Compressive Sleeping Wireless Sensor Networks With Active Node Selection

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***Abstract*—In this paper, we propose an active node selection framework for compressive sleeping wireless sensor networks (WSNs) in order to improve the signal acquisition performance and network lifetime. The node selection can be seen as a specialized sensing matrix design problem where the sensing matrix consists of selected rows of an identity matrix. By capitalizing on a genie-aided reconstruction procedure, we formulate the active node selection problem into an optimization problem, which is then approximated by a constrained convex relaxation plus a rounding scheme. The proposed approach also exploits the partially known signal support, which can be obtained from the previous signal reconstruction. Simulation results show that our proposed active node selection approach leads to an improved reconstruction performance and network lifetime in comparison to various node selection schemes for compressive sleeping WSNs.**

I. Introduction

OVER the past two decades, the rapid development of technologies in sensing, computing and communication has made it possible to employ wireless sensor networks (WSNs) to continuously monitor physical phenomena in a variety of applications, for example air quality monitoring, wildlife tracking, biomedical monitoring and disaster detection. As the number and the resolution of the sensors grow, the performance bottleneck is the sensor node (SN), which usually has limited battery power, memory, computational capability, wireless bandwidth and physical size [1], [2].

Compressive sensing (CS) [3], [4] enables one to reconstruct compressible signals from a small set of linear measurements. It leverages the compressibility of natural signals to trade-off the convenience of data acquisition against the computational complexity of data reconstruction. Thus, it is suitable for data acquisition and reconstruction in a WSN which is typically constituted of a smart fusion center (FC) with high power and computational capability and several dumb front-end SNs with limited energy storage and computing capability [5]–[8].

Conventional CS exploits the signal representation with a sparse structure and applies random measurements. The use of additional signal knowledge such as a part of the signal support enables one to further reduce the number of measurements required for reconstruction. For example, Vaswani and Lu use the support estimate from the previous time instant of a time sequence of sparse spatial signals as the known part and recursively reconstruct the time sequence signals in [9], [10]; Chen et al. use a Fréchet mean approach to estimate the signal support from multiple correlated signals and then leverage the support estimate to enhance the reconstruction in [7]. The recovery conditions for signal reconstruction with partially known support is analyzed in [11]. Furthermore, the CS performance can be enhanced by replacing the random sensing matrices with optimized ones. Various principles have been applied to optimize the sensing matrices in [12]–[14], and all of them lead to an improved performance compared with the use of random sensing matrices.

In this paper, we focus on a scenario where signals detected by a group of SNs distributed in the field have high spatial correlations. We propose a novel active node selection framework, where a subset of the SNs are selected to sense and communicate to the FC, while the remainder are in the sleep mode for saving energy, and the signals are reconstructed at the FC following the CS principle. As compared to the random node selection employed in conventional compressive sleeping WSNs [5], the proposed active node selection is optimized by exploiting the partially known signal support, and thus leads to an improved performance and network lifetime. Our contributions can be summarized as follows:

- We propose a novel active node selection framework for the compressive sleeping WSNs, where the FC performs an optimized selection of SNs. Node selection can be seen as a special sensing matrix design problem where the sensing matrix consists of selected rows of an identity matrix, and consequently none of the existing approaches for sensing matrix optimization in the literature [12]–[14] can be directly applied to solve this problem.
- We formulate the node selection as an optimization problem which aims to improve the reconstruction performance for a certain number of active SNs (or conversely, to reduce the number of samples for a certain target reconstruction quality) and to avoid too frequently selecting any particular SN as to prolong network lifetime.
- We approximate the problem of active node selection by a constrained convex relaxation plus a rounding scheme.

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The related convex problem can be solved by efficient iterative algorithms.

The following notation is used. Lower-case and upper-case letters denote numbers, boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and calligraphic upper-case letters denote sets. The superscripts $(\cdot)^T$ and $(\cdot)^{-1}$ denote the transpose and the inverse of a matrix, respectively. The trace of a matrix is denoted by $\mathtt{Tr}(\cdot)$. x_i denotes the ith element of $\mathbf{x}$, $X_{i,i}$ denotes the ith diagonal element of $\mathbf{X}$, and $\mathbf{X}_{\mathcal{J}}$ denotes the submatrix of $\mathbf{X}$ by selecting columns with indexes in the set $\mathcal{J}$. $\mathcal{J}^c$ denotes the complement of set $\mathcal{J}$. $\mathbb{E}_{\mathbf{x}}(\cdot)$ denotes expectation with respect to the distribution of the random vector $\mathbf{x}$. $\binom{n}{m}$ denotes the number of m combinations from a given set of n elements. $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denotes the multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. The ℓ_0 norm, the ℓ_1 norm, and the ℓ_2 norm of vectors, are denoted by $\|\cdot\|_0$, $\|\cdot\|_1$, and $\|\cdot\|_2$, respectively. The Frobenius norm of a matrix $\mathbf{X}$ is denoted by $\|\mathbf{X}\|_F$.

II. Signal Acquisition Via Compressive Sleeping WSNs

A. Conventional Compressive Sensing

We consider the conventional CS model where a discrete signal $\mathbf{f} \in \mathbb{R}^n$ is measured by linear projections, given by:

$$\mathbf{y} = \boldsymbol{\Phi}\mathbf{f} + \mathbf{z}, \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^m$ $(m < n)$ denotes the vector of measurements, $\boldsymbol{\Phi} \in \mathbb{R}^{m \times n}$ denotes the projection matrix, and $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_m)$ denotes the noise term for the measuring process. The discrete signal $\mathbf{f}$ is assumed with a sparse representation $\mathbf{x} \in \mathbb{R}^n$ under some basis $\boldsymbol{\Psi} \in \mathbb{R}^{n \times n}$, which can be written as $\mathbf{f} = \boldsymbol{\Psi}\mathbf{x}$. Here, sparse means that only s $(s \ll n)$ elements in vector $\mathbf{x}$ are non-zeros while all the other elements are zeros, i.e., $\|\mathbf{x}\|_0 = s$. Then the CS model can be described as

$$\mathbf{y} = \boldsymbol{\Phi}\boldsymbol{\Psi}\mathbf{x} + \mathbf{z} = \mathbf{A}\mathbf{x} + \mathbf{z}, \tag{2}$$

where $\mathbf{A} = \boldsymbol{\Phi}\boldsymbol{\Psi} \in \mathbb{R}^{m \times n}$ denotes the equivalent sensing matrix.

The typical signal reconstruction process behind conventional CS approaches involves solving the following optimization problem to recover the original signal:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}\|_1 \\ \text{s.t.} \quad & \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 \le \epsilon, \end{aligned} \tag{3}$$

where ϵ is an estimate of the noise level. It has been demonstrated that only $m = \mathcal{O}(s \log \frac{n}{s})$ measurements [15] are required for robust reconstruction in the CS framework.

A theoretical question in CS is what conditions should the equivalent sensing matrix $\mathbf{A}$ satisfy in order to reconstruct the signal representation $\mathbf{x}$. The most popular conditions used for evaluating the equivalent sensing matrix includes the null space property, the restricted isometry property (RIP) and mutual coherence property [16]. In comparison to the null space property and the RIP, which are computationally intractable, the calculation of the mutual coherence of a matrix is relatively inexpensive, which is given by:

$$\mu = \max_{1 \le i,j \le n, i \ne j} \frac{|\mathbf{A}_i^T \mathbf{A}_j|}{\|\mathbf{A}_i\|_2 \cdot \|\mathbf{A}_j\|_2}. \tag{4}$$

In [16], Donoho et al. demonstrated that if $\mu < \frac{1}{4s-1}$ and $\mathbf{A}$ has normalized columns, then the error of the reconstructed signal representation $\hat{\mathbf{x}} \in \mathbb{R}^n$ in (3) is upper-bounded by

$$\|\hat{\mathbf{x}} - \mathbf{x}\|_2^2 \le \frac{(\epsilon + \hat{\epsilon})^2}{1 - \mu(4s-1)}, \tag{5}$$

where $\hat{\epsilon} \ge \|\mathbf{z}\|_2^2$. Thus, it is desired to use an equivalent sensing matrix with small mutual coherence, which leads to an improved reconstruction error bound in (5).

In addition to the mutual coherence, which is the maximum coherence of all column pairs, various coherence properties of the equivalent sensing matrix are applied to achieve an optimized projection matrix design in the literature. For example, to obtain an optimized projection matrix with improved reconstruction performance, Elad propose to reduce the coherence of the t largest column pairs [12], and the summation of the square of all the coherence values of the column pairs is considered in [13]. In [14], we propose to use tight frames as the optimal equivalent sensing matrix, where the coherence of column pairs also plays an important role in deriving the design.

B. Compressive Sensing With Partially Known Support

The number of projections required to recover the original signal can be reduced if some additional knowledge beyond the sparse signal structure is given. For signals changing slowly in time, the partially known support information can be obtained from reconstruction of the signal in the previous time slot. The support can also be estimated according to the signal sparsifying model, e.g., the importance of the components of the signal representation under principal component analysis (PCA) are in decreasing order [17].

The modified CS reconstruction proposed in [9], [10] aims to find a signal whose support contains the smallest number of nonzero elements out of support $\mathcal{J}$. The reconstruction process involves solving the following optimization problem to recover the original signal representation:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \|\mathbf{x}_{\mathcal{J}^c}\|_1 \\ \text{s.t.} \quad & \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 \le \epsilon. \end{aligned} \tag{6}$$

Compared with the conventional CS reconstruction in (3), the modified CS reconstruction in (6) has better recovery performance owing to the employment of additional information, i.e., the partially known signal support $\mathcal{J}$. The gain of the modified CS reconstruction is affected by the accuracy of the information of the partial support $\mathcal{J}$ regarding the actual signal support [9], [10].

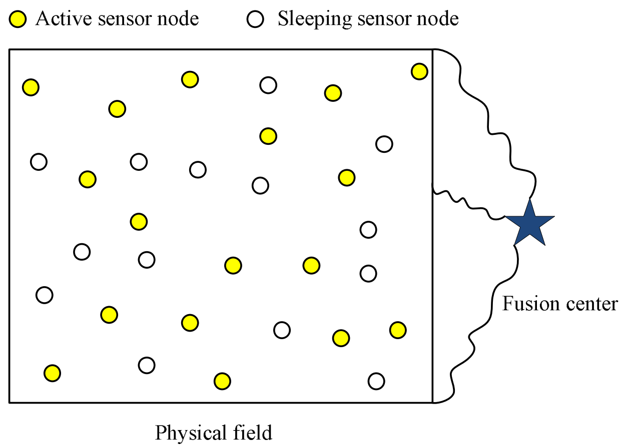

Fig. 1. A WSN with single hop communication.

C. Compressive Sleeping WSNs

Typical WSNs consist of a large number of SNs distributed in the field to collect information of interest for geographical monitoring, industrial monitoring, security and climate monitoring. In these WSNs, signals sensed in the physical field usually have high spatial correlations, and the signals can be represented as sparse vectors under some basis, which can be determined a-priori, or by an adaptive approach updated via PCA [17] or by dictionary learning.

We consider a WSN architecture for collecting physical field information with n SNs and a fusion center (FC) as shown in Fig. 1. SN i $(i = 1, \ldots, n)$ has a monitored parameter f_i to report to the FC. We also consider a node sleeping strategy as in [5] such that only m $(m < n)$ nodes are active and all the others nodes are turned off in order to reduce the energy consumption. The original signal $\mathbf{f} \in \mathbb{R}^n$, which represents the physical field information is then reconstructed at the FC using the measurements of active SNs. The SNs are activated to collect and transmit data in a demand-driven manner, i.e., triggered by a request from the FC[1].

III. The Active Node Selection Framework

In this section, we provide the active node selection framework for compressive sleeping WSNs in order to minimize the total number of SNs needed be activated. The proposed approach, in conjunction with the consideration of network characteristics, can be applied to improve the network lifetime, which will be given in Section V.

A. Problem Formulation

In the compressive sleeping WSN, the entries of the projection matrix $\mathbf{\Phi} \in \mathbb{R}^{m\times n}$ are all zeros except for m unity entries for different rows, and the column indexes of these unity entries corresponds to the SNs' states, i.e., sleeping or active. For example, if SN i is active, then there is a unity element in the ith column of $\mathbf{\Phi}$, otherwise all the elements of the ith column are zeros. Therefore, the projection matrix $\mathbf{\Phi}$ represents the states of SNs.

We define an $n \times n$ diagonal matrix $\tilde{\mathbf{\Phi}}$ such that

$$\tilde{\Phi}_{i,i} = \begin{cases} 1, & \text{SN } i \text{ is activated} \\ 0, & \text{otherwise} \end{cases}. \tag{7}$$

Then, $\tilde{\mathbf{\Phi}}$ can be written as a row-permutation of the concatenation of the sensing matrix $\mathbf{\Phi}$ and a matrix of $n-m$ rows of zeros, which is given by

$$\tilde{\mathbf{\Phi}} = \mathbf{\Pi} \begin{bmatrix} \mathbf{\Phi} \\ \mathbf{0}_{(n-m)\times n} \end{bmatrix}, \tag{8}$$

where $\mathbf{\Pi}$ is a row-permutation matrix, and $\mathbf{\Phi}^T\mathbf{\Phi} = \tilde{\mathbf{\Phi}}$.

The goal of the proposed framework relates to the minimization of the mean square error (MSE) and penalty of the node selection subject to appropriate constraints, which is given as follows

$$\min_{\tilde{\Phi}_{i,i}} \quad \mathbb{E}_{\mathbf{z}}\left(\|\mathcal{F}(\mathbf{y}, \tilde{\mathbf{\Phi}}, \mathbf{\Psi}, \mathcal{J}) - \mathbf{x}\|_2^2\right) + \beta \mathtt{Tr}(\tilde{\mathbf{\Phi}}\mathbf{P}) \tag{9a}$$

$$\text{s.t.} \quad \tilde{\Phi}_{i,i} \in \{0, 1\}, \ i = 1, \ldots, n, \tag{9b}$$

$$\mathtt{Tr}(\tilde{\mathbf{\Phi}}) = m, \tag{9c}$$

where $\mathcal{F}(\cdot)$ denotes an estimator of the signal sparse representation, $\mathbf{P} \in \mathbb{R}^{n\times n}$ is a diagonal matrix with $P_{i,i} \geq 0$ and $\beta > 0$.

The term $\mathtt{Tr}(\tilde{\mathbf{\Phi}}\mathbf{P})$ in (9a) denotes the penalty function of the weighted active node selection matrix $\tilde{\mathbf{\Phi}}\mathbf{P}$ where weight $P_{i,i}$ $(i = 1, \ldots, n)$ is the penalty for selecting SN i. Various design targets could result in different interpretations of the penalty term. For example, SNs could consume different amounts of energy to communicate their readings to the FC owing to the distinct transmission pathloss, where the penalty term $\tilde{\mathbf{\Phi}}\mathbf{P}$ could reflect the constraint on the total energy consumed by the selected SNs by setting $P_{i,i}$ to be the energy consumption of the ith SN. For another example, it could be desirable to wake up those SNs having adequate levels of stored energy rather than those SNs already low in stored energy, so an SN with inadequate energy is associated with a relatively large penalty. In this paper, we exploit this penalty term with the aim of prolonging the network lifetime, and this approach will discussed in more detail in Section IV.

The derivation of such a node selection design is very difficult though, because the squared reconstruction error term in (9a) depends upon the actual estimator. In view of the lack of a closed-form tractable squared reconstruction error expression for actual estimators, we use a genie-aided reconstruction procedure that is assumed to know the actual sparse representation support and performs least squares (LS) estimation based on prior knowledge of the support. The genie-aided reconstruction has been used for the design of projection matrices [14] and in the analysis of the performance of various reconstruction approaches [19], [20]. Here we assume that the estimated signal representation support $\mathcal{J}$ is the actual support and is used in the genie-aided reconstruction performance.

[1]For example, by using a duty-cycle MAC protocol [18], SNs can periodically wake up to listen to broadcast messages from the FC that list the active nodes.

With the use of the signal representation support $\mathcal{J}$, the solution of the LS estimation can be given by

$$\begin{aligned}\hat{\mathbf{x}} =&(\mathbf{\Psi}_{\mathcal{J}}^T\mathbf{\Phi}^T\mathbf{\Phi}\mathbf{\Psi}_{\mathcal{J}})^{-1}\mathbf{\Psi}_{\mathcal{J}}^T\mathbf{\Phi}^T\mathbf{y}\\ =&(\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}})^{-1}\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}\mathbf{x} + (\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}})^{-1}\mathbf{\Psi}_{\mathcal{J}}^T\mathbf{\Phi}^T\mathbf{z},\end{aligned} \tag{10}$$

and the MSE of the LS estimation is

$$\mathbb{E}_{\mathbf{z}}\big(\|\hat{\mathbf{x}}-\mathbf{x}\|_2^2\big) = \sigma^2\mathtt{Tr}\big((\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}})^{-1}\big). \tag{11}$$

According to the original active node selection problem in (9), we put forth the following optimization problem:

$$\begin{aligned}\min_{\tilde{\Phi}_{i,i}} \quad & \mathtt{Tr}\big((\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}})^{-1}\big) + \beta\mathtt{Tr}(\tilde{\mathbf{\Phi}}\mathbf{P})\\ \text{s.t.} \quad & \tilde{\Phi}_{i,i} \in \{0,1\},\ i=1,\dots,n,\\ & \mathtt{Tr}(\tilde{\mathbf{\Phi}}) = m.\end{aligned} \tag{12}$$

Unfortunately, the optimization problem in (12) is non-convex. As the variables $\tilde{\Phi}_{i,i}$ are binary integers, a straightforward way to solve (12) is to perform an exhaustive search over $\binom{n}{m}$ different combinations of m active nodes. However, the complexity of the exhaustive search is impractical for a large number of SNs.

B. Active Node Selection Via Convex Relaxation

In this subsection, we formulate the active node selection problem in (12) as a relaxed convex optimization problem that can be solved efficiently using numerical methods such as interior-point algorithms. To simplify the original problem, we relax the binary integer constraints so that $\tilde{\Phi}_{i,i}$ can be in the range from 0 to 1. Then the relaxed problem can be expressed as follows

$$\min_{\tilde{\Phi}_{i,i}} \quad \mathtt{Tr}\big((\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}})^{-1}\big) + \beta\mathtt{Tr}(\tilde{\mathbf{\Phi}}\mathbf{P}) \tag{13a}$$

$$\text{s.t.} \quad 0 \le \tilde{\Phi}_{i,i} \le 1,\ i=1,\dots,n, \tag{13b}$$

$$\mathtt{Tr}(\tilde{\mathbf{\Phi}}) = m. \tag{13c}$$

Proposition 1: Let $\mathbf{\Psi}_{\mathcal{J}}$ be an $n \times s$ matrix with rank s and $\tilde{\Phi}_{i,i}$ $(i=1,\dots,n)$ be nonnegative numbers. Then the objective function in (13a) is convex in $\tilde{\Phi}_{i,i}$ $(i=1,\dots,n)$.

Proof: We first prove that the expression $\mathtt{Tr}\big((\mathbf{X})^{-1}\big)$ is convex in $\mathbf{X}$ if $\mathbf{X}$ is a positive definite symmetric matrix. Let

$$g(t) = \mathtt{Tr}\big((\mathbf{X}+t\mathbf{V})^{-1}\big), \tag{14}$$

where $\mathbf{X} \succ 0$ and $\mathbf{V}$ is a symmetric matrix. Now we can rewrite $g(t)$ as

$$\begin{aligned}g(t) &= \mathtt{Tr}\big((\mathbf{X}+t\mathbf{V})^{-1}\big)\\ &= \mathtt{Tr}\big(\mathbf{X}^{-1} - t(\mathbf{X}+t\mathbf{V})^{-1}\mathbf{V}\mathbf{X}^{-1}\big)\\ &= \mathtt{Tr}\big(\mathbf{X}^{-1} - t\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1} + t^2\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1}\\ &\quad - t^3\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1} + \dots\big),\end{aligned} \tag{15}$$

where the first equality can be proved by using the Searle set of identities [21]. Then we have the second derivative that can be derived as

$$\lim_{t\to 0}\frac{\partial^2 g(t)}{\partial t^2} = \mathtt{Tr}\big(\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1}\mathbf{V}\mathbf{X}^{-1}\big) = \mathtt{Tr}\big(\mathbf{W}^T\mathbf{X}^{-1}\mathbf{W}\big), \tag{16}$$

where $\mathbf{W} = \mathbf{V}\mathbf{X}^{-1}$. As $\mathbf{X}$ is positive definite, then we have $\mathbf{W}^T\mathbf{X}^{-1}\mathbf{W} \succ 0$. Therefore, $\mathtt{Tr}\big((\mathbf{X})^{-1}\big)$ is convex in $\mathbf{X}$.

As $\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}}$ is invertible and $\tilde{\Phi}_{i,i}$ $(i=1,\dots,n)$ are nonnegative, we have $\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}} \succ 0$. Thus, we can conclude that $\mathtt{Tr}\big((\mathbf{\Psi}_{\mathcal{J}}^T\tilde{\mathbf{\Phi}}\mathbf{\Psi}_{\mathcal{J}})^{-1}\big)$ is convex in $\tilde{\Phi}_{i,i}$ $(i=1,\dots,n)$ in view of the fact that $\mathtt{Tr}\big((\mathbf{X})^{-1}\big)$ is convex in $\mathbf{X}$ if $\mathbf{X}$ is a positive definite and the concavity of a function is preserved under an affine transformation. ∎

This relaxation of an optimization problem with binary integers to a convex form makes the problem much easier to solve than the original integer program. After finding the solution of (13), the m largest $\tilde{\Phi}_{i,i}$ can be chosen and the corresponding indexes relate to the selected nodes. This relaxation has also been used for antenna selection in multi-antenna wireless communication systems [22], and sensor selection in parameter estimation [23].

Note once again that the proposed node selection framework for compressive sleeping WSNs capitalizes on the genie-aided reconstruction procedure, while the support information is only partially known in practical reconstructions. The effect of the estimation accuracy regarding the actual support is investigated in Section V.

IV. Prolonging The Network Lifetime

In this section, we apply the proposed active node selection to improve the lifetime of compressive sleeping WSNs.

Network lifetime is determined by the time instant when the network cannot support application-specific functions. It can be the instant when the first SN runs out of energy, a specified fraction of SNs run out of energy, or the network partitions. In [24], Chen and Zhao study the network lifetime which applies to any definition of the network lifetime and holds independently of the underlying network model, and derive the average network lifetime $\mathbb{E}(L)$ formula, which is given by

$$\mathbb{E}(L) = \frac{\xi_0 - \mathbb{E}(E_u)}{P_c + \rho\mathbb{E}(E_t)}, \tag{17}$$

where ξ_0 is the initial total network energy, P_c is the constant continuous power consumption over the whole network, $\mathbb{E}(E_u)$is the average total unused energy in the network when it fails, ρ is the average data reporting rate, and $\mathbb{E}(E_t)$ is the average transmission energy consumed by all sensors.

According to (17), in order to prolong the network lifetime, one desires to reduce the unused energy and the transmission energy consumption. Define ξ_i and η_i as the required energy for transmission of a message and the node energy storage for SN i, respectively. Chen and Zhao propose a greedy node selection approach that the SN with maximum value of $\eta_i - \xi_i$ is selected. In this paper, we formulate the penalty $P_{i,i}$ by

$$P_{i,i} = -\frac{\eta_i}{\xi_i} \tag{18}$$

This penalty promotes the usage of an SN with large energy storage η_i and/or small transmission cost ξ_i. This penalty expression, in conjunction with the proposed framework (12), aims to prolong the network lifetime and improve the signal reconstruction performance. The SNs associated with the m largest $\tilde{\Phi}_{i,i}$ of the following problem are selected to be activated

$$\begin{aligned} \min_{\tilde{\Phi}_{i,i}} \quad & \mathtt{Tr}\big((\mathbf{\Psi}_{\mathcal{J}}^T \tilde{\mathbf{\Phi}} \mathbf{\Psi}_{\mathcal{J}})^{-1}\big) - \beta \sum_{i=1}^{n} \frac{\eta_i \tilde{\Phi}_{i,i}}{\xi_i} \\ \text{s.t.} \quad & 0 \le \tilde{\Phi}_{i,i} \le 1, \ i = 1, \ldots, n, \\ & \mathtt{Tr}(\tilde{\mathbf{\Phi}}) = m. \end{aligned} \tag{19}$$

V. Performance Results

We now compare the performance of proposed node selection approaches with other approaches in compressive sleeping WSNs using synthetic data. We consider a WSN with $n = 80$ SNs, and we generate the sparse signal representations $\mathbf{x}$ randomly for different time slots, where the non-zero components are drawn from independent identically distributed (i.i.d.) Gaussian distributions $\mathcal{N}(0, 1)$. The equivalent sensing matrices $\mathbf{A}$ are also generated randomly for different time slots, where the elements are drawn from the i.i.d. Gaussian distribution $\mathcal{N}(0, 1)$. The received measurements are corrupted by additive zero-mean Gaussian noise with variance 0.01. We assume all the SNs consume the same amount of energy to transmit one measurement to the FC[2], i.e., $\xi_1 = \ldots = \xi_n$. The signals are reconstructed by solving the conventional ℓ_1 minimization[3] in (3), and the results are averaged over 1000 trials.

The following approaches are compared:

1) Random selection: the active SNs are selected randomly;
2) Chen and Zhao's approach: SNs are selected by the value of remaining energy storage for transmission, i.e., $\eta_i - \xi_i$;
3) Proposed approach: SNs are selected via solving the optimization problem in (19).

We first evaluate the signal reconstruction accuracy using the root-mean-square error (RMSE), i.e., $\mathbb{E}\big(\|\hat{\mathbf{x}} - \mathbf{x}\|_2^2\big)$, for different active node selection approaches without considering the penalty relating to the network lifetime, i.e., $\beta = 0$. We consider both the ideal support estimate, i.e., the estimate is exactly the actual signal support, and two non-ideal scenarios where 20% and 50% elements of the support estimate are incorrect respectively. Fig. 2 shows that the proposed active node selection approach outperforms the random selection and Chen and Zhao's approach in terms of the reconstruction accuracy, i.e., the RMSE, for various numbers of activated SNs in each time slot. We also observe that the gain of the proposed approach is affected by the accuracy of the support estimate. For example, 38 SNs are required to be activated to achieve an RMSE smaller than 0.02 with 20% incorrect support estimate, while only 34 SNs are required in the case of an ideal support estimate.

[2]This assumption is motivated by the fact that the PHY/MAC layers are not adapting the modulation, coding and retransmission strategies during the active time (which is the case with the low-energy IEEE 802.15.4 PHY and typical MAC-layers such as nullMAC in the Contiki operating system). One could also apply other energy consumption model for transmission. However, it is sufficient to provides insights on the comparison of various approaches via this model.

[3]Note that the conventional ℓ_1 minimization does not exploit the support estimate and thus the gain of the proposed approach results from the node selection rather than the use of an advanced algorithm. With the use of the modified CS reconstruction in (6), additional gains can be obtained.

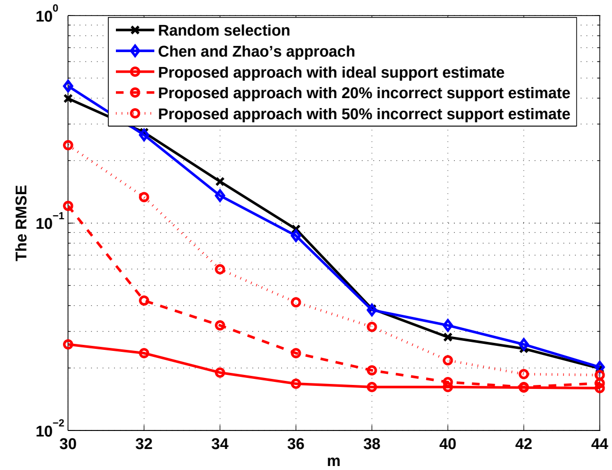

Fig. 2. The RMSE performance of different active node selection approaches vs. number of activated SNs ($\beta = 0$).

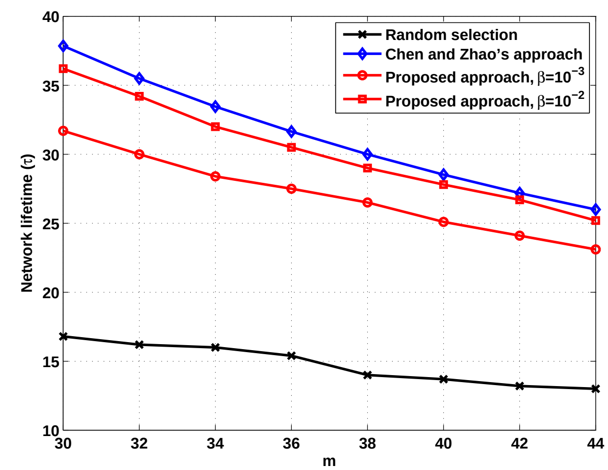

Fig. 3. The network lifetime vs. the number of activated SNs m (non-ideal support estimate).

We also investigate the network lifetime for different node selection schemes in compressive sleeping WSNs. In this experiment, We define the network lifetime by the time instant when the first SN fails and ignore the energy consumed by SNs in the sleeping mode. The initial battery storage of each SN is randomly generated by following a uniform distribution in the range of $[10\tau,\ 20\tau]$, where τ is the amount of energy consumed in one activation time slot for each of the SNs. The network lifetime is reached when any one of the SNs runs out of energy. We consider the non-ideal support estimate case where only 50% elements of the support estimate are correct. Fig. 3 shows the network lifetime for different numbers of active SNs. The network lifetime tends to decrease with a

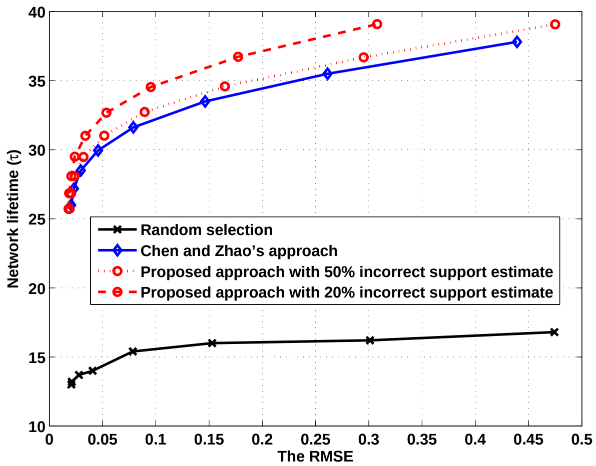

Fig. 4. The trade-off between the RMSE and the network lifetime. ($\beta = 10^{-3}$)

growing number of active SNs. In comparison to the random node selection, the network lifetime has been significantly improved by using the proposed active node selection approach. In addition, it is observed that the lifetime gain is also affected by the weight β that represents the level of penalty for selecting SNs with different levels of energy storage. Chen and Zhao's approach, which aims to prolong network lifetime without considering reconstruction performance, achieves the longest network lifetime.

We now illustrate the trade-off between the network lifetime and the signal reconstruction accuracy, in particular, the RMSE, for compressive sleeping WSNs. With different numbers of activated SNs m, we have distinct pairs of the RMSE and the network lifetime, which are plotted in Fig. 4. We note that the network lifetime tends to increase with rising levels of RMSE, i.e., one has to pay the cost of a reduced network lifetime in order to increase the signal reconstruction accuracy. We observe that the proposed approach leads to the best lifetime-performance curve in comparison to random selection and Chen and Zhao's approach. Fig. 4 also shows that the gain of the proposed approach is affected by the accuracy of the support estimation.

VI. Conclusion

In this paper, we propose a novel active node selection framework for compressive sleeping WSNs to improve the signal acquisition performance and network lifetime. The proposed node selection is performed at the FC and can be solved efficiently by iterative algorithms. The superiority of our proposed approach in relation to the random node selection in the conventional CS framework is revealed by our experimental study.

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