On Robustness of Massive MIMO Systems Against Passive Eavesdropping under Antenna Selection

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Abstract—In massive MIMO wiretap settings, the base station can significantly suppress eavesdroppers by narrow beamforming toward legitimate terminals. Numerical investigations show that by this approach, secrecy is obtained at no significant cost. We call this property of massive MIMO systems "secrecy for free" and show that it not only holds when all the transmit antennas at the base station are employed, but also when only a single antenna is set active. Using linear precoding, the information leakage to the eavesdroppers can be sufficiently diminished, when the total number of available transmit antennas at the base station grows large, even when only a fixed number of them are selected. This result indicates that passive eavesdropping has no significant impact on massive MIMO systems, regardless of the number of active transmit antennas.

Index Terms—Massive MIMO systems, physical layer security, antenna selection.

I. INTRODUCTION

Recently, massive Multiple-Input Multiple-Output (MIMO) systems have emerged as a promising enabling technology to address the explosive growth of data traffic in the next generation of wireless networks (5G) [1]. Reliable and secure transmission of data in 5G is of paramount importance for system designers. In this respect, physical layer security, complemented with cryptographic approaches in upper layers of the network, provides a well-integrated secure platform by exploiting the imperfection of the communication channels [2]. The pioneering work on physical layer security goes back to Wyner who studied a point to point wiretap channel in [3] and showed that confidential message transmission is possible as long as the eavesdropper observes a degraded version of the signal received at the legitimate terminal. Wyner's result was later extended to several other settings including MIMO wiretap channels [4], [5].

In massive MIMO systems, the large number of antennas can provide more security by narrow beamforming toward legitimate receivers. In this case, the signal strength at the eavesdropper, which is located somewhere outside the main beam, is much lower than the strength of the signal received at the legitimate terminal. This observation was demonstrated

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in [6] via numerical investigations where the authors showed that passive eavesdropping in not-too-dense networks has little impacts on the secrecy performance. Similar to other performance gains of massive MIMO, this robustness against passive eavesdropping is obtained at the expense of high hardware cost and complexity imposed by the large number of antennas in these systems. The impact of passive eavesdropping on the secrecy performance in dense networks has been further investigated in [7].

Several approaches were proposed in the literature to alleviate the cost-complexity issue of massive MIMO, e.g., [8]–[10]. A promising solution is antenna selection in which the transmission is carried out through a subset of antennas; see [11] and [12], and the references therein. In addition to its main objective, i.e., reducing the overall Radio Frequency (RF)cost, antenna selection has shown to enhance the performance in some scenarios. In [12] and [13], it was demonstrated that in several MIMO settings, energy efficiency is not an increasing function of the number of transmit antennas, and thus, it can be improved by switching off some of them. The authors in [14], moreover, showed that in MIMO wiretap settings, the secrecy performance, when no precoding is utilized at the Base Station (BS), is optimized when only a few transmit antennas are set active. The security benefits of optimal single antenna selection was further discussed in [15].

Objectives and Contributions

Considering the implementational issues in massive MIMO systems, this study aims to answer the following question: Are massive MIMO wiretap settings robust against passive eavesdropping when only a subset of transmit antennas are set active? To this end, we investigate the robustness against passive eavesdropping by defining the concept of "secrecy for free". It is then shown that even when only a single transmit antenna is set active, by simple linear precoding, the information leakage to the eavesdropper vanishes, as the total number of available antennas at the BS increases. This result indicates that with a large transmit antenna array, regardless of the number of active antennas, secrecy can be achieved at no significant cost. Our analysis moreover provides rigorous justifications for the earlier numerical observations on the robustness of massive MIMO systems against passive eavesdropping, e.g., [6], considering a more generic setup.

Notations

Throughout the paper, scalars, vectors and matrices are represented by non-bold, bold lower case and bold upper case letters, respectively. The set of real numbers is denoted by \mathbb{R} and the complex plane is shown by \mathbb{C} . \mathbf{H}^{H} , \mathbf{H}^* and \mathbf{H}^{T} indicate the Hermitian, complex conjugate and transpose of \mathbf{H} , respectively. $\log{(\cdot)}$ is the binary logarithm. We denote the statistical expectation by \mathbb{E} , and the non-negative part of x by $[x]^+ = \max\{0, x\}$. The beta distribution with the shape parameters α and β is denoted by $\mathcal{B}(\alpha, \beta)$; moreover, $\mathcal{N}(\eta, \sigma^2)$ and $\mathcal{C}\mathcal{N}(\eta, \sigma^2)$ represent the real and complex Gaussian distribution with mean η and variance σ^2 , respectively.

II. PROBLEM FORMULATION

We consider downlink transmission in a multiuser MIMO wiretap setting. In this setting, a BS with M antennas intends to transmit confidential messages to K legitimate users while the channel being overheard by an eavesdropper. For simplicity, we assume that the receiving terminals, i.e., the legitimate users and the eavesdropper, are single-antenna. By a same approach taken in [9], the analysis can be extended to scenarios with multi-antenna receiving terminals. The BS is equipped with $L \leq M$ RF-chains. Hence, in each coherence time interval, only L transmit antennas are set active.

The uplink channel from user terminal k to the BS is represented by $\mathbf{h}_k \in \mathbb{C}^M$ and reads

$$\mathbf{h}_k = \sqrt{\beta_k} \, \mathbf{g}_k \tag{1}$$

where $k \in \{1,\ldots,K\}$ denotes the legitimate users and k=e indicates the eavesdropper. The entries of $\mathbf{g}_k \in \mathbb{C}^M$ denote fast fading coefficients between user k and the transmit antennas, and β_k models path loss and shadowing. It is assumed that β_k is constant over several coherence time intervals and is known in priori. This is the case in most practical scenarios. As the result, the legitimate uplink channel can be written as

$$\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K] = \mathbf{G} \mathbf{\Gamma}^{1/2} \tag{2}$$

where $G = [g_1, \ldots, g_K]$ and Γ is a $K \times K$ diagonal matrix with $[\Gamma]_{kk} = \beta_k$. We further assume that the system operates in standard Time-Division Duplexing (TDD) mode meaning that the uplink and downlink channels are reciprocal.

At the beginning of each coherence time interval, the BS employs a selection algorithm to select L transmit antennas. We denote the set of the selected antennas with

$$\mathbb{L} = \{\ell_1, \dots, \ell_L\} \tag{3}$$

where $1 \leq \ell_j \leq M$ for $j=1,\ldots,L$. The effective legitimate uplink channel after antenna selection is therefore described by $\tilde{\mathbf{H}} \in \mathbb{C}^{L \times K}$ which is constructed from \mathbf{H} by collecting the ℓ_j -th rows of \mathbf{H} for $j=1,\ldots,L$. Similarly, the effective uplink channel from the eavesdropper to the BS is denoted by $\tilde{\mathbf{h}}_{\mathrm{e}} \in \mathbb{C}^L$ whose entries are the entries of \mathbf{h}_{e} indexed by \mathbb{L} .

A. Secure Transmission under Antenna Selection

For $k=1,\ldots,K$, let u_k represent the confidential message aimed to be received by legitimate user k. u_k is encoded into the codeword $[s_k(1),\ldots,s_k(N)]$ where N is the code-length. The encoded vector $\mathbf{s}(n) = [s_1(n),\ldots,s_K(n)]^\mathsf{T}$ is then given to the BS for transmission in the n-th time instant over the L selected transmit antennas. For this aim, the BS constructs the transmit signal $\mathbf{x}(n) \in \mathbb{C}^L$ from $\mathbf{s}(n)$ using a linear precoder. This means that, for $n \in \{1,\ldots,N\}$,

$$x(n) = \sqrt{P} \mathbf{W} s(n), \tag{4}$$

where P constrains the transmit power and $\mathbf{W} \in \mathbb{C}^{L \times K}$ is the shaping matrix satisfying $\mathbb{E} \operatorname{Tr} \{ \mathbf{W} \mathbf{W}^{\mathsf{H}} \} = 1$. Assuming that $s_k(n) \sim \mathcal{CN}(0,1)$ for $k \in \{1,\ldots,K\}$, the total transmit power in (4) is constrained to P.

By transmitting x(n) over the selected antennas, the legitimate terminals receive

$$\boldsymbol{y}(n) = \tilde{\mathbf{H}}^{\mathsf{T}} \boldsymbol{x}(n) + \boldsymbol{n}_{\mathrm{m}}(n), \tag{5}$$

where $y(n) := [y_1(n), \dots, y_K(n)]^\mathsf{T}$ with $y_k(n)$ denoting the signal received by the k-th legitimate user in the time instant n, and $n_m(n) \in \mathbb{C}^{K \times 1}$ is independent and identically distributed (i.i.d.) zero-mean complex Gaussian noise with variance σ_m^2 . The eavesdropper moreover receives

$$z(n) = \tilde{\mathbf{h}}_{e}^{\mathsf{T}} \boldsymbol{x}(n) + n_{e}(n) \tag{6}$$

where $n_{\rm e}(n) \sim \mathcal{CN}(0, \sigma_{\rm e}^2)$.

B. Achievable Secrecy Rate

In the absence of the eavesdropper, the maximum achievable rate of user k is bounded from below by [16]

$$\mathcal{R}_k^{\mathrm{m}} = \log(1 + \mathrm{SINR}_k^{\mathrm{m}}) \tag{7}$$

where $SINR_k^m$ is the Signal to Interference plus Noise Ratio (SINR) at the legitimate terminal k and is given by

$$SINR_k^{m} = \frac{\rho_{m} |\tilde{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_k|^2}{1 + \rho_{m} \sum_{j=1, j \neq k}^{K} |\tilde{\mathbf{h}}_k^{\mathsf{T}} \mathbf{w}_j|^2}.$$
 (8)

Here, $\rho_{\rm m} = P/\sigma_{\rm m}^2$, and $\tilde{\mathbf{h}}_k$ and \mathbf{w}_k are $L \times 1$ vectors which denote the kth column of $\tilde{\mathbf{H}}$ and \mathbf{W} , respectively.

For user k, the maximum achievable secrecy rate is defined as the maximum rate at which the BS can transmit information to legitimate user k such that no information about u_k is leaked to the eavesdropper. This rate is lower-bounded by [5]

$$\mathcal{R}_k^{\mathrm{s}} = \left[\mathcal{R}_k^{\mathrm{m}} - \mathcal{R}_k^{\mathrm{e}} \right]^+ \tag{9}$$

where \mathcal{R}_k^e represents the maximum achievable rate over the eavesdropper's channel under the worst-case assumption that the eavesdropper is able to cancel out the interference of other legitimate terminals¹. Consequently, \mathcal{R}_k^e is given by

$$\mathcal{R}_k^{\mathrm{e}} = \log(1 + \mathrm{SNR}_k^{\mathrm{e}}) \tag{10}$$

¹This is not necessarily the case, and therefore, (9) gives a lower bound.

where SNR_k^e is the SINR at the eavesdropper while overhearing u_k and reads

$$SNR_k^e = \rho_e |\tilde{\mathbf{h}}_e^\mathsf{T} \mathbf{w}_k|^2 \tag{11}$$

with $\rho_{\rm e} = P/\sigma_{\rm e}^2$. By substituting into (9), $\mathcal{R}_k^{\rm s}$ reads

$$\mathcal{R}_k^{\rm s} = \log\left(\frac{1 + {\rm SINR}_k^{\rm m}}{1 + {\rm SNR}_k^{\rm e}}\right). \tag{12}$$

C. Relative Secrecy Cost

Secrecy at the physical layer is obtained at the expense of reduction in the data rate. As (9) indicates, this cost depends on the quality of the eavesdropper's channel. We quantify this cost by defining the measure "relative secrecy cost" as follows.

Definition 1 (Relative secrecy cost): Assume L antennas are selected out of M available transmit antennas. Let $\mathcal{R}_k^{\mathrm{s}}(M,L)$ and $\mathcal{R}_k^{\mathrm{m}}(M,L)$ denote the secrecy rate to user k and the achievable rate to user k in the absence of the eavesdropper, respectively. The relative secrecy cost for user k is defined as

$$C_k(M, L) := 1 - \frac{\mathcal{R}_k^{\mathrm{s}}(M, L)}{\mathcal{R}_k^{\mathrm{m}}(M, L)}.$$
 (13)

From the definition of the relative secrecy cost, one simply observes that $0 \leq \mathsf{C}_k(M,L) \leq 1$ where the lower bound holds when $\mathcal{R}_k^{\mathrm{m}}(M,L) = \mathcal{R}_k^{\mathrm{s}}(M,L)$ and the upper bound is achieved when $\mathcal{R}_k^{\mathrm{s}}(M,L) = 0$. $\mathsf{C}_k(M,L)$ determines the fraction of available rate being used to secure the transmission. We show that this cost converges to zero in massive MIMO wiretap setups even under antenna selection. We refer to this phenomenon as "secrecy for free" in massive MIMO systems in the presence of passive eavesdroppers.

III. SECRECY FOR FREE

Secrecy for free intuitively means that the achievable rate in the absence of the eavesdropper and the achievable secrecy rate in the presence of the eavesdropper are nearly the same when the number of transmit antennas grows large. To formulate this property, we state the following definition.

Definition 2 (Asymptotic secrecy for free): Let $\rho_{\rm m}=P/\sigma_{\rm m}^2$ and $\rho_{\rm e}=P/\sigma_{\rm e}^2$ be bounded from above. For a given number of active transmit antennas L, the multiuser MIMO wiretap setting with passive eavesdropper illustrated in Section II is said to asymptotically achieve secrecy for free when

$$\lim_{M \uparrow \infty} \mathsf{C}_k(M, L) = 0 \tag{14}$$

for any $k \in \{1, ..., K\}$.

In a multiuser MIMO system, in which secrecy is achieved asymptotically for free, $\mathcal{R}_k^{\mathrm{s}} \approx \mathcal{R}_k^{\mathrm{m}}$ when the number of transmit antennas is large. This means that the BS can confidentially transmit messages to each legitimate user with almost no loss in terms of the achievable rate.

In what follows, we show that this property holds in general in massive MIMO wiretap setups with passive eavesdroppers even when only a fixed number of transmit antennas are set active. Throughout our investigations, we consider a Rayleigh fast fading model for the channels. This means that the entries of \mathbf{g}_k for $k \in \{1, \dots, K, e\}$ are independent complex Gaussian with zero mean and unit variance. We moreover consider Maximum Ratio Transmission (MRT) precoding at the BS, which is typical for massive MIMO systems [17]. This means that we set $\mathbf{w}_k = \tilde{\mathbf{h}}_k^* / \|\tilde{\mathbf{h}}_k\|$ for $k \in \{1, \dots, K\}$. The analysis is readily extended to other linear precoders as well as bi-unitarily invariant channel matrices² by a similar approach.

A. Secrecy For Free under Full Transmit Complexity

We begin with the case of full transmit complexity in which all transmit antennas are set active. In this case, $\tilde{\mathbf{h}}_k = \mathbf{h}_k$ for $k \in \{1, \dots, K, e\}$ and $\mathbf{w}_k = \mathbf{h}_k^* / \|\mathbf{h}_k\|$. Thus, the SINR at the legitimate terminal k reads

$$\frac{1}{M} SINR_k^{m} = \frac{1}{M} \frac{\rho_{m} \|\mathbf{h}_k\|^2}{1 + \rho_{m} \sum_{j=1, j \neq k}^{K} \frac{|\mathbf{h}_k^{\mathsf{T}} \mathbf{h}_j^{*}|^2}{\|\mathbf{h}_j\|^2}}$$
(15a)

$$= \frac{1}{M} \frac{\frac{\rho_{\rm m}}{M} \beta_k \|\mathbf{g}_k\|^2}{\frac{1}{M} + \rho_{\rm m} \sum_{j=1, j \neq k}^{K} \frac{\frac{1}{M^2} \beta_k |\mathbf{g}_k^{\mathsf{T}} \mathbf{g}_j^{*}|^2}{\frac{1}{M} \|\mathbf{g}_j\|^2}}$$
(15b)

$$\xrightarrow{\dagger} \rho_{\rm m} \beta_k \tag{15c}$$

where \longrightarrow indicates convergence in mean square, and \dagger comes from channel hardening [19] and the favorable propagation property [20] of massive MIMO systems. In fact, from channel hardening, we have $\|\mathbf{g}_k\|^2/M \longrightarrow \mathbb{E}\{\|\mathbf{g}_k\|^2\}/M = 1$ as M grows large. Moreover, the favorable propagation property of the channel for large M implies that $|\mathbf{g}_k^\mathsf{T}\mathbf{g}_j^*|/M \longrightarrow 0$ for any $k \neq j$.

Denoting the achievable rate with M transmit antennas over the legitimate channel k by ${}^3\mathcal{R}_k^{\mathrm{m}}(M)$, we conclude from (15c) that

$$\mathcal{R}_k^{\mathrm{m}}(M) - \log\left(1 + \rho_{\mathrm{m}}\beta_k M\right) \longrightarrow 0.$$
 (16)

Similarly, by invoking channel hardening and the favorable propagation property, we have for $\mathrm{SNR}_k^\mathrm{e}$

$$\frac{1}{M} \text{SNR}_k^{\text{e}} = \frac{1}{M} \rho_{\text{e}} \frac{|\mathbf{h}_{\text{e}}^{\mathsf{T}} \mathbf{h}_k^*|^2}{\|\mathbf{h}_k\|^2}$$
 (17a)

$$= \frac{\frac{\rho_{\mathbf{e}}}{M^2} \beta_{\mathbf{e}} |\mathbf{g}_{\mathbf{e}}^{\mathsf{T}} \mathbf{g}_{k}^{*}|^2}{\frac{1}{M} \|\mathbf{g}_{k}\|^2} \longrightarrow 0.$$
 (17b)

From (15c) and (17b), we can write

$$\left(\frac{1 + \text{SINR}_{k}^{\text{m}}}{1 + \text{SNR}_{k}^{\text{e}}}\right) / \left(1 + \rho_{\text{m}}\beta_{k}M\right) \longrightarrow 1.$$
(18)

Considering (12), (18) leads us to conclude that

$$\mathcal{R}_k^{\mathrm{s}}(M) - \log\left(1 + \rho_{\mathrm{m}}\beta_k M\right) \longrightarrow 0.$$
 (19)

²The random matrix $\mathbf{H} \in \mathbb{C}^{M \times K}$ is *bi-unitarily invariant*, if for any pair of independent unitary matrices $\mathbf{U} \in \mathbb{C}^{M \times M}$ and $\mathbf{V} \in \mathbb{C}^{K \times K}$, the entries of \mathbf{H} and $\mathbf{U}\mathbf{H}\mathbf{V}^H$ have same distribution [18].

 3 We have dropped the argument L for the case of full transmit complexity as in this case L=M.

Here, $\mathcal{R}_k^s(M)$ is the achievable secrecy rate with full transmit complexity. From (16) and (19), we have

$$\lim_{M \uparrow \infty} \mathsf{C}_k(M) = 1 - \lim_{M \uparrow \infty} \frac{\log(1 + \rho_{\mathrm{m}}\beta_k M)}{\log(1 + \rho_{\mathrm{m}}\beta_k M)} = 0 \tag{20}$$

where $\mathsf{C}_k(M)$ represents the relative secrecy cost under full complexity, i.e., L=M. From (20), one observes that secrecy in massive MIMO wiretap settings with passive eavesdroppers is achieved for free under full complexity, and therefore, for large M, we have $\mathcal{R}_k^{\mathrm{s}} \approx \mathcal{R}_k^{\mathrm{m}}$. The intuition behind this result is that in this setting, the precoder can accurately focus the transmission beam on the legitimate terminals, due to the large number of transmit antennas. This beam becomes significantly narrow, as the number of transmit antennas grows large, and therefore, the leakage to the eavesdropper vanishes.

Remark 1: Here, secrecy for free is achieved by simple MRT precoding. This means that in a massive MIMO setting, the BS can exclude the eavesdropper without knowing the channel state information of the eavesdropper.

B. Secrecy For Free under Antenna Selection

With a large number of transmit antennas, two scenarios for antenna selection can be considered:

- 1) The number of selected antennas L grows large with the total number of antennas M, such that L/M is kept fixed.
- 2) The number of selected antennas is kept fixed, e.g. L = 1, while the total number of antennas growing large.

In the first scenario, by a similar approach as in Section III-A, it is shown that secrecy is asymptotically achieved for free. In fact, in this case, as the number of active antennas grows proportional with the total number of transmit antennas, the precoder can narrow its beam toward the legitimate users, and thus, the leakage to the eavesdropper vanishes in the large-system limit⁴. We therefore concentrate on the second scenario in this section and show that even for a fixed number of active antennas, secrecy is achieved asymptotically for free.

For the sake of presentation, we set K=1. Nevertheless, all results and findings extend to arbitrary K in a straightforward way. In this case, $\mathbf{H}=\mathbf{h}_1\sqrt{\beta_1}\mathbf{g}_1$ where we use the notation

$$\mathbf{h}_1 = [h_1, \dots, h_M]^\mathsf{T},\tag{21a}$$

$$\mathbf{g}_1 = [g_1, \dots, g_M]^\mathsf{T}.\tag{21b}$$

To select a subset of transmit antennas, the following algorithm is employed at the beginning of each coherence time interval: The BS sorts channel coefficients h_1, \ldots, h_M such that

$$|h_{\ell_1}|^2 \ge \dots \ge |h_{\ell_M}|^2$$
 (22)

and selects the L strongest antennas, i.e., $\mathbb{L} = \{\ell_1, \dots, \ell_L\}$.

Under this antenna selection algorithm, the SINR at the legitimate terminal reads

$$SINR_{1}^{m} = \rho_{m} \|\tilde{\mathbf{h}}\|^{2} = \rho_{m} \beta_{1} \sum_{\ell=1}^{L} |g_{j_{\ell}}|^{2} = \rho_{m} \beta_{1} \Xi$$
 (23)

where we define

$$\Xi := \sum_{\ell=1}^{L} |g_{j_{\ell}}|^2. \tag{24}$$

In the context of order statistics [21], Ξ is known as a trimmed sum. The large-system distribution of a trimmed sum has been given in [22]. Note that by the large-system limit in this case, we mean that L is kept fixed and only M grows large. In [22], the asymptotic distribution of a trimmed sum is derived for a general distribution of summands. Noting that the summands in Ξ are exponentially distributed, one can invoke the main theorem of [22] and write

$$\Xi \sim \mathcal{N}\left(L\left(1 + \psi \log \frac{M}{L}\right), L\left(2 - \frac{L}{M}\right)\right)$$
 (25)

where $\psi = 1/\log e \approx 0.6931$. From (25), one observes that as M grows large, Ξ converges in distribution to a Gaussian random variable whose mean increases with $\log M$ and whose variance converges to a constant. Consequently, one can write

$$\lim_{M \uparrow \infty} \mathbb{E} \left\{ \frac{\text{SINR}_1^{\text{m}}}{\log M} \right\} = \psi \rho_{\text{m}} \beta_1 L \tag{26a}$$

$$\lim_{M \uparrow \infty} \mathbb{E} \left\{ \left| \frac{\text{SINR}_{1}^{m}}{\log M} - \mathbb{E} \left\{ \frac{\text{SINR}_{1}^{m}}{\log M} \right\} \right|^{2} \right\} = 0$$
 (26b)

which implies that

$$\frac{\text{SINR}_1^{\text{m}}}{\log M} \longrightarrow \psi \rho_{\text{m}} \beta_1 L \tag{27}$$

or equivalently

$$\mathcal{R}_1^{\mathrm{m}}(M, L) - \log\left(1 + \psi \rho_{\mathrm{m}} \beta_1 L \log M\right) \longrightarrow 0$$
 (28)

where $\mathcal{R}_1^{\mathrm{m}}(M,L)$ denotes the achievable rate over the main channel when L out of M available transmit antennas are selected via the selection algorithm.

Considering the eavesdropper, SNR₁^e is written as

$$SNR_1^e = \rho_e \frac{|\tilde{\mathbf{h}}_e^\mathsf{T} \tilde{\mathbf{h}}_1^*|^2}{\|\tilde{\mathbf{h}}_1\|^2}$$
 (29a)

$$= \rho_{\mathbf{e}} \beta_{\mathbf{e}} \|\tilde{\mathbf{g}}_{\mathbf{e}}\|^2 \cos^2 \theta \tag{29b}$$

where θ denotes the Hermitian angle between $\mathbf{\tilde{g}}_1$ and $\mathbf{\tilde{g}}_e$ and is defined as

$$\theta := \cos^{-1} \left(\frac{|\tilde{\mathbf{g}}_{e}^{\mathsf{T}} \tilde{\mathbf{g}}_{1}^{*}|}{\|\tilde{\mathbf{g}}_{e}\| \|\tilde{\mathbf{g}}_{1}\|} \right). \tag{30}$$

To determine the large-system limit of SNR_1^e , we consider the following lines of justifications:

(a) As \mathbf{h}_1 and \mathbf{h}_e are statistically independent, any ordered sorting on the entries of \mathbf{h}_1 results in a random permutation of \mathbf{h}_e . Therefore, the entries of $\tilde{\mathbf{h}}_e$ are statistically

⁴To show this argument, one can start with a random selection algorithm which selects a fixed fraction of transmit antennas at random. Taking exactly same steps as in Section III-A, it is shown that secrecy is also achieved for free in this case. As the result, other algorithms achieve secrecy asymptotically for free in the first scenario, since all algorithms are superior to random selection.

similar to the entries of a random selection. In other words, from the eavesdropper's point of view, the antenna selection algorithm is random selection. This fact implies that the entries of $\tilde{\mathbf{g}}_{\rm e}$ are independent complex Gaussian with zero mean and unit variance.

- (b) Note that the illustrated selection algorithm sorts the channel coefficients with respect to the magnitudes. This fact, implies that the direction of $\tilde{\mathbf{g}}_1$ is statistically similar to the direction of a randomly selected vector. Therefore, the Hermitian angle between $\tilde{\mathbf{g}}_1$ and $\tilde{\mathbf{g}}_e$ is distributed similar to the hermitian angle between two independent Gaussian random vectors⁵ of size L.
- (c) The distribution of the squared cosine of the Hermitian angle between two independent Gaussian vectors of length L is $\mathcal{B}(1, L-1)$; see [23, Appendix C].
- (d) Since $\tilde{\mathbf{g}}_{e} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{L})$, the normalization $\mathbf{v}_{e} = \tilde{\mathbf{g}}_{e} / \|\tilde{\mathbf{g}}_{e}\|$ is independent of $\|\tilde{\mathbf{g}}_{e}\|$. Therefore, the random variable $\cos \theta = |\mathbf{v}_{e}^{\mathsf{T}} \tilde{\mathbf{g}}_{1}^{*}| / \|\tilde{\mathbf{g}}_{1}\|$ is independent of $\|\tilde{\mathbf{g}}_{e}\|$, as well.

Considering the above lines of justifications, one can conclude that $\cos^2\theta \sim \mathcal{B}\left(1,L-1\right)$ and is independent of $\|\mathbf{\tilde{g}_e}\|^2$. $\|\mathbf{\tilde{g}_e}\|^2$ is moreover a chi-square random variable with 2L degrees of freedom, mean L and variance L. Consequently, the expected value of $\mathrm{SNR}_1^{\mathrm{e}}$ reads

$$\mathbb{E}\left\{\mathrm{SNR}_{1}^{\mathrm{e}}\right\} = \rho_{\mathrm{e}}\beta_{\mathrm{e}} \,\mathbb{E}\left\{\left\|\tilde{\mathbf{g}}_{\mathrm{e}}\right\|^{2}\right\} \mathbb{E}\left\{\cos^{2}\theta\right\} \tag{31a}$$

$$= \rho_{\rm e}\beta_{\rm e} \ L \ \frac{1}{L} = \rho_{\rm e}\beta_{\rm e}. \tag{31b}$$

To determine the variance of SNRe, we note that

$$\mathbb{E}\left\{|\mathrm{SNR}_{1}^{\mathrm{e}}|^{2}\right\} = \rho_{\mathrm{e}}^{2}\beta_{\mathrm{e}}^{2}\,\mathbb{E}\left\{\|\tilde{\mathbf{g}}\|^{2}\right\}^{2}\mathbb{E}\left\{\cos^{2}\theta\right\}^{2} \tag{32a}$$

$$= \rho_{\rm e}^2 \beta_{\rm e}^2 \ L(L+1) \ \frac{2}{L(L+1)} = 2 \rho_{\rm e}^2 \beta_{\rm e}^2 \ \ (32b)$$

which results in

$$\mathbb{E}\left\{|\text{SNR}_{1}^{\text{e}}|^{2}\right\} - \mathbb{E}\left\{\text{SNR}_{1}^{\text{e}}\right\}^{2} = \rho_{\text{e}}^{2}\beta_{\text{e}}^{2}.$$
 (33)

From (31b) and (33), it is concluded that SNR_1^e takes random values around $\rho_e\beta_e$ with the finite variance $\rho_e^2\beta_e^2$. Thus,

$$\lim_{M \uparrow \infty} \mathbb{E} \left\{ \frac{\text{SNR}_1^{\text{e}}}{\log M} \right\} = 0, \tag{34a}$$

$$\lim_{M \uparrow \infty} \mathbb{E} \left\{ \left| \frac{\text{SNR}_1^{\text{e}}}{\log M} - \mathbb{E} \left\{ \frac{\text{SNR}_1^{\text{e}}}{\log M} \right\} \right|^2 \right\} = 0, \quad (34b)$$

or equivalently, we can write

$$\frac{\text{SNR}_1^{\text{e}}}{\log M} \longrightarrow 0. \tag{35}$$

Considering (27) along with (35), we have

$$\mathcal{R}_1^{\mathrm{s}}(M,L) - \log\left(1 + \psi \rho_{\mathrm{m}} \beta_1 L \log M\right) \longrightarrow 0$$
 (36)

where $\mathcal{R}_{1}^{s}(M,L)$ denotes the achievable secrecy rate when L transmit antennas are selected.

Invoking the arguments in (28) and (36), the asymptotic relative secrecy cost $C_1(M, L)$ in this case reads

$$\lim_{M\uparrow\infty}\mathsf{C}_1(M,L) = 1 - \lim_{M\uparrow\infty}\frac{\log\left(1 + \psi\rho_\mathrm{m}L\log M\right)}{\log\left(1 + \psi\rho_\mathrm{m}L\log M\right)} = 0. \eqno(37)$$

From (37), it is observed that even by selecting a *fixed* number of antennas, secrecy is achieved for free as the *total* number of available antennas grows unboundedly large. This observation intuitively comes from this fact that the growth in the total number of transmit antennas improves the channel quality of the selected antennas while the eavesdropper's channel remaining unchanged. Hence, as M grows large, the leakage to the eavesdropper becomes negligible compared to the achievable rate over the main channel, and we have $\mathcal{R}^s \approx \mathcal{R}^m$.

Remark 2: The large-system analysis considers a fixed number of active antennas. This means that the result is valid even when a *single* antenna is selected. The characterization for single transmit antenna selection can be more precisely addressed using the extreme value distribution given by Fisher-Tippet law [21]; see the studies in [24], [25] for some particular examples. Nevertheless, both the approaches lead to this conclusion that the secrecy cost vanishes as M grows large.

Remark 3: Considering the cases of full transmit complexity and antenna selection with fixed L, one observes that in either cases \mathcal{R}_k^e does not grow large with M while the achievable rate over the main channel scales in terms of M. The growth in the achievable rate \mathcal{R}_k^m is however of different orders in these cases: With full complexity, the growth order is $\log M$, while under antenna selection, \mathcal{R}_k^m grows proportional to $\log \log M$. This fact indicates that the secrecy cost converges to zero with lower speed, when a fixed number of antennas are selected.

Remark 4: By either changing the precoding scheme, e.g. zero forcing scheme, or using a superior selection algorithm, e.g. algorithm in [26], the secrecy performance in this setting is improved. This fact indicates that although the analyses are given for MRT precoding and a specific selection algorithm, the results guarantee the achievability of secrecy for free for a large class of algorithms as well as other precodings.

IV. NUMERICAL INVESTIGATIONS

To confirm our analyses, we consider some numerical examples. Fig. 1 illustrates the robustness against passive eavesdropping under full transmit complexity, i.e., L=M. Here, K=4 legitimate users and a single eavesdropper is considered. The channels are i.i.d. Rayleigh fading with zero mean and unit variance. It is assumed that the users are uniformly distributed in the cell, and the path loss is compensated at the receiving terminals, i.e., $\beta_k=1$ for all k. The Signal to Noise Ratio (SNR) at each legitimate terminal and the eavesdropper has been set to $\log \rho_{\rm m}=0$ dB and $\log \rho_{\rm e}=-10$ dB, respectively. For this setting, the achievable rate over the

 $^{^5}$ In the large-system limit, this statement can be rigorously justified by considering a sequence $\{\theta_M\}$ and showing that it converges in distribution to the Hermitian angle of two independent Gaussian vectors of the same size as M grows large. Similar discussions can be found in [9].

⁶Note that SINR_k^m grows linearly with M in this case; see (15c).

⁷Under antenna selection, SINR^m grows with log M; see (27).

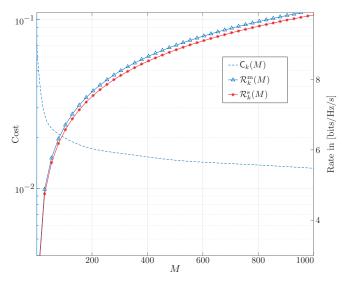


Fig. 1: The relative secrecy cost, achievable rate and secrecy rate for each legitimate user versus the number of antennas at the BS. Here, K=4, $\log \rho_{\rm m}=0$ dB and $\log \rho_{\rm e}=-10$ dB.

legitimate channel $\mathcal{R}_k^{\mathrm{m}}(M)$ as well as the achievable secrecy rate for each legitimate user $\mathcal{R}_k^{\mathrm{s}}(M)$ has been plotted as a function of the number of antennas at the BS⁸ M.

As Fig. 1 shows, the achievable secrecy rate closely tracks the rate achieved over the legitimate channel in the absence of the eavesdropper, i.e., $\mathcal{R}_k^{\mathrm{m}}(M)$. The relative secrecy cost $\mathsf{C}_k(M)$ has been further sketched in Fig. 1. One should note that $\mathsf{C}_k(M)$ is *relative*, meaning that it does not directly scale with $\mathcal{R}_k^{\mathrm{m}}(M) - \mathcal{R}_k^{\mathrm{s}}(M)$, but with the ratio $[\mathcal{R}_k^{\mathrm{m}}(M) - \mathcal{R}_k^{\mathrm{s}}(M)]/\mathcal{R}_k^{\mathrm{m}}(M)$. As it is observed in the figure, the relative secrecy cost drops rapidly with respect to the number transmit antennas which confirms the analysis in Section III-A.

To investigate the robustness against passive eavesdropping under antenna selection, we have considered a MIMO wiretap setting with a single legitimate terminal and an eavesdropper in Fig. 2. Here, the same channel model as in Fig. 1 is assumed. Moreover, the SNRs at the legitimate terminal and the eavesdropper have been set $\log \rho_{\rm m}=0$ dB and $\log \rho_{\rm e}=-15$ dB, respectively. In this figure, the achievable rates $\mathcal{R}_1^{\rm m}(M,L)$ and $\mathcal{R}_1^{\rm s}(M,L)$, as well as the relative secrecy cost $\mathrm{C}_1(M,L)$, have been plotted against the total number of transmit antennas 9 M for two cases of single transmit antenna selection, i.e. L=1, and L=4. It is assumed that the algorithm illustrated in Section III-B is employed for antenna selection. For the sake of comparison the results for full transmit complexity have been sketched as well.

As Fig. 2 depicts even with a single active antenna at the BS, the achievable rate over the legitimate channel and the secrecy rate meet at large values of M. From the figure, it

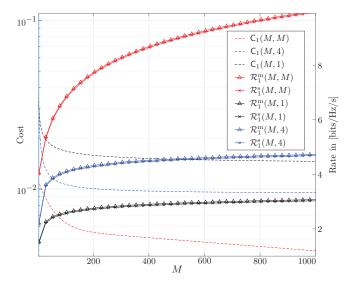


Fig. 2: The relative secrecy cost, achievable rate and secrecy rate versus the total number of transmit antennas M for different number of RF-chains, i.e., L=1 and L=4, at the BS. Here, K=1, $\log \rho_{\rm m}=0$ dB and $\log \rho_{\rm e}=-15$ dB.

is observed that under antenna selection, the relative secrecy cost converges to zero slower than the case with full transmit complexity. In fact, the slope of $C_1(M,M)$ at large values of M is considerably larger than the slope of $C_1(M,1)$ and $C_1(M,4)$. As discussed in Remark 3, this observation comes from the two following facts: 1) The achievable rate $\mathcal{R}_k^{\mathrm{m}}(M,L)$, for a fixed L, grows large significantly slower than $\mathcal{R}_k^{\mathrm{m}}(M,M)$, i.e., the rate achieved by full transmit complexity 10 , and 2) the information leakage under antenna selection does not vanishes as fast as the case with all transmit antennas being active.

V. CONCLUSION

This paper has studied the robustness of massive MIMO wiretap settings against passive eavesdropping. Our investigations have shown that even when the BS employs a *fixed* number of its transmit antennas, including the case with a *single* active antenna, the information leakage to the eavesdropper vanishes as the *total* number of transmit antennas grows large. This fact indicates that in massive MIMO systems, regardless of the number of active antennas, secrecy is achieved almost "for free". Our analytic results guarantee the robustness of massive MIMO settings against passive eavesdropping for a large class of selection algorithms and precodings.

From numerical simulations, it is known that in contrast to setups with *passive* eavesdroppers, massive MIMO systems are not robust against *active* eavesdropping [6]. The large-system characterization of MIMO wiretap settings under active eavesdropping attacks is therefore an interesting direction for future work. The work in that direction is currently ongoing.

⁸Note that in this case, due to the uniform distribution of the users in the cell, $\mathcal{R}_k^{\mathrm{m}} = \mathcal{R}_j^{\mathrm{m}}$ and $\mathcal{R}_k^{\mathrm{s}} = \mathcal{R}_j^{\mathrm{s}}$ for all $k \neq j$.

⁹Note that the curves for the case of antenna selection start on the x-axis from M=L, since we have $M\geq L$.

 $^{^{10} \}mathrm{In}$ fact, $\mathcal{R}_k^{\mathrm{m}}(M,L),$ for a fixed L, grows proportional to $\log\log M,$ while $\mathcal{R}_k^{\mathrm{m}}(M,M)$ grows with $\log M.$

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