On the Performance of Network-Assisted Device-to-Device Discovery

Dionysis Xenakis, Marios Kountouris, Lazaros Merakos, Nikos Passas, and Christos Verikoukis

***Abstract*—Device-to-Device (D2D) communications enable direct exchange of localized traffic between cellular users in proximity. Nonetheless, the employment of D2D communications heavily relies on the capability of the cellular devices to infer on their proximity. In this paper, we analyze the performance of network-assisted D2D discovery in random cellular networks and derive optimal deployment strategies for transforming the today's cellular network, which is optimized for coverage and capacity, into a D2D-centric network where the discovery and communications between cellular devices will be orchestrated by the core network. In this direction, we develop an analytical framework that exploits existing knowledge of the cellular network topology and effectively estimates the probability that two tagged devices are in proximity, a.k.a. D2D discovery probability. Also, we identify conditions under which the D2D discovery probability is maximized and assess the key performance tradeoffs inherent to network-assisted D2D discovery.**

***Index Terms*—Device-to-device (D2D) discovery, network-assistance, Poisson point process, core network.**

I. Introduction

Device-to-device (D2D) communications have recently drawn significant attention due to the unprecedented demand for direct exchange of localized traffic between nearby cellular devices. Relevant applications span from social networking and public safety communications to cellular offloading and localized exchange of control data in the Smart Grid [1]. Two functions are instrumental for proximity-based communications: D2D discovery and D2D communications. D2D discovery involves the process by which a D2D-enabled user detects other D2D-enabled devices in its vicinity, while D2D communications include the utilization of a physical link between D2D-enabled users to directly exchange data without routing packets through the cellular access network.

D2D discovery and communications are within the scope of the 3rd Generation Partnership Project (3GPP) for the Long Term Evolution (LTE) Release 12 system [2], a.k.a. LTE-Advanced. 3GPP focuses on two types of D2D discovery: direct and network-assisted D2D discovery. The direct discovery is solely based on the capabilities of the D2D-enabled devices to autonomously discover, or indicate their presence to, other D2D-enabled devices without network involvement. In the contrary, network-assisted discovery, often referred to as Evolved Packet Core (EPC)-level discovery, relies on the core network's capability to determine the proximity of the D2D-enabled devices. Both methods have their own advantages and unique challenges. On the one hand, direct D2D discovery can be deployed even when the D2D-enabled devices are outside network coverage, whereas network-assisted discovery enables network operators to better handle the energy, signaling, and interference burden occurred by D2D discovery.

A key advantage of network-assisted discovery is its potential for more accurate proximity estimation between D2D-enabled devices by exploiting existing knowledge of the cellular network layout. Besides, the EPC is at least aware of the Base Station (BS) with which the cellular devices associate with, also coined as the associated BS [3]. This knowledge combined with additional information on the spatial relation between the respective associated BSs, such as the inter-site distance and the relative position of the D2D peers with respect to their associated BSs, i.e. distance and angle, enable more effective D2D discovery at the EPC. Besides, the LTE-Advanced system already supports a suite of relevant measurements [4].

Poisson Point Processes (PPPs), which have been extensively used for the analysis of multi-tier cellular networks [6]–[8], are increasingly used to analyze the performance of D2D communications. In [9], the authors analyze the performance of two fundamental spectrum sharing schemes and provide design guidelines for D2D communication in the uplink of cellular networks. The D2D proximity is based on the physical distance between the D2D peers. The work in [10] considers D2D communication in Poisson networks with time/frequency hopping to randomize interference and analytical expressions for the SINR and throughput, as well as optimal strategies for time/frequency hopping, are derived. The authors in [11] investigate how mobility and network assistance affect the performance of multicast D2D transmissions in Poisson networks. Optimal network assistance strategies are discussed towards minimizing the retransmission times of multicast messages given certain constraints.

Different from previous works, in this paper we focus on the performance of *network-assisted D2D discovery* and develop an analytical framework that enables the EPC to evaluate the probability that two tagged devices are in proximity conditioned on existing knowledge of the network topology. We further focus on the scenarios where the EPC is at least aware of the inter-site distance between the associated BSs of the two D2D peers, denoted by D. In such scenarios, the EPC can directly relate the locations of the D2D peers without requiring additional information on their exact locations. Besides, the

D. Xenakis, L. Merakos, and N. Passas are with the Department of Informatics and Telecommunications, University of Athens, Athens, Greece, Email: {nio,merakos,passas}@di.uoa.gr.

M. Kountouris is with the Mathematical and Algorithmic Sciences Lab, France Research Center, Huawei Technologies Co. Ltd., Email: marios.kountouris@huawei.com

C. Verikoukis is with the Centre Tecnologic de Telecomunicacions de Catalunya, Barcelona, Spain, Email: cveri@cttc.es

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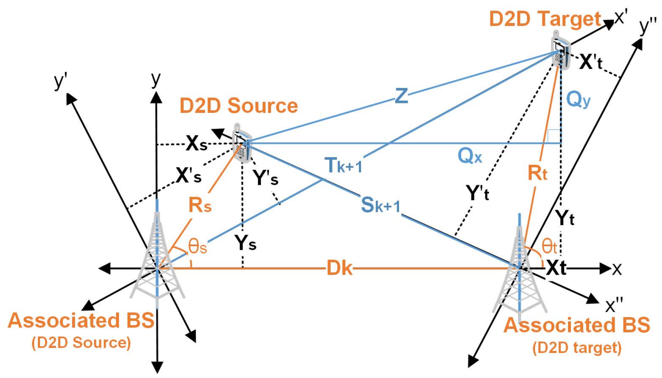

Fig. 1. System model parameters and related RVs

distance between cellular BSs typically remains fixed and does not depend on the D2D peers.

The remainder of this paper is organized as follows. In Section II, we present our system model. In Section III, we derive the D2D discovery probability given different knowledge of the cellular network layout. In Section IV, we investigate how the BS density affects the D2D discovery probability and derive approximate expressions for the optimal BS density. The impact of the key system parameters on the D2D discovery performance is assessed in Section V, where we additionally provide useful design guidelines for network-assisted D2D discovery. Section VI concludes our work.

II. System Model

We consider a D2D-enabled cellular network, where the cellular BSs are distributed according to a homogeneous PPP Φ_B with intensity λ_B in the Euclidean plane. Without assuming a specific distribution for the users, we consider that a) the Point Process (PP) Φ_U describing the user locations is stationary and isotropic, and b) the x and y coordinates of a tagged user are independent of those of other users in Φ_U. We consider that all users associate with the nearest BS in Φ_B [6] and focus on the network-assisted D2D discovery process between a tagged user, referred to as *D2D source*, and a (specific) target D2D-enabled user, referred to as *D2D target*. We further focus on the scenario where the network is capable of i) identifying the associated BS of the D2D peers, and ii) utilize UE and BS positioning measurements to enhance the performance of D2D discovery.

Table I lists the measurements considered in this paper and highlights how they can be derived in LTE. Note that we do not assume that all of these measurements are available to the EPC. Instead, we investigate how certain combinations of these measurements (location information parameters) can enhance the network-assisted D2D discovery process. Fig. 1 depicts all parameters and random variables (RVs) involved in our analysis. The transmit power P_t at the D2D source and the receiver sensitivity P_r at the D2D target are assumed to be fixed and known to the EPC.

Assumption 1. *Given that the distance D is fixed and known, the angle θ_s is uniformly distributed in $(-\pi, \pi]$ and independent of the distance R_s. Similarly, given that D is fixed and known, the angle θ_t is uniformly distributed in $(-\pi, \pi]$ and independent of the distance R_t.*

TABLE I
Location Information Parameters

Parameter	Notation	Estimation methodology
Distance between the D2D source and its associated BS	R_s	By using Timing Advance measurements at the respective BS (standard capability in LTE/LTE-A [4]).
Angle between the D2D source and its associated BS	θ_s	By performing Angle of Arrival (AoA) measurements at the associated BS of the D2D source with respect to the reference direction from the associated BS of the D2D source to the associated BS of the D2D target. AoA is a standard measurement capability in LTE/LTE-A [4].
Distance between the D2D target and its associated BS	R_t	In a similar manner with the estimation of R_s.
Angle between the D2D target and its associated BS	θ_t	By exploiting AoA measurements at the associated BS of the D2D target, similarly to the estimation of θ_s.
Inter-site distance between the associated BS of the D2D source and the associated BS of the D2D target	D	By utilizing knowledge from the network planning phase or by performing Timing Advance measurements for the associated BS of the D2D target. This estimation can also be based on the Reference Signal Received Power (RSRP) from the associated BS of the D2D target [4].

Assumption 1 states that, conditioned on a fixed distance D, the D2D peers are uniformly distributed around their associated BSs having no bias on residing towards a specific direction, or adapting the distance to their associated BS based on their angle to other BSs. Note that Assumption 1 does not imply that the position of a tagged D2D peer relative to its associated BS is in general independent of the BS process Φ_B. Instead, it states that conditioned on a fixed D, the angle of a D2D peer relative to its associated BS is uniform circular and independent of the distance separating them.

III. D2D Discovery Probability

As highlighted in [5], the performance of D2D discovery is tightly coupled with the definition of proximity between the D2D peers. In the sequel, we consider that two D2D-enabled devices are in proximity whenever the *long-term averaged* received signal power from the D2D source is greater than or equal to the receiver sensitivity at the D2D target. Assuming that the pathloss is inversely proportional to the distance between the D2D peers and governed by a pathloss exponent a, the D2D discovery probability is defined as

$$\mathcal{A}_{\boldsymbol{J}} \triangleq P\left[P_t Z^{-a} \geq P_r \middle| \boldsymbol{J}\right], \tag{1}$$

where $\boldsymbol{J}$ denotes the set of location information parameters that are known to the EPC, P_t the transmit power at the D2D source, P_r the receiver sensitivity at the D2D target, and Z the distance between the D2D peers. By rearranging (1), it can be easily shown that the D2D discovery probability corresponds to the value of the cumulative density function (cdf) of the distance Z at point $\left(\frac{P_t}{P_r}\right)^{\frac{1}{a}}$ conditioned on the knowledge available in $\boldsymbol{J}$. In light of the above remark, in this section we derive the probability distribution of the distance Z given four distinct combinations of location information.

Theorem 1. *The conditional pdf $f_{Z|D}(z)$ of the distance Z between two D2D peers, given that the associated BS of the D2D source and the associated BS of the D2D target are separated by a distance D, is given by*

$$f_{Z|D}(z) = \pi\lambda_B z e^{-\frac{\pi\lambda_B}{2}\left(z^2+D^2\right)} I_0\left[\pi\lambda_B z D\right], \tag{2}$$

where $I_0[x]$ is the modified Bessel function. The corresponding ccdf $\bar{F}_{Z|D}(z)$ is given by

$$\bar{F}_{Z|D}(z) = Q_1\left[\sqrt{\pi\lambda_B}D, \sqrt{\pi\lambda_B}z\right], \tag{3}$$

where $Q_M[a,b]$ is the Marcum-Q function.

Proof. See Appendix A. □

Theorem 1 provides an analytical tool for handling the uncertainty on the distance between two D2D peers, given location information that typically remains fixed over time: the inter-site distance D. In the next theorem, we derive closed-form expressions for the conditional pdf and ccdf of the distance Z given additional knowledge on the relative position of the D2D target.

Theorem 2. *The conditional pdf $f_{Z|D,R_t,\theta_t}(z)$ of the distance Z between two D2D peers, given that a) the associated BS of the D2D source and the associated BS of the D2D target are separated by a distance D, and b) the relative position of the D2D target with respect to its associated BS equals to $[R_t = T, \theta_t = \varrho]$ in polar coordinates, is given by (4). The corresponding ccdf $\bar{F}_{Z|D,R_t,\theta_t}(z)$ is given by (5).*

Proof. Theorem 2 can be proved in a similar manner with Theorem 1 by working in the Cartesian plane $x'y'$ and taking into account that $Z = \sqrt{(X_t' + T_{k+1})^2 + Y_t'^2}$ (Fig. 1). Note that T_{k+1} is a fixed parameter that is given by $T_{k+1} = \sqrt{D^2 + T^2 - 2DT\cos\varrho}$. □

The expressions in Theorem 2 further reduce the uncertainty on the distance between the D2D peers by incorporating additional knowledge on the relative position of the D2D target with respect to its associated BS. However, in contrast with the acquisition and caching of the D parameter, the relative position of the D2D target is expected to vary over time, requiring monitoring measurements by the associated BS. Interestingly, Theorems 1 and 2 imply that the distance Z follows a Ricean distribution. In Corollary 1, we present the conditional pdf and ccdf of the distance Z in the scenario where, instead of the relative position of the D2D target, the EPC is aware of the relative position of the D2D source.

Corollary 1. *The conditional pdf $f_{Z|D,R_s,\theta_s}(z)$ and ccdf $\bar{F}_{Z|D,R_s,\theta_s}(z)$ of the distance Z between two D2D peers, given that a) the associated BS of the D2D source and the associated BS of the D2D target are separated by a distance D, and b) the relative position of the D2D source with respect to its associated BS equals to $[R_s = S, \theta_s = \varphi]$ in polar coordinates, are given by (4) and (5), respectively, for $T = S$ and $\varrho = \pi - \varphi$.*

Proof. Corrollary 1 can be proved in a similar manner with Theorem 1 by working in the Cartesian plane $x''y''$ and taking into account that $Z = \sqrt{(X_t' + S_{k+1})^2 + Y_t'^2}$ (Fig. 1). Note that S_{k+1} is a fixed parameter that is given by $T_{k+1} = \sqrt{D^2 + S^2 - 2DS\cos\varphi}$. □

Proposition 1 includes a formula for calculating the distance Z when the EPC is aware of all necessary parameters, i.e. $\{D, R_t, \theta_t, R_s, \theta_s\}$.

Proposition 1. *The distance Z between two D2D peers, given that a) the associated BSs of the two D2D peers is D, b) the relative position of the D2D target with respect to its associated BS equals to $[R_t = T, \theta_t = \varrho]$, and c) the relative position of the D2D source with respect to its associated BS equals to $[R_s = S, \theta_s = \varphi]$, is given by*

$$Z = \sqrt{(D + T\cos\varrho - S\cos\varphi)^2 + (T\sin\varrho - S\sin\varphi)^2}. \tag{6}$$

Proof. The proof is derived by substituting D, $X_t = T\cos(\varrho)$, $Y_t = T\sin(\varrho)$, $X_s = S\cos\varphi$ and $Y_s = S\sin\varphi$ in (9) (see Appendix A). □

Note that the scenario where the D2D peers associate with the same BS applies for $D = 0$.

IV. Optimal Network Deployment

In this section, we provide guidelines for optimal network deployment as a means to optimize the probability of successful network-assisted D2D discovery. This kind of analysis is of high practical interest as it provides useful insights on how to transform the today's cellular network, which is optimized for coverage and capacity, into a D2D-centric network where the discovery and communication between the cellular devices will be orchestrated by the EPC. Moreover, the presented analysis provides useful guidelines for the installation of additional BSs (over a prescribed geographical area) so as to maximize the capability of the EPC to infer on the outcome of D2D discovery. Besides, maximizing the capability of the EPC to estimate proximity between two tagged cellular devices, e.g. an anchor point and a target device with unknown location, is of paramount importance in public emergency networks and private network installations for automated navigation/ control, e.g. industrial installations, underground facilities and in-building parking systems.

Under this viewpoint, we examine the monotonicity of the D2D discovery probability $\mathcal{A}_{\boldsymbol{J}}$ with respect to the BS density λ_B and provide analytical expressions for the optimal BS density when relevant. Note that since $\mathcal{A}_{\boldsymbol{J}}$ is given by the cdf of the distance Z at $\left(\frac{P_t}{P_r}\right)^{\frac{1}{a}}$, by definition $\mathcal{A}_{\boldsymbol{J}}$ is a) proportional to the transmit power P_t and b) inversely proportional to the receiver sensitivity P_r and the pathloss exponent a.

Theorem 3. *Let $q = D\left(\frac{P_t}{P_r}\right)^{-\frac{1}{a}}$. Given the value of the inter-site distance D, the D2D discovery probability $\mathcal{A}_D$ increases with λ_B for $q < 1$. However, for $q > 1$ there exists*

$$f_{Z|D,R_t,\theta_t}(z) = 2\pi\lambda_B z e^{-\pi\lambda_B(z^2+D^2+T^2-2DT\cos(\pi-\varrho))} I_0\left[2\pi\lambda_B z\sqrt{D^2+T^2-2DT\cos(\pi-\varrho)}\right] \tag{4}$$

$$\bar{F}_{Z|D,R_t,\theta_t}(z) = Q_1\left[\sqrt{2\pi\lambda_B\left(D^2+T^2-2DT\cos(\pi-\varrho)\right)}, z\sqrt{2\pi\lambda_B}\right] \tag{5}$$

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an optimal BS density λ_B^* that maximizes the D2D discovery probability and satisfies the following property:

$$I_0\left[\frac{\pi\lambda_B^* D^2}{q}\right] - qI_1\left[\frac{\pi\lambda_B^* D^2}{q}\right] = 0. \tag{7}$$

The optimal BS density can be analytically approximated as

$$\lambda_B^* \approx \frac{q(1+3q+\sqrt{39q^2-6q-17})}{16\pi D^2(q-1)}. \tag{8}$$

Proof. See Appendix E. □

The parameter $\left(\frac{P_t}{P_r}\right)^{\frac{1}{a}}$ corresponds to the maximum distance for successful D2D discovery between the D2D peers. On the other hand, the distance between a user and the associated BS is inversely proportional to λ_B since, by definition, it is Rayleigh distributed with parameter $\frac{1}{2\pi\lambda_B}$. Therefore, as the BS density increases, the distance Z between the D2D peers tends to reach the inter-site distance D between their associated BSs, i.e. a higher λ_B reduces uncertainty on the user position around the associated BS. Based on these observations, Theorem 3 can be interpreted as follows: as λ_B increases, the distance Z between the D2D peers tends to be statistically closer to the inter-site distance D, which for $q < 1$ is by definition lower than the maximum range for successful D2D discovery, i.e. $D < \left(\frac{P_t}{P_r}\right)^{\frac{1}{a}}$. However, for $q > 1$, the inter-site distance D is greater than the maximum D2D discovery range and, above a certain BS density, the distance Z tends to be statistically greater than the D2D discovery range.

Notably, Theorem 3 can be extended to the scenario where the EPC is additionally aware of the relative positions of the D2D peers with respect to their associated BS. In particular, if the EPC is aware of the relative position of the D2D source, Theorem 3 applies for $q = \sqrt{D^2+S^2-2DS\cos\varphi}\left(\frac{P_t}{P_r}\right)^{-\frac{1}{a}}$. If the EPC is aware of the relative position of the D2D target, Theorem 3 applies for $q = \sqrt{D^2+T^2-2DT\cos(\pi-\varrho)}\left(\frac{P_t}{P_r}\right)^{-\frac{1}{a}}$. We omit the proof due to space limitations.

V. Numerical Results and Design Guidelines

In this section, we study the impact of the key system parameters on the D2D discovery probability and derive useful guidelines for the design of network-assisted D2D discovery.

Fig. 2 depicts the impact of the BS density λ_B on the D2D discovery probability given location information that includes at least the inter-site distance D. We focus on two inter-site distances $D = 600$ m and $D = 900$ m which, in combination with the parameters shown in Fig. 2, result in D2D discovery success ($\mathcal{A}_{D,R_t,\theta_t,R_s,\theta_s} = 1$) and failure ($\mathcal{A}_{D,R_t,\theta_t,R_s,\theta_s} = 0$), respectively. When the EPC is aware of only the inter-site distance D, the D2D discovery probability $\mathcal{A}_D$ increases with λ_B for $D = 600$ m, since $q < 1$ (as shown in Theorem 3). On the other hand, for $D = 900$ m, which corresponds to $q > 1$, there exists an optimal BS density that maximizes the D2D discovery probability and is well approximated by (8) (highlighted with the red star). Similar properties are shown when the EPC has additional knowledge on the relative positions of the D2D target or the D2D source, i.e. $\mathcal{A}_{D,R_t,\theta_t}$ and $\mathcal{A}_{D,R_s,\theta_s}$, respectively. The approximations on the optimal BS density for $\mathcal{A}_{D,R_t,\theta_t}$ and $\mathcal{A}_{D,R_s,\theta_s}$, indicated by the blue and green star, respectively, are also shown to be close to the λ_B parameter that maximizes the respective D2D discovery probabilities. Recall that the approximation accuracy can be increased by using more terms from (26) and (27).

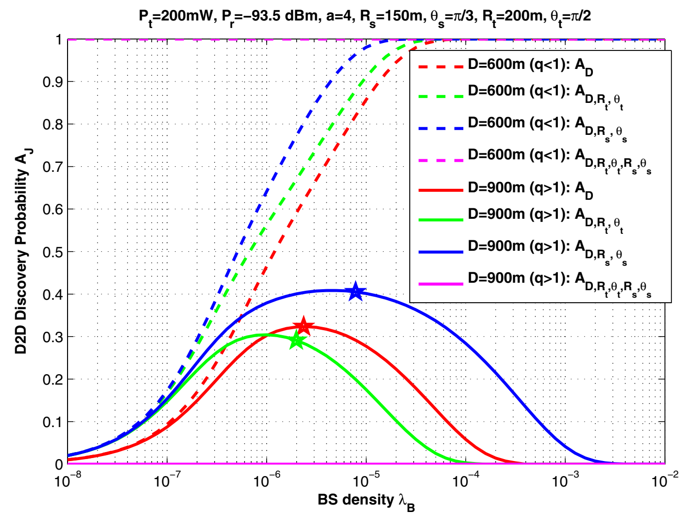

Fig. 2. D2D Discovery Probability given D vs. BS density

The results in Fig. 2 indicate that conditioned on knowledge of the relative position of the D2D source, or the D2D target, the statistical behavior of the D2D discovery probability and the optimal λ_B^* significantly alters compared to $\mathcal{A}_D$, especially when $q > 1$ (the ratio of the Marcum-Q arguments in the ccdf results is higher than one). This follows from the fact that, if not known at the EPC, the locations of the D2D peers are considered to follow a symmetric normal distribution around their associated BS, i.e. Rayleigh-distributed distance combined with uniformly distributed angle. However, when a D2D peer is located in between the two associated BSs, and its location is known to the EPC, the probability of successful D2D discovery increases. This relation can be easily verified in Fig. 2, by comparing the results for $\mathcal{A}_{D,R_s,\theta_s}$ and $\mathcal{A}_D$ and taking into account that $R_s = 200$ m and $\theta_s = \pi/3$ (Fig. 1).

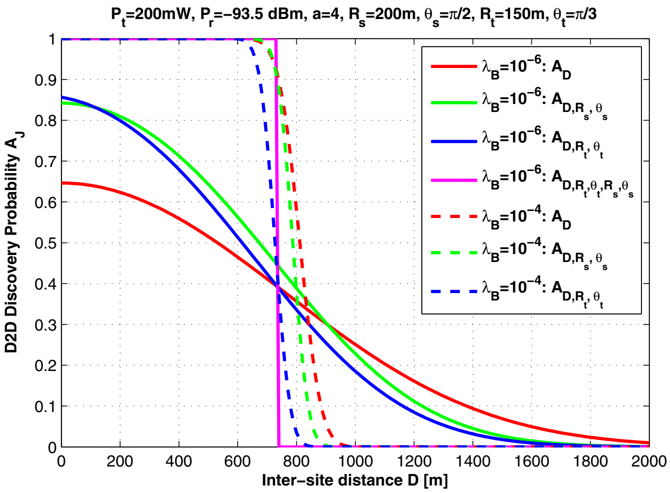

Fig. 3. D2D Discovery Probability given D vs. Inter-site distance D

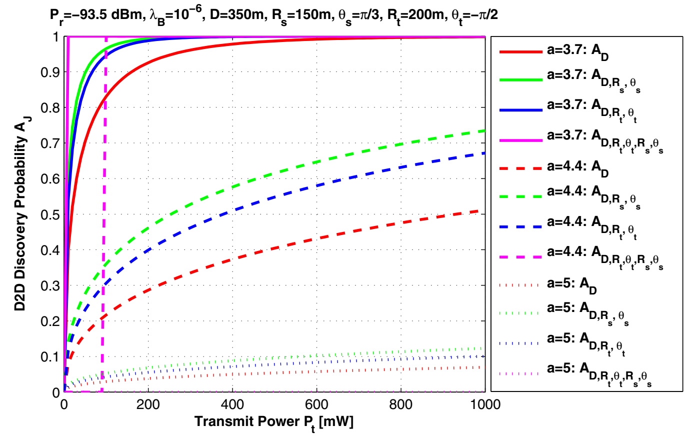

Fig. 4. D2D Discovery Probability given D vs. Transmit power P_t

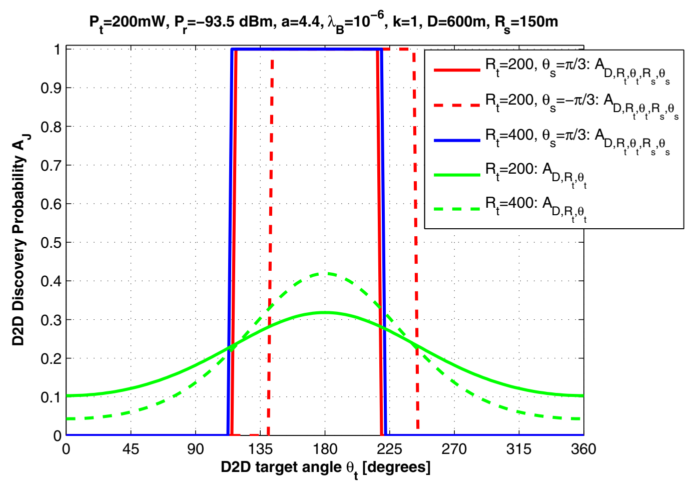

Fig. 5. D2D Discovery Probability vs. D2D target angle θ_t

We now examine the impact of the inter-site distance D on the D2D discovery probability given knowledge of at least the distance D (Fig. 3). First, we observe that given the same set of location parameters, a higher BS density prolongs the tail of the D2D discovery probability, owing to the increased uncertainty on the D2D source and/or D2D target positions around their associated BS. This is also expected if we consider that for the given set of system parameters, the maximum range for successful D2D discovery is equal to $\left(\frac{P_t}{P_r}\right)^{\frac{1}{a}} = 813.3$ m, whereas the average distance between a user and its associated BS is equal to 157m and 1570m for $\lambda_B = 10^{-4}$ and $\lambda_B = 10^{-6}$, respectively, i.e. expected value of the Rayleigh distribution with parameter $\frac{1}{\sqrt{2\pi\lambda_B}}$. Hence, above a certain BS density, the performance of D2D discovery is primarily affected by the inter-site distance D and not the relative positions of the D2D peers, which can be approximated by the position of their associated BSs. This approach can reduce the overhead required for user positioning while leaving the D2D discovery probability unaffected.

For $\lambda_B = 10^{-4}$, we observe that the D2D discovery probability $\mathcal{A}_D$ is higher compared to the one given additional knowledge on $[R_t, \theta_t]$, i.e. $\mathcal{A}_{D,R_t,\theta_t}$. This can be explained as follows: conditioned on $[R_t = 200m, \theta_t = \pi/3]$ (Fig. 1), the distance Z between the D2D peers is statistically higher compared to the scenario with no knowledge on $[R_t, \theta_t]$ (Fig. 1), where the position of the D2D target is considered to follow a symmetrical normal distribution around its associated BS $\left(\sigma^2 = \frac{1}{2\pi\lambda_B}\right)$. This effect is more prominent for $\lambda_B = 10^{-4}$, where the uncertainty on position of the D2D source is significantly reduced compared to the one for $\lambda_B = 10^{-6}$. Similar arguments can be used to compare $\mathcal{A}_D$ and $\mathcal{A}_{D,R_s,\theta_s}$.

D2D discovery will be performed under unfavorable channel conditions, due to the lower height of the transmitter-receiver pair, the increased number of obstacles between the D2D peers, and the low transmit power required to avoid interference with other cellular connections. Under this viewpoint, in Fig. 4 we plot the impact of the transmit power P_t on the D2D discovery probability under high pathloss exponents and given information for at least the inter-site distance D. As expected, the D2D discovery probability increases with P_t under all combinations of location information (Section IV). However, the (positive) impact of increasing P_t on the D2D discovery probability strongly depends on the pathloss exponent governing the D2D channel. For example, for $a = 3.7$, we observe that $P_t = 200$ mW suffices to attain a D2D discovery probability higher than 90% for all combinations of location information parameters. On the other hand, for $a = 4.4$, the D2D discovery probability can be greatly improved with a slight increase in the transmit power whereas, for $a = 5$, it remains roughly unaffected. Note that, for the given set of parameters, additional knowledge on $[R_s, \theta_s]$ and/or $[R_t, \theta_t]$ significantly alters the statistical behavior of the D2D discovery probability, especially for $a = 4.4$.

To summarize, the employment of network-assisted D2D discovery can significantly reduce unnecessary transmissions of D2D discovery signals that increase the network interference and deplete the battery at the mobile terminals. To this direction, the presented results can be used to assist the D2D source upon selecting an appropriate transmit power for a prescribed D2D discovery probability target, by exploiting fundamental location information at the EPC.

In Fig. 5 we plot the statistical behavior of the D2D discovery probability with respect to θ_t (Fig. 1), for all combinations that include θ_t. Obviously, the statistical behavior of the remainder D2D discovery probabilities remains unchanged with respect to θ_t. Given full knowledge on the network layout, we observe that the D2D discovery can be either successful or not. However, depending on the fixed parameters, there exists a θ_t interval within which the D2D discovery is always successful, i.e. $A_{D,R_t,\theta_t,R_s,\theta_s} = 1$. Moreover, we observe that the probability $\mathcal{A}_{D,R_t,\theta_t,R_s,\theta_s}$ for $\theta_s = \pi/3$ is a mirror function of $\mathcal{A}_{D,R_t,\theta_t,R_s,\theta_s}$ for $\theta_s = -\pi/3$ with respect to 180^o, which corresponds to the direction towards the associated BS of the D2D source. For the given parameters in Fig. 5, an increase to the distance R_s symmetrically expands the θ_t interval where $\mathcal{A}_{D,R_t,\theta_t,R_s,\theta_s} = 1$ towards both directions. This result follows from the given θ_s under scope, since for $\theta_s = \pi/3$ a higher distance R_s reduces the distance Z between the D2D peers.

When the EPC is aware of only the parameters $\{D, R_t, \theta_t\}$, the corresponding D2D discovery probability is higher for all angles θ_t that reside closer to the associated BS of the D2D source (green line), i.e. 180^o. On the other hand, an increase to the distance R_t enlarges the D2D discovery probability for θ_t towards the same direction and reduces it for the ones residing towards the opposite one (green dashed). Both these results are expected if we consider that in the absence of knowledge of $[R_s, \theta_s]$, the D2D source is considered to follow a symmetric normal distribution around its associated BS.

The knowledge on the relative positions of the D2D peers majorly impacts the D2D discovery performance, especially in sparse to medium network deployments where the uncertainty on the relative positions of the users (around their associated BSs) is high. The results in Fig. 5 also indicate that, under certain conditions, the estimation accuracy for the angles θ_t and θ_s can be relaxed without affecting the performance of network-assisted D2D discovery. Such an approach could significantly reduce the overheads required for accurately measuring the AoA of the D2D peers at the associated BSs.

VI. Conclusion

In this paper, we analyzed the statistical behavior of the distance between two D2D peers conditioned on existing knowledge for the cellular network deployment. The ccdf expressions were used to analyze the performance of network-assisted D2D discovery and provide useful insights on how different levels of location awareness affect its performance. We also examined how unplanned cellular network densification affects the performance of network-assisted D2D discovery and provided analytical expressions for the optimal BS density that maximizes the D2D discovery probability. Accordingly, we investigated the key performance tradeoffs inherent to the network-assisted D2D discovery and provided useful guidelines for its design in random spatial networks. Among others, the present results can be used to select the transmit power at the D2D source for a given D2D discovery probability target, reduce unnecessary D2D discovery signals, identify the optimal BS density for network-assisted D2D discovery, and relax the accuracy of user positioning while leaving the D2D discovery probability unaffected.

Appendix

A. Proof of Theorem 1

Since the user and BS point processes are stationary and isotropic, we work in the Cartesian plane xy centered at the position of the associated BS of the D2D source and having as a positive x-axis the direction from the associated BS of the D2D source to the associated BS of the D2D target (Fig. 1). Let X_s and X_t be the projections of the distances R_s and R_t on the x-axis, respectively, and Y_s and Y_t the respective projections in the y-axis. Note that $[X_s, Y_s]$ are the Cartesian coordinates of the D2D source in the xy plane, whereas the respective ones for the D2D target are $[D + X_t, Y_t]$. The distance Z between the D2D source and the D2D target is given by

$$Z = \sqrt{(D + X_t - X_s)^2 + (Y_t - Y_s)^2}. \tag{9}$$

We now define two auxiliary RVs: $Q_x = D + X_t - X_s$ and $Q_y = Y_t - Y_s$. Given that the coordinates of the D2D source do not depend on the ones of the D2D target (Section II), Q_x equals to the difference of two independent normal RVs plus a fixed value D. Thus, Q_x is normally distributed with mean D and variance $2b^2 = \frac{1}{\pi\lambda_B}$, i.e. $Q_x \sim \mathcal{N}(D, \frac{1}{\pi\lambda_B})$. In a similar manner, we can show that Q_y is a normal RV with zero mean and variance $2b^2$, i.e.: $Q_y \sim \mathcal{N}(0, \frac{1}{\pi\lambda_B})$. Now, since the RV X_t, X_s, Y_t, and Y_s are shown to be mutually independent it readily follows that Q_y is independent of Q_x. Accordingly, the joint distribution of Q_x and Q_y is given by

$$f_{Q_x,Q_y}(x, y) = \frac{\lambda_B}{2} e^{-\frac{\pi\lambda_B}{2}\left((x-D)^2+y^2\right)}. \tag{10}$$

Now, let ΔA_z denote the region of the plane such that $z < \sqrt{x^2 + y^2} < z + \mathrm{d}z$. Then, the region ΔA_z is a circular ring with inner radius z and thickness $\mathrm{d}z$. By working in polar coordinates, i.e. $x = z\cos\xi$, $y = z\sin\xi$, and $\mathrm{d}x\mathrm{d}y = z\mathrm{d}z\mathrm{d}\xi$, we get

$$\begin{aligned} f_{Z|D_k}(z)\mathrm{d}z &= \int_{\Delta A_z} f_{Q_x,Q_y}(x, y)\mathrm{d}x\mathrm{d}y \qquad (11) \\ &= \frac{\lambda_B}{2}\int_0^{2\pi} e^{-\frac{\pi\lambda_B}{2}\left(z\cos\xi - D)^2 + (z\sin\xi)^2\right)} z\mathrm{d}z\mathrm{d}\xi. \qquad (12) \end{aligned}$$

Therefore,

$$\begin{aligned} f_{Z|D_k}(z) &= \frac{\lambda_B z}{2} e^{-\frac{\pi\lambda_B}{2}\left(z^2+D^2\right)} \int_0^{2\pi} e^{\pi\lambda_B z D\cos\xi}\mathrm{d}\xi \qquad (13) \\ &= \pi\lambda_B z e^{-\frac{\pi\lambda_B}{2}\left(z^2+D^2\right)} I_0\left[\pi\lambda_B z D\right], \qquad (14) \end{aligned}$$

where (14) follows from the integral representation of the modified Bessel function of the first and zero-th order: $I_0[x] = \frac{1}{2\pi}\int_0^{2\pi} e^{x\cos\xi}\mathrm{d}\xi$. Having derived the conditional pdf $f_{Z|D_k}(z)$, we proceed with the derivation of the conditional ccdf $\bar{F}_{Z|D_k}(z)$ as follows:

$$\begin{aligned} \bar{F}_{Z|D_k}(z) &= \int_z^\infty f_{Z|D_k}(x)\mathrm{d}x \qquad (15) \\ &= \int_z^\infty \pi\lambda_B x e^{-\frac{\pi\lambda_B}{2}\left(x^2+D^2\right)} I_0\left[\pi\lambda_B x D\right]\mathrm{d}x \qquad (16) \end{aligned}$$

$$= \int_{\sqrt{\pi\lambda_B}z}^{\infty} q e^{-\frac{q^2+(\sqrt{\pi\lambda_B}D)^2}{2}} I_0\left[q\left(\sqrt{\pi\lambda_B}D\right)\right]\mathrm{d}q \quad (17)$$

$$= Q_1\left[\sqrt{\pi\lambda_B}D, \sqrt{\pi\lambda_B}z\right], \quad (18)$$

where $Q_M[a,b] = \int_b^\infty x\left(\frac{x}{a}\right)^{M-1} e^{-\frac{x^2+a^2}{2}} I_{M-1}[ax]\mathrm{d}x$ is the Marcum-Q function, $I_{M-1}[x]$ the modified Bessel function of the first kind and $(M-1)$-th order, (16) is derived by substituting (14), (17) by employing a change of variables $q = \frac{x}{\sqrt{\pi\lambda_B}}$, and (18) by the definition of the Marcum-Q function.

B. Proof of Theorem 3

By using Theorem 1 and (1), it follows that $\mathcal{A}_D = 1 - Q_1\left[\sqrt{\pi\lambda_B}D, \sqrt{\pi\lambda_B}\left(\frac{P_t}{P_r}\right)^{\frac{1}{a}}\right]$. Observe now that the ratio of the arguments in the Marcum-Q function is equal to $q = D\left(\frac{P_t}{P_r}\right)^{-\frac{1}{a}}$. By using the transform for the Marcum-Q function given in [[12], (4.16)] for $q < 1$, the D2D discovery probability $\mathcal{A}_D$ can be rewritten as follows:

$$\mathcal{A}_D = 1 - \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{(1+q\sin\theta)e^{-\frac{\pi\lambda}{2}\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}(1+2q\sin\theta+q^2)}}{1+2q\sin\theta+q^2}\mathrm{d}\theta. \quad (19)$$

Now, by differentiating with respect to λ_B we get:

$$\frac{\partial\mathcal{A}_D}{\partial\lambda_B} = \frac{\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}}{4}\int_{-\pi}^{\pi}(1+q\sin\theta)e^{-\frac{\pi\lambda}{2}\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}(1+2q\sin\theta+q^2)}\mathrm{d}\theta \quad (20)$$

$$= \frac{\pi\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}}{2e^{\frac{\pi\lambda}{2}\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}(1+q^2)}} \cdot \left(I_0\left[\pi\lambda_B\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}q\right] - qI_1\left[\pi\lambda_B\left(\frac{P_t}{P_r}\right)^{\frac{2}{a}}q\right]\right) \quad (21)$$

Since all parameters in Eq (21) are positive real numbers and by definition $I_0[x] > I_1[x]\ \forall x > 0$, for $q < 1$ the sign of Eq (21) is always positive. Hence, $\frac{\partial\mathcal{A}_D}{\partial\lambda_B} > 0$, which implies that for $q < 1$ the D2D discovery probability is monotonically increasing with respect to λ_B. Let us now examine the scenario where $q > 1$. By using [[12], (4.19)] for $\zeta = \frac{1}{q}$, we can rewrite $\mathcal{A}_D$ as follows:

$$\mathcal{A}_D = -\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{(\zeta^2+\zeta\sin\theta)e^{-\frac{\pi\lambda}{2}D^2(1+2\zeta\sin\theta+\zeta^2)}}{1+2\zeta\sin\theta+\zeta^2}\mathrm{d}\theta. \quad (22)$$

By differentiating with respect to λ_B we obtain

$$\frac{\partial\mathcal{A}_D}{\partial\lambda_B} = \frac{D^2}{4}\int_{-\pi}^{\pi}(\zeta^2+\zeta\sin\theta)e^{-\frac{\pi\lambda}{2}D^2(1+2\zeta\sin\theta+\zeta^2)}\mathrm{d}\theta \quad (23)$$

$$= \frac{\pi D^2}{2e^{\frac{\pi\lambda}{2}D^2(1+\zeta^2)}}\left(\zeta I_0\left[\pi\lambda_B D^2\zeta\right] - I_1\left[\pi\lambda_B D^2\zeta\right]\right). \quad (24)$$

Now, since all parameters in (24) are positive real numbers and the existence of an optimal point requires $\frac{\partial\mathcal{A}_D}{\partial\lambda_B} = 0$, from Eq (24) it readily follows that the optimal BS density λ_B^* satisfies

$$I_0\left[\frac{\pi\lambda_B^* D^2}{q}\right] - qI_1\left[\frac{\pi\lambda_B^* D^2}{q}\right] = 0. \quad (25)$$

If the (common) argument in the two modified Bessel functions in (25) is sufficiently large, typically higher than 2, then each Bessel function can be approximated as

$$I_0[x] = \frac{e^x}{\sqrt{2\pi x}}\left(1+\frac{1}{8x}\left(1+\frac{9}{2(8x)}\left(1+\ldots\right)\right)\right), \quad (26)$$

$$I_1[x] = \frac{e^x}{\sqrt{2\pi x}}\left(1-\frac{3}{8x}\left(1+\frac{5}{2(8x)}\left(1++\ldots\right)\right)\right). \quad (27)$$

Obviously, the more terms we use from (26) and (27), the more accurate the approximation. Assuming that $\frac{\pi\lambda_B^* D^2}{q}$ is sufficiently large, we use the first three terms of (26) and (27) to approximate the Bessel functions in (25). By substituting the approximations in (25) and elaborating with the result, we reach to the following quadric equation with respect to λ_B^*:

$$-128{\lambda_B^*}^2 D^4\pi^2(q-1) + 16\lambda_B^* D^2\pi q(1+3q) + 3q^2(3+5q) = 0 \quad (28)$$

For $q > 1$ and $D > 0$, (28) has the unique solution given in (8).

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