The Impact of Antenna Height Difference on the Performance of Downlink Cellular Networks

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Abstract—Capable of significantly reducing cell size and enhancing spatial reuse, network densification is shown to be one of the most dominant approaches to expand network capacity. Due to the scarcity of available spectrum resources, nevertheless, the over-deployment of network infrastructures, e.g., cellular base stations (BSs), would strengthen the inter-cell interference as well, thus in turn deteriorating the system performance. On this account, we investigate the performance of downlink cellular networks in terms of user coverage probability (CP) and network spatial throughput (ST), aiming to shed light on the limitation of network densification. Notably, it is shown that both CP and ST would be degraded and even diminish to be zero when BS density is sufficiently large, provided that practical antenna height difference (AHD) between BSs and users is involved to characterize pathloss. Moreover, the results also reveal that the increase of network ST is at the expense of the degradation of CP. Therefore, to balance the tradeoff between user and network performance, we further study the critical density, under which ST could be maximized under the CP constraint. Through a special case study, it follows that the critical density is inversely proportional to the square of AHD. The results in this work could provide helpful guideline towards the application of network densification in the next-generation wireless networks.

I. INTRODUCTION

Among the possible approaches to fulfill the unprecedented capacity goals of the future wireless networks, network densification has been shown to be the one with the greatest potential [1]. The basic principle behind network densification is to deploy base stations (BSs) or access points (APs) with smaller coverage to enable local spectrum reuse [2], [3]. As such, mobile users are served with short-distance transmission links, thereby facilitating enormous spatial multiplexing gain and enhancing network capacity. The benefits of network densification are substantially verified via the experimental results from Qualcomm [4]. Specifically, it is shown that over 1000-fold network capacity gain can be harvested by deploying 144 self-organizing small cells into one macrocell, as compared to the macro-only case. Despite the merits, however, the experimental results in [4] also show that the benefits of network densification in terms of network capacity enhancement start to diminish when the number of deployed small cells is sufficiently large. In other words, network densification may gradually drain the spatial multiplexing gain as well. Therefore, the limitation of network densification remains to be fully explored.

The research on how network densification impacts the capacity of wireless networks has received extensive attention

in the literature. In [5], [6], the performance of single-tier cellular networks and multi-tier heterogeneous networks has been investigated, respectively. Remarkably, it is shown that the network spatial throughput (ST), an important indicator of network capacity, would linearly increase with the densification of cellular BSs in both single- and multi-tier networks. As an encouraging result, it indicates that the potential spatial multiplexing gain can be sustainably achieved provided that a sufficient number of BSs are deployed. Nevertheless, the analysis in [5], [6] is made on the premise that only non-line-of-sight (NLOS) paths exist between the transmitters (Tx's) and the intended receivers (Rx's). Due to the shorter transmission distance in dense deployment, lineof-sight (LOS) paths are more likely to appear as well. On this account, authors in [7]-[9] have captured the impact of LOS/NLOS transmissions on the performance of downlink cellular networks. In particular, it has been observed that the user coverage probability (CP) tends to decay at some density and network ST grows sublinearly or even decreases with the increase of BS density [9]. This is mainly due to the fact that the inter-cell interference power is likely to overwhelm the desired signal power when LOS paths exist between interfering BSs and the intended downlink user. Especially, when BS density further increases, more interfering BSs would have LOS paths to the intended user, thereby degrading user and system performance. The results reveal the limitation of network densification. Furthermore, besides the scaling law analysis, authors in [10] have quantified the density, beyond which network ST experiences a notable decrease.

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In the aforementioned research, the 2-D distance is applied to approximate the distance between the antennas of Tx's and Rx's. In sparsely deployed networks where Tx's and Rx's are far from each other, such approximation is of high accuracy and thus valid. When Tx's and Rx's are in proximity, however, it is apparent that the approximation will lose the accuracy (see Fig. 1). Hence, it is of great importance to investigate the performance of ultra-dense networks (UDN) with antenna height difference (AHD) of Tx's and Rx's. Besides, it is shown from [7]–[10] that the increase of network capacity (system performance) is at the cost of the deterioration of user performance (e.g., CP). Since user performance is an important indicator to evaluate the performance of network densification, it is crucial to balance the tradeoff between user and network performance.

Motivated by above discussions, we investigate the funda-

mentals of network densification in downlink cellular networks with the aid of stochastic geometry. To explore the impact of AHD between BSs and downlink users, we study the scaling laws of both CP (user performance) and network ST (system performance) under a generalized multi-slope pathloss model (MSPM). Surprisingly, it is shown that, considering AHD, both CP and network ST would be degraded by network overdensification and even asymptotically approach zero when BS density is sufficiently large. The results are opposite to that derived without considering AHD [7]–[10]. Moreover, to guarantee the quality of service (QoS) of users, we further analyze the critical density that could maximize the network ST under the CP constraint. It is observed that the critical density is much smaller (e.g., 10% or even less under the typical settings) than the density, under which network ST is maximized without the CP constraint. The above results could provide helpful insights and guidelines towards the planning and deployment of future wireless networks.

For the remainder of this paper, we first describe the system model in Section II, followed by a preliminary analysis on CP and ST under a multi-slope pathloss model in Section III. Afterward, we study the CP and ST scaling laws in Section IV and investigate the critical density under the CP constraint in Section V. Finally, conclusions are given in Section VI.

II. SYSTEM MODEL

A. Network Model

Consider a downlink cell network (see Fig. 1), where BSs (with constant transmit power P) and downlink users are distributed in a two-dimension plane \mathbb{R}^2 , in line with two independent Homogeneous Poisson Point Processes (HPPPs), $\Pi_{\rm BS} = \{ \mathrm{BS}_i \, \big| \mathrm{BS}_i \in \mathbb{R}^2 \} \text{ and } \Pi_{\rm U} = \{ \mathrm{U}_j \, \big| \mathrm{U}_j \in \mathbb{R}^2 \}$ $(i, j \in \mathbb{N})$, respectively. It is assumed that all the BSs (downlink users) are equipped with antennas of identical heights. Meanwhile, denote Δh as the AHD between BSs and users. Downlink users are associated with the geometrically nearest BSs so as to obtain the strongest average signal strength. It is assumed that the user density $\lambda_{\rm U}$ is much greater than the BS density λ , i.e., $\lambda_U \gg \lambda$, to ensure that all the BSs are connected and activated. In each time slot, BSs would randomly select one of the associated users to serve. Besides, a saturated data model is considered such that users always require data to download from the serving BSs.

Channel power gain consists of two components: pathloss and small-scale fading. To comprehensively characterize the impact of LOS and NLOS components, we have adopted an MSPM, i.e.,

$$l_N\left(\{\alpha_n\}_{n=0}^{N-1}; x\right) = K_n x^{-\alpha_n}, \ R_n \le x < R_{n+1}$$
(1)

where $K_0 = 1$, $K_n = \prod_{i=1}^n R_i^{\alpha_i - \alpha_{i-1}}$ $(n \ge 1)$, $0 = R_0 < R_1 < \cdots < R_N = \infty$ and $0 \le \alpha_0 \le \alpha_1 \le \cdots \le \alpha_{N-1}$ $(\alpha_{N-1} > 2 \text{ for practical concerns [8]}).$

From (1), it follows that different pathloss exponents are used to characterize the attenuation rates of signal power within different regions. For instance, when N = 2, MSPM



Figure 1. Illustration of downlink cellular networks. Downlink users are connected to the geometrically nearest BSs. When BSs are associated with more than one user, one of them are randomly selected by BSs to serve. Instead of the 2-D distance r_i between BSs and downlink users, the 3-D distance d_i between the antennas of them is considered, involving the AHD Δh . As an example, $d_0 = \sqrt{r_0^2 + \Delta h^2}$ for typical downlink BS₀-U₀.

degenerates into the dual-slope pathloss model (DSPM) [8], [11]

$$l_{2}(\alpha_{0}, \alpha_{1}; x) = \begin{cases} x^{-\alpha_{0}}, & x \leq R_{1} \\ K_{1}x^{-\alpha_{1}}, & x > R_{1} \end{cases}$$
(2)

where $K_1 = R_1^{\alpha_1 - \alpha_0}$. The DSPM in (2) is applied when an LOS path and a ground-reflected path exist between Tx and the intended Rx. As such, signal power attenuates slowly (with rate α_0) within a *corner distance* R_1 , while attenuates much more quickly (with rate α_1) with distance out of R_1 . When N = 1, MSPM further degenerates into the most widely used single-slope pathloss model (SSPM) [5], [11]

$$l_1(\alpha_0; x) = x^{-\alpha_0}, \ x \in [0, \infty).$$
 (3)

For small-scale fading, although it is more suitable to use Rice fading when LOS paths exist between Tx's and Rx's, insightful results could hardly be obtained due to the complicated form. Instead, Rayleigh fading with zero mean and unit variance $h \sim C\mathcal{N}(0,1)$ is applied to model smallscale fading for mathematical tractability. In additional, as will be shown in Section IV, the application of Rayleigh fading will not impact the results on CP and ST scaling laws via the comparison between numerical and simulation results.

B. Performance Metrics

We adopt CP and ST to reflect user performance and system performance, respectively. To be specific, following the signal-to-interference ratio (SIR) at the typical downlink user U_0^{1} , CP is defined as

$$\mathsf{CP}\left(\lambda\right) = \mathbb{P}\left\{\mathsf{SIR}_{\mathsf{U}_0} > \tau\right\},\tag{4}$$

where τ denotes the decoding threshold. Based on CP in (4), we further define network ST as

$$\mathsf{ST}(\lambda) = \lambda \mathbb{P}\left\{\mathsf{SIR}_{U_0} > \tau\right\} \log_2\left(1+\tau\right), \ \left[\mathsf{bits}/\left(\mathsf{s} \cdot \mathsf{Hz} \cdot \mathsf{m}^2\right)\right]$$
(5)

 $^1Without loss of generality, we evaluate the CP of downlink pair BS_0-U_0. Meanwhile, as spectrum resources could be universally reused, inter-cell interference dominates the performance of downlink networks. Hence, the impact of noise is ignored.$

which could characterize the number of bits that are successfully conveyed over unit time, frequency and area. Hence, ST serves as an indicator to network capacity.

Notation: If ${}_{2}F_{1}(\cdot, \cdot, \cdot, \cdot)$ is defined as the standard Gaussian hypergeometric function, denote $\omega_{1}(x, \alpha_{n}) = {}_{2}F_{1}\left(1, 1 - \frac{2}{\alpha_{n}}, 2 - \frac{2}{\alpha_{n}}, -x\right)$ and $\omega_{2}(x, \alpha_{n}) = {}_{2}F_{1}\left(1, \frac{2}{\alpha_{n}}, 1 + \frac{2}{\alpha_{n}}, -x\right)$ in the rest of the paper.

III. ANALYSIS OF CP AND ST WITH AHD

In this section, we first give preliminary analysis of CP and ST under the MSPM in (1). Particularly, the impact of the AHD on the network performance is highlighted.

From (4), CP is defined based on the SIR evaluated at U_0 . Therefore, we first characterize the SIR at U_0 as

$$SIR_{U_0} = PH_{U_0, BS_0} l_N \left(\{ \alpha_n \}_{n=0}^{N-1}; d_0 \right) / I_{IC}, \qquad (6)$$

where $I_{\text{IC}} = \sum_{\text{BS}_i \in \tilde{\Pi}_{\text{BS}}} PH_{\text{U}_0, \text{BS}_i} l_N \left(\{\alpha_n\}_{n=0}^{N-1}; d_i \right)$ denotes

the inter-cell interference suffered by U_0 , $\Pi_{BS} = \Pi_{BS} \setminus BS_0$, d_i denotes the distance from the antenna of BS_i to that of U_0 , and H_{U_0,BS_i} denotes the corresponding channel power gain caused by small-scale fading. Meanwhile, if we denote r_i as the distance from BS_i to U_0 , then $d_i = \sqrt{r_i^2 + \Delta h^2}$. Note that $H_{U_0,BS_i} \sim \exp(1)$ since Rayleigh fading $h \sim C\mathcal{N}(0,1)$ is applied to model small-scale fading.

From (6), we can obtain the following results on CP and ST in Proposition 1.

Proposition 1. Considering the AHD between BSs and downlink users, the ST in downlink cellular networks under MSPM in (1) is given by $ST_N(\lambda) = \lambda CP_N(\lambda) \log_2(1+\tau)$, where $CP_N(\lambda)$ is given by (7) at the top of Page 4. In (7), $C_1 = \frac{2\tau\omega_1(\tau,\alpha_0)}{\alpha_0-2}$, $d_0 = \sqrt{r_0^2 + \Delta h^2}$ and the probability density function (PDF) of r_0 is derived from the contact distribution [12]

$$f_{r_0}(x) = 2\pi\lambda x \exp\left(-\pi\lambda x^2\right), \ x \ge 0.$$
(8)

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Proof: Please refer to Appendix A.

Despite its complicated form, the result in Proposition 1 could provide a numerical approach to capture the relationship between system parameters and performance metrics, namely, CP and ST, under MSPM. Meanwhile, according to the special case in (7), where N = 1, it follows that both CP and ST would exponentially decrease with Δh^2 . In other words, the results, without considering the impact of the AHD, greatly over-estimate the performance of downlink network. In addition, when N = 2 and MSPM degenerates into DSPM, the results on CP and ST could be further simplified according to the following corollary.

Corollary 1. Considering the AHD between BSs and downlink users, the ST in downlink cellular networks under DSPM in (2) is given by $ST_2(\lambda) = \lambda CP_2(\lambda) \log_2(1 + \tau)$, where

$$\mathsf{CP}_{2}(\lambda) = \mathbb{E}_{r_{0} \in [0,R_{1})} \left[e^{-\pi\lambda(\delta_{1}(\alpha_{0},d_{0},\tau,R_{1})+\delta_{2}(\alpha_{0},\alpha_{1},d_{0},\tau,R_{1}))} \right] \\ + \mathbb{E}_{r_{0} \in [R_{1},\infty)} \left[e^{-\pi\lambda\delta_{3}(\alpha_{1},d_{0},\tau)} \right].$$
(9)

 $\begin{array}{ll} In & (9), \ d_0 &= \sqrt{r_0^2 + \Delta h^2}, \ \delta_1 \left(\alpha_0, d_0, \tau, R_1 \right) &= \\ R_1^2 \omega_2 \left(\frac{R_1^{\alpha_0}}{\tau d_0^{\alpha_0}}, \alpha_0 \right) - d_0^2 \omega_2 \left(\frac{1}{\tau}, \alpha_0 \right), \ \delta_2 \left(\alpha_0, \alpha_1, d_0, \tau, R_1 \right) &= \\ \frac{2\tau d_0^{\alpha_0} R_1^{2 - \alpha_0}}{\alpha_1 - 2} \omega_1 \left(\frac{\tau d_0^{\alpha_0}}{R_1^{\alpha_0}}, \alpha_1 \right), \ \delta_3 \left(\alpha_1, d_0, \tau \right) &= \frac{2\tau d_0^2}{\alpha_1 - 2} \omega_1 \left(\tau, \alpha_1 \right) \\ and \ the \ PDF \ of \ r_0 \ is \ given \ by \ (8). \end{array}$

Proof: The proof can be completed by setting N = 2 in (7) with easy manipulation, and thus omitted due to space limitation.

Based on Proposition 1 and Corollary 1, we illustrate the impact of AHD on CP and network ST in detail. In particular, Fig. 2 shows the CP and ST as a function of Δh of BSs and downlink users under different BS densities. It can be seen from Fig. 2 that both CP and ST would be degraded by Δh . This indicates that, although the existence of Δh would weaken both desired and interference signal power, the decrease of the desired signal power overwhelms that of the interference signal powers. Meanwhile, it is shown that the impact of Δh on CP and ST is relatively small under sparse BS deployment, while the impact is significant under dense BS deployment. Hence, in dense wireless networks, where the user antenna heights are basically small, it is preferable to deploy small cell BSs with smaller antenna heights so as to reduce the AHD, thereby ensuring the user performance as well as system performance.

As shown in Fig. 2, it is evident that the existence of Δh leads to the performance degradation in terms of CP and ST, especially in the fully densified networks. Therefore, we have to further explore the influence of Δh on the scaling laws of CP and ST in the following.

IV. CP AND ST SCALING LAWS

In this part, before investigating the fundamental limitation of network densification by analyzing the CP and ST scaling laws, results on $\omega_1(x, y)$ are first given in the following Lemma.

Lemma 1. For y > 2, $\omega_1(x, y)$ is a decreasing function of x.

Proof: Please refer to the proof for Lemma 1 in [13]. \Box On the basis of Lemma 1 and Proposition 1, we show the CP and ST scaling laws in Theorem 1.

Theorem 1. When AHD exists between BSs and downlink users, CP and ST scale with BS density λ as $\mathsf{CP}_N(\lambda) \sim e^{-\kappa\lambda}$ and $\mathsf{ST}_N(\lambda) \sim \lambda e^{-\kappa\lambda}$ (κ is a constant), respectively, under MSPM.

Proof: Please refer to Appendix B.

It is shown from Theorem 1 that both user and system performance would be degraded when BS density is sufficiently large. This is essentially different from the results in [7]–[10], where the impact of AHD has not been taken into account in the scaling law analysis. Particularly, we show the difference in Fig. 3.

Fig. 3 shows the CP and ST as a function of BS density λ under different Δh . It is shown in Fig. 3a that, when $\Delta h = 0$ m, CP almost keeps constant with the increasing λ under SSPM, and slowly decreases with the increasing λ under DSPM (compared to the DSPM case under $\Delta h > 0$ m).

$$\mathsf{CP}_{N}\left(\lambda\right) = \begin{cases} \frac{1}{1+C_{1}} \exp\left(-\pi\lambda C_{1} \bigtriangleup h^{2}\right), & N = 1\\ \sum_{n=0}^{N-1} \mathbb{E}_{r_{0} \in [R_{n}, R_{n+1})} \left\{ \exp\left[-\pi\lambda \left(\bar{R}_{n+1}^{2}\omega_{2}\left(\frac{\bar{R}_{n+1}^{\alpha}}{\tau d_{0}^{\alpha}}, \alpha_{n}\right) - d_{0}^{2}\omega_{2}\left(\tau^{-1}, \alpha_{n}\right) + \sum_{i=n+1}^{N-1} \left(\bar{R}_{i+1}^{2}\omega_{2}\left(\frac{\bar{R}_{i+1}^{\alpha}}{\tau K_{i} d_{0}^{\alpha}}, \alpha_{i}\right) - \bar{R}_{i}^{2}\omega_{2}\left(\frac{\bar{R}_{i}^{\alpha}}{\tau K_{i} d_{0}^{\alpha}}, \alpha_{i}\right)\right) \right) \right] \right\}, \qquad N > 1$$





Figure 3. CP and ST varying with BS density λ . For system settings, set P = 23dBm and $\tau = 0$ dB. For SSPM, set $\alpha_0 = 4$. For DSPM, set $\alpha_0 = 1.5$, $\alpha_1 = 4$ and $R_1 = 10$ m. To reflect the impact of LOS paths on signal propagation, we set $v_{\rm NC} = 1$ and $v_{\rm DoF} = 12$ for Rice fading.

Figure 2. CP and ST varying with AHD Δh . For system settings, set P = 23dBm and $\tau = 0$ dB. For SSPM, set $\alpha_0 = 4$. For DSPM, set $\alpha_0 = 1.5$, $\alpha_1 = 4$ and $R_1 = 10$ m. Lines and markers denote numerical and simulation results, respectively, in this figure and the remaining figures in this paper.

In consequence, network ST would linearly/sublinearly grow with λ , as shown in Fig. 3b. In contrast, both CP and ST asymptotically approach zero when λ is sufficiently large under $\Delta h > 0$ m. In practice, AHD would exist between BSs and cellular users, even when small cell BSs are densely deployed. Therefore, the results, which ignore the impact of AHD, have over-estimated the benefits of network densification, while those in Theorem 1 could shed light on the fundamental limitation of network densification.

To verify the validity of the scaling law analysis under Rayleigh fading, we also evaluate the performance of downlink networks under Rice fading via simulation results in Fig. 3. Specifically, the channel power gain under Rice fading channels follows the non-central χ^2 distribution with non-centrality parameter $v_{\rm NC}$ and degrees of freedom $v_{\rm DoF}$. A larger $v_{\rm DoF}$ indicates more scattered components. It can be seen from Fig. 3 that, although gaps exist between the results under Rice and Rayleigh fadings, it is apparent that the CP and ST scaling laws under Rice fading are identical as those under Rayleigh fading.

In addition, it is observed from Fig. 3 that the improvement of system performance is at the cost of the degeneration of user experience. For instance, when $\Delta h > 0$ m, network ST grows with BS density at $\lambda = 1 \times 10^3 \text{BS/km}^2$ (see Fig. 3a), under which CP already starts to diminish with λ (see Fig. 3b). Therefore, besides ensuring the system performance, it is also critical to guarantee the user experience when planning the deployment of cellular networks, the detail of which will be described in the next section.

V. CRITICAL DENSITY UNDER CP CONSTRAINT

In this section, a CP requirement ε is set to guarantee the QoS of users as

$$\mathsf{CP}(\lambda) = \mathbb{P}\{\mathsf{SIR}_{U_0} > \tau\} > \varepsilon.$$
(10)

From (10), it is intuitive that whether or not the constraint could be satisfied greatly depends on the deployment density of BSs. Nonetheless, as observed from Fig. 3a, the maximal CP that can be achieved reaches 0.56, irrespective of the BS density. Therefore, besides BS density, other parameters such as pathloss exponents, decoding threshold, etc., may impact whether the CP requirement can be met as well. In this light, we first analyze the necessary condition to acquire the CP requirement. Afterward, we derive the critical density, under which network ST can be maximized under the pre-set CP requirement.

It is worth noting that, to provide helpful insights towards the impact of system parameters on necessary regions and critical density, the results derived in this section are built on the SSPM in (3). In the following theorem, the results on the necessary condition are first given.

Theorem 2. Under SSPM in (3), the necessary condition to satisfy the CP requirement in (10) is given by

$$\frac{2\tau\omega_1\left(\tau,\alpha_0\right)}{\alpha_0-2} < \varepsilon^{-1} - 1. \tag{11}$$

Proof: Please refer to Appendix C.

Theorem 2 provides a direct approach on how to reasonably adjust system parameters to meet the pre-set CP requirement of downlink users. Meanwhile, the right-hand-side of (11), i.e., $g(\varepsilon) = \varepsilon^{-1} - 1$, implies that $g(\varepsilon)$ exponentially decreases with ε and approaches 0 when $\varepsilon \to 1$. Therefore, it is more difficult to meet the CP requirement especially when ε grows larger. Aided by Theorem 2, we further obtain the critical BS density in the following corollary.

Corollary 2. With the CP constraint ε , the critical BS density λ^* , under which network ST is maximized, is given by

$$\lambda^* = \frac{(\alpha_0 - 2) \ln \left[\varepsilon^{-1} \left(1 + \frac{2\tau\omega_1(\tau, \alpha_0)}{\alpha_0 - 2} \right)^{-1} \right]}{2\pi\tau\omega_1(\tau, \alpha_0) \bigtriangleup h^2}.$$
 (12)

Without the CP constraint, the critical BS density λ^{\dagger} , under which network ST is maximized, is given by

$$\lambda^{\dagger} = \frac{\alpha_0 - 2}{2\pi\tau\omega_1\left(\tau, \alpha_0\right) \bigtriangleup h^2}.$$
(13)

Proof: It is straightforward to obtain λ^* following (27) in Appendix C and λ^{\dagger} by solving $\frac{\partial ST_1(\lambda)}{\partial \lambda} = 0$, where $ST_1(\lambda)$ is given by Proposition 1.

The influence of system parameters on critical densities is captured using the closed-form expression in Corollary 2. Especially, it is observed that both λ^* and λ^{\dagger} are inversely proportional to the square of AHD, i.e., Δh^2 . Meanwhile, we extend the results into the case with DSPM applied. Specifically, we plot the critical density as a function of Δh under both SSPM and DSPM in Fig. 4. Due to space



Figure 4. Critical densities λ^* and λ^{\dagger} varying with the AHD Δh . For system settings, set P = 23dBm and $\tau = 0$ dB. For SSPM, set $\alpha_0 = 5$. For DSPM, set $\alpha_0 = 1.5$, $\alpha_1 = 5$ and $R_1 = 10$ m.

limitation, numerical results on the critical densities under DSPM are not presented and only simulation results (drawn by markers) are given.

We observe from Fig. 4 that the CP constraint greatly limits the maximal deployment density of BSs in downlink networks. For instance, the critical density is reduced by 3.6 and even 9.7 folds when $\varepsilon = 0.5$ and $\varepsilon = 0.6$, respectively, given $\Delta h = 2m$ under the settings in Fig. 4. Meanwhile, critical densities λ^* and λ^{\dagger} would exponentially decrease with Δh under single-slope and dual-slope models. Therefore, the above results also reveal the essential impact of AHD on the BS deployment in downlink cellular network. In particular, it indicates that, in densely deployed scenarios (e.g., stadium and open gathering), the antenna height of small cell BSs should be lowered, thereby facilitating the maximization of network ST while ensuring the QoS of downlink users.

VI. CONCLUSION

In this paper, we have explored the fundamental limits of network densification in downlink cellular networks under a generalized multi-slope pathloss model. Specifically, considering the AHD between BSs and downlink users, it is shown that the network ST first increases, then decreases with network densification and finally approaches zero when BSs are overdeployed. Meanwhile, it is observed that the CP of downlink users starts to diminish with the BS density when network ST is increased. Therefore, to strike a balance between user and system performance, we have derived the critical density, under which network ST can be maximized with the preset CP constraint. The results in this work could provide helpful guidance for the network deployment and application of network densification in future wireless networks.

APPENDIX

A. Proof for Proposition 1

Substitute (6) into (4), we have

$$\begin{aligned} \mathsf{CP}_{N}\left(\lambda\right) &= \mathbb{P}\left\{H_{\mathrm{U}_{0},\mathrm{BS}_{0}} > s_{N}I_{\mathrm{IC}}\right\} \\ &\stackrel{(\mathrm{a})}{=} \mathbb{E}_{d_{0},\tilde{\Pi}_{\mathrm{BS}},H_{\mathrm{U}_{0},\mathrm{BS}_{i}}}\left[\prod_{\mathrm{BS}_{i}\in\tilde{\Pi}_{\mathrm{BS}}}e^{-s_{N}PH_{\mathrm{U}_{0},\mathrm{BS}_{i}}l_{N}\left(d_{i}\right)}\right] \\ &\stackrel{(\mathrm{b})}{=} \mathbb{E}_{d_{0},\tilde{\Pi}_{\mathrm{BS}}}\left[\prod_{\mathrm{BS}_{i}\in\tilde{\Pi}_{\mathrm{BS}}}\frac{1}{1+s_{N}Pl_{N}\left(d_{i}\right)}\right], \end{aligned}$$
(14)

where $s_N = \frac{\tau}{Pl_N(d_0)}$. In (14), (a) and (b) are due to $H_{U_0,BS_i} \sim \exp(1)$ and the independence of H_{U_0,BS_i} . Aided by the probability generating functional (PGFL) of Poisson point process (PPP) [12], $CP_N(\lambda)$ in (14) further turns into

$$CP_{N}(\lambda) = \mathbb{E}_{d_{0}}\left[e^{-\lambda \int_{d_{0}}^{\infty} \left(1 - \frac{1}{1 + s_{N}P^{l}N(x)}\right)d(\pi x^{2})}\right],$$
$$= \mathbb{E}_{d_{0}}\left[e^{-2\pi\lambda \int_{d_{0}}^{\infty} x \left(1 - \frac{1}{1 + s_{N}P^{l}N(x)}\right)dx}\right].$$
(15)

Given N = 1, it is straightforward to obtain $s_1 = \frac{\tau d_0^{\alpha_0}}{P}$ and

$$CP_{1}(\lambda) = \mathbb{E}_{d_{0}} \left[\exp \left(-\frac{2\pi\lambda\tau\omega_{1}(\tau,\alpha_{0})}{\alpha-2} d_{0}^{2} \right) \right]$$
$$= \mathbb{E}_{r_{0}} \left[\exp \left(-\frac{2\pi\lambda\tau\omega_{1}(\tau,\alpha_{0})}{\alpha-2} \left(r_{0}^{2} + \Delta h^{2} \right) \right) \right]$$
$$\stackrel{(a)}{=} \frac{1}{1+C_{1}} \exp \left(-\pi\lambda C_{1} \bigtriangleup h^{2} \right), \tag{16}$$

where (a) follows because the PDF of r_0 is given by (8). Given N > 2 and $d_0 \in [\bar{B}, \bar{B}]_{++}$ with $\bar{B} = \sqrt{r^2 + R^2}$

Given
$$N > 2$$
 and $d_0 \in [R_n, R_{n+1})$ with $R_n = \sqrt{r_0^2 + R_n^2}$,

$$\int_{d_0}^{\infty} x^{k-1} \left(1 - \frac{1}{1 + s_N P l_N(x)}\right) dx \text{ in (15) turns into}$$

$$\int_{d_0}^{\infty} x \left(1 - \frac{1}{1 + s_N P l_N(x)}\right) dx$$

$$= \int_{d_0}^{\bar{R}_{n+1}} x \left(1 - \frac{1}{1 + \tau d_0^{\alpha_n} x^{-\alpha_n}}\right) dx$$

$$+ \sum_{i=n+1}^{N-1} \int_{\bar{R}_i}^{\bar{R}_{i+1}} x \left(1 - \frac{1}{1 + \tau K_i d_0^{\alpha_n} x^{-\alpha_i}}\right) dx$$

$$= \frac{1}{2} \left[\bar{R}_{n+1}^2 \omega_2 \left(\frac{\bar{R}_{n+1}^{\alpha_n}}{\tau d_0^{\alpha_n}}, \alpha_n\right) - d_0^2 \omega_2 (\tau^{-1}, \alpha_n)\right]$$

$$+ \sum_{i=n+1}^{N-1} \left[\frac{\bar{R}_{i+1}^2}{2} \omega_2 \left(\frac{\bar{R}_{i+1}^{\alpha_i}}{\tau K_i d_0^{\alpha_n}}, \alpha_i\right) - \frac{\bar{R}_i^2}{2} \omega_2 \left(\frac{\bar{R}_i^{\alpha_i}}{\tau K_i d_0^{\alpha_n}}, \alpha_i\right)\right]$$

Hence, the proof is completed.

B. Proof for Theorem 1

From Proposition 1, it follows that the proof for the scaling laws of CP and ST under the SSPM is straightforward and thus omitted due to limitation. Then, we focus on the proof for the case with N > 2, and some useful notations are first given in the following.

Denote $g_1(x)$ and $g_2(x)$ as two functions on the subset of real numbers. We write $g_1(x) = \Omega(g_2(x))$ if $\exists m > 0, x_0$,

 $\forall x > x_0, \ m |g_2(x)| \le |g_1(x)|, \ \text{and} \ g_1(x) = \mathcal{O}(g_2(x)) \text{ if } \exists m > 0, \ x_0, \ \forall x > x_0, \ |g_1(x)| \le m |g_2(x)|.$ Given N > 2 the CP in (7) can be expressed as

Given N > 2, the CP in (7) can be expressed as

Then, it can be directly obtained that

$$\mathsf{CP}_{N}(\lambda) > \mathbb{E}_{r_{0} \in [R_{N-1}, R_{N})} \left[e^{-2\pi\lambda \int_{d_{0}}^{\infty} x \left(1 - \frac{1}{1 + s_{N} P l_{N}(x)}\right) dx} \right].$$
(18)

As $d_0 = \sqrt{r_0^2 + \Delta h^2}$, $\bar{R}_{N-1} = \sqrt{R_{N-1}^2 + \Delta h^2}$ and $R_N = \infty$, when $d_0 \in [\bar{R}_{N-1}, \infty)$, $s_N = \frac{\tau}{PK_{N-1}d_0^{-\alpha_{N-1}}}$ and $l_N(x) = K_{N-1}x^{-\alpha_{N-1}}$, the integral in (18) turns into $\int_{d_0}^{\infty} x \left(1 - \frac{1}{1 + \tau d_0^{\alpha_{N-1}}x^{-\alpha_{N-1}}}\right) dx$ $= \tau \omega_1(\tau, \alpha_{N-1}) d_0^2$

$$=\tau\omega_1\left(\tau,\alpha_{N-1}\right)\left(r_0^2+\Delta h^2\right).$$
(19)

Next, we derive the lower bound of $\mathsf{CP}_N(\lambda)$ as

С

$$P_{N}(\lambda) > \mathsf{CP}_{N}^{\mathsf{L}}(\lambda)$$

$$= \mathbb{E}_{r_{0} \in [R_{N-1},\infty)} \left[e^{-2\pi\lambda\tau\omega_{1}(\tau,\alpha_{N-1})\left(r_{0}^{2}+\Delta h^{2}\right)} \right]$$

$$\stackrel{(a)}{=} \frac{e^{-\pi\lambda\left[R_{N-1}^{2}+2\tau\omega_{1}(\tau,\alpha_{N-1})\left(R_{N-1}^{2}+\Delta h^{2}\right)\right]}}{1+2\tau\omega_{1}(\tau,\alpha_{N-1})}, \quad (20)$$

where (a) is due to the PDF of r_0 given in (8). Therefore, it can be shown that $\exists \frac{1}{1+2\tau\omega_1(\tau,\alpha_{N-1})} > 0, \forall \lambda > 0$,

$$\left| \mathsf{CP}_{N}^{\mathrm{L}}\left(\lambda\right) \right| \geq \frac{e^{-\pi\lambda \left[R_{N-1}^{2}+2\tau\omega_{1}\left(\tau,\alpha_{N-1}\right)\left(R_{N-1}^{2}+\Delta h^{2}\right)\right]}}{1+2\tau\omega_{1}\left(\tau,\alpha_{N-1}\right)}.$$
 (21)

Hence, $\mathsf{CP}_N^{\mathsf{L}}(\lambda) = \Omega\left(e^{-\pi\lambda\left[R_{N-1}^2 + 2\tau\omega_1(\tau,\alpha_{N-1})\left(R_{N-1}^2 + \Delta h^2\right)\right]}\right)$ holds true.

In the following, we analyze the upper bound of $\mathsf{CP}_N(\lambda)$. When $r_0 \in [R_n, R_{n+1})$ or equivalently $d_0 \in [\bar{R}_n, \bar{R}_{n+1})$ $(n = 0, 1, \dots, N-2)$, $s_N = \frac{\tau d_0^{\alpha_n}}{PK_n}$ holds. As such, $\int_{d_0}^{\infty} x \left(1 - \frac{1}{1+s_N Pl_N(x)}\right) dx$ in the first term of (17) can be manipulated as

$$\int_{d_{0}}^{\infty} x \left(1 - \frac{1}{1 + s_{N} P l_{N}(x)}\right) dx$$

$$\stackrel{(a)}{=} \int_{\bar{R}_{N-1}}^{\infty} x \left(1 - \frac{1}{1 + \frac{\tau K_{N-1}}{K_{n} d_{0}^{-\alpha_{n}}} x^{-\alpha_{N-1}}}\right) dx$$

$$= \frac{\tau K_{N-1} \bar{R}_{N-1}^{2-\alpha_{N-1}} d_{0}^{\alpha_{n}}}{K_{n} (\alpha_{N-1} - 2)} \omega_{1} \left(\frac{\tau K_{N-1} d_{0}^{\alpha_{n}}}{K_{n} R_{N-1}^{\alpha_{N-1}}}, \alpha_{N-1}\right)$$

$$\stackrel{(b)}{=} \frac{\tau K_{N-1} \bar{R}_{N-1}^{2-\alpha_{N-1}} \Delta h^{\alpha_{n}}}{K_{n} (\alpha_{N-1} - 2)} \omega_{1} \left(\frac{\tau K_{N-1}}{K_{n}}, \alpha_{N-1}\right)$$

$$= q_{1} (n), \qquad (22)$$

where (a) follows due to $d_0 < \overline{R}_{N-1}$, and (b) follows because $d_0 > \Delta h$, $d_0^{\alpha_n} < R_{N-1}^{\alpha_{N-1}}$ and $\omega_1(x, \alpha_{N-1})$ is a decreasing function of x (see Lemma 1). Using (22) and the PDF of r_0 in (8), we have

$$\mathbb{E}_{r_{0}\in[R_{0},R_{N-1})}\left[e^{-2\pi\lambda\int_{d_{0}}^{\infty}x\left(1-\frac{1}{1+s_{N}P^{l}_{N}(x)}\right)dx}\right]$$

$$<\sum_{n=0}^{N-2}\mathbb{E}_{r_{0}\in[R_{n},R_{n+1})}\left[e^{-2\pi\lambda q_{1}(n)}\right]$$

$$=\sum_{n=0}^{N-2}e^{-2\pi\lambda q_{1}(n)}\left(e^{-\pi\lambda R_{n}^{2}}-e^{-\pi\lambda R_{n+1}^{2}}\right).$$
 (23)

When $r_0 \in [R_{N-1}, \infty)$, the second term of (17) is already given by $\mathsf{CP}_N^{\mathsf{L}}(\lambda)$ in (20). Hence, it is easy to obtain

$$\mathsf{CP}_{N}(\lambda) < \sum_{n=0}^{N-2} e^{-2\pi\lambda q_{1}(n)} \left(e^{-\pi\lambda R_{n}^{2}} - e^{-\pi\lambda R_{n+1}^{2}} \right) + \mathsf{CP}_{N}^{\mathsf{L}}(\lambda)
< \sum_{n=0}^{N-2} e^{-2\pi\lambda q_{1}(n)} e^{-\pi\lambda R_{n}^{2}} + \mathsf{CP}_{N}^{\mathsf{L}}(\lambda)
< \sum_{n=0}^{N-2} e^{-2\pi\lambda q_{1}(n)} + e^{-\pi\lambda R_{N-1}^{2}}
= \mathsf{CP}_{N}^{\mathsf{U}}(\lambda).$$
(24)

In (24), If $n \in \mathbb{C}$ ($\mathbb{C} = \{0, 1, \dots, N-2\}$), which enables $2q_1(n) > R_{N-1}^2$, then the inequality $e^{-2\pi\lambda q_1(n)} < e^{-\pi\lambda R_{N-1}^2}$ holds. Then, $\mathsf{CP}_N^{\mathsf{U}}(\lambda)$ in (24) turns into

$$\mathsf{CP}_{N}^{\mathsf{U}}(\lambda) = \sum_{n=0}^{N-2} e^{-2\pi\lambda q_{1}(n)} + e^{-\pi\lambda R_{N-1}^{2}} < N e^{-\pi\lambda R_{N-1}^{2}},$$

which indicates that $\exists N > 0, \forall \lambda > 0$,

$$\left|\mathsf{CP}_{N}^{\mathrm{U}}\left(\lambda\right)\right| < N e^{-\pi\lambda R_{N-1}^{2}}.$$
(25)

If $n \in \mathbb{C}^{\dagger}$ ($\mathbb{C} \subseteq \{0, 1, \dots, N-2\}$), which enables $2q_1(n) \leq R_{N-1}^2$, then we denote $n = N^{\dagger}$, which makes $e^{-2\pi\lambda q_1(N^{\dagger})} \geq e^{-2\pi\lambda q_1(n)}$ ($0 \leq n \leq N-2$). It is apparent that $e^{-2\pi\lambda q_1(N^{\dagger})} \geq e^{-\pi\lambda R_{N-1}^2}$ holds as well. Then, we have

$$\sum_{n=0}^{N-2} e^{-2\pi\lambda q_1(n)} + e^{-\pi\lambda R_{N-1}^2} < N e^{-2\pi\lambda q_1(N^{\dagger})}.$$

In this case, $\exists N > 0, \forall \lambda > 0$,

$$\left|\mathsf{CP}_{N}^{\mathsf{U}}\left(\lambda\right)\right| < Ne^{-2\pi\lambda q_{1}\left(N^{\dagger}\right)}.\tag{26}$$

Combining (25) and (26), $\mathsf{CP}_N^{\mathrm{U}}(\lambda) = \mathcal{O}\left(e^{-\pi\lambda R_{N-1}^2}\right)$ or $\mathsf{CP}_N^{\mathrm{U}}(\lambda) = \mathcal{O}\left(e^{-2\pi\lambda q_1\left(N^{\dagger}\right)}\right)$ holds true.

According to the above proof for the scaling laws of $\operatorname{CP}_N^{\mathrm{U}}(\lambda)$ and $\operatorname{CP}_N^{\mathrm{L}}(\lambda)$, it is easy to show that there exists a constant κ , which makes $\operatorname{CP}_N(\lambda)$ scale with λ as $e^{-\kappa\lambda}$. Therefore, based on the definition of ST in (5), $\operatorname{ST}_N(\lambda)$ scales with λ as $\lambda e^{-\kappa\lambda}$.

C. Proof for Theorem 1

Substituting the special case of CP (N = 1) in (7) into (10), we have $\frac{1}{1+C_1} \exp\left(-\pi\lambda C_1 \bigtriangleup h^2\right) > \varepsilon$. Through easy manipulation, the following inequality can be obtained

$$\lambda < -\frac{\ln\left[\varepsilon\left(1+C_{1}\right)\right]}{\pi C_{1} \bigtriangleup h^{2}}.$$
(27)

To make the inequality in (27) valid, we have to guarantee $\ln [\varepsilon (1 + C_1)] < 0$. Hence, the proof is complete.

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