

Rate-Achieving Policy in Finite-Horizon Throughput Region for Multi-User Interference Channels

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Abstract—This paper studies a wireless network consisting of multiple transmitter-receiver pairs sharing the same spectrum where interference is regarded as noise. Previously, the throughput region of such a network was characterized for either one time slot or an infinite time horizon. This work aims to close the gap by investigating the throughput region for transmissions over a finite time horizon. We derive an efficient algorithm to examine the achievability of any given rate in the finite-horizon throughput region and provide the rate-achieving policy. The computational efficiency of our algorithm comes from the use of A* search with a carefully chosen heuristic function and a tree pruning strategy. We also show that the celebrated max-weight algorithm which finds all achievable rates in the infinite-horizon throughput region fails to work for the finite-horizon throughput region.

Index Terms—Throughput region, finite time horizon, rate-achieving policy, A* search algorithm, max-weight algorithm.

I. INTRODUCTION

A. Motivation and Related Work

Analyzing the throughput region under any given modulation and coding strategy is an important issue for studying the network capacity from a network-layer perspective [1]. Such studies commonly assume that the interference in the network is treated as noise, hence the capacity of each link is determined by signal-to-interference-plus-noise ratio (SINR). In this work, we take the same network-layer approach and study the throughput region of a wireless network having multiple transmitter-receiver pairs. The key difference between point-to-point systems and multi-user networks is the consideration of multiple time slots. For point-to-point systems, knowing the achievable rate and the rate-achieving transmission policy in one time slot is sufficient to derive the rate-achievable results for any number of time slots. However, this is not the case for multi-user networks, where the throughput region over multiple time slots is different from that in a single time slot. In fact, the multi-slot throughput region is generally larger than the single-slot throughput region [2], [3].

A number of studies investigated the achievable rates in multi-user wireless networks over an infinite number of time slots. The seminal work for infinite-horizon throughput region¹

was introduced in [4], [5] and further generalized in [1], [2], [6]–[10]. These studies revealed the relationship between the exogenous data rate, which is the rate at which data arrives in the data queue of each transmitter, and the infinite-horizon throughput region formed by all the achievable rates over an infinite number of time slots. If a given exogenous rate is in the infinite-horizon throughput region, there exists a rate-achieving transmission policy to result in a stable data queue condition. It is also shown that the infinite-horizon throughput region is convex [1], [2], [4], [5].

Despite the theoretical importance of the infinite-horizon throughput region result, it does not provide sufficient insights into the throughput region or rate-achieving policy over a finite horizon, i.e., a finite number of time slots. In wireless networks, the network traffic, channel condition and even network topology change with time [2]. Transmission should always be designed for a finite time duration, i.e., a relatively small number of time slots, such that the network and channel information used in the design is not outdated when the actual transmission happens. In addition, achieving real-time quality of service (QoS) also requires design over a finite horizon instead of an infinite horizon. To the best of our knowledge, the finite-horizon throughput region of a multi-user wireless network has not yet been investigated.

B. Our Contributions

In this work, we investigate the finite-horizon throughput region of a wireless network consisting of multiple transmitter-receiver pairs. Our approach is not to completely characterize the finite-horizon throughput region because unlike the infinite counterpart, it is non-convex and the complexity of finding all achievable rates increases exponentially with the number of time slots. Instead, we provide a method to determine (i) whether an arbitrarily given rate is achievable, and (ii) if so, what the rate-achieving transmission policy is. We formulate the problem of finding the rate-achieving policy in terms of the transmission-time-minimization problem and provide an efficient solution based on an interference-free based heuristic function. We prove this heuristic function is admissible so that the celebrated A* search algorithm can be implemented [11], which largely improves the computational efficiency.

¹In this paper, the term ‘infinite horizon’ refers to an infinite number of time slots and the term ‘finite horizon’ refers to a finite number of time slots.

We also highlight a fundamental difference between finite-horizon throughput region and the previously studied infinite-horizon throughput region. Specifically, we show that the well-known max-weight algorithm [4] which can achieve all rates in the interior of the infinite-horizon throughput region fails to find the achievable rate in the interior of the finite-horizon throughput region. This suggests that the existing methods dealing with the rate-achieving policies for infinite horizon cannot be directly applied to study the case of finite horizon.

C. Notation

Throughout this paper, for a vector $\mathbf{a} = [a^{(1)}, \dots, a^{(N)}]^{\text{tr}}$ (where tr denotes the transpose operator), $(\mathbf{a})^+$ denotes $\max\{a^{(n)}, 0\}$ for all $n \in \{1, \dots, N\} =: \mathcal{N}$. The cardinality of a set \mathcal{A} is $|\mathcal{A}|$. For $\mathbf{x}_1 = [x_1^{(1)}, \dots, x_1^{(N)}]^{\text{tr}}$ and $\mathbf{x}_2 = [x_2^{(1)}, \dots, x_2^{(N)}]^{\text{tr}}$, $\mathbf{x}_1 \succeq (\succ, \preceq, \prec) \mathbf{x}_2$ represents $x_1^{(n)} \succeq (\succ, \preceq, \prec) x_2^{(n)}$ for all $n \in \mathcal{N}$. $\overline{\mathbb{R}}_+^N$ (\mathbb{R}_+^N) means $\{\mathbf{x} \in \mathbb{R}^N : \mathbf{x} \succeq (\succ) \mathbf{0}\}$. And $\mathbf{0}$ stands for the zero vector.

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

A. System Model

Assume there are N transmitter-receiver pairs sharing the same bandwidth in a wireless network, as shown in Fig. 1. Specifically, Tx_n and Rx_n denote the transmitter and receiver of the n^{th} communication pair. The power gain of the channel between Tx_n and Rx_m is denoted by h_{nm} . All power gains remain constant for a given finite time horizon. The time is slotted, and each time slot is the period of transmitting and receiving a codeword. We consider a finite time horizon of T time slots, which is no longer than the channel coherent time, and the duration of each slot is τ .

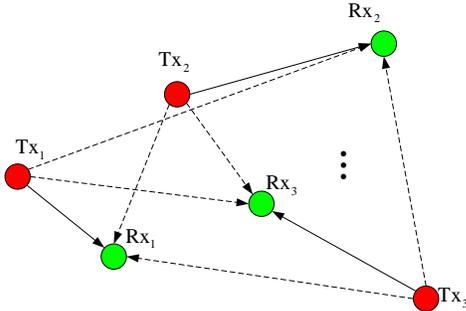


Fig. 1. Transmitter-receiver pairs in a wireless network. The solid and dashed lines indicate the desired data signals and the interference signals, respectively.

In each time slot, every transmitter-receiver pair chooses to transmit or not. That is, for time slot $t \in \{1, \dots, T\} =: \mathcal{T}$, the transmitter Tx_n ($n \in \{1, \dots, N\} =: \mathcal{N}$) can choose its transmit power $s_t^{(n)}$ from the transmit-power set $\mathcal{S}^{(n)}$, in which 0 is included for representing no transmission. Since the number of available power options in a practical communication system is usually finite, we model $\mathcal{S}^{(n)}$ as a finite set. Furthermore, we label $\mathbf{s}_t = [s_t^{(1)}, \dots, s_t^{(N)}]^{\text{tr}}$, and $\mathcal{S} := \mathcal{S}^{(1)} \times \dots \times \mathcal{S}^{(N)}$. Hence, $\mathbf{s}_t \in \mathcal{S}$ and we call \mathcal{S} the transmit-power-vector set.

For time slot t , the SINR for each transmitter-receiver pair is determined by

$$\gamma_n(\mathbf{s}_t) = \frac{h_{nn}s_t^{(n)}}{W_n + \sum_{m \neq n} h_{mn}s_t^{(m)}}, \quad n, m \in \mathcal{N}, \quad (1)$$

where W_n is the power of additive white Gaussian noise for Rx_n during transmission. The capacity of N transmitter-receiver pairs by applying power vector \mathbf{s}_t is²

$$\mathbf{C}(\mathbf{s}_t) = \left[\log_2 \left(1 + \frac{\gamma_1(\mathbf{s}_t)}{\Gamma_1} \right), \dots, \log_2 \left(1 + \frac{\gamma_N(\mathbf{s}_t)}{\Gamma_N} \right) \right]^{\text{tr}} \quad (2)$$

where $\Gamma_n \geq 1$ ($n \in \mathcal{N}$) represents generally any gap to capacity [12] due to practical finite blocklength coding and practical modulation schemes. We absorb $1/\Gamma_n$ into h_{nn} and thus (2) can be rewritten as

$$\mathbf{C}(\mathbf{s}_t) = [\log_2(1 + \gamma_1(\mathbf{s}_t)), \dots, \log_2(1 + \gamma_N(\mathbf{s}_t))]^{\text{tr}}. \quad (3)$$

We say a rate $\boldsymbol{\mu}_t \in \overline{\mathbb{R}}_+^N$ (in time slot t) is achievable when $\boldsymbol{\mu}_t \preceq \mathbf{C}(\mathbf{s}_t)$. For time slot t , all achievable rates form a one-slot throughput region

$$\Lambda_{[1],t} = \bigcup_{\mathbf{s}_t \in \mathcal{S}} \{\boldsymbol{\mu}_t : \mathbf{0} \preceq \boldsymbol{\mu}_t \preceq \mathbf{C}(\mathbf{s}_t)\}. \quad (4)$$

Note that $\Lambda_{[1],t}$ are the same for all t , and thus, for simplicity, we label $\Lambda_{[1],1} = \dots = \Lambda_{[1],T} = \Lambda_{[1]}$.

Similar to the one-slot throughput region, the finite-horizon throughput region for T time slots is defined as follows.

Definition 1 (Finite-Horizon Throughput Region). *The T -slot throughput region $\Lambda_{[T]}$ is the set of average rates that can be achieved in T time slots, i.e.,*

$$\Lambda_{[T]} = \left\{ \boldsymbol{\mu}_{[T]} : \boldsymbol{\mu}_{[T]} = \frac{1}{T} \sum_{t=1}^T \boldsymbol{\mu}_t, \quad \boldsymbol{\mu}_t \in \Lambda_{[1]} \right\}. \quad (5)$$

We also define the weak Pareto frontier and Pareto frontier, which are very helpful in the later parts of the paper.

Definition 2 (Weak Pareto Frontier and Pareto Frontier). *For a set \mathcal{A} , the weak Pareto frontier is*

$$\mathcal{B} = \{b \in \mathcal{A} : \{a \in \mathcal{A} : a \succ b\} = \emptyset\}, \quad (6)$$

and the Pareto Frontier is

$$\overline{\mathcal{B}} = \{b \in \mathcal{A} : \{a \in \mathcal{A} : a \succeq b\} = \{b\}\}. \quad (7)$$

It should be noted that $\overline{\mathcal{B}} \subseteq \mathcal{B}$.

With Definition 2, we define the weak Pareto frontier and Pareto frontier of $\Lambda_{[1]}$ as $\mathcal{M}_{[1]}$ and $\overline{\mathcal{M}}_{[1]}$, respectively. Similarly, $\mathcal{M}_{[T]}$ and $\overline{\mathcal{M}}_{[T]}$ stand for the weak Pareto frontier and Pareto frontier of $\Lambda_{[T]}$. Fig. 2 gives a pictorial illustration on $\Lambda_{[T]}$, $\mathcal{M}_{[T]}$ and $\overline{\mathcal{M}}_{[T]}$. It is clear that the finite-horizon throughput region is generally non-convex.³ This is in contrast to the infinite-horizon throughput region which is convex.

²As discussed in the introduction, we do not consider information-theoretic capacity. The capacity definition in (2) is given in [1], [2] and implicitly assumes that the interference is treated as noise.

³Note that the throughput region is different from that using the time-sharing method in [13], where the length of the ‘‘time slot’’ can be arbitrarily selected which is impractical.

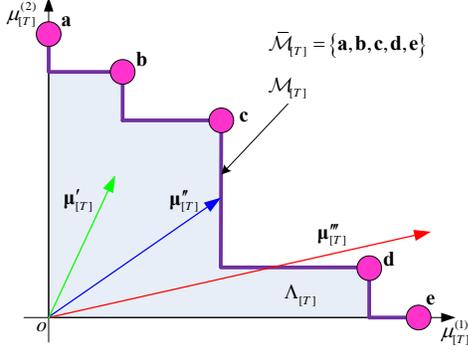


Fig. 2. The Pareto frontier and weak Pareto frontier of throughput region. For two transmitter-receiver pairs, we have $\mu_{[T]} = [\mu_{[T]}^{(1)}, \mu_{[T]}^{(2)}]^{\text{tr}}$ and the T -slot throughput region is in a two-dimensional space. The purple (thick) line segments stand for the weak Pareto frontier $\mathcal{M}_{[T]}$ for $\Lambda_{[T]}$, and the points a, b, c, d, e collectively make the Pareto frontier $\overline{\mathcal{M}}_{[T]}$ of $\Lambda_{[T]}$. $\mu'_{[T]}$ is in $\Lambda_{[T]} \setminus \mathcal{M}_{[T]}$, and $\mu''_{[T]}$ is in $\mathcal{M}_{[T]}$, but $\mu'''_{[T]}$ is not in $\Lambda_{[T]}$.

B. Problem Description

This work focuses on how to achieve any given rate in $\Lambda_{[T]}$. To achieve a rate, say $\mu_{[T]}$, we need to determine the *transmission rate* and *power* in every time slot, which gives the rate-achieving policy. The exact definition of the rate-achieving policy is given as follows.

Definition 3 (Rate-Achieving Policy). *For a given transmit-power-vector set \mathcal{S} and a finite horizon of T time slots, $\forall \mu_{[T]} \in \Lambda_{[T]}$, the rate-achieving policy for $\mu_{[T]}$ is a sequence of rate-power pairs*

$$\mathcal{P}_T = (\mu_t, \mathbf{s}_t)_{t=1}^T, \mathbf{s}_t \in \mathcal{S}, \quad (8)$$

with the capacity constraint⁴ $\mu_t \preceq \mathbf{C}(\mathbf{s}_t)$ such that $\mu_{[T]}$ can be achieved, i.e.,

$$\mu_{[T]} = \frac{1}{T} \sum_{t=1}^T \mu_t. \quad (9)$$

The main task of this paper is to develop a computationally efficient way to find the rate-achieving policy for any achievable rate. Our result will also tell whether a given rate is achievable or not.

III. MAIN RESULTS

To find the rate-achieving policy, we define the following equivalent transmission-time-minimization problem (see Problem 1). The main idea for establishing this equivalent problem is: achieving a given average rate $\mu_{[T]}$ over T time slots is the same as transmitting $\tau T \mu_{[T]}$ amount of data within T time slots, where τ is the length of each time slot.

⁴For each transmitter-receiver pair in every time slot, the transmission rate should not exceed the corresponding capacity.

Problem 1. *For a given transmit-power-vector set \mathcal{S} and a finite horizon of T time slots, the equivalent transmission-time-minimization problem is*

$$\begin{aligned} & \text{minimize} && p \\ & (\mathbf{s}_t)_{t=1}^p, \mathbf{s}_t \in \mathcal{S} \\ & \text{subject to} && \mathbf{Q}_t = (\mathbf{Q}_{t-1} - \tau \mathbf{C}(\mathbf{s}_t))^+, t \in \{1, \dots, p\}, \quad (10) \\ & && \mathbf{Q}_0 = \tau T \mu_{[T]}, \\ & && \mathbf{Q}_p = \mathbf{0}, \end{aligned}$$

where p denotes the number of time slots for completing the transmission and is a variable dependent on $(\mathbf{s}_t)_{t=1}^p$. Additionally, $\mathbf{Q}_t = [Q_t^{(1)}, \dots, Q_t^{(N)}]^{\text{tr}}$ in (10), and each $Q_t^{(n)} \in \mathbb{R}_+$ is the length of an equivalent virtual data queue in transmitter Tx_n after \mathbf{s}_t is applied in time slot t ($t \in \{1, \dots, p\}$). The vector \mathbf{Q}_0 contains the lengths of the initial data queues before applying \mathbf{s}_1 . The vector $\mu_{[T]} = [\mu_{[T]}^{(1)}, \dots, \mu_{[T]}^{(N)}]^{\text{tr}}$ is the given data rate to be achieved. A solution of optimal design parameters (not unique for $T > 1$) is denoted as $(\mathbf{s}_t^*)_{t=1}^p$. We label the optimal objective as p^* , which stands for the minimum number of time slots to clear the data queue. The corresponding vector of data-queue sequence under the optimal solution is denoted by $(\mathbf{Q}_t^*)_{t=1}^p$.

In the rest of this section, we will give detailed discussions on how to derive the rate-achieving policy (see Section III-A) based on the solution of Problem 1. A computationally efficient algorithm for solving Problem 1 will be presented in Section III-B.

A. Deriving the Rate-Achieving Policy

In this subsection, we derive the rate-achieving policy for any given achievable rate. It should be noted that our method is complete, i.e., for any given achievable rate, the corresponding rate-achieving policy can be obtained. In contrast, the classical max-weight algorithm [4] is not complete, which is discussed at the end of this subsection.

First, we present the rate-achieving policy for all rates in the T -slot throughput region as follows.

Theorem 1 (Rate-Achieving Policy). *Given a transmit-power-vector set \mathcal{S} and a finite horizon of T time slots, then:*

- i) *If $\mu_{[T]} \in \Lambda_{[T]}$, then $p^* \leq T$, and the rate-achieving policy is $\mathcal{P}_T = (\mu_t, \mathbf{s}_t)_{t=1}^T$ with*

$$(\mu_t, \mathbf{s}_t) = \begin{cases} \left(\frac{\mathbf{Q}_{t-1}^* - \mathbf{Q}_t^*}{\tau}, \mathbf{s}_t^* \right) & 1 \leq t \leq p^*, \\ (\mathbf{0}, \mathbf{0}) & p^* < t \leq T, \end{cases} \quad (11)$$

where $(\mathbf{s}_t^*)_{t=1}^{p^*}$ is an optimal solution to Problem 1 and \mathbf{Q}_t^* is the corresponding data queue vector in time slot k when applying the optimal solution.

- ii) *If $\mu_{[T]} \notin \Lambda_{[T]}$, then solving Problem 1 gives $p^* > T$.*

Proof: i) $\forall \mu_{[T]} \in \Lambda_{[T]}$, then the data queue can be cleared with some $p \leq T$, which implies $p^* \leq p \leq T$ holds. Based on $p^* \leq T$, we prove that (11) is exactly the rate-achieving

policy for $\boldsymbol{\mu}_{[T]}$. By (11), the average rate over T slots is

$$\frac{1}{T} \sum_{t=1}^T \frac{\mathbf{Q}_{t-1}^* - \mathbf{Q}_t^*}{\tau} = \frac{\mathbf{Q}_0}{\tau T} = \frac{\tau T \boldsymbol{\mu}_{[T]}}{\tau T} = \boldsymbol{\mu}_{[T]}, \quad (12)$$

which means the rate is achieved by rate sequence $((\mathbf{Q}_{t-1}^* - \mathbf{Q}_t^*)/\tau)_{t=1}^{p^*}$. Additionally, since the following holds for every $t \in \{1, \dots, p^*\}$

$$\frac{\mathbf{Q}_{t-1}^* - \mathbf{Q}_t^*}{\tau} \preceq \mathbf{C}(\mathbf{s}_t^*), \quad (13)$$

the capacity constraints (see Definition 3) are satisfied. Therefore, $\boldsymbol{\mu}_{[T]}$ can be achieved by the policy \mathcal{P}_T .

ii) $\forall \boldsymbol{\mu}_{[T]} \notin \Lambda_{[T]}$, $p > T$ always holds, so does $p^* > T$. ■

Remark 1. This theorem implies that by solving Problem 1 for any given rate, we are able to: (i) directly tell whether the rate is achievable or not by looking at the value of the optimal objective of Problem 1; and (ii) obtain the rate-achieving policy in a closed form based on the solution to Problem 1, if the rate is achievable. Hence, the complexity of finding the rate-achieving policy is the same as that of solving Problem 1.

Remark 2. The max-weight algorithm⁵ is a commonly used method to find rate-achieving policies over an infinite time horizon. A natural question is: can we use the max-weight algorithm to derive the rate-achieving policy over a finite horizon of T time slots? We claim that the max-weight algorithm cannot always give feasible rate-achieving policies for achievable rates over a finite horizon.

A simple and explicit example is given in Fig. 3, which illustrates that the max-weight algorithm is not complete in finding rate-achieving policy even for one-slot throughput region: Assume that we want achieve a rate $\boldsymbol{\mu}_{[1]}$ within one time slot, i.e., $T = 1$. To achieve this rate, the designed algorithm should find a pair $(\boldsymbol{\mu}_1, \mathbf{s}_1)$ such that $\boldsymbol{\mu}_{[1]} = \boldsymbol{\mu}_1 \preceq \mathbf{C}(\mathbf{s}_1)$ holds. However, the max-weight algorithm cannot return such a rate. This is because it always sets the transmission rate in a single time slot to be the one having the maximum inner product with the remaining virtual data queue. In the special case of $T = 1$, the remaining virtual data queue is $\mathbf{Q}_0 = \boldsymbol{\mu}_{[1]}$. From Fig. 3, we can see that $\boldsymbol{\mu}_1 = \mathbf{C}(\mathbf{s}'_1)$ will be selected as the transmission rate, since it has the maximum projection $\|\vec{oa}\|$ on \mathbf{Q}_0 . However, transmitting at the rate of $\mathbf{C}(\mathbf{s}'_1)$ cannot achieve $\boldsymbol{\mu}_{[1]}$, or more precisely, the required rate of the first transmitter-receiver pair is not achieved. In contrast, setting the transmission rate to $\mathbf{C}(\mathbf{s}''_1)$ is sufficient to achieve $\boldsymbol{\mu}_{[1]}$ (recall that $\mathbf{Q}_0 = \boldsymbol{\mu}_{[1]}$), even though $\mathbf{C}(\mathbf{s}''_1)$ has a smaller projection on \mathbf{Q}_0 , because $\|\vec{ob}\| < \|\vec{oa}\|$.

Therefore, the max-weight algorithm does not always work even for the simplest case of one time slot. The same argument can be extended to examine the transmission policy returned by the max-weight algorithm in the final time slot of a general T -slot scenario. Therefore, we conclude that the max-weight

algorithm is not complete in finding rate-achieving policies for any finite-horizon throughput region.

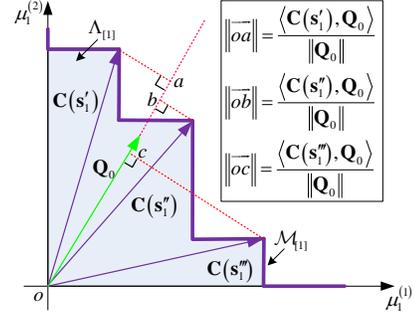


Fig. 3. The application of the max-weight algorithm in one time slot. For two transmitter-receiver pairs, we have $\boldsymbol{\mu}_1 = [\mu_1^{(1)}, \mu_1^{(2)}]^\text{tr}$ and the one-slot throughput region $\Lambda_{[1]}$ is a two-dimensional region. The purple (thick) line segments stand for the weak Pareto frontier $\mathcal{M}_{[1]}$ for $\Lambda_{[1]}$, and the Pareto frontier is $\overline{\mathcal{M}}_{[1]} = \{\mathbf{C}(\mathbf{s}'_1), \mathbf{C}(\mathbf{s}''_1), \mathbf{C}(\mathbf{s}'''_1)\}$. Without loss of generality and for the simplicity of analysis, we assume the length of a time slot $\tau = 1$.

B. Solving the Transmission-Time-Minimization Problem

In Section III-A, the results are based on the solution to Problem 1. In this subsection, we discuss how to efficiently solve Problem 1.

To solve (10) in Problem 1, intuitively, we could use dynamic programming to search from $\mathbf{Q}_p = \mathbf{0}$ to $\mathbf{Q}_0 = \tau T \boldsymbol{\mu}_{[T]}$ (backward) or employ other uninformed search strategies [11]. However, in such searching methods, the number of leaf nodes in the search tree grows exponentially with the depth of the tree and has a large branch factor. To be more specific, the branching factor is $|\mathcal{S}|$. For example, if we start the search from $\mathbf{Q}_0 = \tau T \boldsymbol{\mu}_{[T]}$, for the first step, we need to calculate all

$$\mathbf{Q}_1 = (\mathbf{Q}_0 - \tau \mathbf{C}(\mathbf{s}_1))^+, \quad (14)$$

for all $\mathbf{s}_1 \in \mathcal{S}$. Thus, the number of leaf nodes is $|\mathcal{S}|$ for the depth $t = 1$. Similarly, for every possible \mathbf{Q}_1 in (14), we have $|\mathcal{S}|$ possible \mathbf{Q}_2 , and thus the number of leaf nodes for $t = 2$ is $|\mathcal{S}|^2$. As such, the number of leaf nodes for depth $t = p^*$ (the optimal transmission time) is $|\mathcal{S}|^{p^*}$. The complexity of such searching methods is $O(|\mathcal{S}|^{p^*})$.

In this subsection, we use the following three steps to significantly improve the computational efficiency in solving Problem 1 and arrive at an lower complexity $O(|B|^{p^*})$, where B is very small compared to $|\mathcal{S}|$.

Step 1: Firstly, we reduce the branching factor from $|\mathcal{S}|$ to $|\overline{\mathcal{M}}_{[1]}|$, which is given in Proposition 1.

Proposition 1 (Branching Factor Reduction). *There exists a sequence $(\mathbf{s}_t)_{t=1}^{p^*}$, where $\mathbf{C}(\mathbf{s}_t) \in \overline{\mathcal{M}}_{[1]}$, $t \in \{1, \dots, p^*\}$, such that $(\mathbf{s}_t)_{t=1}^{p^*}$ is an optimal solution of Problem 1.*

Proof: Let $(\mathbf{s}_t^*)_{t=1}^{p^*}$ be an optimal solution of Problem 1, we have $\mathbf{Q}_{p^*} = \mathbf{0}$, which implies

$$\tau T \boldsymbol{\mu}_{[T]} \preceq \tau \sum_{t=1}^{p^*} \mathbf{C}(\mathbf{s}_t^*). \quad (15)$$

⁵This algorithm was given in a seminal work in [4] and it can achieve all rates in the interior of the infinite-horizon throughput region (not including the boundary rate).

Let $(\mathbf{s}_t)_{t=1}^{p^*}$ be the sequence with $\mathbf{C}(\mathbf{s}_t) \in \overline{\mathcal{M}}_{[1]}$, and $\mathbf{C}(\mathbf{s}_t^*) \preceq \mathbf{C}(\mathbf{s}_t)$ ($t \in \{1, \dots, p^*\}$). Thus, (15) can be rewritten as

$$\tau T \boldsymbol{\mu}_{[T]} \preceq \tau \sum_{t=1}^{p^*} \mathbf{C}(\mathbf{s}_t^*) \preceq \tau \sum_{t=1}^{p^*} \mathbf{C}(\mathbf{s}_t), \quad (16)$$

which implies $\mathbf{Q}_{p^*} = \mathbf{0}$ when applying $(\mathbf{s}_t)_{t=1}^{p^*}$. Therefore, $(\mathbf{s}_t)_{t=1}^{p^*}$ is an optimal solution of Problem 1. ■

Remark 3. Proposition 1 tells that we only need to consider the transmit powers corresponding to the rate on the Pareto frontier of the one-slot throughput region, instead of all possible transmit powers. Hence, the transmit-power-vector set \mathcal{S} in Problem 1 can be substituted by $\overline{\mathcal{S}}$, called the refined transmit-power-vector set, such that $\mathbf{C}(\mathbf{s}_t) \in \overline{\mathcal{M}}_{[1]}$ holds for all $\mathbf{s}_t \in \overline{\mathcal{S}}$. Therefore, the branching factor is $|\overline{\mathcal{S}}| = |\overline{\mathcal{M}}_{[1]}|$.

Step 2: More importantly, A* search is employed to further improve the searching efficiency while maintaining the optimality (see [11]) for Problem 1. A brief description is given here on the application of A* search in solving Problem 1, while we refer the readers to [11] for a complete description of the A* search algorithm.

For A* search (or any searching algorithm in general), ‘node’ is a fundamental concept. In our case, a node is given by $(\mathbf{Q}_t, (\mathbf{s}_i)_{i=1}^t)$, which depends on \mathbf{Q}_t the state, and $(\mathbf{s}_i)_{i=1}^t$ the path to achieve this state from the initial node $(\mathbf{Q}_0, \emptyset)$. A* search requires five components to be implemented:

- **Initial node.** The node starting the search, which is $(\mathbf{Q}_0, \emptyset)$.
- **Action space.** The set of actions that move from a node to all possible child nodes. In our case, the action space is $\overline{\mathcal{S}}$ according to Remark 3.
- **Goal.** The condition for stopping the search. In our case, the goal is $\mathbf{Q}_p = \mathbf{0}$ or simply denoted as $\mathbf{0}$.
- **Step cost.** The step cost is the cost for each searching step. In Problem 1, the step cost is $c_t = 1$, $t \in \{1, \dots, p\}$.
- **Evaluation function.** It records the path cost (the summation of all previous step costs) from the past and estimates the path cost in the future. To be more specific, for a given node $(\mathbf{Q}_t, (\mathbf{s}_i)_{i=1}^t)$, the evaluation function is

$$F(\mathbf{Q}_t, (\mathbf{s}_i)_{i=1}^t) = G((\mathbf{s}_i)_{i=1}^t) + H(\mathbf{Q}_t), \quad (17)$$

where $G((\mathbf{s}_i)_{i=1}^t)$ returns the path cost from the initial node to node $(\mathbf{Q}_t, (\mathbf{s}_i)_{i=1}^t)$. A heuristic function $H(\mathbf{Q}_t)$ estimates the path cost from $(\mathbf{Q}_t, (\mathbf{s}_i)_{i=1}^t)$ to the goal $\mathbf{0}$. A* search always expands the node with smallest F .

It should be noted that the core of A* search is to construct an admissible heuristic function⁶, since other parts of A* can be determined by the definition of the problem. In this work, we propose the interference-free based heuristic function:

$$H^I(\mathbf{Q}_t) = \max_{n \in \mathcal{N}} \frac{Q_t^{(n)}}{\tau \log_2(1 + \gamma'_n(s_{\max}^{(n)}))}, \quad (18)$$

⁶That is, $H(\mathbf{Q}_t) \leq H^*(\mathbf{Q}_t)$ holds for every \mathbf{Q}_t , where $H^*(\mathbf{Q}_t)$ is the actual cost from \mathbf{Q}_t to the goal $\mathbf{0}$.

where $t \in \{1, \dots, p\}$, $s_{\max}^{(n)} := \max \mathcal{S}^{(n)}$, and

$$\gamma'_n(s_{\max}^{(n)}) = \frac{h_{nn}s_{\max}^{(n)}}{W_n}, \quad n \in \mathcal{N}. \quad (19)$$

This heuristic function is interference-free based, since compared to (1), (19) does not consider the interference from other transmitters. The following proposition states that $H^I(\mathbf{Q}_t)$ is admissible, which means A* search can be employed.

Proposition 2 (Admissibility of Interference-Free Based Heuristic Function for Problem 1). *Let the actual cost to reach the goal $\mathbf{Q}_p = \mathbf{0}$ be $H^*(\mathbf{Q}_t) = p - t$, where $t \in \{1, \dots, p\}$. Then $H^I(\mathbf{Q}_t) \leq H^*(\mathbf{Q}_t)$ holds for every \mathbf{Q}_t .*

Proof: $\forall \mathbf{Q}_t$, let $\bar{\mathbf{s}}_k = [\bar{s}_k^{(1)}, \dots, \bar{s}_k^{(n)}]$, $k \in \{t+1, \dots, p\}$ be any possible action (transmit-power vector) after \mathbf{Q}_{k-1} . $\forall n \in \mathcal{N}$, we have

$$H^*(\mathbf{Q}_t) = p - t = \sum_{k=t+1}^p 1 \geq \sum_{k=t+1}^p \frac{Q_{k-1}^{(n)} - Q_k^{(n)}}{\tau \log_2(1 + \gamma_n(\bar{\mathbf{s}}_k))}. \quad (20)$$

Additionally, since $\bar{s}_k^{(n)} \leq s_{\max}^{(n)}$, the following holds

$$\gamma_n(\bar{\mathbf{s}}_k) = \frac{h_{nn}\bar{s}_k^{(n)}}{W_n + \sum_{m \neq n} h_{mn}\bar{s}_k^{(m)}} \leq \frac{h_{nn}s_{\max}^{(n)}}{W_n} = \gamma'_n(s_{\max}^{(n)}). \quad (21)$$

Thus, (20) can be further bounded from below as

$$\begin{aligned} H^*(\mathbf{Q}_t) &\geq \max_{n \in \mathcal{N}} \sum_{k=t+1}^p \frac{Q_{k-1}^{(n)} - Q_k^{(n)}}{\tau \log_2(1 + \gamma'_n(s_{\max}^{(n)}))} \\ &= \max_{n \in \mathcal{N}} \left\{ \frac{Q_t^{(n)} - Q_p^{(n)}}{\tau \log_2(1 + \gamma'_n(s_{\max}^{(n)}))} \right\} = H^I(\mathbf{Q}_t). \end{aligned} \quad (22)$$

Therefore, $H^I(\mathbf{Q}_t) \leq H^*(\mathbf{Q}_t)$ holds. ■

Step 3: Last but not least, we propose a pruning strategy to further improve the searching efficiency of A* search: After selecting a node to expand, labelled by $(\mathbf{Q}_{t_1}, (\mathbf{s}_i)_{i=1}^{t_1})$, we delete those nodes with $t \geq t_1$ but $(\mathbf{C}(\mathbf{s}_i))_{i=1}^t \preceq (\mathbf{C}(\mathbf{s}_i))_{i=1}^{t_1}$ in the fringe (or called open set, more details can be found in [11]), since those nodes’ subtrees are suboptimal or can be replaced with the new node $(\mathbf{Q}_{t_1}, (\mathbf{s}_i)_{i=1}^{t_1})$.

To sum up, the method for solving Problem 1 is given in Algorithm 1, in which our pruning strategy is implicitly included in the A* search algorithm.

Algorithm 1 Solving Problem 1 with A* Search

Input: T : number of time slots; N : the number of transmitter-receiver pairs; $\boldsymbol{\mu}_{[T]}$: the given rate to be achieved; $\overline{\mathcal{S}}$: refined transmit-power-vector set.

Output: $(\mathbf{s}_t^*)_{t=1}^{p^*}$: the optimal solution of Problem 1; p^* : the optimal objective of Problem 1.

- 1: $\mathbf{Q}_0 = \tau T \boldsymbol{\mu}_{[T]}$;
 - 2: $[(\mathbf{s}_t^*)_{t=1}^{p^*}, p^*] = \text{A}^*((\mathbf{Q}_0, \emptyset), \overline{\mathcal{S}}, \mathbf{0}, c_t, F(\cdot))$;
 - 3: **return** $(\mathbf{s}_t^*)_{t=1}^{p^*}$ and p^* .
-

We use the concept of effective branching factor⁷ (EBF) to measure the searching efficiency of the proposed solution to Problem 1. For a fixed p^* , the relationship between U (the total number of expanded nodes) and the EBF is

$$U = \sum_{t=1}^{p^*} B^t. \quad (23)$$

where B is the EBF. We can see that B polynomially increases with U , which means the smaller the EBF is, the better our algorithm performs. In Section IV, we will present numerical results on EBF to measure the searching efficiency.

IV. NUMERICAL RESULTS

In this section, we present numerical results to corroborate our analytical results. First, we give two illustrative examples with different channel conditions: an example of a given rate falling in the throughput region (i.e., achievable rate) and an example of a given rate falling out of the throughput region. Consider a network with $N = 3$ transmitter-receiver pairs within $T = 5$ time slots. The transmit-power sets of these 3 transmitter-receiver pairs are $\mathcal{S}^{(1)} = \mathcal{S}^{(2)} = \mathcal{S}^{(3)} = \{0, 2\}$, which actually represent an on-off transmission scheme. The noise powers are $W_1 = W_2 = W_3 = 0.1$, and the length of a time slot τ is normalized to 1. Under the following two different channel conditions, we want to achieve the rate $\boldsymbol{\mu}_{[5]} = [1, 1, 1]^{\text{tr}}$:

- Consider channel power gains $h_{11} = 0.5$, $h_{22} = 0.6$, $h_{33} = 0.7$, and $h_{12} = h_{21} = h_{13} = h_{31} = h_{23} = h_{32} = 0.2$. By using Theorem 1 and solving Problem 1 with the proposed A* search algorithm, the rate-achieving policy is $\mathcal{P}_5 = (\boldsymbol{\mu}_t, \mathbf{s}_t)_{t=1}^5$, where $\boldsymbol{\mu}_1 = [3.4594, 0, 0]^{\text{tr}}$, $\boldsymbol{\mu}_2 = \boldsymbol{\mu}_3 = [0, 1.7655, 1.9260]^{\text{tr}}$, $\boldsymbol{\mu}_4 = [1.0780, 1.2224, 1.1480]^{\text{tr}}$, $\boldsymbol{\mu}_5 = [0.4626, 10.2465, 0]^{\text{tr}}$, and $\mathbf{s}_1 = [2, 0, 0]^{\text{tr}}$, $\mathbf{s}_2 = \mathbf{s}_3 = [0, 2, 2]^{\text{tr}}$, $\mathbf{s}_4 = [2, 2, 2]^{\text{tr}}$, $\mathbf{s}_5 = [2, 2, 0]^{\text{tr}}$.
- Consider channel power gains $h_{11} = h_{22} = h_{33} = 0.2$, and $h_{12} = h_{21} = h_{13} = h_{31} = h_{23} = h_{32} = 0.5$. Solving Problem 1 gives $p^* = 8 > 5$. Hence, Theorem 1 tells that the rate $[1, 1, 1]^{\text{tr}}$ is not achievable in $T = 5$ time slots.

Next, we conduct Monte Carlo simulations to examine the computational efficiency. All the system parameters including the given rate to be achieved $\boldsymbol{\mu}_{[5]} = [1, 1, 1]^{\text{tr}}$ remain the same instead of the channel power gains. Here, we consider many possible realizations of the fading channels. Specifically, we use Nakagami- m fading with $m \in \{1, \dots, 5\}$ to generate 10000 realizations of the channel for each communication and interference link (hence we have 10000 different scenarios). The average EBF (effective branching factor) is given in Table I. Assuming $p^* = T = 5$, then the average total number of nodes (except for the starting node) of the original tree (computed using (23)) without applying any of the three steps in Section III is $\sum_{t=1}^5 8^t = 37449$. But using our A* search with pruning, e.g., for $m = 3$, the average number of expanded

⁷It is a popular metric for characterizing the quality of searching method, e.g., see Section 3.6.1 in [11].

TABLE I
AVERAGE EBF UNDER DIFFERENT NAKAGAMI- m FADING

	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$
EBF	3.5557	3.5757	3.6116	3.6334	3.6502

nodes is only $\sum_{t=1}^5 3.6116^t \approx 849$. This shows a significant improvement in the computational efficiency.

V. CONCLUSION

For the first time, this work studied the throughput region of a wireless multi-user interference channel over a finite time horizon. We provided a computationally efficient algorithm that determines whether a rate is achievable in a given finite number of time slots, and if so this algorithm provides the rate-achieving policy (a sequence of rate-power pairs) to achieve that rate. We started by formulating an equivalent transmission-time-minimization problem whose optimal solution provides a closed-form expression for the rate-achieving policy. In order to efficiently solve the transmission-time-minimization problem, we applied three steps: i) branch factor reduction; ii) A* search algorithm with a carefully chosen admissible heuristic function; and iii) pruning strategy. Simulation results demonstrated the efficiency of the proposed method in improving the computational efficiency.

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