# Fair Channel Allocation and Access Design for Cognitive Ad Hoc Networks

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Abstract-We investigate the fair channel assignment and access design problem for cognitive radio ad hoc network in this paper. In particular, we consider a scenario where ad hoc network nodes have hardware constraints which allow them to access at most one channel at any time. We investigate a fair channel allocation problem where each node is allocated a subset of channels which are sensed and accessed periodically by their owners by using a MAC protocol. Toward this end, we analyze the complexity of the optimal brute-force search algorithm which finds the optimal solution for this NP-hard problem. We then develop low-complexity algorithms that can work efficiently with a MAC protocol algorithm, which resolves the access contention from neighboring secondary nodes. Also, we develop a throughput analytical model, which is used in the proposed channel allocation algorithm and for performance evaluation of its performance. Finally, we present extensive numerical results to demonstrate the efficacy of the proposed algorithms in achieving fair spectrum sharing among traffic flows in the network.

*Index Terms*—Channel assignment, MAC protocol, cognitive ad hoc network, fair resource allocation.

## I. INTRODUCTION

Cognitive radio has recently emerged as an important research field, which promises to fundamentally enhance wireless network capacity in future wireless system. To exploit spectrum opportunities on a given set of channels of interest, each cognitive radio node must typically rely on spectrum sensing and access mechanisms. In particular, an efficient spectrum sensing scheme aims at discovering spectrum holes in a timely and accurate manner while a spectrum access strategy coordinates the spectrum access of different cognitive nodes so that high spectrum utilization can be achieved. These research themes have been extensively investigated by many researchers in recent years [1]-[9]. In [1], a survey of recent advances in spectrum sensing for cognitive radios has been reported.

There is also a rich literature on MAC protocol design and analysis under different network and QoS provisioning objectives. In [2], a joint spectrum sensing and scheduling scheme is proposed where each cognitive user is assumed to possess two radios. A beacon-based cognitive MAC protocol is proposed in [4] to mitigate the hidden terminal problem while effectively exploiting spectrum holes. Synchronized and channel-hopping based MAC protocols are proposed in [5] and [6], respectively. Other multi-channel MAC protocols [7], [8] are developed for cognitive multihop networks. However, these existing papers do not consider the setting where cognitive radios have access constraints that we investigate in this paper.

In [9], we have investigated the channel allocation problem considering this access constraint for a collocated cognitive network where each cognitive node can hear transmissions from other cognitive nodes (i.e., there is a single contention domain). In this paper, we make several fundamental contributions beyond [9]. First, we consider the large-scale cognitive ad hoc network setting in this paper where there can be many contention domains. In addition, the conflict constraints become much more complicated since each secondary node may conflict with several neighboring primary nodes and vice versa. These complex constraints indeed make the channel assignment and the throughput analysis very difficult. Second, we consider a fair channel allocation problem under the maxmin fairness criterion [13] while throughput maximization is investigated in [9]. Third, we propose optimal brute-force search and low-complexity channel assignment algorithms and analyze their complexity. Finally, we develop a throughput analytical model, which is used in the proposed channel allocation algorithms and for performance analysis.

#### **II. SYSTEM MODEL AND PROBLEM FORMULATION**

#### A. System Model

We consider a cognitive ad-hoc network where there are  $M_s$  flows exploiting spectrum opportunities in N channels for their transmissions. Each secondary flow corresponds to one cognitive transmitter and receiver and we refer to secondary flows as secondary users (SU) in the following. We assume there are  $M_p$  primary users (PU) each of which can transmit their own data on these N channels. We assume that each SU can use at most one channel for his/her data transmission. In addition, time is divided fixed-size cycle where SUs perform sensing on assigned channels at the beginning of each cycle to explore available channels for communications. For simplicity, we assume that there is no sensing error although the analysis presented in this paper can be extended to consider sensing errors. It is assumed that SUs transmit at a constant rate which is normalized to 1 for throughput calculation purposes.

To model the interference among SUs in the secondary network, we form a contention graph  $\mathcal{G} = \{\mathcal{N}, \mathcal{L}\}$ , where  $\mathcal{N} = \{1, 2, \ldots, M_s\}$  is the set of nodes (SUs) representing SUs and the set of links  $\mathcal{L} = \{1, 2, \ldots, L\}$  representing contention relationship among SUs. In particular, there is a link between two SUs in  $\mathcal{L}$  if these SUs cannot transmit packet data on the same channel at the same time, which is illustrated in Fig. 1. To model the activity of PUs on each channel, let us define  $p_{ij}^p$  as the probability that PU *i* does not transmit on channel *j*. We stack these probabilities and define  $\mathbb{P}_i = (p_{i1}^p, \ldots, p_{iN}^p), i \in [1, M_p]$ , which captures the activity of PU *i* on all channels. In addition, let us define

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Fig. 1. The contention graph.

 $\mathbb{P}^p = (\mathbb{P}_1, \dots, \mathbb{P}_{M_p})$  where  $\mathbb{P}_i$  is the vector representing activities of PU *i*.

We now model the contention relationship among SUs and between PUs and SUs. Specifically, we assume that  $\mathcal{U}_i^n$  be the set of neighboring SUs that conflict with SU *i* (i.e., there is a link connecting each SU in  $\mathcal{U}_i^n$  to SU *i* in the contention graph). Also, assume that SU *k* has a set of neighboring PUs denoted as  $\mathcal{U}_k^p$ , which is the subset of  $1, \ldots, M_p$  so that if any PU in the set  $\mathcal{U}_k^p$  transmit on a particular channel then SU *k* is not allowed to transmit on this channel to protect the primary transmission. Assuming that the activities of different PUs on any channel are independent then the probability that channel *j* is available for SU *k* indicates can be written as  $p_{kj} = \prod_{i \in \mathcal{U}_k^p} p_{ij}^p$  since channel *j* is available for SU *k* if all conflicting PUs in  $\mathcal{U}_k^p$  do not use channel *j*.

### B. Problem Formulation

We are interested in performing channel assignment that maximizes the minimum throughput among all SUs (i.e., maxmin fairness [13]). Let  $T_i$  denote the throughput achieved by SU *i*. Let  $x_{ij}$  describe the channel assignment decision where  $x_{ij} = 1$  if channel *j* is assigned to SU *i* and  $x_{ij} = 0$ , otherwise. Then, the max-min channel assignment problem can be written as

$$\max_{\mathbf{x}} \min_{i} T_i \tag{1}$$

where **x** is the channel assignment vector whose elements are  $x_{ij}$ . For the case where each SU is allocated a distinct set of channels, i.e., we have  $\sum_{i=1}^{M_s} x_{ij} = 1$ , for all j. Under this non-overlapping channel assignments, let  $S_i$  be the set of channels assigned to SU i. Recall that  $p_{ij}$  is the probability that channel j is available at SU i. Then,  $T_i$  can be calculated as  $T_i = 1 - \prod_{j \in S_i} \overline{p}_{ij} = 1 - \prod_{j=1}^{N} (\overline{p}_{ij})^{x_{ij}}$  where  $\overline{p}_{ij} = 1 - p_{ij}$  is the probability that channel j is not available for SU i [9]. In fact,  $1 - \prod_{j \in S_i} \overline{p}_{ij}$  is the probability that there is at least one channel available for SU i. Because each SU can use at most one available channel, its maximum throughput is 1.

In general, it would be beneficial if each channel is allocated to several SUs in a common neighborhood to exploit the multiuser diversity. Under both non-overlapping and overlapping channel assignments, it can be observed that the channel assignment problem with the objective defined in (1) is a nonlinear integer program, which is an NP-hard problem (interest readers can refer to [12] for detailed treatment of this hardness result).

# C. Optimal Algorithm and Its Complexity

We describe a brute-force search (i.e., exhaustive search) to determine the optimal channel assignment solution. Specifically, we can enumerate all possible channel assignment solutions then determine the best one by comparing their achieved throughput. While throughput can be calculated quite easily for the non-overlapping channel assignments as being presented in Section II-B, developing a throughput analytical model for an overlapping channel assignment solution is indeed challenging task, which is performed in Section III-B2 of this paper.

We now quantify the complexity of the optimal brute-force search algorithm. Let us consider SU *i* (i.e.,  $i \in \{1, \ldots, M_s\}$ ). Suppose we assign it *k* channels where  $k \in \{1, \ldots, N\}$ ). Then, there are  $C_N^k$  ways to do so. Since *k* can take any values in  $k \in \{1, \ldots, N\}$ , the total number of ways to assign channels to SU *i* is  $\sum_{k=1}^{N} C_N^k \approx 2^N$ . Hence, the total number of ways to assign channels to all SUs is  $(2^N)^{M_s} = 2^{NM_s}$ . Recall that we need to calculate the throughputs achieved by  $M_s$  SUs for each potential assignment to determine the best one. Therefore, the complexity of the optimal bruteforce search algorithm is  $\mathcal{O}(2^{NM_s})$ . Given the exponentially large complexity of this brute-force search, we will develop low-complexity channel assignment algorithms, namely nonoverlapping and overlapping assignment algorithms.

#### III. CHANNEL ALLOCATION AND ACCESS DESIGN

## A. Non-overlapping Channel Assignment

We develop a low-complexity algorithm for non-overlapping channel assignment in this section. Recall that  $S_i$  is the set of channels assigned for secondary user *i*. In the non-overlapping channel assignment scheme, we have  $S_i \cap S_j = \emptyset$ ,  $i \neq j$ where SUs *i* and *j* are neighbors of each other (i.e., there is a link connecting them in the contention graph  $\mathcal{G}$ ). Note that one particular channel can be assigned to SUs who are not neighbors of each other. This aspect makes the channel assignment different from the collocated network setting considered in [9]. Specifically all channels assigned for different SUs should be different in [9] under non-overlapping channel assignment since there is only one contention domain for the collocated network investigated in [9].

The greedy channel assignment algorithm iteratively allocates channels to one of the minimum-throughput SUs so that we can achieve maximum increase in the throughput for the chosen SU. Detailed description of the proposed algorithm is presented in Algorithm 1. In each channel allocation iteration, each minimum-throughput SU *i* calculates its increase in throughput if the best available channel (i.e., channel  $j_i^* = \arg \max p_{ij}$ ) is allocated. This increase in throughput can be  $j \in S_a$ 

calculated as  $\Delta T_i = T_i^a - T_i^b = p_{ij_i^*} \prod_{j \in S_i} (1 - p_{ij})$  [9]. In step 4, there may be several SUs achieving the minimum

In step 4, there may be several SUs achieving the minimum throughput. We denote this set of minimum-throughput SUs as  $S^{\min}$ . Then, we assign the best channel that results in the maximum increase of throughput among all SUs in the set  $S^{\min}$ . We update the set of available channels for each SU after each allocation. Note that only neighboring SUs compete for the same channel; hence, the update of available

# Algorithm 1 NON-OVERLAPPING CHANNEL ASSIGNMENT

- 1: Initialize SU *i*'s set of available channels,  $S_i^a$  $\{1, 2, \ldots, N\}$  and  $\mathcal{S}_i := \emptyset$  for  $i = 1, 2, \ldots, M_s$  where  $S_i$  denotes the set of channels assigned for SU *i*.
- 2: continue := 1
- 3: while continue = 1 do
- Find the set of SUs who currently achieve the min- $4 \cdot$ imum throughput  $S^{\min} = \operatorname{argmin} T_i^b$  where  $S^{\min} =$  $\{i_1,\ldots,i_m\} \subset \{1,\ldots,M_s\}$  is the set of minimumthroughput SUs.
- if  $\underset{i_l \in S^{\min}}{\mathcal{OR}} \left( S^a_{i_l} \neq \emptyset \right)$  then 5:
- For each SU  $i_l \in S^{\min}$  and channel  $j_{i_l} \in S^a_{i_l}$ , find  $\Delta T_{i_l} = T^a_{i_l} T^b_{i_l}$  where  $T^a_{i_l}$  and  $T^b_{i_l}$  are the throughputs after and before assigning channel  $j_{i_l}$ ; 6: and we set  $\Delta T_{i_l} = 0$  if  $\mathcal{S}^a_{i_l} = \emptyset$

7: 
$$\left\{i_{l}^{*}, j_{i_{l}^{*}}^{*}\right\} = \operatorname*{argmax}_{i_{l} \in \mathcal{S}^{\min}, j_{i_{l}} \in \mathcal{S}_{i_{l}}^{a}} \Delta T_{i_{l}}\left(j_{i_{l}}\right)$$

- 8:
- Assign channel  $j_{i_l^*}^*$  to SU  $i_l^*$ . Update  $S_{i_l^*} = S_{i_l^*} \cup j_{i_l^*}^*$  and  $S_k^a = S_k^a \setminus j_{i_l^*}^*$  for all 9:  $k \in \mathcal{U}_{i_i^*}^n$ .
- 10: else
- Set continue := 011:
- 12: end if
- 13: end while

channels for the chosen minimum-throughput SU is only performed for its neighbors. This means that we can exploit spatial reuse in a large cognitive ad hoc network. It can be verified that if the number of channels is sufficiently large (i.e.,  $N > \max_i |\mathcal{U}_i^n|$ ), then the proposed non-overlapping channel assignment achieves throughput close to 1 for all SUs.

## **B.** Overlapping Channel Assignment

1) MAC Protocol: Overlapping channel assignment can improve the minimum throughput but we need to design a MAC protocol to resolve access contention among different SUs. Note that a channel assignment solution needs to be determined only once while the MAC protocol operates repeatedly using the chosen channel assignment solution in each cycle. Let  $S_i$  be the set of channels solely assigned for SU *i* and  $\mathcal{S}_i^{\text{com}}$  be the set of channels assigned for SU *i* and some other SUs. These two sets are referred to as separate set and *common set* in the following. Let denote  $S_i^{tot} = S_i \cup S_i^{com}$ , which is the set of all channels assigned to SU *i*.

Assume that there is one control channel, which is always available and used for access contention resolution. We consider the following MAC protocol run by any particular SU i, which belongs the class of synchronized MAC protocol [11].<sup>1</sup> The MAC protocol operates a cyclic manner where synchronization and sensing phases are employed before the channel contention and transmission phase in each cycle. After sensing the assigned channels in the sensing phase, if a particular SU i finds at least one channel in  $S_i$  available,

then it chooses one of these available channels randomly for communication. If this is not the case, SU i will choose one available channel in  $\mathcal{S}_i^{\text{com}}$  randomly (if any). Then, it chooses a random backoff value which is uniformly distributed in [0, W - 1] (i.e., W is the contention window) and starts decreasing its backoff counter while listening on the control channel.

If it overhears transmissions of RTS/CTS from any other SUs, it will freeze from decreasing its backoff counter until the control channel is free again. As soon as a SU's backoff counter reaches zero, its transmitter and receiver exchange RTS/CTS messages containing the chosen available channel for communication. If the RTS/CTS message exchange fails due to collisions, the corresponding SU will quit the contention and wait until the next cycle. In addition, by overhearing RTS/CTS messages of neighboring SUs, which convey information about the channels chosen for communications, other SUs compared these channels with their chosen ones. Any SU who has his/her chosen channel coincides with the overheard channels guits the contention and waits until the next cycle. Note that in the considered cognitive ad hoc setting each SU i only competes with its neighbors in the set  $\mathcal{U}_i^n$ , which is different from the setting investigated in [9].

2) Throughput Analysis: To analyze the throughput achieved by one particular SU i, we consider all possible sensing outcomes for the considered SU i on its assigned channels. We will consider the following cases.

• Case 1: If there is at least one channel in  $S_i$  available, then SU *i* will exploit this available channel and achieve the throughput of one. Here, we have

$$T_i \{ \text{Case 1} \} = \Pr \{ \text{Case 1} \} = 1 - \prod_{j \in S_i} \bar{p}_{ij}$$

- Case 2: We consider scenarios where all channels in  $S_i$ are not available; there is at least one channel in  $S_i^{\text{com}}$ available, and SU i chooses the available channel j for transmission. Suppose that channel j is shared by SU iand  $\mathcal{MS}_j$  neighboring SUs (i.e.,  $\mathcal{MS}_j = |\mathcal{U}_j|$  where  $\mathcal{U}_i$  denotes the set of these  $\mathcal{MS}_i$  neighboring SUs). Recall that all  $\mathcal{MS}_j$  SUs conflict with SU *i* (i.e., they are not allowed to transmit data on the same channel with SU *i*). There are four possible groups of SUs  $i_k$ ,  $k = 1, \ldots, \mathcal{MS}_i$  sharing channel j, which are described in the following
  - Group I: channel j is not available for SU  $i_k$ .
  - Group II: channel j is available for SU  $i_k$  and SU  $i_k$  has at least 1 channel in  $S_{i_k}$  available.
  - Group III: channel j is available for SU  $i_k$ , all channels in  $S_{i_k}$  are not available and there is another channel j' in  $\mathcal{S}_{i_k}^{\text{com}}$  available for SU  $i_k$ . In addition, SU  $i_k$  chooses channel  $j' \neq j$  for transmission in the contention stage.
  - **Group IV**: channel *j* is available for SU  $i_k$ , all chan-\_ nels in  $S_{i_k}$  are not available. Also, SU  $i_k$  chooses channel j for transmission in the contention stage. Hence, SU  $i_k$  competes with SU *i* for channel *j*.

<sup>&</sup>lt;sup>1</sup>Since we focus on the channel assignment issue in this paper, we do not attempt different alternative MAC protocol designs. Interest readers can refer to [11] for detailed treatment of this issue.

Let  $\mathcal{U}_{j,i}^p$  be the set of PUs who are neighbors of SUs in  $\mathcal{U}_j$ . Then, the throughput achieved by SU *i* can be written as

$$T_{i} ( \text{ Case } 3) = (1 - \delta) \Theta_{i} \sum_{A_{1}=0}^{\mathcal{MS}_{j}} \sum_{A_{2}=0}^{\mathcal{MS}_{j}-A_{1}} \sum_{A_{3}=0}^{\mathcal{MS}_{j}-A_{1}-A_{2}} \frac{1}{1 + A_{4}}$$
$$\sum_{c_{1}=1}^{C_{\mathcal{MS}_{j}}^{A_{1}}} \sum_{c_{2}=1}^{C_{\mathcal{MS}_{j}-A_{1}}} \sum_{c_{3}=1}^{C_{3}=1} \Theta_{j} \Phi_{1}(A_{1}) \Phi_{2}(A_{2}) \Phi_{3}(A_{3})$$

where  $A_4 = \mathcal{MS}_j - A_1 - A_2 - A_3$  and  $\delta$  denotes the MAC protocol overhead, which will be derived in Section III-B4. In this derivation, we consider all possible cases where SUs in  $\mathcal{U}_j$  are divided into four groups defined above with sizes  $A_1$ ,  $A_2$ ,  $A_3$ , and  $A_4$ , respectively. For one such particular case, let  $\mathcal{U}_{j,i}^{p,1}$  be the set of PUs who are only neighbors of SUs in group I with size  $A_1$  and  $\mathcal{U}_{j,i}^{p,2} = \mathcal{U}_{j,i}^p \setminus \mathcal{U}_{j,i}^{p,1}$  be the set of remaining PUs in  $\mathcal{U}_{j,i}^p$ . In addition, let  $\mathcal{U}_{j,i}^{p,3}$  be the set of PUs who are neighbors of SUs in group III and IV with sizes  $A_3$ and  $A_4$ , respectively. The terms  $\Theta_i$ ,  $\Theta_j$ ,  $\Phi_1(A_1)$ ,  $\Phi_2(A_2)$ , and  $\Phi_3(A_3)$  in the above derivation are

- $\Theta_i$  is the probability that all channels in  $S_i$  are not available and SU *i* chooses an available channel *j* in  $S_i^{\text{com}}$  for transmission.
- $\Theta_j$  is the probability that all PUs in  $\mathcal{U}_{j,i}^{p,2}$  do not use channel j.
- $\Phi_1(A_1)$  denotes the total probability of all cases for PUs in  $\mathcal{U}_{j,i}^{p,1}$  such that channel j is not available for all  $A_1$  SUs in group I.
- $\Phi_2(A_2)$  represents the probability that there is at least one available channel in the separate set for each of the  $A_2$  SUs in Group II.
- $\Phi_3(A_3)$  describes the total probability of all cases for PUs in  $\mathcal{U}_{j,i}^{p,3}$  such that each SU in group III chooses other available channel  $j' \neq j$  for transmission and each SU in group IV chooses channel j for transmission.

In this formula, we have considered all possible events and combinations that can happen for neighboring SUs of the underlying SU *i*. Note that only  $A_4$  SUs in Group IV compete with SU *i* for channel *j* by using the proposed MAC protocol. Therefore, SU *i* wins this contention with probability  $1/(1 + A_4)$ . In addition, the throughput is reduced by a factor  $1 - \delta$  where  $\delta$  is the MAC protocol overhead. Due to the space constraint, detailed derivation of these rather complicated probabilities are presented in the online technical report. Summarizing all considered cases, the throughput achieved by SU *i* is given as

$$T_i = T_i \{ \text{Case 1} \} + T_i \{ \text{Case 2} \}.$$
 (2)

This throughput derivation is used for channel assignment and performance evaluation of the proposed algorithms.

3) Configuration of Contention Window: We show how to calculate contention window W so that collision probabilities among contending SUs are sufficiently small. Note that the probability of the first collision among potential collisions is largest because the number of contending SUs decreases for successive potential collisions. Derivation of these collision probabilities for the cognitive ad-hoc networks is more complicated than that for collocated networks considered in [9] since the interference constraints are more complicated.

We calculate contention window  $W_k$  for each SU k considering the contention with its neighbors. Let us calculate  $\mathcal{P}_{c,k}$ as a function of  $W_k$  assuming that there are m secondary SUs in the contention phase. Without loss of generality, assume that the random backoff times of m SUs are ordered as  $r_1 \leq r_2 \leq \ldots \leq r_m$ . The conditional probability of the first collision if there are m SUs in the contention stage can be written as

$$\mathcal{P}_{c,k}^{(m)} = \sum_{j=2}^{m} \Pr\left(j \text{ users collide}\right)$$
$$= \sum_{j=2}^{m} \sum_{l=0}^{W_k-2} C_m^j \left(\frac{1}{W_k}\right)^j \left(\frac{W_k-l-1}{W_k}\right)^{m-j} (3)$$

where each term in the double-sum represents the probability that j users collide when they choose the same backoff value equal to l. Hence, the probability of the first collision can be calculated as

$$\mathcal{P}_{c,k} = \sum_{m=2}^{M_k^n} \mathcal{P}_{c,k}^{(m)} \times \Pr\left\{m \text{ users contend}\right\},\tag{4}$$

where  $M_k^n = |\mathcal{U}_k^n| + 1$  is the total number of SUs (including SU k and its neighbors),  $\mathcal{P}_{c,k}^{(m)}$  is given in (3) and  $\Pr\{m \text{ users contend}\}\$  is the probability that m SUs contend with SU k in the contention phase. To compute  $\mathcal{P}_{c,k}$ , we now derive  $\Pr\{m \text{ users contend}\}\$ .

We can divide the set of neighbors of SU k into two groups. In particular, there are m SUs contending with SU k while the remaining  $M_k^n - m$  SUs do not join the contention phase. There are  $C_{M_k^n}^m$  such combinations for a particular value of m where it happens with the following probability

$$\Pr\{m \text{ users contend}\} = \sum_{n=1}^{C_{M_n}^{m_n}} \mathcal{P}_{\text{con}}^{(n)}$$
(5)

where  $\mathcal{P}_{con}^{(n)}$  is the probability of one particular case where m SUs contend with SU k. We can divide the set of remaining  $M_k^n - m$  SUs who do not join the contention into two subgroups, namely SUs who could not find any available channels in their allocated channels  $\mathcal{S}_{i_2}^{\text{tot}}$  (first subgroup) and SUs who find some available channels in their separate sets  $\mathcal{S}_{i_1}$  (second subgroup).

Now, let  $\Lambda_n$  be one particular set of m SUs in the first group and  $A_1$  denote the number of SUs in the first subgroup of the remaining  $M_k^n - m$  SUs. Then, we can calculate  $\mathcal{P}_{con}^{(n)}$  as follows:

$$\mathcal{P}_{\rm con}^{(n)} = \prod_{i_1 \in \Lambda_n} \left[ \prod_{l_1 \in \mathcal{S}_{i_1}} \overline{p}_{i_1 l_1} \right] \tag{6}$$

$$\sum_{A_1=0}^{M_k^n - m} \sum_{c_1=1}^{C_{M_k^n - m}} \prod_{i_2 \in \Omega_{c_1}^{(1)}} \prod_{l_2 \in \mathcal{S}_{i_2}} \overline{p}_{i_2 l_2} \prod_{i_3 \in \Omega_{c_1}^{(2)}} \left( 1 - \prod_{l_3 \in \mathcal{S}_{i_3}} \overline{p}_{i_3 l_3} \right)$$
(7)

 $\sum_{n^{(1)}=1}^{\beta^{(1)}} \sum_{q^{(1)}=1}^{C_{\beta^{(1)}}^{n^{(1)}}} \cdots \sum_{n^{(m)}=1}^{\beta^{(m)}} \sum_{q^{(m)}=1}^{C_{\beta^{(m)}}^{n^{(m)}}} \prod_{i_4 \in \mathcal{U}_{c_1}^{p_1}} \prod_{l_4 \in \Lambda_{c_1}^{(1)}} p_{i_4 l_4}^p \prod_{l_5 \in \Lambda_{c_1}^{(2)}} \overline{p}_{i_4 l_5}^p.$ (8)

The term inside [.] in (6) represents the probability that all channels in the separate sets  $\mathcal{S}_{i_1}$  for all SUs  $i_1 \in \Lambda_n$  are not available so that these SUs contend to access available channels in  $\mathcal{S}_{i_1}^{\text{com}}$ . The term in (7) denotes the probability that each of  $A_1$  SUs in the first subgroup (i.e., in the set  $\Omega_{c_1}^{(1)}$ ) find no available channels in their separate sets and each of the  $M_k^n - m - A_1$  SUs in the second subgroup (i.e., in the set  $\Omega_{c_1}^{(2)}$ ) find at least one available channel in their separate sets (therefore, these SUs will not perform contention). Here,  $c_1$ is the index of one particular case where there are  $A_1$  SUs in the first subgroup and a particular set  $\Lambda_n$ . The last term in (8) denotes the probability of the event representing the status of all PUs who are neighbors of SUs in the set  $\mathcal{U}_{k}^{n}$  (i.e., neighbors of SU k) so that there are exactly m contending SUs in the set  $\Lambda_n$  and  $A_1$  SUs in the first subgroup. In (8) we consider all possible scenarios where for each SU  $i \in \Lambda_n$ , there are  $n^{(i)}$  available channels among  $\beta^{(i)} = |\mathcal{S}_i^{\text{com}}|$  channels in the set  $\mathcal{S}_i^{\text{com}}$  where  $q^{(i)}$  represents the index of one such particular case. Corresponding to such  $(n^{(i)}, q^{(i)}), \mathcal{U}_{c_1}^p$  denotes the set of PUs who are neighbors of SUs in  $\mathcal{U}_k^n$  so that indeed m underlying SUs perform contention.

By substituting  $\mathcal{P}_{con}^{(n)}$  calculated above into (5), we can calculate the collision probability in  $\mathcal{P}_{c,k}$  in (4). From this, we can determine  $W_k$  as follows:

$$W_k = \min \left\{ W_k \text{ such that } \mathcal{P}_{c,k}(W_k) \le \epsilon_{P_k} \right\}$$
(9)

where  $\epsilon_{P_k}$  controls the collision probability and overhead tradeoff and for clarity we denote  $\mathcal{P}_{c,k}(W_k)$ , which is given in (4) as a function of  $W_k$ . Then, we will determine the contention window for all SUs as  $W = \max_k W_k$ .

4) Calculation of MAC Protocol Overhead: Let r be the average value of the backoff value chosen by any SU. Then, we have r = (W - 1)/2 because the backoff counter value is uniformly chosen in the interval [0, W - 1]. As a result, average overhead can be calculated as follows:

$$\delta\left(W\right) = \frac{\left[W-1\right]\theta/2 + t_{\mathsf{RTS}} + t_{\mathsf{CTS}} + 3t_{\mathsf{SIFS}} + t_{\mathsf{SEN}} + t_{\mathsf{SYN}}}{\mathsf{T}_{\mathsf{cycle}}}$$

where  $\theta$  is the time corresponding to one backoff unit;  $t_{\text{RTS}}$ ,  $t_{\text{CTS}}$ ,  $t_{\text{SIFS}}$  are the corresponding time of RTS, CTS and SIFS (i.e., short inter-frame space) messages;  $t_{\text{SEN}}$  is the sensing time;  $t_{\text{SYN}}$  is the transmission time of the synchronization message; and  $T_{\text{cycle}}$  is the cycle time.

5) Overlapping Channel Assignment Algorithm: In the overlapping channel assignment algorithm described in Algorithm 2, we run Algorithm 1 to obtain the non-overlapping channel assignment solution in the first phase and perform overlapping channel assignments by allocating channels that have been assigned to a particular SU to other SUs in the second phase. We calculate the increase-of-throughput metric for all potential channel assignments that can improve the throughput of minimum-throughput SUs. To calculate the increaseof-throughput, we use the throughput analytical model in Subsection III-B2, where the MAC protocol overhead,  $\delta < 1$ is derived from III-B4. After running Algorithm 1 in the first phase, each SU *i* has the set of assigned non-overlapping channels,  $S_i$ , and it initiates the set of overlapping channels as  $\mathcal{S}_i^{\text{com}} = \emptyset$ ,  $i = 1, \dots, M_s$ . Recall that the set of all assigned channels for SU *i* is  $S_i^{\text{tot}} = S_i \cup S_i^{\text{com}}$ . Let  $S_{i^*}^{\text{Uni}}$  is the set of

# Algorithm 2 OVERLAPPING CHANNEL ASSIGNMENT

- 1: After running Algorithm 1, each SU *i* has  $S_i$ ,  $S_i^{\text{com}} = \emptyset$  and  $S_i^n$ ,  $i = 1, \dots, M_s$ .
- 2: continue := 1.
- 3: while continue = 1 do

4: Find 
$$T_{\min}$$
 and  $i^* = \operatorname*{argmin}_{i \in \{1, \dots, M_s\}} T_i^o$ .

5: 
$$\mathcal{S}_{i^*}^{\text{Uni}} = \bigcup_{l \in \mathcal{U}_{i^*}^n} \mathcal{S}_l^{\text{tot}}.$$

6: 
$$S_{i^*}^{\text{scp}} = \underset{l \in \mathcal{U}_{i^*}}{\text{SETXOR}} (S_l^{\text{tot}}).$$

7: 
$$\mathcal{S}_{i^*}^{\text{Int}} = \mathcal{S}_{i^*}^{\text{Um}} \setminus \mathcal{S}_{i^*}^{\text{sep}} \setminus \mathcal{S}_{i^*}^{\text{com}}$$

8: Find all minimum-throughput SUs and find the best channels from either  $S_{i^*}^{\text{Sep}}$  or  $S_{i^*}^{\text{Int}}$  for these minimum-throughput SUs to improve the overall minimum throughput.

9: if 
$$\bigcup_{i \in \{1,...,M_s\}} S_i^{\text{com,temp}} \neq \emptyset$$
 then  
0: Assign  $S_i^{\text{com}} = S_i^{\text{com,temp}}$  and  $S_i = S_i^{\text{temp}}$ .  
1: else  
2: Set continue := 0.  
3: end if

14: end while

all channels that have been assigned for SU *i*\*'s neighboring SUs. Also, let  $S_{i*}^{\text{Sep}}$  be the set of all channels assigned solely for each individual neighbor of SU *i*\* (i.e., each channel in  $S_{i*}^{\text{Sep}}$  is allocated for only one particular SU in  $U_{i*}^n$ ). Therefore,  $S_{i*}^{\text{Int}}$  defined in step 7 of Algorithm 2 is the set of "intersecting channels", which are shared by at least two neighbors of SU *i*\*. Here, SETXOR(**A**,**B**) would return the set of all elements in **A** or **B** but not the common elements of both **A** and **B**.

In each iteration, we determine the set of SUs which achieve the minimum throughput. Then, we need to search over two sets  $S_{i^*}^{\text{Sep}}$  or  $S_{i^*}^{\text{Int}}$  to find the best channel for each of these minimum-throughput SUs. Note that allocation of channels in  $S_{i^*}^{\text{Int}}$  to minimum-throughput SUs can indeed decrease the achievable throughput of their owners (i.e., SUs which own these channels before the allocation). Therefore, channel allocations in step 8 are only performed if the minimum throughput can be improved. In step 9,  $S_i^{\text{com,temp}}$  is the potential set of channels for SU *i*. Algorithm 2 terminates when there is no assignment that can improve the minimum throughput. Due to the space constraint, detailed description of step 8 is omitted.

6) Complexity Analysis: In each iteration of Algorithm 1, the number of minimum-throughput SUs is at most  $M_s$  and there are at most N channel candidates which can be allocated for each of them. Therefore, the complexity involved in each iteration is upper bounded by  $M_sN$ . We can also determine an upper bound for the number of iterations, which is  $M_sN$ . This is simple because each SU can be allocated at most N channels and there are  $M_s$  SUs. Therefore, the complexity of Algorithm 1 is upper bounded by  $M_s^2N^2$ . In Algorithm 2, we run Algorithm 1 in the first phase and perform overlapping channel assignments in the second phase. The complexity of this second phase can also be upper-bounded by  $M_s^2N^2$ . Therefore, the complexity of both Algorithms 1 and 2 can be upper-bounded by  $\mathcal{O}(M_s^2N^2)$ , which is much lower than that



Fig. 2. The scenario with 3 SUs and 2 PUs.



Fig. 3. Throughput versus the number of channels,  $p_{ij}^p = 0.6$  and 0.8, Non: Non-overlapping, Ove: Overlapping, The: Theory, Sim: Simulation, Opt:Optimal.(a)  $M_p = 2$ ,  $M_s = 3$  (b)  $M_p = 5$ ,  $M_s = 8$ 



Fig. 4. (a) Throughput versus  $p_{ij}^p$ , N = 7 and 9. (b) Throughput achieved by each SU,  $M_p = 5$ ,  $M_s = 8$ ,  $p_{ij}^p = 0.8$ , N = 8.

of the brute-force search algorithm presented in Section II-C.

#### **IV. NUMERICAL RESULTS**

To obtain numerical results, we choose the length of control packets as follows: RTS including PHY header 288 bits, CTS including PHY header 240 bits, which correspond to  $t_{\text{RTS}} = 48\mu s$ ,  $t_{\text{CTS}} = 40\mu s$  for transmission rate of 6 Mbps, which is the basic rate of 802.11a/g standards [14]. Other parameters are chosen as follows: cycle time  $T_{\text{cycle}} = 3ms$ ;  $\theta = 20\mu s$ ,  $t_{\text{SIFS}} = 28\mu s$ , target collision probability  $\epsilon_P = 0.03$ ;  $t_{\text{SEN}}$  and  $t_{\text{SYN}}$  are assumed to be negligible so they are ignored. Note that these values of  $\theta$  and  $t_{\text{SIFS}}$  are typical (e.g., see [10]).

To compare the performance of optimal brute-force search and our proposed algorithms, we consider a small network shown in Fig. 2 where we choose  $M_s = 3$  SUs,  $M_p = 2$  PUs and  $p_{ij}^p = 0.6$  and 0.8. Fig. 3(a) shows that the minimum throughputs achieved by Algs 2 are very close to that obtained the optimal search, which confirms the merit of this lowcomplexity algorithm. Also, the simulation results match the analytical results very well, which validates the proposed throughput analytical model. Figs. 3(b), 4(a), and 4(b) illustrate the minimum throughputs achieved by our proposed algorithms for a larger network shown in Fig. 1. In particular, Fig. 3(b) shows the minimum throughput versus the number of channels for  $p_{ij}^p$  equal to 0.6 and 0.8. This figure confirms that Alg. 2 achieves significantly larger throughput than that due to Alg. 1 thanks to overlapping channel assignments.

Fig. 4(a) illustrates the minimum throughput versus  $p_{ij}^p$ . It can be observed that as  $p_{ij}^p$  increases, the minimum achievable throughput indeed increases. This figure also shows that the minimum throughput for N = 9 is greater than that for N = 7. This means our proposed algorithms can efficiently exploit available spectrum holes. In Fig. 4(b), we illustrate the throughputs achieved by different SUs to demonstrate the fairness performance. It can be observed that the differences between the maximum and minimum throughputs under Alg. 2 are much smaller than that due to Alg. 1. This result implies that Alg. 2 not only achieves better throughput but also results in improved fairness compared to Alg. 1.

## V. CONCLUSION

We have investigated the fair channel allocation problem in cognitive ad hoc networks. Specifically, we have presented both optimal brute-force search and low-complexity algorithms and analyzed their complexity and throughput performance through analytical and numerical studies.

#### REFERENCES

- T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys Tutorials*, vol. 11, no. 1, pp. 116–130, 2009.
- [2] H. Su, and X. Zhang, "Cross-layer based opportunistic MAC protocols for QoS provisionings over cognitive radio wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 118–129, Jan. 2008.
- [3] H. Nan, T.-I. Hyon, and S.-J. Yoo, "Distributed coordinated spectrum sharing MAC protocol for cognitive radio," in *IEEE DySPAN*'2007.
- [4] C. Cordeiro, and K. Challapali, "C-MAC: A cognitive MAC protocol for multi-channel wireless networks," in *IEEE DySPAN*'2007.
- [5] Y.R. Kondareddy, and P. Agrawal, "Synchronized MAC protocol for multi-hop cognitive radio networks," in *Proc. IEEE ICC*'2008.
- [6] H. Su and X. Zhang, "Channel-hopping based single transceiver MAC for cognitive radio networks," in *Proc. CISS*'2008.
- [7] M. Timmers, S. Pollin, A. Dejonghe, L. Van der Perre, and F. Catthoor, "A distributed multichannel MAC protocol for multihop cognitive radio networks," *IEEE Trans. Veh. Tech.*, vol. 59, no. 1, pp. 446–459, Jan. 2010
- [8] S. C. Jha, U. Phuyal, M. M. Rashid, and V. K. Bhargava, "Design of OMC-MAC: An opportunistic multi-channel MAC with QoS provisioning for distributed cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 10, pp. 3414–3425, Oct. 2011.
- [9] L. T. Tan and L. B. Le, "Channel assignment for throughput maximization in cognitive radio networks," in *Proc. IEEE WCNC'2012*.
- [10] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," in *IEEE J. Sel. Areas Commun.*, vol. 18, no. 3, pp. 535-547, Mar. 2000.
- [11] J. Mo, H. So, and J. Walrand, "Comparison of multichannel MAC protocols," *IEEE Trans. Mobile Computing*, vol. 7, no. 1, pp. 50–65, Jan. 2008.
- [12] J. Lee and S. Leyffer eds., "Mixed integer nonlinear programming," *The IMA Volumes in Mathematics and its Applications*, vol. 154, Springer, 2012.
- [13] L. Tassiulas and S. Sarkar, "Maxmin fair scheduling in wireless ad hoc networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 163–173, Jan. 2005.
- [14] 802.11 Standard. Online: http://www.ieee802.org/11