

# Optimal Scheduling in Interference Limited Fading Wireless Networks

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**Abstract**—We consider the problem of minimum-length scheduling of point-to-point links in a spatial TDMA (STDMA)-based wireless network with Rayleigh fading of both desired and interference signals. The problem formulation integrates the activation of multiple sets of links in the network, while taking into account their explicit statistical variations. We assume uniform (fixed) transmission power at all nodes and propose an algorithm based on a column generation approach, which takes into consideration the signal-to-interference and noise ratio (SINR) constraints at the receivers in order to generate a link schedule that minimizes the schedule length. For the formulated problem, we show that this column generation based approach can converge to a globally optimal solution.

## I. INTRODUCTION

Scheduling in wireless networks avoids collisions and re-transmissions due to collisions, that are typical in contention-based access control methods. While time division multiple access (TDMA)-based scheduling schemes can guarantee delay bounds and are easier to implement, their efficiency can be further improved both in terms of delay guarantees as well as achieving higher capacities by allowing the TDMA time-slots to be shared by simultaneous transmissions that are geographically separated. This improvement is appropriately termed as Spatial-TDMA or STDMA [19], in which time is divided into frames that are organized cyclically. An STDMA schedule describes the activation details for each time slot in such a way that links assigned to the same slot can communicate simultaneously, i.e., they satisfy their respective signal-to-interference and noise ratio (SINR) constraints.

Traditional scheduling algorithms, whether centralized [3], [7], [8], [14] or distributed [5], [6], [22], assume that the wireless channel fading statistics are quasi-stationary and therefore, base their scheduling on the knowledge of the (fixed) channel gains of all the links. In other words, they implicitly assume that the scheduling updates are made every time the fading state of the channels (i.e., the channel gains) change, and the fading state changes happen at a much larger timescale in comparison with data transfer. This might not be practical in certain scenarios where the communication channels exhibit fast fading, meaning, the fades can change within milliseconds, which could result in very frequent schedule changes that can consume a significant amount of energy just to maintain the right schedule. In this paper, we propose a scheduling

algorithm that need not be updated whenever the channel changes from one fading state to the other. Instead, we explicitly take into account the statistical variation of the SINR of each link and optimally allocate the resources in order to minimize the schedule length.

Scheduling in wireless ad hoc networks has been considered in [12] by Hajek and Sasaki and in [6], [7] by Ephremides and Truong. Several innovative algorithms have been proposed in the literature for generating minimum-length wireless link schedules. Existing work on centralized STDMA link scheduling algorithms can be broadly classified into two categories. The *Protocol Interference Model* [11] based scheduling algorithms describes interference constraints according to a conflict graph, where nodes within a certain distance can communicate as long as the receiver is separated by at least a distance  $d$  from any other active transmitter. On the other hand, the *Physical Interference Model* [3], [4] based scheduling algorithms directly considers the signal-to-interference-plus-noise ratio (SINR) constraints at the receivers by accounting for all the secondary transmissions as interference. The fundamental difference between these two schemes is that while the graph based scheduling algorithms [13], [21] overcompensate for interference in the vicinity of transmitting and receiving nodes, and therefore, result in a lower throughput, the physical interference model is considered to be more accurate and also more complicated. Therefore scheduling algorithms based on the physical model can result in higher throughput.

Other STDMA-based scheduling algorithms include the following: Bjorklund *et al.* [2] showed that even the most basic planning problems in wireless networks, such as node and link assignment, are NP-hard. They formulated the so-called node and link assignment optimization problems, which assign at least one time slot to each node or link such that the number of time slots is minimized using set-covering formulations, and developed a column generation approach for solving the resulting linear programming relaxations. These assignment problems were also discussed in detail by Gronkvist [10]. However, in all these works, the wireless channels were assumed to be fixed i.e., fading was not considered, and except in [14] specific traffic demands on links were also not taken into account.

On the other hand, resource allocation for fading channels

is a popular topic of study in information theory. Delay constrained scheduling over a fading channel has been studied, in particular, towards minimizing average delay [24], as well as dealing with hard delay deadlines [9], [18]. However, the analysis is most often performed over an individual wireless channel, rather than in a network setting, and the optimizing quantity is most often the Shannon capacity.

#### A. Our Contribution

Our earlier work in the area of wireless link scheduling [15], [16] focused on computing the minimum-length schedule that is required to satisfy a set of specified traffic demands in a wireless network, such that a given SINR is exceeded at the receivers of all simultaneously active links. In this paper, we extend our previous work by explicitly considering the statistical variations in the communication channel. We specifically assume that the communication channels between the nodes are Rayleigh fading wireless channels. The solution approach of generating the minimum-length schedule is based on column generation, while taking into account the “link success” probability at each link. This link success probability can be interpreted as the fraction of time a link  $\{i, j\}$  between source node  $i$  and destination node  $j$ , experiences a channel where the SINR exceeds the threshold, while the remaining fraction experiences outage due to fading.

The paper is organized as follows. In Section II, we discuss the network and communication model that is used in the formulation of the minimum-length scheduling problem. We then propose a column generation based solution procedure in Section III. We discuss the computation of a lower bound for the minimum-length scheduling problem in Section IV along with other relevant computational considerations, and in Section V, we provide final concluding remarks.

## II. PROBLEM FORMULATION

### A. Network and Communication Model

We model the wireless network as a set of nodes  $\mathcal{N}$ , where each node is assumed to have a fixed position. Each node can act as a transmitter and receiver, and also relay data to other nodes across the network. A set of (directed) links  $\mathcal{E}$  constitutes the network topology, and link  $\{i, j\} \in \mathcal{E}$  indicates that node  $i$  can communicate directly with node  $j$ , i.e., if the corresponding signal-to-noise ratio (SNR) in the absence of any other interference source is beyond a certain threshold.

Let  $P_i$  be the transmission power for node  $i$ ,  $G_{ij}$  the gain of the radio channel between nodes  $i$  and  $j$ , and  $\eta_j$  be the thermal noise at receiver  $j$ . The channel gain is calculated by a widely used model  $G_{ij} = d_{ij}^{-\alpha}$ , where  $d_{ij}$  is the distance between nodes  $i$  and  $j$ , and  $\alpha$  is the path loss index. Let  $F_{ij}$  model the Rayleigh fading between nodes  $i$  and  $j$ . Then the received power at receiver  $j$  due to transmitter  $i$  is given by  $G_{ij}F_{ij}P_i$ .

The Rayleigh fading parameters  $F_{ij}$  are assumed to be independent exponentially distributed random variables with unit mean. In a Rayleigh fading environment, the received

signal envelope has a Rayleigh distribution, and the received signal power has an exponential distribution. That is, the power received at  $j$  is an exponentially distributed random variable with mean value  $E[G_{ij}F_{ij}P_i] = G_{ij}P_i$ . Here, we assume that both the desired signals and the interference causing signals are subject to Rayleigh fading. Therefore we refer to this as Rayleigh/Rayleigh fading environment. By extension, other fading environments can also be envisioned, such as Rician/Rayleigh, Nakagami/Rayleigh, etc.

In the physical model without fading, the SINR at receiver  $j$  due to transmission from node  $i$  in the presence of other transmissions is given by:

$$\text{SINR}_{ij} = \frac{G_{ij}P_i}{\eta_j + \sum_{k \neq i,j} G_{kj}P_k}. \quad (1)$$

When fading is considered, the SINR is given by,

$$\text{SINR}_{ij} = \frac{G_{ij}F_{ij}P_i}{\eta_j + \sum_{k \neq i,j} G_{kj}F_{kj}P_k}. \quad (2)$$

The capacity of the wireless channel associated with a link  $(i, j)$  is a function of SINR on the channel. We assume that data is coded separately for each link and that the receivers consider unintended receptions as noise. In this case, each link  $(i, j)$  can be viewed as a single-user Gaussian channel, with the Shannon capacity over a frequency band  $W$  being given by:

$$C_{ij} = W \log_2(1 + \text{SINR}_{ij}). \quad (3)$$

In practice, however, it is understood that most communication schemes will achieve lower rates, which depend on specified BER constraints, modulation, coding schemes, and detector structure. We are not concerned here with the capacity issue and use (3) only selectively for bounding purposes.

### B. Scheduling

Given a set of links  $M$ , all links in  $M$  can be activated concurrently if such simultaneous activation does not violate the minimum SINR required for communication, i.e., the SINR threshold is satisfied across all links in  $M$ . A set  $M$  satisfying this condition is called a feasible matching, or simply, a *matching*.

A schedule is an indexed collection  $\mathcal{S} = (M^s, \lambda^s, s \in S)$ , where the index set  $S$  is an arbitrarily large but finite set,  $M^s$  is a matching for each  $s$ , and  $\lambda^s \geq 0$  is the duration of activation associated with the matching  $M^s$ . The schedule length  $\tau$  of the schedule  $\mathcal{S}$  is defined as

$$\tau = \sum_s \lambda^s.$$

This schedule  $\mathcal{S}$  can be implemented in a Spatial-TDMA (or STDMA) framework, in which the time axis is divided into time-slots (whose lengths take on continuous values  $\lambda^s$ ), and a link may be active in one or several time-slots based on how many matchings contain this link.

### C. Rayleigh/Rayleigh Fading Environment

In the Rayleigh/Rayleigh fading environment, and considering a matching  $M$ , the signal-to-interference and noise ratio of a link  $\{i, j\} \in M$ , i.e.,  $SINR_{ij}$  is a random variable with a complex distribution, since it is the ratio of an exponential random variable to a sum of exponential random variables, with different means. However, we can derive an analytical expression for its density [20].

Suppose  $X_0, \dots, X_n$  are independent exponentially distributed random variables with respective means  $E[X_i] = 1/\lambda_i, \forall i = 0, \dots, n$ , and  $\eta > 0, \gamma > 0$  are constants. Then, we have that

$$\begin{aligned} \text{Prob}(X_0 > \eta + \gamma \sum_{i=1}^n X_i) &= \int_{x_1=0}^{\infty} \dots \int_{x_n=0}^{\infty} \text{Prob}(X_0 > \eta + \gamma \sum_{i=1}^n x_i) \\ &\quad \prod_{i=1}^n (\lambda_i e^{-\lambda_i x_i}) dx_1 \dots dx_n \\ &= \int_{x_1=0}^{\infty} \dots \int_{x_n=0}^{\infty} e^{-\lambda_0(\eta + \gamma(x_1 + \dots + x_n))} \\ &\quad \prod_{i=1}^n (\lambda_i e^{-\lambda_i x_i}) dx_1 \dots dx_n \\ &= e^{-\lambda_0 \eta} \prod_{i=1}^n \int_{x_i=0}^{\infty} \lambda_i e^{-(\lambda_0 \gamma + \lambda_i) x_i} dx_i \\ &= e^{-\lambda_0 \eta} \prod_{i=1}^n \frac{1}{1 + \frac{\lambda_0 \gamma}{\lambda_i}}. \end{aligned}$$

Looking back at (2), we can now conclude that

$$\begin{aligned} A_{ij} &= \text{Prob}(SINR_{ij} > \gamma) \\ &= \text{Prob}\left(G_{ij} F_{ij} P_i > \eta + \gamma \sum_{k \neq i, j} G_{kj} F_{kj} P_k\right) \\ &= e^{-\eta/(G_{ij} P_i)} \prod_{k \neq i, j} \frac{1}{1 + \frac{\gamma G_{kj} P_k}{G_{ij} P_i}}. \end{aligned} \quad (4)$$

This probability  $A_{ij}$ , denoted as the link success probability, can be interpreted as the fraction of time the link  $\{i, j\} \in M$  experiences a channel where the SINR exceeds the threshold, while the remaining fraction experiences outage due to fading. For a given matching, it can be seen that this probability vector  $\{A_{ij}\}$  can be easily computed, if  $G_{ij}$  and  $P_i$  are known. In this paper, we assume that all the nodes transmit at a fixed transmission power  $P_{max}$ .

### D. The Minimum-Length Scheduling Problem with Fading

A schedule  $\mathcal{S} = (M^s, \lambda^s, s \in S_I \subseteq Z^+)$  is a collection of matchings  $M^s$  and corresponding nonnegative durations  $\lambda^s$ , such that each of the matchings is feasible, and the end-to-end demands of all sessions are satisfied. Our objective is to minimize the schedule length  $\tau = \sum_s \lambda^s$ .

Each link  $\{i, j\} \in \mathcal{E}$  has a specific traffic demand of  $f_{ij}$  bits per frame that need to be transmitted across the link, and the entire information transfer across all the links can be completed in a time interval of length  $\tau$  as follows. Each matching  $M^s$  indexed by  $s \in S_I$ , is active for a duration of  $\lambda^s$ , and each link  $\{i, j\}$  that is part of the matching  $M^s$  transmits at a rate of  $c_{ij}^s$  bits/sec, which is computed based on the SINR at receiver  $j$ , as described in (3). Thus a link  $\{i, j\}$  is active during all the slots for which  $\{i, j\} \in M^s$ , and the overall data

that is transmitted in the duration for which the link  $\{i, j\}$  is active, must be at least  $f_{ij}$ .

Based on the interpretation presented above, and given the set of all possible feasible matchings for a network denoted by  $\mathcal{M}$ , where  $M^s \in \mathcal{M}, s \in S_I$ , the Minimal Length Scheduling Problem with Fading [MLSPF] can be formulated as follows. [MLSPF]:

$$\text{Minimize: } \tau = \sum_{1 \leq s \leq |\mathcal{M}|} \lambda^s \quad (5)$$

subject to:

$$\begin{aligned} \sum_{1 \leq s \leq |\mathcal{M}|} c_{ij}^s A_{ij}^s \lambda^s &\geq f_{ij}, \quad \forall \{i, j\} \in \mathcal{E} \\ \lambda^s &\geq 0, \quad \forall s = 1, \dots, |\mathcal{M}| \end{aligned} \quad (6)$$

### III. COLUMN GENERATION

For realistic problem instances, [MLSPF] could have far too many matchings (in the form of columns in the problem formulation) to be enumerated before the LP problem can be formulated. Since most of them might not be used in an optimal solution anyway, it is useful to develop techniques that could reduce the size of the search space. A column generation approach [1] is a viable alternative in such a situation. In this approach, the original problem is decomposed into a master problem and a subproblem. The strategy of this decomposition procedure is to operate iteratively on two separate, but easier-to-solve, problems. The master problem passes down a set of cost coefficients to the subproblem, and receives a new column (i.e., a matching) based on these cost coefficients from the subproblem. In the optimization literature, the subproblem is also referred to as the *pricing* problem. The key idea of the column generation algorithm is to sequentially improve the objective value of the master problem by identifying new matchings and adding them to the master problem [MP]. For further information regarding column generation, please refer to [16], and the references therein.

#### A. Master Problem

The master problem [MP] is a restriction of the original problem [MLSPF], which uses only a subset of columns (matchings) indexed by  $s \in S_I$ . The master problem [MP] is first initialized with any feasible schedule  $\mathcal{S}$  that satisfies the link demands of all the links in  $\mathcal{E}$ , for example, a TDMA schedule. The master problem [MP] is formulated as follows. [MP(S)]:

$$\text{Minimize: } \tau = \sum_{s \in S_I} \lambda^s \quad (7)$$

subject to:

$$\begin{aligned} \sum_{s \in S_I} c_{ij}^s A_{ij}^s \lambda^s &\geq f_{ij}, \quad \forall \{i, j\} \in \mathcal{E} \\ \lambda^s &\geq 0, \quad \forall s \in S_I \end{aligned} \quad (8)$$

Since this LP formulation optimizes over a subset of all feasible matchings, it is a restriction of the original problem [MLSPF]. Hence, an optimal solution to [MP] provides an upper bound  $UB$  for the [MLSPF].

### B. Generating Feasible Matchings: Fixed Transmit Power

During every iteration, when the master problem [MP] is solved, we need to either verify that the current solution is optimal, or else identify a new matching that can improve the current solution, i.e., we need to identify a new column to enter into the so-called *basis*. Recall that each matching constitutes one column in [MP]. Based on the theory of linear programming and the revised simplex algorithm [1], this can be achieved by examining whether any new column (that is not currently in [MP]), has a negative reduced cost. Denoting the dual variables (also known as *prices*) corresponding to (8) by  $(\bar{\omega}_{ij})$ , the reduced cost  $\bar{z}_s$  for any column  $s$  in [MP] can be expressed as:

$$\bar{z}_s = 1 - \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^s A_{ij}^s. \quad (9)$$

Therefore, in order to find a new column having the most *negative reduced cost*, we solve the subproblem defined as:

$$\text{Minimize}_{s \in \mathcal{M}} \bar{z}_s,$$

or equivalently,

$$\text{Maximize}_{s \in \mathcal{M}} \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^s A_{ij}^s. \quad (10)$$

Because the reduced cost  $\bar{z}_s$  for  $s \in S_I$  is nonnegative, the focus in (10) is automatically on  $s \in \mathcal{M} \setminus \mathcal{S}$  i.e., the set of columns in  $\mathcal{M}$  that are not a part of  $\mathcal{S}$ . This subproblem can be referred to as the *scheduling subproblem*, because it aids in identifying a new matching that could be a part of an optimal schedule. Based on the solution obtained for this scheduling subproblem, a non-negative reduced cost implies that the current solution to [MP] is indeed an optimal solution to [MLSPF]. Otherwise, the new matching that is identified by the subproblem is included in the current schedule  $\mathcal{S}$ , and [MP] is re-optimized. We now present the formulation of the sub-problem.

Recall that the source nodes of all active links in the matching are set to use their maximum power  $P_{max}$ , with the condition that the SINR of all the active links in the matching exceeds a *fixed* threshold  $\gamma$ . Associated with  $\gamma$  is a transmission rate  $c_{ij}$  at which each active link  $\{i, j\}$  would be allowed to transmit. Therefore, based on (refeq:sinrf), in the case of fixed transmit power  $A_{ij}$  is expressed as follows:

$$A_{ij} = e^{-\eta/(G_{ij}P_{max})} \prod_{k \neq i,j} \frac{1}{1 + \frac{\gamma G_{kj}}{G_{ij}}}.$$

Just as in the case of the minimum-length scheduling problem without fading, we define  $x_{ij}$  as a binary variable associated with the link  $\{i, j\}$ , for each  $\{i, j\} \in \mathcal{E}$ , such that

$$x_{ij} = \begin{cases} 1, & \text{if link } \{i, j\} \text{ belongs to the new matching} \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

Given a set of dual variables  $(\omega_{ij})$  (obtained from the master problem [MP]), we can generate a new matching having

the most negative reduced cost, by solving the following subproblem, in the case of fixed transmit powers, referred to as [SP]<sup>1</sup>.

[SP]:

$$\text{Maximize: } \sum_{\{i,j\} \in \mathcal{E}} \omega_{ij} c_{ij} A_{ij} x_{ij}$$

subject to:

$$A_{ij} = e^{-\eta/(G_{ij}P_{max})} \prod_{k \neq i,j: x_{kj}=1} \frac{1}{1 + \frac{\gamma G_{kj} x_{ij}}{G_{ij}}} \quad (12)$$

$$(\eta_j + \sum_{k \neq i,j} G_{kj} P_{max} - \gamma^{-1} G_{ij} P_{max}) x_{ij} + \sum_{k, m \neq i,j} G_{kj} P_{max} x_{km} \leq \sum_{k \neq i,j} G_{kj} P_{max}, \quad \forall \{i, j\} \in \mathcal{E} \quad (13)$$

$$\sum_{j: \{i,j\} \in \mathcal{E}} x_{ij} + \sum_{j: \{j,i\} \in \mathcal{E}} x_{ji} \leq 1, \quad \forall i \in \mathcal{N} \quad (14)$$

$$x_{ij} \in \{0, 1\}, \quad \forall \{i, j\} \in \mathcal{E}.$$

In [SP], the constraint set (13) guarantees that the SINR threshold is satisfied for all active links that belong to the matching, while the constraint set (14) consists of transmission constraints that ensure that for each node, at most one incident link is activated.

As it stands, it is difficult to solve the sub-problem [SP] directly, because of the non-linear terms in constraint (12). Therefore, in the rest of this section, we show how (12) can be relaxed to a linear constraint set.

Note that the objective function has the terms  $A_{ij} x_{ij}$  and that  $A_{ij}$  is also part of (12). Hence, we can let  $y_{ij} = A_{ij} x_{ij}$  and write this objective function as

$$\text{Maximize: } \sum_{\{i,j\} \in \mathcal{E}} \omega_{ij} c_{ij} y_{ij} \quad (15)$$

and rewrite (12) as,

$$y_{ij} = x_{ij} e^{-\eta/(G_{ij}P_{max})} \prod_{k \neq i,j: x_{kj}=1} \frac{1}{1 + \frac{\gamma G_{kj} x_{ij}}{G_{ij}}} \quad (16)$$

Now, for each  $\{i, j\} \in \mathcal{E}$ , we transform (16) as follows. First, note that

$$y_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \\ e^{-\eta/(G_{ij}P_{max})} \prod_{k \neq i,j: x_{kj}=1} \frac{G_{ij}}{\gamma G_{kj} + G_{ij}}, & \text{if } x_{ij} = 1. \end{cases} \quad (17)$$

Define the parameter  $\beta_{ijk} = G_{ij}/(\gamma G_{kj} + G_{ij}), \forall k \neq i, j, \forall \{i, j\} \in \mathcal{E}$ . Then (17) can be written as

$$y_{ij} = \begin{cases} 0, & \text{if } x_{ij} = 0 \\ e^{\frac{-\eta}{G_{ij}P_{max}}} \prod_{k \neq i,j} [\beta_{ijk} x_{kj} + (1 - x_{kj})], & \text{if } x_{ij} = 1. \end{cases} \quad (18)$$

<sup>1</sup>Due to space constraints, some of the details leading to the formulation of the sub-problem [SP] that were already discussed in [15] are not presented here.

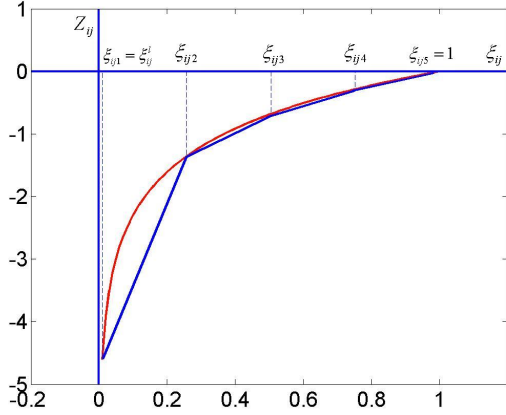


Fig. 1. Piecewise linear approximation for (21d,e) using  $H = 4$  segments.

Noting that the maximization in (15) would automatically prefer higher values of  $y_{ij}$ , we can formulate (18) as follows

$$0 \leq y_{ij} \leq x_{ij} \quad (19a)$$

$$y_{ij} \leq \xi_{ij} \quad (19b)$$

$$\xi_{ij} = e^{-\eta/(G_{ij}P_{max})} \prod_{k \neq i,j} [\beta_{ijk}x_{kj} + (1 - x_{kj})], \quad (19c)$$

where, we have used the fact that  $\xi_{ij} \leq 1$  since  $\beta_{ijk} \leq 1$ . Now, in order to linearize (19c) and yet to keep the problem size manageable for large  $|N|$ , let us define

$$z_{ij} \equiv \ln(\xi_{ij}) = \frac{-\eta}{G_{ij}P_{max}} + \sum_{k \neq i,j} [\ln(\beta_{ijk})]x_{kj} \quad (20)$$

where the right-hand side follows by noting that for binary  $x_{kj}$ , we get  $\ln[\beta_{ijk}x_{kj} + (1 - x_{kj})] = \ln(\beta_{ijk})x_{kj}$ . Hence the reformulated version of (12) is given by

$$0 \leq y_{ij} \leq x_{ij} \quad (21a)$$

$$y_{ij} \leq \xi_{ij} \quad (21b)$$

$$z_{ij} = \frac{-\eta}{G_{ij}P_{max}} + \sum_{k \neq i,j} [\ln(\beta_{ijk})]x_{kj} \quad (21c)$$

$$z_{ij} = \ln(\xi_{ij}) \quad (21d)$$

$$\xi_{ij}^l \leq \xi_{ij} \leq \xi_{ij}^u, \quad (21e)$$

where the implied bounds in (21e) are added for convenience in the next step, and where, at the beginning, we have

$$\xi_{ij}^l \equiv e^{-\eta/(G_{ij}P_{max})} \prod_{k \neq i,j} \beta_{ijk}, \text{ and } \xi_{ij}^u \equiv 1. \quad (22)$$

Note that the system (21) is linear except for the relationship (21d). To (approximately) linearize (21d), we adopt a piecewise linear grid approximation as depicted in Figure 1, based on some  $H$  segments defined via grid points  $\xi_{ijh}$ ,  $h = 1, \dots, H + 1$  such that the corresponding  $\ln[\xi_{ijh}]$ -values are equispaced along the abscissa. Hence, we define

$$\xi_{ijh} = \exp\left[\left(\frac{H - h + 1}{H}\right) \ln[\xi_{ij}^l]\right], \quad h = 1, \dots, H + 1 \quad (23)$$

and then replace (21d) and (21e) by the following:

$$\xi_{ij} = \sum_{h=1}^H [\lambda_{h1}^{ij} \xi_{ijh} + \lambda_{h2}^{ij} \xi_{ij(h+1)}] \quad (24a)$$

$$z_{ij} = \sum_{h=1}^H [\lambda_{h1}^{ij} \ln(\xi_{ijh}) + \lambda_{h2}^{ij} \ln(\xi_{ij(h+1)})] \quad (24b)$$

$$\lambda_{h1}^{ij} + \lambda_{h2}^{ij} = \mu_h^{ij}, \quad \forall h = 1, \dots, H \quad (24c)$$

$$\sum_{h=1}^H \mu_h^{ij} = 1 \quad (24d)$$

$$\mu^{ij} \text{ binary}, \quad \lambda^{ij} \geq 0. \quad (24e)$$

In this system (24), the  $\lambda^{ij}$ -variables are convex combination weights attached to the end-points of each segment, and the  $\mu^{ij}$ -variables are defined as  $\mu_h^{ij} = 1$  if segment  $h$  is activated in the piecewise-linear approximation and  $\mu_h^{ij} = 0$  otherwise. By [23], this yields a strong (partial convex hull) representation of the piecewise linear function. Hence for each  $\{i, j\} \in \mathcal{E}$ , (12) can be represented via [(21a) - (21c), plus (24a) - (24e)]. This is an approximation that affects only the objective value in [SP]. Therefore, given a solution to this approximation, we can extract the  $x$ -part of the solution (which is feasible to ((13)-(14))), and substitute that in (12) to evaluate exactly, the associated objective value of [SP]. If this is negative, then we have discovered a new column to be incorporated into the master problem. On the other hand, if this is  $\geq 0$ , then we have a near optimal solution, based on the accuracy of the approximation used for (21d) via [(24a) - (24e)].

#### IV. LOWER BOUND

*Proposition 1:* Let us consider any restricted master problem [MP], and let the optimal dual solutions obtained with respect to (7) be given by  $\bar{\omega} \equiv (\bar{\omega}_{ij}, \{i, j\} \in \mathcal{E})$ , with an optimal objective value  $\nu$ . Then a lower bound  $LB$  for [MLSPF] is given by

$$LB = \frac{\bar{\omega}^T \mathbf{f}}{1 - \nu}.$$

*Proof:* We have, by duality, that any dual solution ( $\omega$ ) to constraint set (8) in [MLSPF] must satisfy the following:

$$\sum_{\{i,j\} \in \mathcal{E}} \omega_{ij} c_{ij}^s A_{ij}^s \leq 1, \quad \forall s \quad (25)$$

$$\omega_{ij} \geq 0, \quad \forall \{i, j\} \in \mathcal{E}.$$

Using the dual vectors  $\bar{\omega}_{ij}$  obtained via the master problem [MP], and solving the subproblem we compute a new column having the most negative reduced cost, i.e.,

$$\nu = \text{Min}\{1 - \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^s A_{ij}^s\},$$

$$\Rightarrow \sum_{\{i,j\} \in \mathcal{E}} \bar{\omega}_{ij} c_{ij}^s A_{ij}^s \leq (1 - \nu), \quad \forall s, \quad (26)$$

where  $\nu \leq 0$ . Moreover, by duality in [MP], we have that

$$\bar{\omega}_{ij} \geq 0, \quad \forall \{i, j\} \in \mathcal{E}.$$

From Equations (25) and (26), we conclude that  $\omega_{ij}$ , given by,

$$\omega_{ij} = \frac{\bar{\omega}_{ij}}{1 - \nu}, \quad \forall \{i, j\} \in \mathcal{E}$$

is a dual feasible solution to [MLSPF], with the objective value given by

$$\omega^T \mathbf{f} = \frac{\bar{\omega}^T \mathbf{f}}{1 - \nu}, \quad (27)$$

which yields a lower bound  $LB$  for [MLSPF]. ■

Computationally, it has been observed in the context of column generation algorithms (see [1], [17]) that one can usually determine solutions to within 1-5 % of optimality fairly quickly. However, for many classes of problems, the tail-end convergence rate in obtaining the optimal solution can be very slow. Therefore, relying on Proposition 1, since we can compute a lower bound at each iteration, in addition to the upper bound  $UB$  (denoted  $\nu$  in Proposition 1) available via the master problem, we can terminate the procedure once the optimality gap (gap between the lower and upper bounds) approaches a value within 1-5 %. It should also be noted that the lower bounds generated based on the discussion above need not be monotone, and one should therefore maintain the best (highest) lower bound in order to compute the optimality gap at every iteration.

## V. CONCLUSION

In this paper, we have addressed the problem of SINR-based minimum-length scheduling in wireless networks in the presence of Rayleigh fading links. We formulated the problem as a cross-layer optimization problem with consideration of link layer and physical layer parameters, in order to generate feasible matchings. We proposed a solution procedure based on column generation, and showed that this method not only actually converges to an optimal solution, but also provides a control on the final optimality gap achieved, if required.

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