

Multipair Massive MIMO Two-Way Full-Duplex Relay Systems with Hardware Impairments

Ying Liu, Xipeng Xue, Jiayi Zhang, Xu Li, Linglong Dai, and Shi Jin

Abstract—Hardware impairments, such as phase noise, quantization errors, non-linearities, and noise amplification, have baneful effects on wireless communications. In this paper, we investigate the effect of hardware impairments on multipair massive multiple-input multiple-output (MIMO) two-way full-duplex relay systems with amplify-and-forward scheme. More specifically, novel closed-form approximate expressions for the spectral efficiency are derived to obtain some important insights into the practical design of the considered system. When the number of relay antennas N increases without bound, we propose a hardware scaling law, which reveals that the level of hardware impairments that can be tolerated is roughly proportional to \sqrt{N} . This new result inspires us to design low-cost and practical multipair massive MIMO two-way full-duplex relay systems. Moreover, the optimal number of relay antennas is derived to maximize the energy efficiency. Finally, Motor-Carlo simulation results are provided to validate our analytical results.

I. INTRODUCTION

The two-way full-duplex (FD) relay system can ideally achieve almost twice of the spectral efficiency (SE) achieved by the traditional two-way half-duplex (HD) scheme, since the relay can transmit and receive signals simultaneously. However, the practical implementation of the two-way FD relay is challenging due to the severe self-interference (SI) caused by FD [1]–[3].

Recently, massive multiple-input multiple-output (MIMO) has been proposed as an efficient approach to suppress the SI of two-way FD relay systems in the spatial domain [4]. Different from most of existing works, which consider systems deploying high-cost ideal hardware components, in this paper we consider a multipair massive MIMO two-way FD relay system with low-cost non-ideal hardware that suffers from hardware impairments. In practical systems, the cost and power consumption increase with the number of radio frequency (RF) chains. In order to achieve higher energy efficiency (EE) and/or lower hardware cost, each RF chain can use some cheap hardware components [5], [6]. However, low-cost hardware is particularly prone to the impairments in transceivers, such as quantization errors of low-resolution

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analog to digital converters (ADCs), I/Q-imbalance, phase noise, and non-linearities [7]–[9].

Although the influence of hardware impairments can be mitigated by some compensation algorithms, residual impairments still exist due to time-varying and random hardware characteristics. The effect of hardware impairments on the massive MIMO two-way FD relay system has only recently been studied in [10], which focused on the decode-and-forward (DF) scheme at the relay. However, the signal processing complexity of the DF scheme is much higher than that of the amplify-and-forward (AF) scheme for the implementation of massive MIMO relay systems [11]. Therefore, the AF scheme is more attractive for practical system design. To the best of authors' knowledge, the performance of AF based multipair massive MIMO two-way FD relay systems with hardware impairments has not been investigated in the literature, partially due to the difficulty of manipulating products of SI and hardware impairments vectors.

Motivated by the aforementioned consideration, a natural question is that, whether the low-cost non-ideal hardware can be deployed at the AF based multipair massive MIMO two-way FD relay system without sacrificing the expected performance gains? In this paper, we try to answer this question with the following contributions:

- An analytical SE approximation of multipair massive MIMO two-way FD relay systems with hardware impairments is derived in closed-form. The effect of both the number of relay antennas and the level of transceiver hardware impairments on the SE has been investigated.
- A hardware scaling law has been presented to show that one can tolerate larger level of hardware impairments as the number of antennas increases. This is an analytic proof that the considered system can be deployed with low-cost hardware components.
- The sum SEs of FD and HD systems have been compared with different levels of hardware impairments. It is interesting to find that the FD system with hardware impairments can achieve the same SE of the HD system with larger loop interferences. Finally, we derive the optimal number of relay antennas to maximize the EE.

II. SYSTEM MODEL

We consider a massive MIMO two-way FD relay system where K pairs of devices on two sides communicate with each other through a single relay T_R . The devices are denoted as T_{A_i} and T_{B_i} , for $i = 1, \dots, K$, respectively. The devices could be sensors that exchange a small amount of information

or small cell base stations which need high throughput links. The relay is equipped with $2N$ antennas, where N antennas are used for transmission, and the other N antennas are used for reception. Each device is equipped with one receive and one transmit antenna. In addition, the relay and devices are assumed to work in FD mode, i.e., they can transmit and receive signals at the same time. We further assume that there is no direct communication link between each pair of devices due to heavy shadowing and/or large path loss. The device is interfered by other devices on the same side.

A. Channel Model

Block fading is considered in this paper. This means that it is an ergodic process with a static channel realization in a coherence block and the realizations in blocks are independent. Then, we define $\mathbf{G}_u \triangleq [\mathbf{g}_{u1}, \dots, \mathbf{g}_{uK}]$ and $\mathbf{H}_u \triangleq [\mathbf{h}_{u1}, \dots, \mathbf{h}_{uK}]$, where $\mathbf{g}_{ui} \in \mathbb{C}^{N \times 1}$ and $\mathbf{h}_{ui} \in \mathbb{C}^{N \times 1}$ ($i = 1, \dots, K$), which denote the uplink channels between T_{A_i} and T_R , T_{B_i} and T_R , respectively. In addition, the downlink channels between T_{A_i} and T_R , T_{B_i} and T_R are given by $\mathbf{G}_d \triangleq [\mathbf{g}_{d1}, \dots, \mathbf{g}_{dK}]$ and $\mathbf{H}_d \triangleq [\mathbf{h}_{d1}, \dots, \mathbf{h}_{dK}]$, where $\mathbf{g}_{di} \in \mathbb{C}^{N \times 1}$ and $\mathbf{h}_{di} \in \mathbb{C}^{N \times 1}$ ($i = 1, \dots, K$), respectively. Furthermore, \mathbf{G}_u , \mathbf{H}_u , \mathbf{G}_d and \mathbf{H}_d are assumed to follow the independent identically distributed (i.i.d.) Rayleigh fading, i.e., the elements of \mathbf{g}_{uK} , \mathbf{h}_{uK} , \mathbf{g}_{dK} , and \mathbf{h}_{dK} are i.i.d. $\mathcal{CN}(\mathbf{0}, \sigma_{g_{ui}}^2)$, $\mathcal{CN}(\mathbf{0}, \sigma_{h_{ui}}^2)$, $\mathcal{CN}(\mathbf{0}, \sigma_{g_{di}}^2)$, and $\mathcal{CN}(\mathbf{0}, \sigma_{h_{di}}^2)$ random variables [6]. Furthermore, \mathbf{G}_u , \mathbf{H}_u , \mathbf{G}_d , and \mathbf{H}_d can be expressed as $\mathbf{G}_u = \mathbf{S}_{gu} \mathbf{D}_{gu}^{1/2}$, $\mathbf{H}_u = \mathbf{S}_{hu} \mathbf{D}_{hu}^{1/2}$, $\mathbf{G}_d = \mathbf{S}_{gd} \mathbf{D}_{gd}^{1/2}$, and $\mathbf{H}_d = \mathbf{S}_{hd} \mathbf{D}_{hd}^{1/2}$, respectively, where \mathbf{S}_{gu} , \mathbf{S}_{hu} , \mathbf{S}_{gd} , and \mathbf{S}_{hd} stand for the small-scale fading and their elements are all i.i.d. $\mathcal{CN}(0, 1)$ random variables. On the other hand, \mathbf{D}_{gu} , \mathbf{D}_{hu} , \mathbf{D}_{gd} , and \mathbf{D}_{hd} are diagonal matrices representing the large-scale fading, and the k th diagonal elements are denoted as $\sigma_{g_{uk}}^2$, $\sigma_{h_{uk}}^2$, $\sigma_{g_{dk}}^2$, and $\sigma_{h_{dk}}^2$, respectively.

Furthermore, let $\mathbf{G}_{RR} \in \mathbb{C}^{N \times N}$ denote the SI matrix between the transmit and receive arrays of the relay due to the FD mode. Each row of \mathbf{G}_{RR} such as \mathbf{G}_{RRi} denotes the channel between i th receive antenna and all transmit antennas of the relay. $\Omega_{k,i}$ is the inter-device interference channel coefficient from i th device to k th device. Note that $\Omega_{k,k}$ denotes the SI at the k th device. The elements of \mathbf{G}_{RR} and $\Omega_{k,i}$ are random variables following the i.i.d. complex Gaussian distribution, e.g., $\mathcal{CN}(\mathbf{0}, \sigma_{LTr}^2)$ and $\mathcal{CN}(\mathbf{0}, \sigma_{k,i}^2)$, respectively [2].

B. Hardware Impairments

As shown in [12], the residual hardware impairments at the transmitter and receiver can be modeled as additive distortion noises that are proportional to the signal power. Thus, the additive distortion term $\boldsymbol{\eta}_r$ describes the residual impairments of receiver at the relay and is proportional to the instantaneous power of received signals at the relay antenna as $\boldsymbol{\eta}_r \sim \mathcal{CN}(\mathbf{0}, \kappa_r^2 \text{diag}(W_{11}, \dots, W_{NN}))$, where W_{ii} is the i th diagonal element of the covariance matrix $\mathbf{W} = \sum_{j=1}^K P_U (\mathbf{h}_{uj} \mathbf{h}_{uj}^H + \mathbf{g}_{uj} \mathbf{g}_{uj}^H) + \frac{P_R}{N} \sum_{j=1}^N \mathbf{G}_{RRj} \mathbf{G}_{RRj}^H$ with P_U being the power constraint of the device and P_R being the transmit power of relay [12]. Furthermore, the proportionality

coefficient κ_r describes the level of hardware impairments and is related to the received error vector magnitude (EVM) [8]. Note that the EVM is a common quality indicator of the signal distortion magnitude, and it can be defined as the ratio of the signal distortion to the signal magnitude. For example, the EVM at relay can be defined as [8, Eq. (5)]

$$EVM_r = \sqrt{\frac{E \left\{ \|\boldsymbol{\eta}_r\|^2 \mid \mathfrak{S} \right\}}{E \left\{ \|\mathbf{x}\|^2 \mid \mathfrak{S} \right\}}} = \sqrt{\frac{\text{tr}(\kappa_r^2 \mathbf{W})}{\text{tr}(\mathbf{W})}} = \kappa_r, \quad (1)$$

where \mathfrak{S} denotes the set of channel realizations (i.e., $\mathbf{g}_u, \mathbf{h}_u \in \mathfrak{S}$). Furthermore, 3GPP LTE suggests that the EVM should be smaller than 0.175 [8].

C. Signal Transmission

At the time instant n , all devices T_{A_i} and T_{B_i} ($i = 1, \dots, K$) transmit their signals $x_{A_i}(n)$ and $x_{B_i}(n)$ to the relay T_R , respectively, and T_R broadcasts its processed previously received signal $\mathbf{y}_t(n)$ to all devices.

First, we assume that $x_{A_i}(n)$ and $x_{B_i}(n)$ are Gaussian distributed signals. Due to the FD mode, T_R also receives the signal, i.e., $\mathbf{y}_t(n)$ which is broadcasted to all devices. Thus, at the time instant n , the received signal at T_R is given by

$$\mathbf{y}_r(n) = \mathbf{A} \mathbf{x}(n) + \mathbf{G}_{RR} \mathbf{y}_t(n) + \boldsymbol{\eta}_r + \mathbf{n}_R(n), \quad (2)$$

where $\mathbf{A} \triangleq [\mathbf{G}_u, \mathbf{H}_u]$, $\mathbf{x}(n) \triangleq [\mathbf{x}_A^T(n), \mathbf{x}_B^T(n)]^T$ with $\mathbf{x}_A(n) \triangleq [x_{A_1}(n), \dots, x_{A_K}(n)]$ and $\mathbf{x}_B(n) \triangleq [x_{B_1}(n), \dots, x_{B_K}(n)]$, and $\mathbf{n}_R(n) \sim \mathcal{CN}(0, \sigma_R^2 \mathbf{I}_N)$ denotes an additive white Gaussian noise (AWGN) vector at T_R .

Then, we analyze the received signal at devices. At the time instant n ($n > 1$), the relay using the simple AF protocol amplifies the previously received signal $\mathbf{y}_r(n-1)$ and broadcasts it to the devices. Therefore, the transmit signal vector at the relay is given by

$$\mathbf{y}'_t(n) = \rho \mathbf{F} \mathbf{y}_r(n-1), \quad (3)$$

where $\mathbf{F} \in \mathbb{C}^{N \times N}$ is the precoding matrix and ρ is the amplification factor. Then T_R broadcasts $\mathbf{y}'_t(n)$ to all devices. However, due to the hardware impairments of RF chains at the transmitter, T_R actually broadcasts $\mathbf{y}_t(n)$ to all devices as

$$\mathbf{y}_t(n) = \mathbf{y}'_t(n) + \boldsymbol{\eta}_t = \rho \mathbf{F} \mathbf{y}_r(n-1) + \boldsymbol{\eta}_t, \quad (4)$$

where $\boldsymbol{\eta}_t \sim \mathcal{CN}(\mathbf{0}, \kappa_t^2 \frac{P_R}{N} \mathbf{I}_N)$ with the proportionality parameters κ_t characterizing the level of hardware impairment at the transmitter. Here we assume that each antenna has the same power and T_R can obtain perfect channel state information (CSI) according to uplink pilots from the devices, and the devices can then obtain CSI through channel reciprocity [13]. Due to the power constraint of the relay P_R , ρ is normalized by the instantaneous received signal power

$$\rho = \sqrt{\frac{P_R}{P_U \|\mathbf{F} \mathbf{A}\|^2 + \frac{P_R}{N} \|\mathbf{F} \mathbf{G}_{RR}\|^2 + \|\mathbf{F} \boldsymbol{\eta}_r\|^2 + \sigma_R^2 \|\mathbf{F}\|^2}}.$$

At the relay, we adopt the low-complexity maximum-ratio (MR) scheme suitable for low-cost massive MIMO deployment [13]. Therefore, the precoding matrix \mathbf{F} can be written as $\mathbf{F} = \mathbf{B}^* \mathbf{A}^H$, where $\mathbf{B} \triangleq [\mathbf{H}_d, \mathbf{G}_d]$, $\mathbf{G}_d \triangleq [\mathbf{g}_{d1}, \dots, \mathbf{g}_{dK}]$ and $\mathbf{H}_d \triangleq [\mathbf{h}_{d1}, \dots, \mathbf{h}_{dK}]$. To our best knowledge, it is very challenging to analyze the residual loop interference power if substituting (4) into (2) iteratively. However, the residual loop interference can be modeled as additional Gaussian noise. This is due to the fact that the loop interference can be significantly degraded and the residual loop interference is too weak by applying loop interference mitigation schemes [4].

Following similar steps in [4], $\mathbf{y}_r(n)$ in (2) can be approximated by a Gaussian noise source $\tilde{\mathbf{y}}_r(n)$ with $\mathbb{E}\{\tilde{\mathbf{y}}_r(n)\tilde{\mathbf{y}}_r^H(n)\} = \frac{P_R}{N}\mathbf{I}_N$. Furthermore, T_{A_i} and T_{B_i} receive the combined signal as

$$Z_{A_i}(n) = \mathbf{g}_{di}^T \mathbf{y}_t(n) + \sum_{i,k \in U_A} \Omega_{i,k} x_{A_k}(n) + n_{A_i}(n), \quad (5)$$

$$Z_{B_i}(n) = \mathbf{h}_{di}^T \mathbf{y}_t(n) + \sum_{i,k \in U_B} \Omega_{i,k} x_{B_k}(n) + n_{B_i}(n), \quad (6)$$

where the noise $n_{A_i}(n)$ and $n_{B_i}(n)$ are AWGN with $n_{A_i}(n) \sim \mathcal{CN}(0, \sigma_{A_i}^2)$ and $n_{B_i}(n) \sim \mathcal{CN}(0, \sigma_{B_i}^2)$, respectively. In the following, we only discuss the analytical result for T_{A_i} . The corresponding result of T_{B_i} can be obtained by replacing T_{A_i} with T_{B_i} . Note that the relay can only receive signal and the transmission part keeps silent at the first time slot ($n=1$), during which the received signals at the relay and devices are respectively given by

$$\mathbf{y}_r(1) = \mathbf{A}\mathbf{x}(1) + \boldsymbol{\eta}_r + \mathbf{n}_R(1) \quad (7)$$

$$Z_{A_i}(1) = \mathbf{g}_{di}^T \mathbf{y}_t(1) + n_{A_i}(1), i = 1, \dots, K. \quad (8)$$

For simplicity, the time label n is omitted in the following [6]. Substituting (2) and (4) into (5), the combined received signal Z_{A_i} can be expressed as

$$\begin{aligned} Z_{A_i} = & \underbrace{\rho \mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui} x_{B_i}}_{\text{desired signal}} + \underbrace{\rho \sum_{j=1, j \neq i}^K (\mathbf{g}_{di}^T \mathbf{F} \mathbf{g}_{uj} x_{A_j} + \mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{uj} x_{B_j})}_{\text{inter-pair interference}} \\ & + \underbrace{\rho \mathbf{g}_{di}^T \mathbf{F} \mathbf{g}_{ui} x_{A_i}}_{\text{self-interference}} + \underbrace{\rho \mathbf{g}_{di}^T \mathbf{F} \mathbf{G}_{RR} \tilde{\mathbf{y}}_r}_{\text{loop interference}} + \underbrace{\sum_{i,k \in U_A} \Omega_{i,k} x_{A_k}}_{\text{inter-device interference by FD mode}} \\ & + \underbrace{\rho \mathbf{g}_{di}^T \mathbf{F} \boldsymbol{\eta}_r + \mathbf{g}_{di}^T \boldsymbol{\eta}_t}_{\text{hardware impairments}} + \underbrace{\rho \mathbf{g}_{di}^T \mathbf{F} \mathbf{n}_R + n_{A_i}}_{\text{compound noise}}, \quad (9) \end{aligned}$$

where we use the set notation of $U_A = \{1, 3, \dots, 2K-1\}$ or $U_B = \{2, 4, \dots, 2K\}$ to represent the devices on both sides of relay. Note that one set of devices (U_A) can not exchange information with the other set (U_B) directly. From (9), we can find that Z_{A_i} is composed of seven terms: the signal that T_{A_i} desires to receive, the inter-pair interference due to other devices' signal, the SI from the device, the loop interference from the relay, the inter-device interference caused by other devices due to FD mode, the distortion noise induced by

hardware impairments at the relay, and the compound noise.

With the power constraint of the relay and perfect CSI, the FD relay can take advantage of massive antennas and simple SI cancellation (SIC) schemes to eliminate the SI [4]. Furthermore, the interference and noise power can be obtained by taking expectation with respect to interference and noise within one coherence block of channel fading. As a result, the SE of T_{A_i} is given by

$$R_{A_i} = \mathbb{E} \{ \log_2 (1 + \text{SINR}_{A_i}) \}, \quad \text{for } i = 1, \dots, K, \quad (10)$$

where SINR_{A_i} denotes the signal-to-interference plus noise ratio (SINR) of A_i and can be expressed as

$$\text{SINR}_{A_i} = \frac{P_U |\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}|^2}{A' + B' + C' + D' + E' + |\mathbf{g}_{di}^T \mathbf{F} \boldsymbol{\eta}_r|^2 + \frac{1}{\rho^2} |\mathbf{g}_{di}^T \boldsymbol{\eta}_t|^2},$$

where $A' \triangleq P_U \sum_{j=1, j \neq i}^K (|\mathbf{g}_i^T \mathbf{F} \mathbf{g}_j|^2 + |\mathbf{g}_i^T \mathbf{F} \mathbf{h}_j|^2)$, $B' \triangleq \sigma_R^2 \|\mathbf{g}_i^T \mathbf{F}\|^2$, $C' \triangleq \frac{\sigma_{A_i}^2}{\rho^2}$, $D' \triangleq |\mathbf{g}_i^T \mathbf{F} \mathbf{G}_{RR} \tilde{\mathbf{y}}_r|^2$, and $E' \triangleq \frac{P_U}{\rho^2} \sum_{i,k \in U_A} \sigma_{i,k}^2$, respectively.

III. PERFORMANCE ANALYSIS

To the best of authors' knowledge, the exact derivation of (10) is really difficult [14]. Herein we consider the asymptotic scenario when $N \rightarrow \infty$, which is the large system limit. Utilizing the convexity of $\log_2(1 + 1/x)$ and Jensen's inequality, the lower bound of R_{A_i} in (10) can be written as

$$R_{A_i} \geq \tilde{R}_{A_i} = \log_2 \left(1 + \frac{1}{\mathbb{E} \{ [\text{SINR}_{A_i}]^{-1} \}} \right). \quad (11)$$

Based on (11) and considering devices at both sides, we can obtain the sum SE of the multipair massive MIMO two-way FD relay system as

$$R_{\text{sum}} = \sum_{i=1}^K (\tilde{R}_{A_i} + \tilde{R}_{B_i}). \quad (12)$$

Note that, in the remainder of the paper, we only show the analytical results for R_{A_i} since the formula of R_{B_i} is symmetric with that of R_{A_i} . In the following, we present the SE of A_i in **Lemma 1**.

Lemma 1. *With hardware impairments and MR processing at the relay, \tilde{R}_{A_i} can be approximated as*

$$\tilde{R}_{A_i} - \log_2 \left(1 + \frac{N}{A_i + B_i + C_i + D_i + E_i + F_i + G_i} \right) \xrightarrow{N \rightarrow \infty} 0, \quad (13)$$

$$\begin{aligned} \text{where } A_i & \triangleq \sum_{j=1, j \neq i}^K \left(\frac{\sigma_{h_{uj}}^2}{\sigma_{h_{ui}}^2} + \frac{\sigma_{h_{uj}}^4 \sigma_{g_{dj}}^2}{\sigma_{h_{ui}}^4 \sigma_{g_{di}}^2} + \frac{\sigma_{g_{uj}}^2}{\sigma_{h_{ui}}^2} + \frac{\sigma_{g_{uj}}^4 \sigma_{h_{dj}}^2}{\sigma_{h_{ui}}^4 \sigma_{g_{di}}^2} \right), \\ B_i & \triangleq \frac{\sigma_R^2}{P_U \sigma_{h_{ui}}^2}, \quad C_i \triangleq \frac{\sigma_{A_i}^2 J}{P_R P_U \sigma_{g_{di}}^4 \sigma_{h_{ui}}^4}, \quad D_i \triangleq \\ & \frac{\kappa_r^2 \left(P_U \sum_{j=1}^K (\sigma_{h_{uj}}^2 + \sigma_{g_{uj}}^2) + P_R \sigma_{Lr}^2 \right)}{P_U \sigma_{h_{ui}}^2}, \quad E_i \triangleq \frac{\kappa_t^2 J}{P_U \sigma_{g_{di}}^2 \sigma_{h_{ui}}^4}, \end{aligned}$$

$$F_i \triangleq \frac{P_R \sigma_{LTr}^2}{P_U \sigma_{h_{iu}}^2}, G_i \triangleq \frac{J \sum_{i,k \in U_A} \sigma_{i,k}^2}{P_R \sigma_{g_{di}}^4 \sigma_{h_{ui}}^4}, \text{ and}$$

$$J \triangleq P_U \sum_{i=1}^K (\sigma_{g_{ui}}^4 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^4) + \frac{\kappa_r^2}{N} \sum_{i=1}^K (\sigma_{g_{ui}}^2 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2) \left(P_U \sum_{j=1}^K (\sigma_{g_{uj}}^2 + \sigma_{h_{uj}}^2) + P_R \sigma_{LTr}^2 \right)$$

Proof. Please refer to Appendix. \square

From **Lemma 1**, it is clear to see that the SE R_{A_i} increases with the number of antennas N . Further insights can be gained by investigating the terms $A_i, B_i, C_i, D_i, E_i, F_i,$ and G_i in (13), respectively. First, we focus on the inter-device interference term A_i caused by the broadcasting signal from the relay. The SE R_{A_i} increases when we enlarge the values of $\sigma_{g_{di}}^2$ and $\sigma_{h_{ui}}^2$, which indicates that reducing the channel fading of the i th device pair. However, R_{A_i} will decrease if we enlarge $\sigma_{g_{uj}}^2$ and $\sigma_{h_{uj}}^2$, for $j \neq i$, which means that reducing the channel fading of other device pairs except the i th device pair. This finding is consistent with the result in [14].

Furthermore, **Lemma 1** reveals that B_i consists of the transmit power of T_{B_i} and the channel fading $\sigma_{h_{ui}}^2$ from T_{B_i} to T_R . Therefore, we can increase the transmit power of T_{B_i} and/or decrease $\sigma_{h_{ui}}^2$ to increase R_{A_i} . Then, from C_i , we can find that R_{A_i} increases when the transmit power for T_R and the transmit power of devices increase, but decreases when ρ becomes large. Moreover, it is clear to see from (13) that the detrimental effect of hardware impairments in D_i, E_i and G_i on the SE R_{A_i} . Finally, the loop interference due to the FD mode in F_i and G_i can also reduce the SE.

In order to show how fast the hardware impairments can increase with N while maintaining the constant rate, we establish an important hardware scaling law in the following corollary.

Corollary 1. *Suppose the hardware impairment parameters are replaced by $\kappa_r^2 = \kappa_{0r}^2 N^z$ and $\kappa_t^2 = \kappa_{0t}^2 N^z$ for an initial value $\kappa_{0r} \geq 0, \kappa_{0t} \geq 0$ and a given scaling exponent $0 < z \leq 1$, the SE \tilde{R}_{A_i} , under MR processing and $N \rightarrow \infty$, converges to a non-zero limit*

$$\begin{cases} \tilde{R}_{A_i} - \log_2 \left(1 + \frac{\sigma_{g_{di}}^2 \sigma_{h_{ui}}^4 N^{1-z}}{\kappa_{0r}^2 \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2 \xi + 2K \kappa_{0t}^2 \bar{\mu}} \right) \xrightarrow{N \rightarrow \infty} 0, 0 < z < 1 \\ \tilde{R}_{A_i} - \log_2 \left(1 + \frac{\sigma_{g_{di}}^2 \sigma_{h_{ui}}^4}{\kappa_{0r}^2 (2K \kappa_{0t}^2 \bar{\mu} + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2) \xi + 2K \kappa_{0t}^2 \bar{\mu}} \right) \\ \xrightarrow{N \rightarrow \infty} 0, z = 1 \end{cases} \quad (14)$$

where $\xi \triangleq \frac{2K\mu}{\sum_{j=1}^K (\sigma_{h_{uj}}^2 + \sigma_{g_{uj}}^2)} + \frac{P_R \sigma_{LTr}^2}{P_U}$, $\mu \triangleq \frac{P_U}{\sum_{j=1}^K (\sigma_{h_{uj}}^2 + \sigma_{g_{uj}}^2)} / (2K)$, $\bar{\mu} \triangleq \frac{P_U}{\sum_{i=1}^K (\sigma_{g_{ui}}^4 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^4)} / (2K)$ and $\bar{\mu} \triangleq \frac{P_U}{\sum_{i=1}^K (\sigma_{g_{ui}}^2 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2)} / (2K)$.

Proof. Substituting $\kappa_r^2 = \kappa_{0r}^2 N^z$ and $\kappa_t^2 = \kappa_{0t}^2 N^z$ into (13), with $N \rightarrow \infty$ and $0 < z \leq 1$, $A_i, B_i, C_i, F_i,$ and G_i tend to zero. Moreover, D_i behaves as $\mathcal{O}(N^z)$, while E_i behaves as $\mathcal{O}(N^z + N^{2z-1})$. To make the numerator and denominator

have the identical scaling, we can finish the proof by fulfilling $1 - \max(z, 2z - 1) \geq 0$ as $0 < z \leq 1$. \square

Corollary 1 reveals that large level of hardware impairments can be compensated by increasing number of antennas at the relay in multipair massive MIMO two-way relaying systems. Furthermore, the EVM at relay is defined as $\text{EVM} = \kappa$ [12]. Considering the condition of $\kappa^2 = \kappa_0^2 N^z$ in **Corollary 1** and $z = 1$, it is easy to have $\text{EVM}^2 = \kappa_0^2 N$, which means the EVM can be increased proportionally to $N^{1/2}$. Thus, for the negligible SE loss, we can replace 8 high-quality antennas with $\text{EVM} = 0.05$ with 128 low-quality antennas with $\text{EVM} = 0.2$. This encouraging result enable reducing the power consumption and cost of the multipair massive MIMO two-way FD relay system.

In the following, we evaluate the EE of the multipair massive MIMO two-way FD relay system when the number of relay antennas becomes large. The EE is defined as the ratio of the sum SE to the total power consumption of the system [13]. Considering the classical architecture where each antenna is connected to one RF chain. The total power consumption of the system can be modeled as [4]

$$P_{\text{total}} = (N + 2K)(P_t + P_r) + P_0 + (2K P_U + P_R) / \varphi, \quad (15)$$

where P_t and P_r are the power of RF chains at the transmitter and receiver, respectively. Moreover, P_0 denotes the power of the static circuits, the term $2K P_U + P_R$ is the total power of the power amplifiers at devices and the relay, and φ denotes the efficiency of the power amplifier in each RF chain. Thus, the EE of the considered system is given by $EE = R_{\text{sum}} / P_{\text{total}}$.

IV. NUMERICAL RESULTS

In this section, the derived results of the multipair massive MIMO two-way FD relay system with hardware impairments and AF schemes are validated through Monte-Carlo simulations by averaging over 10^4 independent channel samples. Similar to previous works [4], [14], we set $P_u = 10$ W, $P_R = 40$ W and normalize $\sigma_R^2 = \sigma_{A_i}^2 = \sigma_{B_i}^2 = 1$ for $i = 1, \dots, K$. Furthermore, without loss of generality, we simply set the same values for both loop and inter-device interferences as $\sigma^2 = \sigma_{LTr}^2 = \sigma_{k,i}^2 = 1$ ($i \in U_A \cup U_B, k = 1, \dots, 2K$) and $\kappa_0 = \kappa_{0r} = \kappa_{0t}$, respectively [2], [6].

The simulated and analytical asymptotic sum SE (13) are plotted as a function of the half number of antennas N at the relay in Fig. 1. The simulation results validate the tightness of the derived large-scale approximations. Moreover, Fig. 1 validates the hardware scaling law established by **Corollary 1**. The SE grows with low levels of hardware impairments ($z = 0.5, 1$). However, the SE curve asymptotically bend toward zero when the scaling law is not satisfied ($z = 1.5$).

Note that the analytical curves plotted in Fig. 1 are not always below the simulated curves. This is due to the reason that we utilize the large number law to derive the SE. When N is relatively small (e.g. $N < 350$), the low order term of N cannot be omitted. Thus, the analytical result is a little

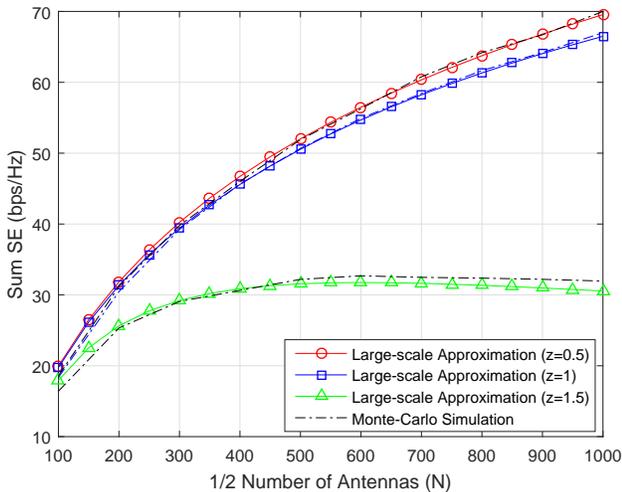


Fig. 1. Hardware scaling law of multipair massive MIMO two-way FD relay systems against different number of antennas N at the relay ($K = 10$, $\kappa_0 = 0.0156$).

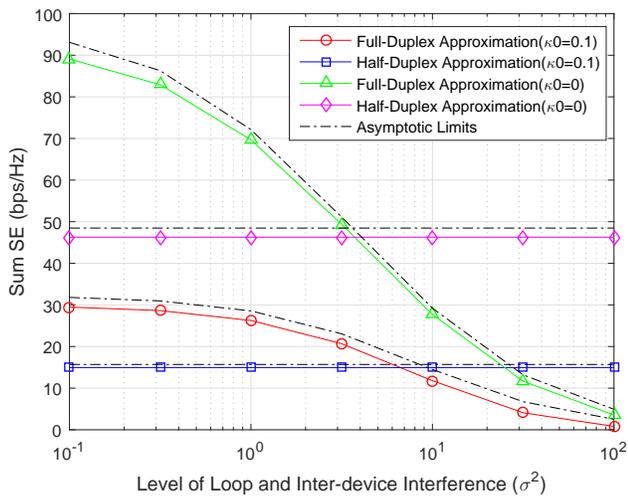


Fig. 2. Sum SE of multipair massive MIMO two-way FD relay systems with hardware impairments against different levels of loop and inter-device interference ($N = 1000$, $z = 1$, $K = 10$).

larger than the corresponding simulation result. However, even if N is smaller, the curves of large-scale approximation and Monte-Carlo simulations are close [14]. Compared with the DF scheme in [10], the signal processing can achieve a smaller sum SE. However, the complexity of AF based system at the relay is much lower than the DF based system.

Fig. 2 shows the large-scale approximation (13) and the asymptotic SE limit (14) against the levels of the loop and inter-device interferences σ^2 . The SE of such system in HD model is also plotted as a baseline for comparison. Since the HD system utilizes two phases to transmit and receive signal, the inherent loop and inter-device interference do not exist. Therefore, the SE of HD systems is constant in Fig. 2. The

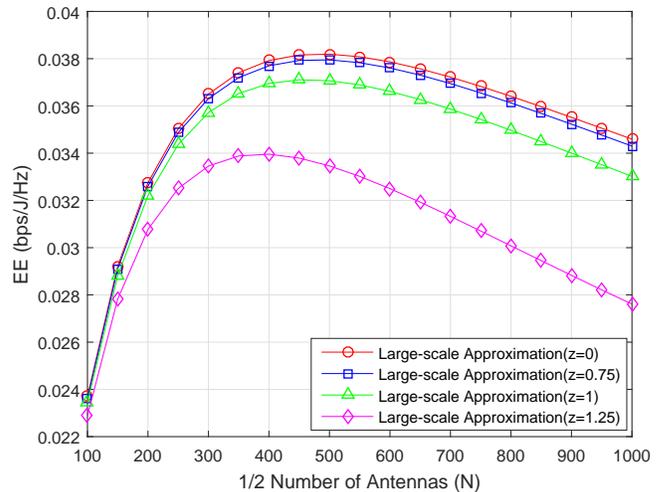


Fig. 3. EE of multipair massive MIMO two-way FD relay systems with hardware impairments against different number of antennas N at the relay ($K = 10$, $\kappa_0 = 0.0156$).

first observation from Fig. 2 is that when $N = 1000$, the asymptotic SE limit of FD systems outperforms the one of HD systems for small and moderate levels of interference, e.g., $\sigma^2 < 10^{1.18}$ for ideal hardware ($\kappa_0 = 0$) and $\sigma^2 < 10^{0.9}$ for non-ideal hardware ($\kappa_0 = 0.1$). This can be explained that only the half time required in the FD mode compared with the HD mode. Interestingly, the SE of the multipair massive MIMO two-way FD relay system with hardware impairments is larger than the one with ideal hardware for small and moderate levels of loop and inter-device interference. However, large value of loop and inter-device interference decreases the sum SE of FD systems. Moreover, the gap of SE curves between FD and HD systems increases with the level hardware impairments κ_0 .

The large-scale approximation of EE as a function of the number of antennas N at the relay is plotted in Fig. 3. Similar to [15], we set $P_t = 1$ W, $P_r = 0.3$ W, $P_0 = 2$ W and $\varphi = 0.35$. It is clear to see that the EE decreases with z due to the distortion noise caused by hardware impairments. Moreover, there exists an optimal number of antennas N_{opt} to reach the corresponding maximum EE. When $N \leq N_{\text{opt}}$, the EE can be improved by increasing N . However, when $N > N_{\text{opt}}$, increasing N will reduce the EE since the addition power consumption of RF chains and static circuits dominate the performance.

V. CONCLUSIONS

In this paper, we investigate the SE and EE of AF-based multipair massive MIMO two-way FD relay systems with hardware impairments. The effect of N and κ on the SE has been investigated by deriving a closed-form large-scale approximate expression. In addition, the optimal number of relay antennas has been derived to maximize the EE. We also find that the SE of the massive MIMO two-way FD system with hardware impairments outperforms that of the HD system

when the level of loop and inter-device interference is small and moderate. Finally, an useful hardware scaling law has been established to prove that low-cost hardware can be deployed at the relay due to the huge degrees-of-freedom brought by massive antennas.

APPENDIX

From (11), we can rewrite $\mathbb{E} \left\{ [\text{SINR}_{A_i}]^{-1} \right\}$ as

$$\begin{aligned} \mathbb{E} \left\{ \frac{1}{\text{SINR}_{A_i}} \right\} &= \frac{P_R}{NP_U} \mathbb{E} \left\{ \frac{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{G}_{RR}\|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} \\ &+ \frac{\sigma_R^2}{P_U} \mathbb{E} \left\{ \frac{\|\mathbf{g}_{di}^T \mathbf{F}\|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} + \frac{1}{\rho^2 P_U} \mathbb{E} \left\{ \frac{|\mathbf{g}_{di}^T \eta_t|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} \\ &+ \frac{\sum_{i,k \in U_A} \sigma_{i,k}^2}{\rho^2} \mathbb{E} \left\{ \frac{1}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} + \frac{\sigma_{A_i}^2}{\rho^2 P_U} \mathbb{E} \left\{ \frac{1}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} \\ &+ \sum_{j=1, j \neq i}^K \left(\mathbb{E} \left\{ \frac{|\mathbf{g}_{di}^T \mathbf{F} \mathbf{g}_{uj}|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} + \mathbb{E} \left\{ \frac{|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{uj}|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} \right) \\ &+ \frac{1}{P_U} \mathbb{E} \left\{ \frac{|\mathbf{g}_{di}^T \mathbf{F} \eta_r|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\}. \end{aligned} \quad (16)$$

According to the law of large numbers, we have

$$\begin{aligned} \frac{1}{N} \mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui} - \frac{1}{N} \|\mathbf{g}_{di}^*\|^2 \|\mathbf{h}_{ui}\|^2 &\xrightarrow[N \rightarrow \infty]{} 0, \\ \frac{1}{N} \mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{uj} - \frac{1}{N} \left(\|\mathbf{g}_{di}^*\|^2 \mathbf{h}_{ui}^H \mathbf{h}_{uj} + \|\mathbf{h}_{uj}\|^2 \mathbf{g}_{di}^T \mathbf{g}_{dj}^* \right) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \frac{1}{N} \mathbf{g}_{di}^T \mathbf{F} \mathbf{g}_{uj} - \frac{1}{N} \left(\|\mathbf{g}_{di}^*\|^2 \mathbf{h}_{ui}^H \mathbf{g}_{uj} + \|\mathbf{g}_{uj}\|^2 \mathbf{g}_{di}^T \mathbf{h}_{dj}^* \right) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \frac{1}{N} \|\mathbf{g}_{di}^T \mathbf{F}\|^2 - \frac{1}{N} \|\mathbf{g}_{di}^*\|^4 \|\mathbf{h}_{di}\|^2 &\xrightarrow[N \rightarrow \infty]{} 0. \end{aligned}$$

When $N \rightarrow \infty$, we further have

$$\begin{aligned} \mathbb{E} \left\{ \frac{\mathbf{h}_{ui}^H \mathbf{h}_{uj} + \|\mathbf{h}_{uj}\|^2 \mathbf{g}_{di}^T \mathbf{g}_{dj}^*}{\|\mathbf{h}_{ui}\|^2 + \|\mathbf{h}_{ui}\|^2 \|\mathbf{g}_{di}^*\|^2} \right\} - \frac{1}{N} \left(\frac{\sigma_{h_{uj}}^2}{\sigma_{h_{ui}}^2} + \frac{\sigma_{h_{uj}}^4 \sigma_{g_{dj}}^2}{\sigma_{h_{ui}}^4 \sigma_{g_{di}}^2} \right) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \frac{\mathbf{h}_{ui}^H \mathbf{g}_{uj} + \|\mathbf{g}_{uj}\|^2 \mathbf{g}_{di}^T \mathbf{h}_{dj}^*}{\|\mathbf{h}_{ui}\|^2 + \|\mathbf{g}_{di}^*\|^2 \|\mathbf{h}_{ui}\|^2} \right\} - \frac{1}{N} \left(\frac{\sigma_{g_{uj}}^2}{\sigma_{h_{ui}}^2} + \frac{\sigma_{g_{uj}}^4 \sigma_{h_{dj}}^2}{\sigma_{h_{ui}}^4 \sigma_{g_{di}}^2} \right) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \frac{1}{\|\mathbf{h}_{ui}\|^2} \right\} - \frac{1}{N \sigma_{h_{ui}}^2} &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \frac{|\mathbf{g}_{di}^T \mathbf{F} \eta_r|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} - \frac{\kappa_r^2 \xi}{NP_U \sigma_{h_{ui}}^2} &\xrightarrow[N \rightarrow \infty]{} 0. \end{aligned} \quad (17)$$

$$\begin{aligned} \mathbb{E} \left\{ \frac{|\mathbf{g}_{di}^T \eta_t|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} - \frac{\kappa_t^2 P_R}{N^4 \sigma_{g_{di}}^4 \sigma_{h_{ui}}^4} &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \frac{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{G}_{RR}\|^2}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} - \frac{\sigma_{LIR}^2}{\sigma_{h_{iu}}^2} &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \frac{1}{\|\mathbf{g}_{di}^T \mathbf{F} \mathbf{h}_{ui}\|^2} \right\} - \frac{1}{N^4 \sigma_{g_{di}}^4 \sigma_{h_{ui}}^4} &\xrightarrow[N \rightarrow \infty]{} 0. \end{aligned} \quad (18)$$

Moreover, we have the following approximations

$$\begin{aligned} \mathbb{E} \left\{ \|\mathbf{F} \mathbf{A}\|^2 \right\} - N^3 \sum_{i=1}^K (\sigma_{g_{ui}}^4 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^4) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \|\mathbf{F} \mathbf{G}_{RR}\|^2 \right\} - N^3 \sigma_{LIR}^2 \sum_{i=1}^K (\sigma_{g_{ui}}^2 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \|\mathbf{F} \eta_r\|^2 \right\} - \kappa_r^2 N^2 \xi \sum_{i=1}^K (\sigma_{g_{ui}}^2 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2) / P_U &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \|\mathbf{F}\|^2 \right\} - \sigma_R^2 N^2 \sum_{i=1}^K (\sigma_{g_{ui}}^2 \sigma_{h_{di}}^2 + \sigma_{g_{di}}^2 \sigma_{h_{ui}}^2) &\xrightarrow[N \rightarrow \infty]{} 0, \\ \mathbb{E} \left\{ \rho^2 \right\} - \frac{P_R}{N^3 J} &\xrightarrow[N \rightarrow \infty]{} 0. \end{aligned} \quad (19)$$

By substituting (18)-(19) into (16), we can complete the proof after some simplifications.

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