l,2}|^2 + |a_{l,3}|^2 + |a_{l,4}|^2)$, constitutes the ST signal space $\mathcal{C}_{\mathbf{G}_2}$, whose diversity product $\zeta_{\mathbf{G}_2}$ is determined by the MED of the set $\mathcal{A}$ formulated in Eq.(2).

III. System Overview

Let us now construct the generalised system equations for an STBC-aided UL MU-MIMO scenario, where the SDMA/OFDM system supports a total of U UL users and employs N number of receive antennas at the BS. For the sake of simplicity, the $\mathbf{G}_2$ scheme using two transmit antennas is employed by each user. The overall equivalent MU-MIMO system equation can be derived in complete analogy to the case of STBC-assisted SU-MIMO systems (e.g. [11, 12]) with the aid of the so-called equivalent channel matrix. After straightforward manipulations, under the assumption that the CIR taps between each of the two transmit antennas of a specific user and the nth receive antenna at the BS remain constant during two consecutive symbol periods, we have:

$$\begin{aligned} \tilde{\mathbf{y}} &= \begin{bmatrix} \tilde{\mathbf{y}}_1 \\ \tilde{\mathbf{y}}_2 \\ \vdots \\ \tilde{\mathbf{y}}_N \end{bmatrix}_{2N\times1} = \begin{bmatrix} \sum_{u=1}^{U} \tilde{\mathbf{H}}_1^{(u)} \cdot \mathbf{x}^{(u)} \\ \sum_{u=1}^{U} \tilde{\mathbf{H}}_2^{(u)} \cdot \mathbf{x}^{(u)} \\ \vdots \\ \sum_{u=1}^{U} \tilde{\mathbf{H}}_N^{(u)} \cdot \mathbf{x}^{(u)} \end{bmatrix}_{2N\times1} + \begin{bmatrix} \tilde{\mathbf{w}}_1 \\ \tilde{\mathbf{w}}_2 \\ \vdots \\ \tilde{\mathbf{w}}_N \end{bmatrix}_{2N\times1} \quad (6) \\ &= \tilde{\mathbf{H}} \cdot \mathbf{x} + \tilde{\mathbf{w}}, \quad (7) \end{aligned}$$

where the overall equivalent channel matrix $\tilde{\mathbf{H}}$ can be expressed as:

$$\tilde{\mathbf{H}} = \begin{bmatrix} \tilde{\mathbf{H}}_1^{(1)} & \tilde{\mathbf{H}}_1^{(2)} & \cdots & \tilde{\mathbf{H}}_1^{(U)} \\ \tilde{\mathbf{H}}_2^{(1)} & \tilde{\mathbf{H}}_2^{(2)} & \cdots & \tilde{\mathbf{H}}_2^{(U)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{H}}_N^{(1)} & \tilde{\mathbf{H}}_N^{(2)} & \cdots & \tilde{\mathbf{H}}_N^{(U)} \end{bmatrix}, \quad (8)$$

with each submatrix $\tilde{\mathbf{H}}_n^{(u)}$ being defined as

$$\tilde{\mathbf{H}}_n^{(u)} = \begin{bmatrix} h_{n1}^{(u)} & h_{n2}^{(u)} \\ h_{n2}^{(u)*} & -h_{n1}^{(u)*} \end{bmatrix}, \quad (9)$$

where $h_{n1}^{(u)}$ and $h_{n2}^{(u)}$ are the UL CIR coefficients between the first and second tranmsit antennas of the uth user to the nth BS receive antenna, repectively. Additionally, the transmitted symbol vector $\mathbf{x}$ of the entire MU-MIMO system is a column vector created by concatenating each user's transmitted symbol vector $\mathbf{x}^{(u)} = F_4(\vec{a}^{(u)}) = [x_1^{(u)} \ x_2^{(u)}]^T$ which is given by:

$$\begin{aligned} \mathbf{x} &= [F_4((\vec{a}^{(1)})^T) \quad F_4((\vec{a}^{(2)})^T) \quad \cdots \quad F_4((\vec{a}^{(U)})^T)]^T \quad (10) \\ &= [F_4((\vec{a}^{(1)})^T \quad (\vec{a}^{(2)})^T \quad \cdots \quad (\vec{a}^{(U)})^T)]^T \quad (11) \\ &= [(\mathbf{x}^{(1)})^T \quad (\mathbf{x}^{(2)})^T \quad \cdots \quad (\mathbf{x}^{(U)})^T]^T. \quad (12) \end{aligned}$$

Thus, by defining $\mathbf{a} = [(\vec{a}^{(1)})^T \ (\vec{a}^{(2)})^T \ \cdots \ (\vec{a}^{(u)})^T]^T$, we have:

$$\mathbf{x} = F_4(\mathbf{a}). \quad (13)$$

Moreover, as observed in Eq.(6), the *equivalent received noise-contaminated signal vector* $\tilde{\mathbf{y}}$ and the *equivalent noise vector* $\tilde{\mathbf{w}}$ is formed by concatenating the N number of two-elements sub-vector $\tilde{\mathbf{y}}_n$ and $\tilde{\mathbf{w}}_n$ respectively, which can be written as:

$$\tilde{\mathbf{y}}_n = [y_{1,n} \quad y_{2,n}^*]^T, \quad (14)$$

where the first element $y_{1,n}$ corresponds to the signal received by the nth antenna during the first symbol period and the second element $y_{2,n}^*$ is the conjugate of the signal received at the same antenna during the second symbol period, while

$$\tilde{\mathbf{w}}_n = [w_{1,n} \quad w_{2,n}^*]^T, \quad (15)$$

where again $w_{1,n}$ denotes the AWGN imposed on the nth receive antenna during the first symbol period and $w_{2,n}^*$ is the conjugate of the AWGN inflicted during the second symbol period. The AWGN encountered during each symbol period has a zero mean and a variance of σ_w^2.

According to [11], an orthogonal STBC scheme eliminates the *Multi-Stream-Interference* (MSI) among the MIMO elements of a specific user, due to the orthogonality of the equivalent channel matrix $\tilde{\mathbf{H}}_n^{(u)}$ in Eq.(9). Therefore, the receiver is capable of performing Maximum-Likelihood (ML) detection based on low-complexity linear processing to achieve full transmit diversity by imposing a negligible extra encoding complexity at the MS in the STBC-SP-assisted SU-MIMO UL scenario [10]. However, in the context of a MU-MIMO system, the resultant overall equivalent channel matrix $\tilde{\mathbf{H}}$ of Eq.(8) is no longer orthogonal, therefore imposing the *Multi-User-Interference* (MUI). Hence, in order to avoid using the traditional brute-force ML detector, we intend to further develop the K-best SD to be used at the BS in the STBC-SP-assisted SDMA/OFDM UL scenario, in order to achieve a near-MAP performance at a moderate complexity.

The schematic of the system is depicted in Fig.1, where the transmitted source bits of the uth user are channel encoded and then interleaved by a random bit-interleaver. The B interleaved bits $\mathbf{b}^{(u)} = b_{0,\cdots,B-1}^{(u)} \in \{0,1\}$ are mapped to an SP modulated symbol $\mathbf{a}^{(u)} \in \mathcal{A}$ by the SP modulator/mapper of Fig. 1, where $B = \log_2 L$.

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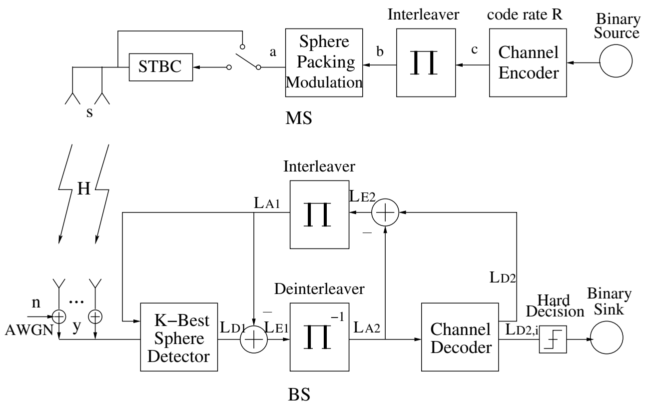

Fig. 1. Schematic of the uplink SP modulated multi-user MIMO system using K-best SD.

The $\mathbf{G}_2$ encoder then maps the SP modulated symbol $\mathbf{a}^{(u)}$ to a space-time signal $\mathbf{C}^{(u)} = \sqrt{\frac{2L}{E_{total}}}\mathbf{G}_2(F_4(\mathbf{a}^{(u)})) \in \mathcal{C}_{\mathbf{G}_2}$ by invoking Eq.(1) and Eq.(4). Finally, the space-time signal $\mathbf{C}^{(u)}$ is transmitted from the two tranmsit antennas of the uth MS during consecutive two time slots.

In Fig.1 the interleaver and deinterleaver pair seen at the BS divides the receiver into two parts, namely the SD (*inner decoder*) and the channel decoder (*outer decoder*). Note that in Fig.1 L_A, L_E and L_D denote the *a priori*, the *extrinsic* and the *a posteriori* Log-Likelihood-Ratios (LLRs), while the subscript '1' and '2' represent the bit LLRs associated with the inner decoder and the outer decoder, respectively. It was detailed throughout [11] and [13] that the iterative exchange of extrinsic information between these serially concatenated receiver blocks results in substantial performance improvements. In this treatise we assume familiarity with the classic turbo detection principles [11, 13].

IV. Sphere Detection Design for Sphere Packing Modulated Systems

A. Bit-Based MAP Detection for SP Modulated MU-MIMO Systems

According to Eq.(7) and Eq.(13), the conditional PDF $p(\tilde{\mathbf{y}}|\mathbf{a})$ for systems using $N_D = 4$-dimensional real-valued SP modulation is given by:

$$p(\tilde{\mathbf{y}}|\mathbf{a}) = \frac{1}{(2\pi\sigma_w^2)^{\frac{N_D}{2}}} e^{-\frac{1}{2\sigma_w^2}||\tilde{\mathbf{y}}-\tilde{\mathbf{H}}\cdot F_4(\mathbf{a})||^2}. \tag{16}$$

Then, using Bayes' theorem, and exploiting the independence of the bits in the vector $\mathbf{b} = [b_1, b_2, \cdots b_{B\cdot U}]$ carried by the received symbol vector $\tilde{\mathbf{y}}$ we can factorize the joint bit-probabilities into their products [14], hence the LLR of bit k for $k = 1, \cdots, B \cdot U$ can be written as:

$$L_D(\mathbf{b}_k|\tilde{\mathbf{y}}) = L_A(\mathbf{b}_k) + \underbrace{\ln \frac{\sum_{\mathbf{a}\in\mathcal{A}^U_{k=1}} p(\tilde{\mathbf{y}}|\mathbf{a}) \cdot e^{\sum_{j=1,j\neq k}^{B\cdot U} \mathbf{b}_j L_A(\mathbf{b}_j)}}{\sum_{\mathbf{a}\in\mathcal{A}^U_{k=0}} p(\tilde{\mathbf{y}}|\mathbf{a}) \cdot e^{\sum_{j=1,j\neq k}^{B\cdot U} \mathbf{b}_j L_A(\mathbf{b}_j)}}}_{L_E(\mathbf{b}_k|\tilde{\mathbf{y}})}, \tag{17}$$

where $\mathcal{A}^U_{k=1}$ and $\mathcal{A}^U_{k=0}$ are subsets of the multi-user SP symbol constellation $\mathcal{A}^U$ where we have $\mathcal{A}^U_{k=1} \triangleq \{\mathbf{a} \in \mathcal{A}^U : b_k = 1\}$, and in a similar fashion, $\mathcal{A}^U_{k=1} \triangleq \{\mathbf{a} \in \mathcal{A}^U : b_k = 0\}$. Using Eq.(16), we arrive at:

$$\begin{aligned} L_D(\mathbf{b}_k|\tilde{\mathbf{y}}) &= L_A(\mathbf{b}_k) \\ &+ \underbrace{\ln \frac{\sum_{\mathbf{a}\in\mathcal{A}^U_{k=1}} e^{[-\frac{1}{2\sigma_w^2}||\tilde{\mathbf{y}}-\tilde{\mathbf{H}}\cdot F_4(\mathbf{a})||^2 + \sum_{j=1,j\neq k}^{B\cdot U} \mathbf{b}_j L_A(\mathbf{b}_j)]}}{\sum_{\mathbf{a}\in\mathcal{A}^U_{k=0}} e^{[-\frac{1}{2\sigma_w^2}||\tilde{\mathbf{y}}-\tilde{\mathbf{H}}\cdot F_4(\mathbf{a})||^2 + \sum_{j=1,j\neq k}^{B\cdot U} \mathbf{b}_j L_A(\mathbf{b}_j)]}}}_{L_E(\mathbf{b}_k|\tilde{\mathbf{y}})}. \end{aligned} \tag{18}$$

B. Sphere Detection Design for Sphere Packing Modulation

1) Transformation of the ML Metric: The well-known ML solution of Eq.(7) is:

$$\hat{\mathbf{a}}_{ML} = arg \min_{\check{\mathbf{a}}\in\mathcal{A}^U} ||\tilde{\mathbf{y}} - \tilde{\mathbf{H}} \cdot F_4(\check{\mathbf{a}})||_2^2, \tag{19}$$

where $F_4(\cdot)$ is defined in Eq.(13) in the context of our multi-user system. By comparing the unconstrained least-square (LS) solution of $\hat{\mathbf{a}}_{ls} = F_4^{-1}(\hat{\mathbf{x}}_{ls}) = F_4^{-1}((\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})\tilde{\mathbf{H}}^H\tilde{\mathbf{y}})$ to all legitimate constrained/sliced solution, namely $\check{\mathbf{a}} \in \mathcal{A}^U$, the ML solution of Eq.(19) can be transformed into:

$$\hat{\mathbf{a}}_{ML} = arg \min_{\check{\mathbf{a}}\in\mathcal{A}^U} F_4(\check{\mathbf{a}} - \hat{\mathbf{a}}_{ls})^H(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})F_4(\check{\mathbf{a}} - \hat{\mathbf{a}}_{ls}). \tag{20}$$

2) Channel Matrix Triangularization: Let us now generate the $(2U \times 2U)$-dimensional upper-triangular matrix $\mathbf{U}$, which satisfies $\mathbf{U}^H\mathbf{U} = \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ with the aid of, for example, the ubiquitous Cholesky factorization [8]. Then, upon defining a matrix $\mathbf{Q}$ with elemetns $\mathbf{Q}_{i,i} \triangleq \mathbf{U}_{i,i}^2$ and $\mathbf{Q}_{i,j} \triangleq \mathbf{U}_{i,j}/\mathbf{U}_{i,i}$ we can rewrite Eq.(20) as:

$$\begin{aligned} &\hat{\mathbf{a}}_{ML} \\ = \;& arg \min_{\check{\mathbf{a}}\in\mathcal{A}^U} F_4(\check{\mathbf{a}} - \hat{\mathbf{a}}_{ls})^H\mathbf{U}^H\mathbf{U}F_4(\check{\mathbf{a}} - \hat{\mathbf{a}}_{ls}), \\ = \;& arg \min_{\check{\mathbf{a}}\in\mathcal{A}^U} \{\sum_{u=1}^{U} \mathbf{Q}_{2u-1,2u-1}[\mathbf{e}_1^{(u)} + \sum_{v=u+1}^{U} \mathbf{Q}_{2u-1,2v-1}\mathbf{e}_1^{(v)} \\ + \;& \sum_{v=u}^{U} \mathbf{Q}_{2u-1,2v}\mathbf{e}_2^{(v)}]^2 + \sum_{u=1}^{U} \mathbf{Q}_{2u,2u}[\mathbf{e}_2^{(u)} \\ + \;& \sum_{v=u+1}^{U} \mathbf{Q}_{2u,2v-1}\mathbf{e}_1^{(v)} + \sum_{v=u+1}^{U} \mathbf{Q}_{2u,2v}\mathbf{e}_2^{(v)}]^2\}, \end{aligned} \tag{21}$$

where $\mathbf{e}^{(u)}$ is the uth two-element sub-vector of the multi-user vector $\mathbf{e} = [(\mathbf{e}^{(1)})^T \cdots (\mathbf{e}^{(u)})^T \cdots (\mathbf{e}^{(U)})^T]^T$, corresponding to the uth user, and is given by:

$$\mathbf{e}^{(u)} = \check{\mathbf{x}}^{(u)} - \hat{\mathbf{x}}_{ls}^{(u)}, \tag{22}$$

where $\check{\mathbf{x}}^{(u)} = [\check{\mathbf{x}}_1^{(u)}, \check{\mathbf{x}}_2^{(u)}]^T = F_4(\vec{a}^{(u)})$, $\vec{a}^{(u)} \in \mathcal{A}$ and $\hat{\mathbf{x}}_{ls}^{(u)} = [\hat{\mathbf{x}}_{ls}^{(u)}{}_1, \hat{\mathbf{x}}_{ls}^{(u)}{}_2]^T = F_4(\hat{\mathbf{a}}_{ls}^{(u)})$. Hence, the sum in $\{\cdot\}$ is the *user-based* accumulated *Partial Euclidean Distance* (PED) between the tentative symbol vector $\check{\mathbf{x}} = [(\check{\mathbf{x}}^{(1)})^T, (\check{\mathbf{x}}^{(2)})^T, \cdots, (\check{\mathbf{x}}^{(U)})^T]^T$ and the search center $\hat{\mathbf{x}}_{ls} = [(\hat{\mathbf{x}}_{ls}^{(1)})^T, (\hat{\mathbf{x}}_{ls}^{(2)})^T, \cdots, (\hat{\mathbf{x}}_{ls}^{(U)})^T]^T$.

3) User-Based Tree Search: Let us now consider the tree search carried out by the K-best SD for conventional modulation schemes, such as BPSK, where each tree level represents an independent data stream corresponding to a certain transmit antenna of a specific user [9], whilst each tree node corresponds to a legitimate complex-valued BPSK symbol[2] in the constellation of domain $\mathbb{C}^1$. Consequently, in the absence of joint ST signal design for the $M_u = 2$ transmit antennas, the BPSK constellations of the two adjacent tree levels corresponding to a specific user are independent and identical. However, when the joint ST signals are transmitted from the $M_u = 2$ transmit antennas of the uth user, they are combined into a joint ST design with the aid of the SP scheme as discussed in Section II-A. The corresponding SP-symbol based search tree structure is depicted in Fig. 2, where the search tree of the modified K-best SD is exemplified in the context of an UL SDMA system supporting $U = 2$ $\mathbf{G}_2$-SP-aided users, where $K = 2$ and a 4-point-SP constellation is employed. Explicitly, the two adjacent tree levels corresponding to the SP-symbols of the jointly designed STBC-SP data streams of a specific user should be considered together

[2] In this treatise, we consider complex-valued BPSK symbols having zero imaginary parts.

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in the tree search process, resulting in multi-dimensional/multi-layer tree nodes in the $\mathbb{C}^2$ SP-symbol domain, which we refer to as *user-based* tree search. The resultant 2-D complex-valued tree node is constituted of two complex-valued BPSK symbols, which are the constituent components of a transformed SP symbol $F_4(\vec{a})$. On the other hand, due to the joint consideration of the two adjacent BPSK tree levels, the number of effective search tree levels is reduced by a factor of two, whilst each symbols becomes quaternary, instead of being binary.

As observed in Fig. 2, since a 4-SP scheme is employed and the number of candidate tree nodes retained at each tree level is $K = 2$, each of the two selected tree nodes having the smallest two accumulated PED values at the previous search-tree level of the survivor path has to be expanded into four child nodes at the current level. Consequently, in analogy to the conventional K-best algorithm [9], both the calculation of the user-based accumulated PEDs as well as the tree pruning process continues in the downward direction of Fig. 2 all the way along the tree, until it reaches the tree-leaf level, producing a candidate list of $\mathcal{L}_{cand} \in \mathcal{A}^U$. This list contains $N_{cand} = K$ number of SP symbol candidate solutions, which are then used for the *extrinsic* bit LLR calculation using Eq.(18). Having a reduced candidate list size assists us in achieving a substantial complexity reduction. Explicitly, after the max-log-approximation, we arrive at:

$$L_E(\mathbf{b}_k|\tilde{\mathbf{y}}) = \max_{\mathbf{a}\in\mathcal{L}_{cand}\cap\mathcal{A}^U_{k=1}} [-\frac{1}{2\sigma_w^2}||\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\cdot F_4(\mathbf{a})||^2 + \sum_{j=1,j\neq k}^{B\cdot U} \mathbf{b}_j L_A(\mathbf{b}_j)] - \max_{\mathbf{a}\in\mathcal{L}_{cand}\cap\mathcal{A}^U_{k=0}} [-\frac{1}{2\sigma_w^2}||\tilde{\mathbf{y}} - \tilde{\mathbf{H}}\cdot F_4(\mathbf{a})||^2 + \sum_{j=1,j\neq k}^{B\cdot U} \mathbf{b}_j L_A(\mathbf{b}_j)]. \quad (23)$$

The K-best SD algorithm designed for $N_D = 4$-dimensional SP modulation scheme is summarized in the Appendix.

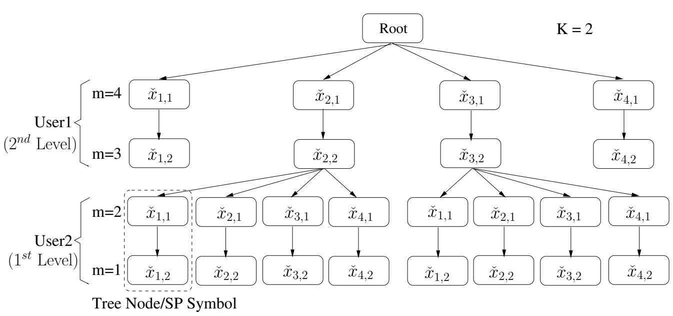


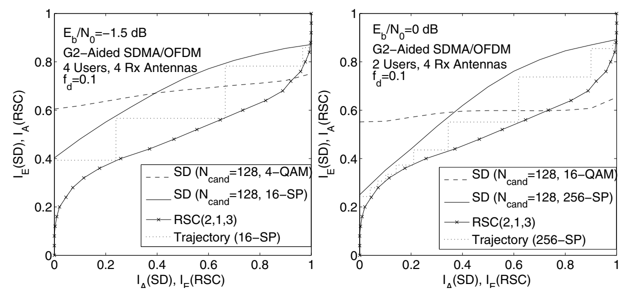
Fig. 2. The Search Tree of K-Best SD in the Scenario of an STBC-SP Aided Uplink SDMA System: the number of users is $U = 2$, the number of transmit antennas per user is $M_u = 2$ and the number of candidates retained at each search tree level is $K = 2$.

V. Results and Discussion

For the sake of investigating the performance of the STBC-SP-assisted multi-user SDMA/OFDM UL system, we compare the SP-modulated system with its conventionally-modulated counterpart in the two scenarios using the system parameters summarised in Table I. Fig. 3(a) and 3(b) depict, respectively, the corresponding EXIT charts [15] used as a convenient visualization technique for analyzing the convergence characteristics of turbo receivers. This technique computes the mutual information (MI) of the output *extrinsic* and input *a priori* components, which are denoted by I_E and I_A respectively, corresponding to the associated bits for each of the iterative SISO blocks of Fig. 1, namely, to the SD and the RSC(2,1,3)

	Scenario I	*Scenario II*
Modulation	4-QAM/16-SP	16-QAM/256-SP
Users Supported	4	2
System		SDMA/OFDM
Sub-Carriers		1024
STBC		$\mathbf{G}_2$
Rx at BS		4
CIR Model		$P(\tau_k) = [0.5\ 0.3\ 0.2]$
Detector/MAP		K-Best List-SD
List Length		$\mathcal{N}_{cand} = K$
Channel Code		Half-Rate RSC(2,1,3) (5/7)
BW Efficiency		4 bits/sec/Hz

TABLE I
Summary of System Parameters



(a) Scenario I (b) Scenario II

Fig. 3. EXIT charts of STBC-SP-aided iterative receiver of Fig. 1 employing the modified K-best SD and the parameters of Table I. The overall system throughput is 8 bits/symbol.

channel decoder. As observed in Fig. 3, the maximum achievable iterative gains of traditional QAM-modulated systems employing the conventional K-best SD using $N_{cand} = K = 128$ are rather limited in comparison to our SP-aided K-best SD specifically designed for SP signals having the same list size of $N_{cand} = 128$. This is because the EXIT curve of the SD used by the conventional 4- and 16-QAM-based system has a relatively low I_E value at $I_A = 1$, in contrast to the corresponding EXIT curve of its SP-modulated counterpart. Nonetheless, we also observe from Fig. 3 that the SD's EXIT curve in the QAM-modulated system emerges from a higher starting point at $I_A = 0$ than that of its SP-modulated counterpart. This leads to a potentially lower BER at relatively low SNRs, where I_A is also low, although the exact detection-convergence behavior is determined by the SD's complexity as well as by the SNR. Observe in Fig. 3 that in principle the employment of SP modulation is capable of eliminating the EXIT curve intercept point at a lower SNR, hence leading to an infinitesimally low BER. However, an open EXIT tunnel can only be formed, if the value of $K = N_{cand}$ as well as that of the SNR is sufficiently high.

Monte Carlo simulations were performed for characterizing the above-mentioned decoding convergence prediction in both *Scenario I* and *Scenario II* of Table I. Figs. 4(a) and 4(b) suggest that the SP-modulated system exhibits a relatively higher BER at low SNRs in both scenarios, which matches the predictions of the EXIT charts seen in Fig. 3. On the other hand, as a benefit of employing the SP modulation, performance gains of 1.5 dB and 3.5 dB can be achieved by 16-SP and 256-SP modulated systems in *Scenario I* and *Scenario II* of Table I, respectively, in comparison to their identical-throughput QAM-based counterparts, given a target BER of 10^{-4} and $N_{cand} = 128$. Furthermore, as observed from Figs. 4(a) and 4(b), an attractive compromise can be achieved between the achievable performance and the complexity imposed by adjusting the list size N_{cand} employed by the K-best SD.

VI. Conclusion

In this paper, a novel K-best SD designed for SP-modulated systems was proposed, which extends the employment of the STBC-

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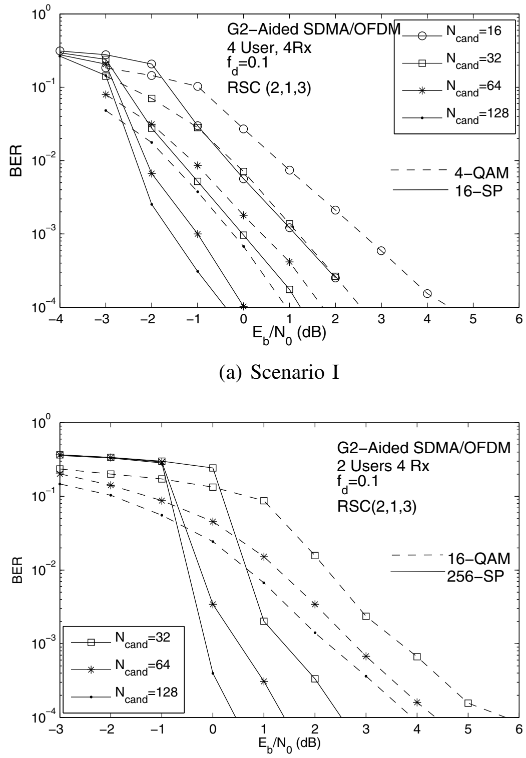

(a) Scenario I

(b) Scenario II

Fig. 4. BER performance of the system of Fig. 1 in *Scenario I* and *Scenario II* of Table I. The overall system throughput is 8 bits/symbol.

SP scheme to the MU-MIMO scenario, while achieving a near-MAP performance at a moderate complexity. Consequently, with the aid of our K-best SD, a significant performance gain can be attained by the SP-modulated system over its conventionally-modulated identical-throughput counterpart in MU-MIMO scenarios. For example, a performance gain of 3.5 dB can be achieved over a 16-QAM benchmarker by the 256-SP scheme in the scenario of a four-receive-antenna SDMA UL system supporting $U = 2$ $\mathbf{G}_2$-assisted users, given a target BER of 10^{-4}.

APPENDIX
K-BEST SD ALGORITHM FOR $N_D = 4$-DIMENSIONAL SP

The preprocessing phase:

1) Obtain the upper-triangular matrix $\mathbf{U}$ via Cholesky factorization on the Grammiam matrix $\mathbf{G} = \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, namely, we have $\mathbf{U} = Chol(\mathbf{G})$.

2) Calculate the search center $\hat{\mathbf{x}}_{ls}$ by:

$$\hat{\mathbf{x}}_{ls} = \mathbf{G}^{-1}\tilde{\mathbf{H}}^H\mathbf{y}. \tag{24}$$

The tree search phase:

The first stage:

1) $m = M$, $\mathbf{d}_M = \hat{\mathbf{x}}_{lsM}$, where M is the total number of transmit antennas supported by the system.

2) Calculate the corresponding PED for each SP symbol $(\check{x}_{l,1},\ \check{x}_{l,2})$, $l = 1, 2, \cdots, L$ in the constellation of $\mathbb{C}^2$ domain as follows:

$$\mathbf{e}_M = \hat{\mathbf{x}}_{lsM} - \check{x}_{l,1}, \tag{25}$$

$$\mathbf{d}_{M-1} = \hat{\mathbf{x}}_{lsM-1} + \frac{\mathbf{U}_{M-1,M}}{\mathbf{U}_{M-1,M-1}}\mathbf{e}_M, \tag{26}$$

$$PED = \mathbf{U}^2_{M-1,M-1}(\mathbf{d}_{M-1} - \check{x}_{l,2}). \tag{27}$$

3) Choose K number of SP symbols $(\check{x}_{k,1},\ \check{x}_{k,2})$, $k = 1, 2, \cdots, K$ that have the K smallest PEDs.

4) For each chosen SP symbol, compute

$$\mathbf{e}_{M-1} = \hat{\mathbf{x}}_{lsM-1} - \check{x}_{k,2}, \tag{28}$$

$$\mathbf{d}_{M-2} = \hat{\mathbf{x}}_{lsM-1} + \frac{\mathbf{U}_{M-2,M-1}}{\mathbf{U}_{M-2,M-2}}\mathbf{e}_{M-1} + \frac{\mathbf{U}_{M-1,M}}{\mathbf{U}_{M-1,M-1}}\mathbf{e}_M \tag{29}$$

The mth stage:

1) $m = m - 2$.

2) For each survived search tree path from the previous tree level, calculate the corresponding PED for each SP symbol $(\check{x}_{l,1},\ \check{x}_{l,2})$ in the constellation of $\mathbb{C}^2$ domain:

$$\mathbf{e}_m = \hat{\mathbf{x}}_{ls\ m} - \check{x}_{l,1}, \tag{30}$$

$$\mathbf{d}_{m-1} = \hat{\mathbf{x}}_{ls\ m-1} + \sum_{j=m}^{M}\frac{\mathbf{U}_{m-1,j}}{\mathbf{U}_{m-1,m-1}}\mathbf{e}_j, \tag{31}$$

$$PED = \mathbf{U}^2_{m-1,m-1}(\mathbf{d}_{m-1} - \check{x}_{l,2}). \tag{32}$$

3) Choose K number of SP symbols $(\check{x}_{k,1},\ \check{x}_{k,2})$, $k = 1, 2, \cdots, K$ that have the K smallest PEDs.

4) For each chosen SP symbol, compute

$$\mathbf{e}_{m-1} = \hat{\mathbf{x}}_{ls\ m-1} - \check{x}_{k,2}, \tag{33}$$

$$\mathbf{d}_{m-2} = \hat{\mathbf{x}}_{ls\ m-1} + \frac{\mathbf{U}_{m-2,m-1}}{\mathbf{U}_{m-2,m-2}}\mathbf{e}_{m-1} + \frac{\mathbf{U}_{m-1,m}}{\mathbf{U}_{m-1,m-1}}\mathbf{e}_m \tag{34}$$

5) If $m - 1 = 1$, obtain the solution by backtracing from the tree leaf having the largest accumulated PED to the tree root. Otherwise, go to step 1 of the mth stage.

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