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On the Use of Multipath Geometry for Wideband Cooperative Localization

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Abstract—The combination of wideband transmission and cooperative techniques enables high-precision location-awareness. Wideband transmission provides fine delay resolution and multipath resolvability, while cooperation among nodes can yield significant performance benefit in harsh or infrastructure-limited environments. In this paper, we propose to exploit the geometric relationship inherent in multipath propagation, i.e., multipath geometry, via cooperation among nodes for localization. We characterize the contribution of this multipath geometry in terms of the nodes' squared position error bound, which is the fundamental limit of localization accuracy. Analytical and numerical results validate the benefit of using multipath geometry in wideband cooperative localization.

Index Terms—Cooperative localization, Cramer-Rao bound, squared position error bound (SPEB), multipath propagation

I. INTRODUCTION

The ability to geographically localize nodes within a network is of pivotal importance for many applications, including search-and-rescue operations, logistics, and blue force tracking [1]–[9]. The global positioning system (GPS) is used worldwide to enable location-awareness, complemented by dedicated technologies for GPS-denied scenarios [10], such as indoors, in urban canyons, and under tree canopies. GPS is known to be ineffective in harsh environments due to the inability of GPS signals to penetrate obstacles, while indoor technologies offer limited precision or require a prohibitive infrastructure cost.

Cooperation among nodes has been shown effective in addressing these problems by allowing the nodes to assist one another for localization (see Fig. 1) [1]–[5]. On the other hand, wide bandwidth and ultra-wide bandwidth (UWB) signals are particular well-suited for localization, since they can provide accurate and reliable range measurements due to their fine delay resolution and robustness in harsh environments [11]–[13]. Fundamental limits of localization accuracy, in terms of the squared position error bound (SPEB) by the information inequality, are derived based directly on the received waveforms rather than specific signal metrics obtained from these waveforms for cooperative networks [3], [4].

Multipath propagation refers to a phenomenon where signals reach the receive antenna via multiple paths, arising from either reflecting off objects or scattering. This phenomenon is considered harmful for localization since path-overlap in



Fig. 1. Cooperative localization in wireless networks: the anchors (A, B, and C) communicate with the agents (1 and 2). Agent 2 can trilaterate its position after ranging with the anchors, while agent 1 cannot. Agent 2 can serve as a virtual anchor for agent 1, thus enabling both agents to be localized.

multipath propagation channels introduces interference in estimating the time-of-arrival, which results in poorer localization performance [7], [8], [14]. However, multipath components (MPCs) can be thought of as direct paths from virtual nodes behind reflecting surfaces or on scattering objects¹ (see Fig. 2). Thus, the arrival times of the signals received at different nodes from the same virtual node are dependent. We refer to this as multipath geometry and will show that this position-based relationship among MPCs can be used to increase localization accuracy. Note that major reflections and scattering, such as those from ceilings and floors, are likely to follow the multipath geometry, although some minor ones may not be as ideal as shown in Fig. 2 (e.g., random scattering). In the latter case, the set of corresponding MPCs can be thought of as direct paths coming from different virtual nodes, and hence the geometric relationship among the MPCs does not exist.

In this paper, we investigate the contribution of the geometric relationship inherent in multipath propagation for localization via cooperation among nodes. In particular, we introduce the notion of virtual node to model multipath geometry, and characterize its benefit from the perspective of fundamental

¹Since wider transmission bandwidths translate to higher multipath resolution, dominant MPCs can be resolved via the use of wide bandwidth signals [11]–[13], [15], [16].



Fig. 2. Multipath geometry: MPCs come from either reflecting off the objects or scattering, and these paths can be considered as direct paths from virtual nodes (virtual node a_1 and a_2 of anchor A).

performance limits,² i.e., the SPEB. The notion of equivalent Fisher information (EFI) has been applied to derive the SPEB, and further analysis on the SPEB provides insights into the role of multipath geometry in different scenarios.

Notation: The notation $\mathbb{E}_{\mathbf{x}}\{\cdot\}$ is the expectation operator with respect to the random vectors \mathbf{x} , and $\mathbf{A} \succeq \mathbf{B}$ denotes that the matrix $\mathbf{A} - \mathbf{B}$ is positive semi-definite. The notation $\operatorname{tr}\{\cdot\}$ is the trace of a square matrix, $[\cdot]^{\mathrm{T}}$ denotes the transpose of its argument, and $[\cdot]_{n \times n, k}$ denotes the *k*th $n \times n$ submatrix (starting from the n(k-1) + 1th element) on the diagonal of its argument. The notation $\|\cdot\|$ is the Euclidean distance, and \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote a real vector of dimension n and an $n \times m$ matrix of real elements, respectively.

II. PROBLEM FORMULATION

In this section, we describe the models for wideband signals and location-aware networks, and formulate the problem of cooperative localization using multipath geometry. We will also briefly review the notions of EFI and SPEB [7], [8], which characterize the fundamental limits on the localization accuracy.

A. System Model

Consider a network consisting of N_b anchors and N_a agents. Each anchor has perfect knowledge of its position, and each agent attempts to estimate its position based on the received wide bandwidth waveforms from neighboring nodes, both anchors and agents (see Fig. 1). The set of agents is denoted by $\mathcal{N}_a = \{1, 2, \dots, N_a\}$, while the set of anchors is $\mathcal{N}_b = \{N_a + 1, N_a + 2, \dots, N_a + N_b\}$. The position of node k is denoted by \mathbf{p}_k . For convenience, we focus on two-dimensional localization where $\mathbf{p}_k \in \mathbb{R}^2$. Extension to the three-dimensional localization is straightforward [7].

The received waveform at the *k*th agent $(k \in \mathcal{N}_a)$ from the *j*th node $(j \in \mathcal{N}_b \cup \mathcal{N}_a \setminus \{k\})$ can be written as [11]–[13]

$$r_{kj}(t) = \sum_{l=1}^{L_{kj}} \alpha_{kj}^{(l)} s\left(t - \tau_{kj}^{(l)}\right) + z_{kj}(t), \quad t \in [0, T_{\text{ob}}), \quad (1)$$

where s(t) is a known wideband waveform, $\alpha_{kj}^{(l)}$ and $\tau_{kj}^{(l)}$ are the amplitude and delay, respectively, of the *l*th path,³ L_{kj} is the number of MPCs, $z_{kj}(t)$ represents the observation noise modeled as additive white Gaussian processes with two-side power spectral density $N_0/2$, and $[0, T_{ob})$ is the observation interval.

For notational convenience, let the numbers of MPCs in the received waveforms coming from the same nodes be the same, i.e., $L_{kj} = L_j$ for all j.⁴ Furthermore, we consider all the first paths be line-of-sight (LOS).⁵ Based on multipath geometry, we denote the positions of the virtual nodes corresponding to the received waveforms from node j be $\mathbf{p}_j^{(l)}$, where $l = 2, \dots, L_j$, with the understanding that $\mathbf{p}_j^{(1)} = \mathbf{p}_j$. Then the relationship between the path delays and the positions of the nodes can be expressed as

$$\tau_{kj}^{(l)} = \begin{cases} \|\mathbf{p}_{j}^{(l)} - \mathbf{p}_{k}\|/c, & \text{Type I}, \\ \left(\|\mathbf{p}_{j}^{(l)} - \mathbf{p}_{k}\| + \|\mathbf{p}_{j}^{(l)} - \mathbf{p}_{j}\|\right)/c, & \text{Type II}, \end{cases}$$
(2)

where c is the speed of propagation, Case I denotes the reflecting paths, and Case II denotes the scattering paths (see Fig. 2).

B. Error Bounds on the Position Estimation

Our analysis is based on the received waveforms of the form given by (1), and we first introduce θ as the vector of unknown parameters

where $\mathbf{P} = [\mathbf{p}_1^{\mathsf{T}}, \cdots, \mathbf{p}_{N_{\mathrm{a}}}^{\mathsf{T}}]^{\mathsf{T}}$ denotes the positions of all agents, $\tilde{\mathbf{p}}_j = [\mathbf{p}_j^{(2) \mathsf{T}}, \cdots, \mathbf{p}_j^{(L_j) \mathsf{T}}]^{\mathsf{T}}$ denotes the positions of the virtual nodes corresponding to node j, and

$$oldsymbol{lpha}_{j} = egin{cases} \left[egin{array}{cc} oldsymbol{lpha}_{1j}^{\mathrm{T}}, & j \in \mathcal{N}_{\mathrm{b}} \,, \ \left[oldsymbol{lpha}_{1j}^{\mathrm{T}}, \cdots, oldsymbol{lpha}_{j-1,j}^{\mathrm{T}}, oldsymbol{lpha}_{j+1,j}^{\mathrm{T}}, \cdots, oldsymbol{lpha}_{N_{\mathrm{a}}j}^{\mathrm{T}}
ight]^{\mathrm{T}}, & j \in \mathcal{N}_{\mathrm{a}} \,, \ \end{array}$$

in which $\alpha_{kj} = [\alpha_{kj}^{(1)}, \cdots, \alpha_{kj}^{(L_j)}]^{\mathrm{T}}$ denotes the amplitudes of the MPCs in the waveforms transmitted from node j. Moreover, we denote \mathbf{r} as the vector representation of all received waveforms, by stacking vectors \mathbf{r}_{kj} , which is obtained from the KL expansion of $r_{kj}(t)$ [17]. We tacitly assume

²In this paper, we consider the contribution of multipath geometry in terms of fundamental performance limits. Practical issues such as MPC identification and localization algorithms are out of the scope of this paper.

³We consider the general case where the wideband channel is not reciprocal. Our results can be easily specialized to the reciprocal case, where we have $L_{kj} = L_{jk}$, $\alpha_{kj}^{(l)} = \alpha_{jk}^{(l)}$, and $\tau_{kj}^{(l)} = \tau_{jk}^{(l)}$ hence $b_{kj}^{(l)} = b_{jk}^{(l)}$, for $l = 1, \dots, L_{kj}$. ⁴The number L_j accounts for all possible MPCs. When certain MPC in

[&]quot;The number L_j accounts for all possible MPCs. When certain MPC in the received waveform at a particular agent does not exist, its amplitude is assigned to be zero.

 $^{^{5}}$ The amplitudes of the first paths in non-line-of-sight (NLOS) signals are assigned to be zero.

that when nodes j and k cannot communicate directly, the corresponding entry \mathbf{r}_{kj} is omitted in \mathbf{r} .

Let $\hat{\theta}$ denote an estimate of the unknown parameter θ based on the observation **r**. From the information inequality [17], it can be shown that [4]

$$\mathbb{E}_{\mathbf{r}}\left\{\|\hat{\mathbf{p}}_{k}-\mathbf{p}_{k}\|^{2}\right\} \succeq \operatorname{tr}\left\{\left[\mathbf{J}_{\theta}^{-1}\right]_{2\times 2,k}\right\},\qquad(3)$$

where $\hat{\mathbf{p}}_k$ is the position estimate for agent k, and \mathbf{J}_{θ} is the Fisher information matrix (FIM) for θ , given by⁶

$$\mathbf{J}_{\boldsymbol{\theta}} = \mathbb{E}_{\mathbf{r}} \left\{ -\frac{\partial^2}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^{\mathrm{T}}} \ln f(\mathbf{r}|\boldsymbol{\theta}) \right\} \,. \tag{4}$$

Evaluation of (4) requires the knowledge of the likelihood function $f(\mathbf{r}|\boldsymbol{\theta})$, which can be expressed as a product of terms given by [17]

$$\begin{split} f(\mathbf{r}_{kj}|\boldsymbol{\theta}) &\propto \exp\left\{\frac{2}{N_0} \int_0^{T_{ob}} r_{kj}(t) \sum_{l=1}^{L_j} \alpha_{kj}^{(l)} s\left(t - \tau_{kj}^{(l)}\right) dt \\ &- \frac{1}{N_0} \int_0^{T_{ob}} \left[\sum_{l=1}^{L_j} \alpha_{kj}^{(l)} s\left(t - \tau_{kj}^{(l)}\right)\right]^2 dt \right\}, \end{split}$$

since noise terms $z_{kj}(t)$ of the received waveforms are independent.

Definition 1 (Squared Position Error Bound [7]): The squared position error bound (SPEB) of agent k is defined to be

$$\mathcal{P}(\mathbf{p}_k) \triangleq \operatorname{tr}\left\{ \left[\mathbf{J}_{\boldsymbol{\theta}}^{-1} \right]_{2 \times 2, k} \right\}.$$
(5)

Note that the right-hand side of (3) provides a lower bound on the squared position error, and hence we define it as a metric (c.f. (5)) to characterize the localization performance.

C. Equivalent Fisher Information Matrix

Since $\mathbf{J}_{\boldsymbol{\theta}}$ is a matrix of very high dimension, while only a small submatrix $[\mathbf{J}_{\boldsymbol{\theta}}^{-1}]_{2N_a \times 2N_a}$ is of interest, we introduce the notion of equivalent Fisher information matrix (EFIM) [7], [8] to avoid inverting the original FIM. We also introduce the concept of ranging information (RI), which will turn out to be the basic building block of the EFIM.

Definition 2 (Equivalent Fisher Information Matrix [7]): Given a parameter vector $\boldsymbol{\theta} = [\boldsymbol{\theta}_1^T \boldsymbol{\theta}_2^T]^T$ where $\boldsymbol{\theta} \in \mathbb{R}^N$ and $\boldsymbol{\theta}_1 \in \mathbb{R}^n$, and a FIM $\mathbf{J}_{\boldsymbol{\theta}}$ of the form

$$\mathbf{J}_{\boldsymbol{\theta}} = \left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^{\mathrm{T}} & \mathbf{C} \end{array} \right],$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$, $\mathbf{B} \in \mathbb{R}^{n \times (N-n)}$, and $\mathbf{C} \in \mathbb{R}^{(N-n) \times (N-n)}$, the equivalent Fisher information matrix (EFIM) for $\boldsymbol{\theta}_1$ is given by

$$\mathbf{J}_{\mathbf{e}}(\boldsymbol{\theta}_1) \triangleq \mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^{\mathrm{T}}.$$
 (6)

⁶Some of the virtual nodes may not be localized due to insufficient audible agents around it, which can lead to a singular FIM. To prevent this, we eliminate such virtual nodes from the parameter set before deriving the FIM.

The EFIM retains all the necessary information to derive the information inequality of the parameter θ_1 , in a sense that $[\mathbf{J}_{\theta}^{-1}]_{n \times n} = [\mathbf{J}_{e}(\theta_1)]^{-1}$, so that the MSE matrix of the estimates of θ_1 satisfies the following inequality:

$$\mathbb{E}_{\mathbf{r}}\left\{\|\hat{\boldsymbol{\theta}}_{1}-\boldsymbol{\theta}_{1}\|^{2}\right\} \succeq \operatorname{tr}\left\{\left[\mathbf{J}_{\mathsf{e}}(\boldsymbol{\theta}_{1})\right]^{-1}\right\}$$

Note that one can apply (6) repeatedly to reduce the dimension of the original FIM until the EFIM of the minimum size is obtained.

Definition 3 (Ranging Information [7]): The ranging information (RI) is a 2×2 matrix of the form $\lambda \mathbf{J}_{r}(\phi)$, where λ is a nonnegative number called the ranging information intensity (RII) and the matrix $\mathbf{J}_{r}(\phi)$ is called the ranging direction matrix (RDM) with the following structure:

$$\mathbf{J}_{\mathrm{r}}(\phi) \triangleq \begin{bmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{bmatrix}$$

Note that $\mathbf{J}_{\mathbf{r}}(\phi)$ has exactly one non-zero eigenvalue equal to 1 and can be represented as $\mathbf{J}_{\mathbf{r}}(\phi) = \mathbf{q}(\phi) \mathbf{q}(\phi)^{\mathrm{T}}$, where $\mathbf{q}(\phi) \triangleq [\cos \phi \sin \phi]^{\mathrm{T}}$.

III. MAIN RESULTS

In this section, we derive the EFIM, or equivalently the SPEBs by (5), for all agents. The EFIM for general cases is involved and provides little insights, and hence we focus on the scenarios in which the MPCs in the received waveforms are resolvable.

A. EFIM for Cooperative Networks

We first derive the EFIM for the cooperative network using multipath geometry in the following theorem. In the following, we omit the proofs for brevity.

Theorem 1: When the MPCs are resolvable, the EFIM for the agents' positions is given by (7), shown at the top of the next page,⁸ where

$$\mathbf{C}_{k,j}^{(l)} = \lambda_{kj}^{(l)} \, \mathbf{J}_{\mathbf{r}}(\phi_{kj}^{(l)}) \,,$$

$$\mathbf{D}_{k,j}^{(l)} = \begin{cases} \lambda_{kj}^{(l)} \, \mathbf{J}_{\mathbf{r}}(\phi_{kj}^{(l)}) \,, & \text{Type I,} \\ \\ \lambda_{kj}^{(l)} \, \mathbf{q}(\phi_{kj}^{(l)}) \, \left[\, \mathbf{q}(\phi_{kj}^{(l)}) + \mathbf{q}(\phi_{kk}^{(l)}) \,\right]^{\mathrm{T}} \,, & \text{Type II,} \end{cases}$$

and

$$\mathbf{E}_{k,j}^{(l)} = \begin{cases} \lambda_{kj}^{(l)} \mathbf{J}_{\mathbf{r}}(\phi_{kj}^{(l)}) , & \text{Type I,} \\ \\ \lambda_{kj}^{(l)} \left[\mathbf{q}(\phi_{kj}^{(l)}) + \mathbf{q}(\phi_{kk}^{(l)}) \right] \left[\mathbf{q}(\phi_{kj}^{(l)}) + \mathbf{q}(\phi_{kk}^{(l)}) \right]^{\mathrm{T}}, \\ \\ & \text{Type II.} \end{cases}$$

⁷This follows immediately from Schur's complement [18].

⁸Unspecified elements in the matrix are equal to zero throughout the paper.

In the above expressions, the RII $\lambda_{kj}^{(l)}$ is given by

$$\lambda_{kj}^{(l)} = \begin{cases} 8\pi^2 \beta^2 / c^2 \cdot \left(\alpha_{k,j}^{(1)\,2} + \alpha_{j,k}^{(1)\,2} \right) / N_0 \,, \\ & k \neq j \in \mathcal{N}_{\mathrm{a}}, l = 1 \,, \\ 0 \,, & k = j \,, \\ 8\pi^2 \beta^2 / c^2 \cdot \alpha_{k,j}^{(l)\,2} / N_0 \,, & \text{otherwise} \,, \end{cases}$$

with β denoting the effective bandwidth of the signal s(t) [17], and $\phi_{kj}^{(l)}$ denotes the angle-of-arrival from \mathbf{p}_k to $\mathbf{p}_j^{(l)}$. *Remark 1:* We draw the following observations about the

theorem.

- The EFIM $J_e(\mathbf{P})$ for all agents' positions is derived in the form of a $2N_{\rm a} \times 2N_{\rm a}$ matrix by applying the notion of EFI. As shown in (7), the EFIM can be decomposed as a sum of three parts, i.e., \mathbf{K}_{A} , \mathbf{K}_{C} , and $\mathbf{M}_{\mathit{i}}^{(l)}$: 1) \mathbf{K}_{A} is the localization information of the direct paths in the received waveforms transmitted from anchors; 2) K_C is the information of the direct paths from cooperating agents; and 3) $\mathbf{M}_{j}^{(l)}$ is the localization information of the *l*th path in the received waveform from node *j*, or virtual node at $\mathbf{p}_i^{(l)}$. Note that corresponding elements in \mathbf{K}_A and \mathbf{K}_{C} are equal to 0 if the LOS path does not exists (NLOS signals).
- The EFIM for cooperative localization has been derived in [4] where multipath geometry is left unexploited. In those scenarios, the corresponding EFIM is given by $\tilde{\mathbf{J}}_{e}(\mathbf{P}) = \mathbf{K}_{A} + \mathbf{K}_{C}$. Compared with (7), the contribution from multipath geometry to the EFIM is characterized by additional terms $\mathbf{M}_{j}^{(l)}$. Since $\sum_{j \in \mathcal{N}_{b} \cup \mathcal{N}_{a}} \sum_{l=2}^{L_{j}} \mathbf{M}_{j}^{(l)}$ is a semi-positive definite matrix, this contribution in EFIM is non-negative, i.e.,

$$\mathbf{J}_{e}(\mathbf{P}) \succeq \mathbf{J}_{e}(\mathbf{P})$$
.

This agrees with the intuition that multipath geometry indicates the relationship among the MPCs in the received waveforms at different agents, and this relationship may provide more information and hence a larger EFIM. Indeed, as we introduce the notion of virtual node to characterize this relationship, those virtual nodes can

be thought of additional "agents" for the cooperative location-aware networks, except that those "agents" are deaf and do not receive waveforms from other nodes.

The next theorem points out that the multipath geometry is not useful unless enough agents are in cooperation.

Lemma 1: Let a 2 × 2 matrix $\mathbf{J} = \sum_{k=1}^{N} \lambda_k \mathbf{J}_{\mathbf{r}}(\phi_k)$, and then its inverse is

$$\mathbf{J}^{-1} = \frac{1}{|\mathbf{J}|} \sum_{k=1}^{N} \lambda_k \, \mathbf{J}_{\mathbf{r}}(\phi_k + \pi/2) \,,$$

where $|\cdot|$ denotes the determinant operator.

By the lemma, we have the following theorem.

Theorem 2: In k-dimensional (k = 1, 2, 3) localization. when MPCs are resolvable, multipath geometry of a virtual node is not useful when less than k+1 cooperating agents receive signals from a virtual node.

Remark 2: In the case of k = 2, i.e., two-dimensional localization, the theorem asserts that multipath geometry is useless if there is only one agent or two cooperating agents. Here is an intuitive explanation. Every virtual node has an unknown position of freedom two, which needs measurements from two agents to be "determined". Hence two agents in cooperation only provide enough information to "locate" the virtual node, and will not benefit from this virtual node for their own position accuracy. On the other hand, if there is a third agent receiving the signal from this virtual node, the information from the measurement will help improve the position estimate of the third agent since the virtual node has been "located" by the first two agents.

B. Absolute and Relative Position Errors

The total localization error of a cooperative network can be decomposed into relative and absolute position errors [19]. The relative portion represents the estimation error of the agents' positions relative to one another (agent network "topology"), while absolute portion represents the estimation error of the entire agent network in an absolute frame, such as translation and rotation.

Theorem 3: When MPCs are resolvable, multipath geometry of type I can only increase the accuracy of relative positions.

Remark 3: It can be shown that Fisher information from virtual nodes is orthogonal to the space of absolute positions. This implies that multipath geometry does not contain translation and rotation information for the entire agent network. The intuitive interpretation is as follows: virtual nodes have neither a priori position knowledge nor measurement from the anchors, and hence the signals "transmitted" from these virtual nodes do not contain any information about the absolute positions. Therefore, we have the statement of Theorem 3.

If the relative positions of agents are known, such as in the case of antenna arrays, Theorem 3 implies that multipath geometry becomes useless for localization. We make it clear in the following corollary.

Corollary 1: When MPCs are resolvable and the relative positions of the agents are known, multipath geometry of type I does not further improve the localization accuracy.

Remark 4: Antennas of the array can be thought of as individual agents, and their relative positions are known a priori. Hence the corollary implies that exploiting multipath geometry will not further improve the localization accuracy for the antenna array, since multipath geometry only helps determine the relative position of agent network topology.

IV. NUMERICAL RESULTS

In this section, we examine several numerical examples and illustrate applications of our analytical results.

We investigate the SPEB performance for cooperative localization as functions of the number of agents. The network configuration is drawn in Fig. 3 with four fixed anchors and a number of agents. The agents uniformly and independently reside in a 10 meter by 10 meter area. Multipath propagation is generated by signals ideally reflecting off from the four walls (case I). For simplicity, signals by multiple reflections are ignored and the MPCs are resolvable. Hence each received waveform has five resolved paths, one direct path and four MPCs coming from the four walls respectively. We also assume a fully connected network with signals that obey free-space path-loss model, such that the RII $\lambda_{kj}^{(l)} \propto 1/||\mathbf{p}_j^{(l)}-\mathbf{p}_k||^2$.

Figure 4 shows the average SPEB of agents as a function of the number of agents in the network through Monte Carlo simulation. The first and second curves from the top correspond to non-cooperative localization and cooperative localization without using MPCs, while the third and fourth curves characterize the average SPEB using one (from Wall I) and all four MPCs, respectively. Here are the observations drawn from the figure. First, cooperation among agents significantly decreases the SPEB, roughly proportional to the number of agents. Second, multipath geometry is shown to further decrease the SPEB. This result is expected as proven in the theorems since essentially more virtual nodes join cooperation. Third, the gain from MPCs does not exist when there are only two agents, which has been shown in Theorem 2.



Fig. 3. Typical network deployment of $N_{\rm b} = 4$ anchors (square) and $N_{\rm a} = 15$ agents (circle). The agents are distributed uniformly and independently on the 10 m × 10 m area. Multipath propagation is generated by signals reflecting off the four walls.



Fig. 4. SPEB as a function of the number of agents in the network for various number of MPCs.

Figure 5 shows the ratio of the average SPEB using MPCs to that without using MPCs as a function of the number of agents with different number of anchors and MPCs. Only Anchor A and C are active for the two-anchor case ($N_b = 2$). First, multipath geometry provides substantial gains: the average SPEB is decreased by 10% and 30% when one and four MPCs are used, respectively. Although more relative improvement for the fouranchor network, multipath geometry provides more absolute improvement for the two-anchor network. This is intuitive since the two-anchor network lacks fixed infrastructure and hence benefits more from the additional virtual nodes.

These numerical examples agree with our analytical results and validate the usefulness of multipath geometry for cooperative localization.



Fig. 5. Ratio of the average SPEB using MPCs to that without using MPCs as a function of the number of agents for various number of anchors and MPCs. Only Anchor A and C are active in the two-anchor network.

V. CONCLUSION

In this paper, we have investigated the geometric relationship inherent in multipath propagation for wideband cooperative localization. Our contribution is three-fold. First, we have introduced the concept of virtual nodes in the context of the multipath geometry, and have applied the notion of equivalent Fisher information to derive the fundamental bounds for agents' position errors. Second, we have shown that multipath geometry can substantially improve the localization accuracy with more than two agents in cooperation, and the gain is more significant when the number of anchors is limited. Third, we have also pointed out that multipath geometry only improves the accuracy of relative positions, and hence this geometry is not helpful if the relative positions of agents are known a priori, such as antenna arrays.

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REFERENCES

- N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, III, R. L. Moses, and N. S. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 54–69, Jul. 2005.
- [2] H. Wymeersch, J. Lien, and M. Z. Win, "Cooperative localization in wireless networks," *Proc. IEEE*, vol. 97, no. 2, pp. 427–450, Feb. 2009, special issue on *Ultra-Wide Bandwidth (UWB) Technology & Emerging Applications.*
- [3] Y. Shen, H. Wymeersch, and M. Z. Win, "Fundamental limits of wideband localization – part II: Cooperative networks," *IEEE Trans. Inf. Theory*, 2009, submitted for publication.

- [4] —, "Fundamental limits of wideband cooperative localization via Fisher information," in *Proc. IEEE Wireless Commun. and Networking Conf.*, Kowloon, HONG KONG, Mar. 2007, pp. 3951–3955.
- [5] M. Z. Win, Y. Shen, and H. Wymeersch, "On the position error bound in cooperative networks: a geometric approach," in *Proc. IEEE Int. Symp.* on Spread Spectrum Techniques & Applications, Bologna, ITALY, Aug. 2008, pp. 637–643.
- [6] D. Dardari, A. Conti, U. J. Ferner, A. Giorgetti, and M. Z. Win, "Ranging with ultrawide bandwidth signals in multipath environments," *Proc. IEEE*, vol. 97, no. 2, pp. 404–426, Feb. 2009, special issue on Ultra-Wide Bandwidth (UWB) Technology & Emerging Applications.
- [7] Y. Shen and M. Z. Win, "Fundamental limits of wideband localization – part I: A general framework," *IEEE Trans. Inf. Theory*, vol. 55, 2009, to appear.
- [8] —, "Fundamental limits of wideband localization accuracy via Fisher information," in *Proc. IEEE Wireless Commun. and Networking Conf.*, Kowloon, HONG KONG, Mar. 2007, pp. 3046–3051.
- [9] D. B. Jourdan, D. Dardari, and M. Z. Win, "Position error bound for UWB localization in dense cluttered environments," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 44, no. 2, pp. 613–628, Apr. 2008.
- [10] A. LaMarca, Y. Chawathe, S. Consolvo, J. Hightower, I. Smith, J. Scott, T. Sohn, J. Howard, J. Hughes, F. Potter *et al.*, "Place Lab: Device positioning using radio beacons in the wild," in *Proc. IEEE Int. Conf. on Pervasive Computing and Commun.*, vol. 3468. Springer, 2005, pp. 116–133.
- [11] M. Z. Win and R. A. Scholtz, "Characterization of ultra-wide bandwidth wireless indoor communications channel: A communication theoretic view," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 9, pp. 1613–1627, Dec. 2002.
- [12] A. F. Molisch, D. Cassioli, C.-C. Chong, S. Emami, A. Fort, B. Kannan, J. Karedal, J. Kunisch, H. Schantz, K. Siwiak, and M. Z. Win, "A comprehensive standardized model for ultrawideband propagation channels," *IEEE Trans. Antennas Propag.*, vol. 54, no. 11, pp. 3151–3166, Nov. 2006, Special Issue on Wireless Communications.
- [13] D. Cassioli, M. Z. Win, and A. F. Molisch, "The ultra-wide bandwidth indoor channel: from statistical model to simulations," *IEEE J. Sel. Areas Commun.*, vol. 20, no. 6, pp. 1247–1257, Aug. 2002.
- [14] Y. Shen and M. Z. Win, "Effect of path-overlap on localization accuracy in dense multipath environments," in *Proc. IEEE Int. Conf. on Commun.*, Beijing, CHINA, May 2008, pp. 4197–4202.
- [15] M. Z. Win and R. A. Scholtz, "Impulse radio: How it works," *IEEE Commun. Lett.*, vol. 2, no. 2, pp. 36–38, Feb. 1998.
- [16] —, "Ultra-wide bandwidth time-hopping spread-spectrum impulse radio for wireless multiple-access communications," *IEEE Trans. Commun.*, vol. 48, no. 4, pp. 679–691, Apr. 2000.
- [17] H. L. Van Trees, Detection, Estimation and Modulation Theory. New York, NY: Wiley, 1968, vol. 1.
- [18] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 1st ed. Cambridge, NY: Cambridge University Press, 1985.
- [19] J. N. Ash and R. L. Moses, "On the relative and absolute positioning errors in self-localization systems," *IEEE Transactions on Signal Processing*, vol. 56, no. 11, pp. 5668–5679, Nov. 2008.