A Novel Low Complexity Differential Energy Detection For Sensing OFDM Sources In Low SNR Environment

Parisa Cheraghi, Yi Ma, Zhengwei Lu, and Rahim Tafazolli
Centre for Communication Systems Research,
University of Surrey, United Kingdom, GU2 7XH
e-mail:{p.cheraghi, y.ma, z.lu, r.tafazolli }@surrey.ac.uk

***Abstract*—This paper presents a novel frequency-domain energy detection scheme based on extreme statistics for robust sensing of OFDM sources in the low SNR region. The basic idea is to exploit the frequency diversity gain inherited by frequency selective channels with the aid of extreme statistics of the differential energy spectral density (ESD). Thanks to the differential stage the proposed spectrum sensing is robust to noise uncertainty problem. The low computational complexity requirement of the proposed technique makes it suitable for even machine-to-machine sensing. Analytical performance analysis is performed in terms of two classical metrics, i.e. probability of detection and probability of false alarm. The computer simulations carried out further show that the proposed scheme outperforms energy detection and second order cyclostationarity based approach for up to 10 dB gain in the low SNR range.**

***Index Terms*—Differential, low SNR, machine-to-machine, OFDM, spectrum sensing.**

I. Introduction

Spectrum sensing is often referred to as a signal-processing technique that can monitor user activities within a certain frequency band. It has been becoming one of key techniques to enable advanced wireless communication systems such as self-organizing networks [1] and cognitive radios [2]. In order to have a successful spectrum sensing, the three requirements of latency, complexity and reliability should be meet. It is to our interest to minimise the latency of the sensing process to achieve higher throughput while low complexity sensing unit reduces the overall energy consumption of the device. Reliability of a spectrum sensing technique is measured in to classical metrics namely, probability of detection (P_D) and probability of false alarm (P_{FA}). The spectrum sensing device is required to achieve P_D of more than 90% whilst P_{FA} does not exceed 10% [3]. This signal-processing apparatus has been well investigated since late 1940s. Amongst the proposed spectrum sensing techniques, matched filtering requires knowledge of users' air-interface while demanding reasonable synchronization [4], thus limiting the flexibility of the signal being detected. Energy detection is capable of offering acceptable performance when the noise uncertainty factor is sufficiently low [5]. Cyclostationarity-based detection approaches can deliver an excellent performance by exploiting the cyclic information, with the trade off of introducing a long latency into the system [6],[7]. The wavelet detection which has recently been introduced in the literature , is capable of performing wide-band sensing with the aid of edge detection [8]. The eigenvalue-decomposition is an other powerful spectrum sensing technique which takes advantage of the orthogonality between the noise and the signal subspace with the aid of multiple antennas. A matter of primary interest is that almost all existing spectrum sensing techniques do not deliver an acceptable performance without introducing high latency or complexity in the system. In particular failing to meet the demanding requirements of machine-to-machine sensing in low SNR range, i.e. $(-25, -10)$ dB where both complexity and latency should be minimised due to energy efficiency necessity. One of the main problems which arises from employing spectrum sensing approaches in low SNR environments is known as the hidden node problem [9]. This would mean that the spectrum sensing technique has to be less susceptible to noise and noise uncertainty in order to differentiate a faded or shadowed signal from a vacant band. This paper is mainly motivated by this problem. This paper presents a novel frequency-domain energy detection scheme based on extreme statistics for robust sensing of OFDM sources in the low SNR region. The basic idea is to exploit the frequency diversity gain inherited by frequency selective channels with the aid of extreme statistics of the differential energy spectral density (ESD). Thanks to the differential stage the proposed spectrum sensing is robust to noise uncertainty problem. The low computational complexity requirement of the proposed technique makes it suitable for even machine-to-machine sensing. Analytical performance analysis is performed in terms of both probability of detection and probability of false alarm. Moreover, computer simulations show that the proposed scheme outperforms energy detection and second order cyclostationarity based approach for up to 10 dB gain in the low SNR range.

II. System Description

Consider an OFDM based primary system employing a total number of N sub-carriers. Let $\{x_{n,k}\}_{k=1}^{N}$ with $E|x_{n,k}|^2 = \sigma^2$ be the complex symbols to be transmitted at the kth sub-carrier of nth OFDM block. At this point the data blocks are

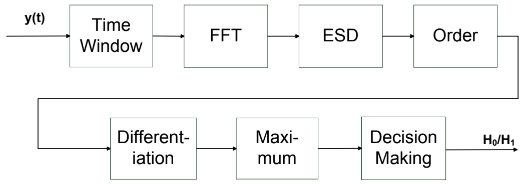

Fig. 1. Block diagram of the proposed technique.

modulated over the whole allocated bandwidth.

Modulating the OFDM symbols belonging to a block on a set of overlapping and orthogonal sub-carriers in this way is equivalent to taking an N-point inverse Fourier transform from the corresponding OFDM-block. To avoid inter-block interference caused by the channel each OFDM block is cyclically extended. Therefore the Cyclic Prefix (CP), whose length is not less than channel length, is padded at preamble of each block. With insertion of CP the OFDM signal is extended to $T = T_g + T_s$ where T_g and T_s are the CP duration and symbol duration respectively. Hence the OFDM signal at baseband can be expressed as:

$$s_n(t) = \sum_{k=1}^{N} x_{n,k} e^{j2\pi k \Delta f t} \quad , \qquad -T_g \le t \le T_s \quad (1)$$

where Δf is the the carrier spacing. T_s and Δf should be chosen such that the orthogonal condition is met, i.e. $T_s \Delta f = 1$. It can be observed that the above signal is the IDFT of the transmitted symbols $\{x_{n,k}\}_{k=1}^{N}$. During the transmission signal will be affected by the communication channel. The impulse response of a wireless communication channel can be expressed as $h(t) = \sum_{l=1}^{L} \alpha_l \delta(t-l)$ where L is the number of taps in the multipath channel and α is the channel complex amplitude. In this setting, the problem of spectrum sensing is equivalent to hypothesis testing. There exists two hypothesizes: $\mathcal{H}_0$, when the signal does not appear; and $\mathcal{H}_1$ when the channel is occupied by the primary signal, i.e.:

$$y\left(\frac{mT}{N}\right) = \begin{cases} v(\frac{mT}{N}), & \mathcal{H}_0 \\ r\left(\frac{mT}{N}\right) e^{-j2\pi\varepsilon\frac{mT}{N}} + v(\frac{mT}{N}), & \mathcal{H}_1 \end{cases} \quad (2)$$

$$r\left(\frac{mT}{N}\right) = \sum_{l=1}^{L} \alpha_l s\left(\frac{mT}{N} - \mu - l\right) \quad (3)$$

where y denotes sampled version of the received signal at the baseband, v AWGN with zero mean, μ the timing offset and ε frequency offset respectively.

III. Proposed Source Detection Technique

Our approach to robust spectrum sensing takes the following four steps: *i*) estimating the ESD across a total of BW Hz in frequency; *ii*) Ordering the calculated ESD in terms of magnitude; *iii*) performing differentiation with respect to the frequency, *iv*) search for the maximum value of the result.

A. ESD Estimation

Let W denote the window length (after extracting the CP symbols) where the energy is calculated over for each frequency bin. The first task is to perform the N-point FFT operation on the windowed signal $y_w\left(\frac{mT}{N}\right)$, i.e. $\tilde{z}_w(k) = \mathbf{F} y_w\left(\frac{mT}{N}\right)$; where $\mathbf{F}$ is an $N \times N$ Fourier transform matrix:

$$F(n,w) = \frac{1}{\sqrt{N}} e^{-j2\pi(n)(l)/N} \quad , \qquad 1 \le n, l \le N \quad (4)$$

Applying the above to (2) we obtain:

$$\tilde{z}_w(k) = \begin{cases} \tilde{v}_w(k), & \mathcal{H}_0 \\ \tilde{h}(k)\tilde{s}_w(k-\varepsilon)e^{-j2\pi\mu k} + \tilde{v}_w(k), & \mathcal{H}_1 \end{cases} \quad (5)$$

where $\tilde{s}$, $\tilde{v}$ and $\tilde{h}$ denote the frequency-domain version of transmitted signal, noise and channel respectively. The ESD calculation can be represented as $Q_k = \frac{1}{W}\sum_{w=1}^{W} |\tilde{z}_w(k)|^2$. Hence:

$$Q_k \begin{cases} E\{Q\} = \sigma_v^2 \ , \quad \sigma_Q^2 = \frac{2\sigma_v^4}{W}, & \mathcal{H}_0 \\ E\{Q\} = \gamma + \sigma_v^2 \ , \quad \sigma_Q^2 = \frac{4}{W}\left(\gamma + \sigma_v^2\right)^2 , & \mathcal{H}_1 \end{cases} \quad (6)$$

$\gamma = |H|^2 \sigma_s^2$; while σ_v^2 and σ_s^2 denote the noise power the average signal power respectively. It is worth bringing to attention that Q_k has a normal distribution with the parameters shown in (6), this approximation can be achieved by applying central limit theorem.

Employment of ESD operation in the proposed approach makes it robust to the possible time offset which can be encountered in any communication system. ESD approximation is based on the magnitude of the signal represented in the frequency domain, this is while the time offset introduces phase distortion which wont affect the ESD operation. Furthermore, frequency offset will shift the received signal in the frequency domain, and in the case of an OFDM signal this will cause energy leakage from subsequent carriers. In low SNR environments energy leakage does not have significant affect since the dominating distortions is resulting from noise. The objective of the proposed technique is to perform spectrum sensing in low SNR which implies that the proposed technique is robust to both frequency and time offsets.

B. Order

After determination of the ESDs they have to be ordered in terms of magnitude ($\uparrow Q_k$). This step does not affect the statistical properties of Q_k. Hence:

$$\uparrow Q_k \ \approx \ \begin{cases} \mathcal{N}\left(\sigma_v^2 \ , \ \frac{2}{W}\sigma_v^4\right), & \mathcal{H}_0 \\ \mathcal{N}\left(\gamma + \sigma_v^2 \ , \ \frac{2}{W}(\gamma + \sigma_v^2)^2\right), & \mathcal{H}_1 \end{cases} \quad (7)$$

where $\mathcal{N}(.)$ denotes the Gaussian distribution for real random variable.

C. Differentiation

At this stage of the algorithm the ESD is differentiated with respect to the frequency. By performing the differentiation which in this case would consist of subtracting two normal distributed variables, i.e. $U_k = \uparrow Q_k - \uparrow Q_{k-1}$. This results in having a normal distribution with a zero mean and a variance which has twice the value of variance mentioned in (7), i.e. $E\{U|\mathcal{H}_0\} = E\{U|\mathcal{H}_1\} = 0$, $\sigma^2_{U|\mathcal{H}_0} = 4\sigma_v^4 \;/\; W$ and $\sigma^2_{U|\mathcal{H}_1} = 8\left(\gamma + \sigma_v^2\right)^2 \;/\; W$.

The motivation behind this differentiation is to find the rate at which the ESD changes with respect to frequency. If the OFDM-based primary system is absent, i.e. $\mathcal{H}_0$ in (2) holds; this would result in negligible difference due to AWGN property which results in approximately constant ESD. On the other hand when the primary system is active, i.e. $\mathcal{H}_1$ in (2) holds; the rate at which the estimated ESD changes is noticeable. This is due to frequency diversity introduced by the channel (h). It should be noted that the differentiation done at this stage will considerably reduced the noise uncertainty factor. This would imply that even though this technique is based on the energy level observation but it is able to overcome the noise uncertainty problem which is the issue which makes the conventional energy detection reluctant in low SNR environment. Hence making it robust in terms of uncertainty factors.

D. Maximum Value

Having U_k the next task is to find the maximum value ($U_{\max}$). Using Von Mises theorem [10] [1] it was found that U_{MAX} follows a Gumbel distribution, i.e. $f_{\max}(x) = \frac{1}{\beta} e^{-\frac{x-\alpha}{\beta}} e^{-e^{-\frac{x-\alpha}{\beta}}}$ where $f(x)$, α and β denote the the PDF, location and scale parameters of the Gumbel distribution respectively. It was proven that $U_{\max}$ follows a Gumbel distribution using Von Mises theorem is met but due to length restrictions of this paper the proof is not included here. The results of applying the theorem are:

$$U_{MAX} \;\approx\; \sigma \;\left(\psi \;-\; \frac{\log\log N + \log 4\pi}{2\psi}\right) \tag{8}$$

where $\psi = \sqrt{2\log N}$. The expected value and the variance of U_{MAX} can be expressed using order statistic properties [11], thus:

$$U_{MAX} \begin{cases} E(U_{MAX}) = \sigma\left(\psi - \zeta + O\left(\frac{1}{\log N}\right)\right) \\ \sigma^2(U_{MAX}) = \frac{\sigma^2}{2\log N}\left(\frac{\pi^2}{6} - S_2\right) + O\left(\frac{1}{\log^2 N}\right) \end{cases} \tag{9}$$

$$\zeta = \frac{\log\log N + \log 4\pi \;+\; 2(S_1 - C)}{2\psi} \tag{10}$$

$S_1 = \sum_{b=1}^{N-1} 1/b$, $S_2 = \sum_{b=1}^{N-1} 1/b^2$; C denotes Euler's constant. The expected value and variance of U_{MAX} can be calculated by multiplying the received power by a constant. This constant is a function of the frequency bins used and the window size. In order to reduce noise uncertainty problems lower variance and higher expected value are preferred. This implies that the proposed method offers a better performance for detecting systems with a higher number of sub-carriers, i.e. bigger value of N. Using the Gumbel distribution properties we can derive the PDF of the U_{MAX}. The location and scale parameter α and β respectively of this distribution are related to the mean and standard deviation by $\alpha = E(x) - C \;/\; \beta$ and $\beta = \sqrt{6}\,\sigma^2 \;/\; \pi$ [11]. Also, the terms with order of $1/\log N$ and $1/\log^2 N$ were eliminated for simplicity. Hence:

$$U_{MAX} \approx \begin{cases} \mathcal{G}\left(\eta\sigma_v^2\left(\psi - \zeta\right) - \varphi\sigma_v^4\left(\frac{\pi^2}{6} - S_2\right)\right. \\ \left. \;,\; \frac{\varphi}{C}\sigma_v^4\left(\frac{\pi^2}{6} - S_2\right)\right) & \mathcal{H}_0 \\ \mathcal{G}\left(\eta\;\lambda\left(\psi \;-\; \zeta\right) \;-\; \varphi\lambda^2\left(\frac{\pi^2}{6} - S_2\right)\right. \\ \left. \;,\; \frac{\varphi}{C}\lambda^2\left(\frac{\pi^2}{6} - S_2\right)\right) & \mathcal{H}_1 \end{cases} \tag{11}$$

$\varphi = 4\sqrt{6}C \;/\; W\pi\psi^2$, $\eta = 2 \;/\; \sqrt{W}$ and $\lambda = |H|^2\sigma_s + \sigma_v^2$. $\mathcal{G}(\beta, \alpha)$ represents a Gumbel distribution with location parameter α and scale parameter β. Once U_{MAX} has been determined the value is compared with the predetermined threshold. The optimum threshold (Υ) can be obtained using the Neyman-Pearson test [12]. According to this test $LLR\left(U_{MAX}\right) \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \Upsilon$, $LLR(.)$ denotes log-likelihood ratio. Equation (12) can be employed to find the optimum threshold value:

$$LLR\left(U_{MAX}\right) = \sum_{n=1}^{N} \log\left[\frac{P\left(U_{MAX}[n]|\mathcal{H}_1\right)}{P\left(U_{MAX}[n]|\mathcal{H}_0\right)}\right]. \tag{12}$$

IV. Performance Analysis

Based on the information stated in the previous section it is possible to derive the closed form for the probability of false alarm (P_{FA}) and probability of missed detection (P_{MD}) [13]. Noise uncertainty factor (ρ) should also be considered while analysing the performance. Since the proposed method is based on the ESD, the noise uncertainty distribution can be summarized in the interval $\sigma_v^2 \in [(1/\rho)\sigma_v)^2, \rho\sigma_v^2]$ [13]:

$$P_{FA} \;\approx\; \max_{\sigma_v^2 \in [(1/\rho)\sigma_v^2, \rho\sigma_v^2]} \mathcal{Q}\left(\frac{\lambda \;-\; \left(\eta\sigma_v^2\left(\psi - \zeta\right) - \varphi\sigma_v^4\phi\right)}{\sqrt{\frac{\varphi}{C}\sigma_v^4\phi}}\right) \tag{13}$$

$$P_{MD} \;\approx\; 1 - \min_{\sigma_v^2 \in [(1/\rho)\sigma_v^2, \rho\sigma_v^2]} \mathcal{Q}\left(\frac{\lambda \;-\; \eta\,\gamma\left(\psi - \zeta\right) - \varphi\gamma^2\phi}{\sqrt{\frac{\varphi}{C}\gamma^2\phi}}\right) \tag{14}$$

$\phi = \pi^2/6 - S_2$, $\mathcal{Q}(.)$ denotes the standard Gumbel complementary CDF, i.e. $\mathcal{Q}(x) = -\ln\left(\ln\left(\frac{\beta}{x-\alpha}\right)\right)$. Both P_{FA} and P_{MD} are significantly affected by values of W and N. Since N is fixed for a given system we can change W in order to achieve the desired P_{FA} and P_{MD}. This feature increases the sensitivity and also makes the proposed technique more immune to noise uncertainty problem. The proposed spectrum sensing takes advantage of the frequency selectivity gain which is imposed by the communication channel during the transmission. Incorporating frequency domain ESD into the proposed technique makes it robust

[1] The Extreme value for an arbitrary distribution X will follow a Gumbel distribution if the PDF of X is non-negative and is differentiable for all x in (x_1, ξ_1) for some x_1 and $\lim_{x\to\xi_1} \frac{d}{dx}\left[\frac{1-F(x)}{f(x)}\right] = 0$ where F(x) and f(x) are the CDF and PDF of random variable x respectively.

to possible time and frequency offsets even in low SNR environments. It was found that employing order statistics in the proposed algorithm can improve the performance further. The proposed technique can offer very high sensitivity in terms of SNR which means it can perform desirably in low SNR range, i.e. SNR< -10 dB. The hidden node problem is one of the main issues in spectrum sensing problems. It can be found in [9] that this issue can be due to many factors, such as significant multipath fading or deep shadowing. This would mean that the sensing technique has to be less susceptible to noise and noise uncertainty in order to be able to differentiate a faded or shadowed primary users' signal from a vacant band. In the case of the proposed technique this problem can be overcome to some extent since it offers high sensitivity. This can be observed by comparing the performance of conventional energy detection [13] and proposed technique both in terms performance analysis shown above and the simulations carried out in the next section.

V. Simulation Results

An OFDM based air interface IEEE 802.11g [14] system was chosen as the primary system. According to this protocol each OFDM block contains 64 sub-carriers with CP of length 16. One OFDM duration is 4.0 μs and each OFDM frame contains 10 OFDM blocks. The channel is assumed to be quasi-static and was modelled as 5-path frequency-selective Rayleigh fading channel with power delay profile (PDP) $E\{|h(l)|^2\} = e^{-l/5} / \left(\sum_{l=0}^{4} e^{-l/5}\right)$, $l = 0, \cdots, 4$. It is further assumed that the sensing terminal speed sets up at 3.0 km/h. In the following figures the results are averaged over 2000 Monte Carlo realizations. Furthermore, the SNR value in the simulation performed are defined as the ratio of the average received signal power to the average noise power

$$SNR \equiv \frac{E\left(\|y(n) - v(n)\|^2\right)}{E\left(\|v(n)\|^2\right)}. \quad (15)$$

The constraint for probability of false alarm is $P_{FA} \leq 0.1$. The threshold is set to meet this constraint.

Fig. 2 compares the performance of the second order cyclostationary based detection and the proposed technique in terms of SNR requirements and latency. As it can be observed the proposed technique requires a shorter observation time. This was expected since one of the main drawbacks of second order cyclostationarity is its requirement for long observation time due to its computational complexity. As it can be observed from Fig. 2 the proposed technique the second order cyclostationarity when the observation length is set to 10 frames by minimum of 10 dB, while this gain increases to 12 dB when the observation length is set to 20 frames. The proposed method is also able to perform considerably better in lower SNR environments. This is while second order cyclostationarity method is effected severely in low SNR environments. The second order cyclostionary based detection relies on the cyclic frequency content of the received signal in order to determine the vacancy of a band, hence it would suffer significant degradation in performance when it comes to frequency selective channels; on the other hand the proposed technique takes advantage of this diversity inhered in the channel. The other characteristic which is shown in Fig. 2 is the fast convergence rate which can be offered by the proposed technique, i.e. the performance is increased by 4 dB when the observation length is increased from 2 to 10 frames, while there is performance resulting from increasing the observation length form 10 to 20 frames is less than 1 dB. This feature makes the proposed technique suitable for machine-to-machine communication where the observation length should be kept as minimum as possible. Fig. 3 compares frequency-domain energy detection to the presented technique for observation length equal to two frames. The threshold setting for energy detection can be found in [5]. The simulation performed for energy detection are based on noise uncertainty factor $U = 0, 0.1, 3$ and the threshold is based on the assumed/estimated noise power, while the real noise power varies with each Monte Carlo realization by a certain degree. The proposed technique outperforms the frequency-domain energy detection in low SNR regions. As shown in Fig. 3 the proposed technique outperforms the frequency-domain by at least 10 dB when the noise uncertainty is set to 0 dB. Furthermore, as it can be observed the performance of the energy detection is considerably dependent on the noise uncertainty factor. This implies that energy detection is unreliable in practical scenarios specially in low SNR environments since it is not possible to obtain the exact noise power. This is due to the higher accuracy which order statistics of the differentiated ESD can offer by benefits from channel frequency diversity while energy detection is only based on the estimated energy of the received signal. The performance of the energy detection degrades determinately as it hits the SNR wall [13]. It is further proved in [13] that increasing the observation length does not affect the performance of the energy detection where the the exact noise power is not known, i.e. noise uncertainty factor $\neq 0$, while it is proved in previous section that the performance of the proposed technique is a function of observation length. It can be further observed from Fig. 3 that the performance of the energy detection degrades severely as it hits the SNR wall, while in the proposed technique the performance degrades in a more stable trend, which makes it more preferable for practical situations. The other main advantage regarding the proposed technique is that the improvement introduced is without the cost of increased computational complexity and hence making the suitable for machine-to-machine communications.

Parameters	Value
N_{FFT}	64
N_{cp}	16
Bandwidth	20MHz
Number of OFDM blocks per frame	10

TABLE I
Simulation Setup

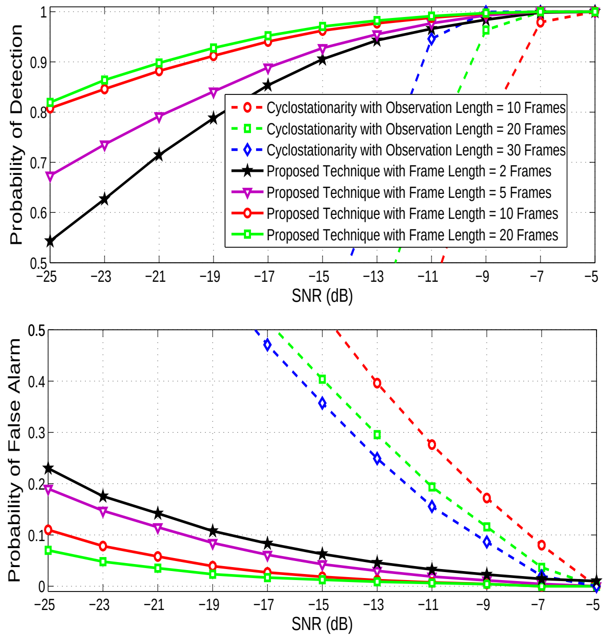

Fig. 2. Performance comparison of the second order cyclostationarity and the proposed technique

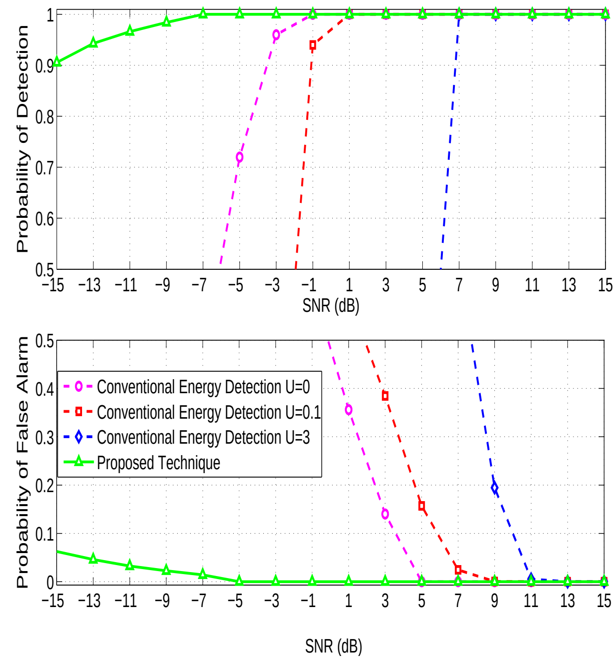

Fig. 3. Performance comparison of the conventional energy detection and the proposed technique. Observation length = 2 frames.

VI. Conclusion

In this paper a novel frequency-domain energy detection scheme based on extreme statistics for robust sensing of OFDM sources in the low SNR region has been presented. The basic idea is to exploit the frequency diversity gain inherited by frequency selective channels with the aid of extreme statistics of the differential energy spectral density (ESD). Thanks to the differential stage the proposed spectrum sensing is robust to noise uncertainty problem. The low computational complexity requirement and robustness in terms of time/frequency offsets of the proposed technique makes it suitable for even machine-to-machine sensing. Analytical performance analysis is performed in terms of two classical metrics, i.e. probability of detection and probability of false alarm. Moreover, computer simulations carried out showed that the proposed scheme outperforms both energy detection and second order cyclostationarity based approach for up to 10 dB gain in the low SNR range

VII. Acknowledgement

This work has been performed in the framework of the ICT project ICT-248894 WHERE 2 and ICT-258512 EXALTED, which are partly funded by the European Union.

References

[1] T. Robertazzi and P. Sarachik, "Self-organizing communication networks," *Communications Magazine, IEEE*, vol. 24, no. 1, pp. 28 – 33, Jan. 1986.

[2] J. MitolaIII and G. Q. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13 –18, Aug 1999.

[3] Federal Communications Commission, "Notice of proposed rule making and order: Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," Feb 2005, pp. ET Docket No. 03–108.

[4] H. S. Chen, W. Gao, and D. G. Daut, "Signature based spectrum sensing algorithms for ieee 802.22 wran," Jun 2007, pp. 6487 –6492.

[5] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523 – 531, Apr 1967.

[6] A. V. Dandawate and G. B Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Transactions on Signal Processing*, vol. 42, no. 9, pp. 2355 –2369, Sep. 1994.

[7] S. M. Cabric, D. Mishra and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," Nov 2004, vol. 1, pp. 772 – 776 Vol.1.

[8] Z. Tian and C. B. Giannakis, "A wavelet approach to wideband spectrum sensing for cognitive radios," *International Conference on CrownCom Proceedings, CrownCom*, Jun. 2006.

[9] A. Ghasemi and E. S. Sousa, "Spectrum sensing in cognitive radio networks: requirements, challenges and design trade-offs," *IEEE Communications Magazine*, vol. 46, no. 4, pp. 32 –39, Apr. 2008.

[10] R. Von Mises, *La distribution de la plus grande de n valeurs*, Rev. Math. Union Interbalkanique, 1936.

[11] H. A. David and H. N. Nagaraja, *Order statistics*, John Wiley and sons Hoboken. New Jersey, 2003.

[12] J. Neyman and E. Pearson, "On the problem of the most efficient tests for statistical hypotheses," *Philosophical Trans. of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, vol. 231, no. 4, pp. 289–337, Feb. 1933.

[13] R. Tandra and A. Sahai, "Snr walls for signal detection," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 4 –17, Feb. 2008.

[14] "Draft ieee standard for information technology telecommunications and information exchange between systems local and metropolitan area networks specific requirements part 11: Wireless medium access control (mac) and physical layer (phy) specifications: Amendment 6: Medium access control (mac) security enhancements (amendment to ieee std 802.11, 1999 edition as amended by ieee std 802.11g-2003 and ieee std 802.11h-2003)," 2004.