Towards a Theory of Societal Co-Evolution: Individualism versus Collectivism

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Abstract

Substantial empirical research has shown that the level of individualism vs. collectivism is one of the most critical and important determinants of societal traits, such as economic growth, economic institutions and health conditions. But the exact nature of this impact has thus far not been well understood in an analytical setting. In this work, we develop one of the first theoretical models that analytically studies the impact of individualism-collectivism on the society. We model the growth of an individual's welfare (wealth, resources and health) as depending not only on himself, but also on the level of collectivism, i.e. the level of dependence on the rest of the individuals in the society, which leads to a co-evolutionary setting. Based on our model, we are able to predict the impact of individualism-collectivism on various societal metrics, such as average welfare, average life-time, total population, cumulative welfare and average inequality. We analytically show that individualism has a positive impact on average welfare and cumulative welfare, but comes with the drawbacks of lower average life-time, lower total population and higher average inequality.

I. Introduction

Why are some societies wealthier or healthier than others? Why do some societies have substantial inequality among their members while others have relatively little? And why do certain societies have a large population while others have a small population? Culture, specifically the level of individualism vs. collectivism in the society, plays an important and even central role in answering the above questions [1] [2].

Landes [3] [4] and many others make the argument for the impact of culture on economic development. Furthermore, in [5] the authors argue that among the different dimensions of culture that affect long run growth, such as individualism-collectivism, masculinity, power distance etc., the single most relevant dimension is individualism-collectivism. Thus understanding the impact of the level of individualism vs. collectivism on a society is of incredible importance in building a model of societal development. In individualistic societies, people tend to depend more on themselves and less on society for growth in life, whereas in collectivistic societies, people tend to contribute to and depend on society to a greater extent. The level of collectivism in the society thus determines how much the growth of an individual is affected by the society, as well as how much the individual affects the development of the society, leading to a co-evolutionary setting. In this paper, collectivism represents a cultural element and not communism or a state (or religion) direction of activity.

There has been substantial research [2] [6] [7] [8] towards analyzing the determinants of societal development. A significant thrust of this research has been on developing theories based on empirical tests [2] [7] [9]. Empirical studies have established the positive impact of individualism on economic parameters, namely GDP per capita and GDP of a country [8]. But there is also more inequality in the societies with higher levels of development both in economic [10] [11] and health conditions [12]. Even though these empirical results exist, developing mathematical models to understand such social systems is very important, because these mathematical models help us predict societal phenomenon and provide useful insights which can otherwise not be obtained just based on empirical tests. However, there are relatively few papers that analytically study the impact of individualism vs. collectivism. In [6] the author develops a mathematical model to show that individualism-collectivism is important in determining the structure of economic institutions in the society. In [8] the authors come up with a mathematical model through which they can predict that the individualistic societies promote more long run economic growth than collectivistic societies.

In this work, we develop a mathematical model of the impact of individualism-collectivism on more general parameters of a society, as opposed to only on economic institutions as in the above papers. Our mathematical model helps us answer questions pertaining to the impact of individualism-collectivism on the socio-economic inequality in the society, the total population that can be sustained in the society and the average life-time of individuals, which cannot be answered with existing models. In our model, individuals are born into the society with a fixed level of intrinsic quality, which determines the rate of change of their welfare. Welfare in our model is an abstract quantity which represents an aggregate of the wealth, resources and health of an individual. An individual in our model will die either if its level of welfare drops too low or due to natural causes. Importantly, the level of collectivism determines the extent to which an individual's welfare is affected by rest of the society and vice-versa. Our objective is to compare societies with different levels of collectivism, levels of welfare required to survive while assuming the societies are identically impacted by other factors, such as economic institutions, government [13] [14] or geography, environment [15]. Our model is simplistic as we abstract away the impact of economic institutions, government, geography and environment however, it still allows us to capture the impact of individualism-collectivism, as well as other forces, such as the level of welfare required to survive on societal metrics, namely average welfare, average life-time, average inequality, and total population. From our model we can make the following predictions:

1. Although there is higher societal support given to individuals with low quality in a collectivistic society, this does not increase the average welfare of individuals in collectivistic societies since the support from the rest of individuals in society comes at the expense of their own welfare levels. This implies lower average welfare levels in a collectivistic society than in an individualistic society.

2. Despite lower average welfare levels, average life-time may be higher in a collectivistic society because the social support given to lower quality individuals will allow them to survive for a longer amount of time. This also means that collectivistic societies can sustain higher population levels.

3. Cumulative welfare, defined as the total wealth, resources and health of a society, is lower in a collectivistic society. Although a collectivistic society supports a larger total population than an individualistic society, this increase is dominated by the decrease in the average welfare.

4. The level of inequality in the society is higher in an individualistic society than in a collectivistic one, because individualistic societies allow agents to reach higher personal welfare while giving less social support to individuals

with low welfare levels.

5. In addition we also study the impact of rate of birth, rate of natural deaths and the minimum welfare level required to survive on the above societal metrics.

Our analytical results are in general agreement with the existing empirical evidence, and we also provide some new predictions that have so far not been tested empirically. We want to emphasize that the study here is very general and potentially has a broader scope. Individualism-collectivism is a trait not particular to humans, and in a broad sense it can capture the collectivistic versus individualistic behavior of different biological species, such as bacteria [16]. Being able to mathematically understand individualism and collectivism is not only useful for societal evolution, but can also be of significant interest in biology.

II. System Model

We consider an infinite horizon continuous-time model with a continuum of individuals living in a society. Each individual is characterized by his intrinsic quality, Q, which models his ability to develop in life, i.e. increase his wealth, resources and health. The intrinsic quality is a random variable which can take either a good or a bad value, i.e. $Q \in \{1, -1\}$, where the probability that $Q = 1$, $P(Q = 1) = \frac{1}{2}$. Due to space limitations, we only treat a simplistic model here, however our results can be extended for more general distributions of quality. We denote the individual's welfare, an abstract quantity representing aggregate wealth, resources and health of individual, at time t from birth as, $X(t)$ and the welfare at birth is zero, $X(0) = 0$. The rate at which the welfare of an individual increases at any time t from birth is determined by the individual's quality as well as the average quality of the rest of society, and is given by $R(t) \triangleq \frac{dX(t)}{dt} = (1-w).Q + w.\bar{\mathbf{Q}}(\mathbf{t})$, where $\bar{\mathbf{Q}}(\mathbf{t})$ is the average quality of all the individuals in the society at time t, and $w \in [0, 1]$ is the level of dependence on society. This weight w is same for all individuals in the society and is a measure of collectivism in the society, i.e. $w = 1$ and $w = 0$ correspond to a purely collectivistic and purely individualistic society respectively. This mutual dependence amongst the individuals leads to a co-evolutionary setting.

The individuals are born into the society at a rate of λ_b mass per unit time, which means that the total mass of individuals entering the society in Δt time is $\lambda_b \Delta t$. Individuals in the society can die either due to natural causes or due to poor welfare levels. The death due to natural causes is modelled as a Poisson arrival process with a rate λ_d starting at the time of birth of the individual, and at the first arrival instance the individual dies, see Fig. 1. The death due to poor welfare levels happens if the welfare levels fall below a threshold, $-r$ which we call the death boundary, see Fig. 1.

Steady State of the Society: As time increases the population increases up to a point where the rate of death equals the rate of birth. Thus the total population will converge to a fixed mass, and the distribution of welfare levels within the society will also converge to a constant. Thus in a steady state: a) the total population mass in the society attains a fixed value, denoted by $Pop(\lambda_b, \lambda_d, r, w)$, at which the rate of birth will equal the rate of death and b) the density of the population at a given welfare level x, $p_{\lambda_d,\lambda_d,r,w}(x)$, see Fig. 2, and the mass of the population with quality $Q = q$, $M(q)$, are also determined. We show below in Theorem 1 that there is always a unique steady state in our model given the exogenous parameters $\{\lambda_b, \lambda_d, r, w\}$, which characterize the society.

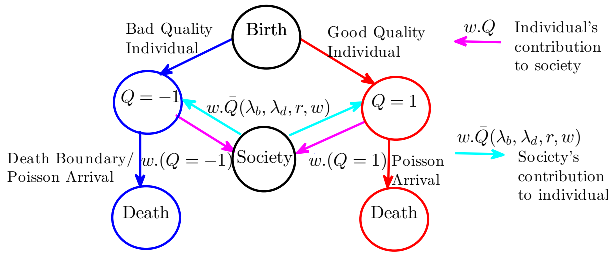

Figure 1. Life-time of good and bad quality individuals.

Theorem 1. Every society has a unique steady state.

The detailed proofs can be found in the appendix (Section V) given at the end.

Lemma 1. Good and bad quality individuals attain positive and negative welfare values respectively in the steady state.

Bad quality individuals can die either due to a Poisson arrival or due to poor welfare levels. As a result the proportion of the bad quality individuals is lower than that of good quality ones, which leads to a positive average quality $\bar{Q}(\lambda_b,\lambda_d,r,w)$. Hence, good quality individuals cannot take negative welfare values. Also, it can be shown that the bad quality individuals cannot take positive welfare values, see the appendix (Section V) at the end for details.

In the unique steady state the population density, $p_{\lambda_d,\lambda_d,r,w}(x)$ decays exponentially in both positive and negative directions, see Fig. 2. We illustrate the life-time of an individual with good (bad) quality, i.e. $Q=1\,(Q=-1)$ in steady state in Fig. 1. The positive (negative) welfare levels are attained by good (bad) quality individuals in the population. The rate at which the welfare of a bad quality individual decays in time is typically lesser than the rate of growth of good quality individuals, (due to the opposing effects of the negative quality and positive societal support for a bad quality individual), this leads to a higher decay in the population density of bad quality individuals as compared to good quality individuals, see Fig. 2. We focus on understanding the impact of the exogenous parameters on the properties of this steady state. To do so we denominate some important societal metrics which help understand the properties of the steady state.

Definition 1. Average quality: The average quality of individuals represents the net impact the society has on rate of growth of welfare of each individual and is defined as $\bar{Q}(\lambda_b,\lambda_d,r,w)=1\frac{M(Q=1)}{Pop(\lambda_b,\lambda_d,r,w)}-1.\frac{M(Q=-1)}{Pop(\lambda_b,\lambda_d,r,w)}$.

Definition 2. Average welfare: The average value of welfare of the population, a measure of the average wealth, resources and health of an individual in the society, is defined as $\bar{X}(\lambda_b,\lambda_d,r,w)=\int_{-r}^{\infty}x\frac{p_{\lambda_b,\lambda_d,r,w}(x)}{Pop(\lambda_b,\lambda_d,r,w)}dx$.

Let T denote the random variable corresponding to the life-time of an individual in steady state. Let R be the rate of growth of the individual in steady state where $R=(1-w).Q+w.\bar{Q}(\lambda_b,\lambda_d,r,w))$ and Q is the quality of the individual. If $R\geq 0$, then the individual's welfare will always be above zero, hence the individual will

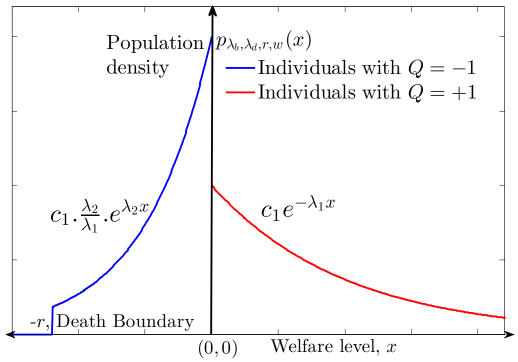

Figure 2. Steady State Distribution of population density as a function of welfare levels.

only die when there is a Poisson arrival. Therefore, T in this case will be an exponential random variable, $T^{'}$, with mean $\frac{1}{\lambda_d}$. If $R<0$ then the death will happen either at time $T_2(\lambda_b,\lambda_d,r,w)=\frac{r}{1-w(1+\bar{Q}(\lambda_b,\lambda_d,r,w))}$, where $T_2(\lambda_b,\lambda_d,r,w)$ is the time taken to reach the death boundary, or if there is a Poisson arrival before $T_2(\lambda_b,\lambda_d,r,w)$. Hence, $T=\min\{T^{'},T_2(\lambda_b,\lambda_d,r,w)\}$,

Definition 3. Average life-time: The average life-time of an individual is defined as the expected value of the life-time (unconditional on individual's quality), $\bar{T}(\lambda_b,\lambda_d,r,w)=E_{\lambda_b,\lambda_d,r,w}[T]$.

The next societal metric is a measure of average inequality in the welfare levels of individuals.

Definition 4. Average inequality: Average inequality, a measure of disparity in the society, is defined as the variance of welfare , $Var_X(\lambda_b,\lambda_d,r,w)=\int_{-r}^{\infty}(x-\bar{X}(\lambda_b,\lambda_d,r,w))^2\frac{p_{\lambda_b,\lambda_d,r,w}(x)}{Pop(\lambda_b,\lambda_d,r,w)}dx$.

Next, we come up with a notion of Cumulative welfare, which is the aggregate amount of welfare in the society, a measure of total wealth and resources.

Definition 5. Cumulative welfare: The cumulative welfare, a measure of total welfare of society accumulated together, is defined as $CF(\lambda_b,\lambda_d,r,w)=Pop(\lambda_b,\lambda_d,r,w)\bar{X}(\lambda_b,\lambda_d,r,w)$.

In the above societal metrics, average life-time, total population and average quality are more intuitive to understand, while average welfare is similar to GDP per capita [8], cumulative welfare is similar to the GDP [8] and average inequality is related to GINI coefficient [10] [11].

III. Results

In this section, we will compare different societal metrics across societies differing either in the level of collectivism, w or the other exogenous parameters.

Lemma 2. a) The average quality $\bar{Q}(\lambda_b,\lambda_d,r,w)$ and the average welfare $\bar{X}(\lambda_b,\lambda_d,r,w)$ of an individual decrease as the level of collectivism w increases. b) $\bar{Q}(\lambda_b,\lambda_d,r,w)$ and $\bar{X}(\lambda_b,\lambda_d,r,w)$ decrease as the rate of natural deaths λ_d increases. c) $\bar{Q}(\lambda_b,\lambda_d,r,w)$ and $\bar{X}(\lambda_b,\lambda_d,r,w)$ decrease as the death boundary$-r$ decreases.

In part a), as the level of collectivism is increased, the support from the society slows the rate at which the welfare of a bad quality individual decays with time, causing a larger proportion of the population to be of low quality. The good quality individuals contribute more to this support as well and as a result their own growth is slowed. As a result, there is a negative impact both on the average quality and average welfare of the individuals. Parts b) and c) are straightforward, see the appendix (Section V) at the end for detail. This lemma is supported by the empirical studies showing lower per capita income in collectivistic societies in comparison to individualistic societies [8].

Theorem 2. a) Total population $Pop(\lambda_b, \lambda_d, r, w)$ increases as the rate of birth λ_b increases. b) $Pop(\lambda_b, \lambda_d, r, w)$ increases as the level of collectivism w increases. c) $Pop(\lambda_b, \lambda_d, r, w)$ increases as the death boundary $-r$ decreases. d) If $w < \frac{1}{2}$ then $Pop(\lambda_b, \lambda_d, r, w)$ increases as the rate of natural deaths λ_d decreases.

Part a) and c) are easier to comprehend, see the appendix (Section V) at the end for details. For part b), as the level of collectivism increases the support from the society slows the rate at which the welfare of a bad quality individual decays with time. As a result, the proportion of individuals dying at the death boundary decreases, which means that the population level at which the mass of population dying equals the mass of population being born is higher. This agrees with the empirical studies which show collectivistic societies have less income per worker and have a larger population [17] [18]. In part d), as the rate at which natural deaths occur decreases, the rate of deaths due to achieving poor welfare levels through hitting the death boundary can increase. However, if the level of dependence on the society is low then the decrease in the rate of natural deaths dominates, and as a result the total population increases such that the mass of deaths equals mass of birth.

Theorem 3: a) Cumulative welfare $CF(\lambda_b, \lambda_d, r, w)$ decreases as the rate of birth λ_b decreases. b) $CF(\lambda_b, \lambda_d, r, w)$ decreases as the rate of natural deaths λ_d increases. c) If $\lambda_d r \leq \epsilon < \frac{1}{2}$ and $w < \frac{1}{2} - \epsilon$ with $\epsilon > 0$, then $CF(\lambda_b, \lambda_d, r, w)$ decreases as the death boundary $-r$ decreases. d) $CF(\lambda_b, \lambda_d, r, w)$ decreases as the level of collectivism w increases.

Since $CF(\lambda_b, \lambda_d, r, w) \propto Pop(\lambda_b, \lambda_d, r, w)$, part a) follows from Theorem 2. For part b), as the rate of natural death increases the average welfare of an individual decreases (Lemma 2) and the total population also decreases (Theorem 2), if the level of collectivism is not high. This shows the result for part b), when the collectivism is not high. However, it can be shown that even if the level of collectivism is high then as well there will be a decrease in cumulative welfare owing to a significant decrease in the average welfare (see the appendix (Section V)). For part c), as the death boundary decreases, the total population in the society increases whereas the average welfare of an individual decreases, leading to opposing effects. Therefore, if the $\lambda_d r$ is sufficiently low then the proportion of the population with bad quality is sufficiently low as well. Also, if the level of collectivism, w is low then then the rate at which the welfare of bad quality individuals decays with time is high, hence the effect of decreasing the death boundary on the average welfare is high. Under these conditions the decrease in average welfare dominates the increase in population. For part d), increasing the level of collectivism increases the total population (Theorem 2), but it decreases the average welfare of an individual (Lemma 2). Interestingly, it can be shown that the decrease in the average welfare of an individual dominates the increase in population (see the appendix (Section V) for technical detail). This result is also aligned with the empirical tests showing higher GDPs for an individualistic society [8].

Theorem 4. a) Average life time $\bar{T}(\lambda_b, \lambda_d, r, w)$ decreases with an increase in rate of natural deaths λ_d. b) If $\lambda_d r > \theta^* = \ln(1 + \frac{\sqrt{2}}{2})$ then $\bar{T}(\lambda_b, \lambda_d, r, w)$ increases with an increase in level of collectivism w else, it first decreases and then increases with an increase in level of collectivism w. c), If $\lambda_d r > \theta^*$, then $\bar{T}(\lambda_b, \lambda_d, r, w)$ increases with a decrease in death boundary $-r$ else, it first decreases and then increases with a decrease in death boundary $-r$.

For part a), an increase in the rate of deaths will affect the life-time of both good and bad quality individuals negatively, thus leading to the result. For part b), increasing the level of collectivism slows the rate at which the welfare of individuals with bad quality decays with time resulting in an increase in their life-time. It also leads to an increase in the proportion of individuals with bad quality, but note that individuals with good quality have a higher life-time than individuals with bad quality. This leads to an opposing effect. However, if $\lambda_d r$ is high i.e. $\lambda_d r > \theta^*$, then the proportion of the individuals with bad quality is high enough, implying that the increase in the life-time of individuals with bad quality has a dominating effect in comparison to the decrease resulting from a decreasing proportion of individuals with good quality. The proportion of the population of bad quality individuals increases with an increase in the level of collectivism. If $\lambda_d r \leq \theta^*$ and the level of collectivism is sufficiently high, there will be a sufficiently high proportion of bad quality individuals, and so if level of collectivism is increased then there will be an increase in the average life-time. However, if the level of collectivism is not high then there will be a decrease in the average life-time with an increase in the level of collectivism. A similar explanation applies to part c). In Fig. 3, it is shown that if $\lambda_d r$ is sufficiently high the average life-time increases with the level of collectivism, otherwise, the average life-time decreases and then increases. It is important at this point to note that in part b), we compare two societies with different levels of collectivism while other parameters remain the same which may include medical facilities, health awareness etc. that are also crucial determinants of life-time. Also, our model does not yet consider the impact of cumulative welfare on the rate of natural deaths λ_d and is an important direction for future research.

Theorem 5. The average inequality $Var_X(\lambda_b, \lambda_d, r, w)$ is always more in an individualistic society $w = 0$ as compared to a collectivistic society $w = 1$. Also if the person only dies a natural death, i.e. $r \to \infty$, then a) $\lim_{r\to\infty} Var_X(\lambda_b, \lambda_d, r, w)$ decreases with an increase in level of collectivism w and b) $\lim_{r\to\infty} Var_X(\lambda_b, \lambda_d, r, w)$ decreases with an increase in rate of natural deaths λ_d.

For part a), the case when an individual only dies a natural death there is a symmetry in the proportion of individuals with good and bad quality. Hence, the average quality of an individual is zero. Therefore, the rate of decay (growth) for an individual with bad (good) quality is $1 - w$. Hence, increasing w slows the rate of decay and growth, thereby allowing individuals to neither take too low or too high welfare values, which leads to a lower average inequality. Having higher levels of inequality in individualistic societies has also been observed in calculations of GINI coefficient for various countries [10], [11]. Also, having higher inequality has been an important factor affecting the health of the society [12], this observation supports our result on the negative impact of individualism on average life-time in Theorem 4. For part b), it is clear that a higher rate of death, λ_d implies

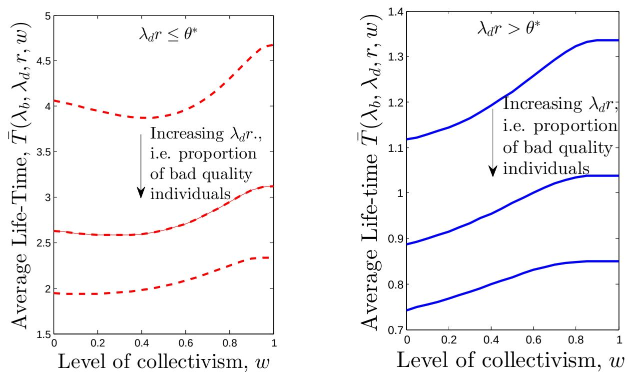

Figure 3. Illustration of part b) of Theorem 4.

that individuals with very high or low welfare levels are less likely to exist, thus leading to lesser inequality.

IV. Conclusion

We propose a mathematical model to study societal co-evolution under the forces of individualism and collectivism. This work serves as an important step towards understanding the exact nature of the impact of individualism-collectivism on various societal facets. Through our model we can show that the average welfare of individuals is higher in an individualistic society, however the average life-time is typically lower in comparison to a collectivistic society. A larger life-time in collectivistic society does allow for a larger population to be sustained, however the cumulative welfare is still lesser. Moreover, the average inequality is more in an individualistic society owing to the lack of social support. Our results show concordance with existing empirical tests.

V. Appendix

Theorem 1: Every society has a unique steady state.

Proof: We will start by deriving the population density at a given welfare level x, $p_{\lambda_b,\lambda_d,r,w}(x)$ and the total population mass $Pop(\lambda_d, \lambda_b, r, w)$ in the steady state and show that they are unique. To do so we first arrive at the expression for the normalized population density $f_{\lambda_b,\lambda_d,r,w}(x)$. The relation between $f_{\lambda_b,\lambda_d,r,w}(x)$, $p_{\lambda_b,\lambda_d,r,w}(x)$ and $Pop(\lambda_d, \lambda_b, r, w)$ is given as, $Pop(\lambda_d, \lambda_b, r, w) = \int_{-\infty}^{\infty} p_{\lambda_b,\lambda_d,r,w}(x)dx$, $f_{\lambda_b,\lambda_d,r,w}(x) = \frac{p_{\lambda_b,\lambda_d,r,w}(x)}{Pop(\lambda_d,\lambda_b,r,w)}$. In steady state the average impact of the society, i.e. $\bar{Q}(\lambda_b, \lambda_d, r, w)$ is determined since the proportion of individuals with $Q = q$, i.e. $M(Q = q)$ do not change. Hence, the rate at which the welfare of an individual grows can take only two values depending on his quality, $R_1 = (1-w).1 + w.\bar{Q}(\lambda_b, \lambda_d, r, w)$, $R_{-1} = (1-w).-1 + w.\bar{Q}(\lambda_b, \lambda_d, r, w)$, here R_1 and R_{-1} are the rate of growth of good and bad quality individual respectively. To derive the densities in steady state, we will first show that in the steady state R_1 and R_{-1} will be positive and negative respectively. Let's assume that R_1 and R_{-1} are both positive, i.e. all the individuals in the society experience a positive growth. In such a case the individuals can only die due to a Poisson arrival. Also, we know that an individual who is born is as likely to be good as he is to be bad. Hence, the population mass at which the rate of death will equal the rate of birth of good/bad quality individual is the same for both the types of individuals, i.e. $M(Q = +1) = M(Q = -1)$.

As a result, the average quality $\bar{Q}(\lambda_b, \lambda_d, r, w) = 0$. Substituting this back in the expressions for the rate we get, $R_1 = (1-w)$ and $R_{-1} = (1-w).-1$. Therefore, R_{-1} is negative this contradicts the supposition that the both the rates are positive. Next, let's assume that both R_1 and R_{-1} are negative. In this case the individuals can die either due to a Poisson arrival or due to hitting the death boundary. In such a case the welfare values attained will only be negative. Let $f^1_{\lambda_b,\lambda_d,r,w}(x)$ correspond to the joint density that the individual of good quality attains a welfare level of x. Similarly, we can define $f^{-1}_{\lambda_b,\lambda_d,r,w}(x)$ to be the joint density for a bad quality individual at a given welfare level of x. In steady state although the density of population in a given welfare level is fixed, however the individuals comprising the density at a given welfare level is not the same owing to change of welfare levels, births and deaths that happen continually. As a result, at any instant of time the mass of individuals that attain a given welfare level will equal the mass of indiduals that leave that welfare level either due to change in welfare or due to dying. Consider an infinitesimal interval h, the mass of the population with quality $Q = 1$ between $x - h$ and x at time t, where $x \leq 0$, is given as, $f^1_{\lambda_b,\lambda_d,r,w}(x).h$. Consider a time interval $t^{'}$ after which this mass of individuals, $f^1_{\lambda_b,\lambda_d,r,w}(x).h$ will either die or will attain a different welfare level between, $y - h$ and y, here $y = x+R_1.t^{'}$. The probability that an individual does not die a natural death in time interval $t^{'}$ is $e^{-\lambda_d t^{'}}$. Hence, the proportion of the mass of individuals who do not die a natural death and a result attain a welfare between $y-h$ and y is $e^{-\lambda t^{'}} f^1_{\lambda_b,\lambda_d,r,w}(x).h = f^1_{\lambda_b,\lambda_d,r,w}(y).h$. This can be expressed as $f^1_{\lambda_b,\lambda_d,r,w}(y) = e^{-\lambda_d \frac{y-x}{R_1}} f^1_{\lambda_b,\lambda_d,r,w}(x)$ and $f^1_{\lambda_b,\lambda_d,r,w}(y) = C_1.e^{-\lambda_d \frac{y}{R_1}}$ where $f^1_{\lambda_b,\lambda_d,r,w}(0) = C_1$. Similarly, for $y \leq 0$ we can get $f^{-1}_{\lambda_b,\lambda_d,r,w}(y) = C_{-1}.e^{\lambda_d \frac{y}{R_{-1}}}$ where $f^{-1}_{\lambda_b,\lambda_d,r,w}(0) = C_{-1}$. Note that both $f^1_{\lambda_b,\lambda_d,r,w}(x)$ and $f^{-1}_{\lambda_b,\lambda_d,r,w}(x)$ are zero for positive welfare values since both good and bad quality individuals are assumed to have a negative rate of growth. Also, the rate at which individuals of good quality and bad quality are born is the same given as $\frac{\lambda_b}{2}$. Hence, we can equate the mass of good (bad) quality individuals which enter the society in time δt, i.e. $\frac{\lambda_b}{2}\delta t$ to the mass of individuals between welfare level of 0 and δx_1 (0 and δx_2), i.e. $C_1 \delta x_1$ ($C_{-1}\delta x_{-1}$). This gives, $C_{-1}R_{-1} = C_1 R_1 = C$. Since the $f^1_{\lambda_b,\lambda_d,r,w}(x)$ and $f^{-1}_{\lambda_b,\lambda_d,r,w}(x)$ are joint density functions the integral of the sum of these joint densities should be 1.

$$
\begin{aligned}
\int_{-\infty}^{\infty} f^1_{\lambda_b,\lambda_d,r,w}(x)dx + \int_{-\infty}^{\infty} f^{-1}_{\lambda_b,\lambda_d,r,w}(x)dx &= 1 \\
\frac{C_1 R_1}{\lambda_d}(1 - e^{\frac{\lambda_d}{R_1}r}) + \frac{C_{-1}R_{-1}}{\lambda_d}(1 - e^{\frac{\lambda_d}{R_{-1}}r}) &= 1 \\
C = \frac{\lambda_d}{2 - e^{\frac{\lambda_d}{R_1}r} - e^{\frac{\lambda_d}{R_{-1}}r}}
\end{aligned}
$$

From this we can calculate the mass of the individuals with $Q = 1$ and $Q = -1$, i.e. $M(Q = +1) = \frac{C}{\lambda_d}(1 - e^{\frac{\lambda_d}{R_1}r})$ and $M(Q = -1) = \frac{C}{\lambda_d}(1 - e^{\frac{\lambda_d}{R_{-1}}r})$. Since $R_1 > R_{-1}$ we can see that $M(Q = +1) > M(Q = -1)$. This yields that the $\bar{Q}(\lambda_b, \lambda_d, r, w) > 0$ and thereby $R_1 > 0$. This contradicts the supposition that both the rates are negative. Also, since $R_1 > R_{-1}$ the only case left is R_1 is positive while R_{-1} is negative. In this case the good and bad quality individuals take positive and negative welfare values respectively. We can calculate the joint densities in the same manner as described above and thus the resulting density is $f^1_{\lambda_b,\lambda_d,r,w}(x) = C_1 e^{-\frac{\lambda_d}{R_1}x}$, $x > 0$ and $f^{-1}_{\lambda_b,\lambda_d,r,w}(x) = C_{-1} e^{\frac{\lambda_d}{R_{-1}}x}$, $x < 0$, with $C_1 R_1 = C_{-1}R_{-1}$. To solve for the constants we need to proceed in a similar manner as above:

$$\begin{aligned}
\int f^1_{\lambda_b,\lambda_d,r,w}(x)dx + \int f^{-1}_{\lambda_b,\lambda_d,r,w}(x)dx &= 1 \\
\frac{C_1 R_1}{\lambda_d} + \frac{C_{-1}R_{-1}}{\lambda_d}(1 - e^{-\frac{\lambda_d}{(1-w).1-wQ(\lambda_b,\lambda_d,r,w).}r}) &= 1 \\
C = \frac{\lambda_d}{2-e^{-\frac{\lambda}{(1-w).1-wQ(\lambda_b,\lambda_d,r,w).}r})}
\end{aligned}$$

For simplification of notation, we introduce auxiliary notation, $\lambda_1 = \frac{\lambda_d}{(1-w)+w.\bar{Q}(\lambda_b,\lambda_d,r,w)}$ and $\lambda_2 = \frac{\lambda_d}{(1-w)-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}$. Hence, the density functions are denoted as follows, $f^1_{\lambda_b,\lambda_d,r,w}(x) = \frac{\lambda_1}{2-e^{-\lambda_2 r}}e^{-\lambda_1 x}$, $x > 0$ and $f^{-1}_{\lambda_b,\lambda_d,r,w}(x) = \frac{\lambda_2}{2-e^{-\lambda_2 r}}e^{\lambda_2 x}$, $x < 0$. Also, we can deduce that the marginal density $f_{\lambda_b,\lambda_d,r,w}(x) = f^1_{\lambda_b,\lambda_d,r,w}(x)$, $x > 0$ and $f_{\lambda_b,\lambda_d,r,w}(x) = f^{-1}_{\lambda_b,\lambda_d,r,w}(x)$, $x < 0$. Using the density computed above we can calculate $M(Q = 1) = \frac{1}{2-e^{-\lambda_2 r}}$ and $M(Q = -1) = \frac{1-e^{-\lambda_2 r}}{2-e^{-\lambda_2 r}}$. Also, the average quality needs to be consistent with the average quality computed using the distributions derived above. This is formally stated as

$$\bar{Q}(\lambda_b, \lambda_d, r, w) = \frac{e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}}{2 - e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}} = \frac{e^{-\lambda_2 r}}{2 - e^{\lambda_2 r}} \tag{1}$$

Next, we compute the total population mass by equating rate of births to the rate of deaths. The rate of deaths is comprised of two terms, the first term is the rate of natural deaths occurring due to Poisson shocks and the next term is the rate of deaths due to hitting the death boundary, $f_{\lambda_b,\lambda_d,r,w}(-r).(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))$ corresponds to the density of the individuals hitting the death boundary per unit time. Hence, the rate of deaths is $\lambda_d.Pop(\lambda_b, \lambda_d, r, w) + f_{\lambda_b,\lambda_d,r,w}(-r).(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w)).Pop(\lambda_b, \lambda_d, r, w),$. Equating rate of births to rate of deaths we get the following.

$$\begin{aligned}
\lambda_b &= (\lambda_d + f_{\lambda_b,\lambda_d,r,w}(-r).(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))Pop(\lambda_b, \lambda_d, r, w) \\
\lambda_b &= (\lambda_d + \lambda_d.\frac{e^{-\lambda_2 r}}{2 - e^{-\lambda_2 r}}).Pop(\lambda_b, \lambda_d, r, w)
\end{aligned}$$

$$Pop(\lambda_b, \lambda_d, r, w) = \frac{\lambda_b}{\lambda_d(1 + \bar{Q}(\lambda_b, \lambda_d, r, w))} \tag{2}$$

Now that we have both the normalized density and the total population's expressions, we can arrive at the expression of the population density $p_{\lambda_b,\lambda_d,r,w}(x)$ which is just a product of the two, formally given as follows.

$$p_{\lambda_b,\lambda_d,r,w}(x) = \begin{cases} Pop(\lambda_b, \lambda_d, r, w).\frac{\lambda_1}{2-e^{-\lambda_2 r}}e^{-\lambda_1 x}, & \text{if } x > 0 \\ Pop(\lambda_b, \lambda_d, r, w).\frac{\lambda_2}{2-e^{-\lambda_2 r}}e^{\lambda_2 x}, & \text{if } x<0 \end{cases}$$

If we can show that there is a unique average quality, $\bar{Q}(\lambda_b, \lambda_d, r, w)$ satisfying (1) then both the total population mass (2) and the population density (3) are uniquely determined. We know that $Q \in \{-1, 1\}$ hence, $\bar{Q}(\lambda_b, \lambda_d, r, w) \in [-1, 1]$. To solve for $\bar{Q}(\lambda_b, \lambda_d, r, w)$, we need to solve $z = g(z)$, where $g(z) = \frac{e^{-\frac{\lambda_d r}{(1-w)-wz.}r}}{2-e^{-\frac{\lambda_d r}{1-w-w.z}r}}$ and $z \in [-1, 1]$. We will first show that there exists a solution in the set, $[-1, 1]$. Let $z_1 = -1$ and $z_2 = \min\{1, \frac{1-w}{w}\}$. If $w < \frac{1}{2}$ then,

$z_2 = 1$ else $z_2 = \frac{1-w}{w}$. $g(z_1) = \frac{e^{-\frac{\lambda_d r}{1-w}}}{2-e^{-\frac{\lambda_d r}{1-w}}}$ and $g(z_1) > z_1$. If $w < \frac{1}{2}$ then $g(z_2) = \frac{e^{-\frac{\lambda_d r}{1-2w}}}{2-e^{-\frac{\lambda_d r}{1-2w}}}$ which is less than or equal to $z_2 = 1$, i.e. $g(z_2) \leq z_2$. Based on this and since the function z and $g(z)$ are continuous in the range $[-1, \frac{1-w}{w})$, there has to be a point in the interval $[-1,1] \subset [-1, \frac{1-w}{w})$ where $g(z) = z$. Also, $g(z)$ is decreasing in the range $[-1, \frac{1-w}{w})$, this can be seen from the expression for $g^{'}(z) = -\frac{\lambda_d r w}{(1-w-wz)^2(2e^{\frac{\lambda_d r}{1-w-wz}}-1)^2} 2e^{\frac{\lambda_d r}{1-w-wz}}$ and z is strictly increasing function. Therefore, $g(z) - z$ is a strictly decreasing function in $[-1, \frac{1-w}{w})$, which implies that the root is unique. When $w = \frac{1}{2}$, $z_2 = 1$ we can see that $g(z_1) > z_1$ holds, but $g(z)$ is not continuous at z_2. This is not a problem as we know that the function is continuous everywhere from $[-1, z_2)$ and $\lim^{'}_{z \to z_2} g(z) = 0$, where $\lim^{'}_{z \to z_2} g(z)$ corresponds to the left hand limit, hence $\lim^{'}_{z \to z_2} g(z) < z_2$. Hence, the same argument as above can be applied. In the case when $w > \frac{1}{2}$ then we will show that there exists a unique solution for $g(z) = z$ in the range $[-1, 1]$. We know that $g(z_1) = \frac{e^{-\frac{\lambda_d r}{1-w}}}{2-e^{-\frac{\lambda_d r}{1-w}}}$, but since $w > \frac{1}{2}$ we need to be careful about the case when $w = 1$. For now we can assume that $\frac{1}{2} < w < 1$. Hence, we know that $g(z_1) > z_1$. Here $z_2 = \frac{1-w}{w}$ and $g(z)$ will not be continuous at z_2. But we can show that $\lim^{'}_{z \to z_2} g(z) = 0$, where $\lim^{'}_{z \to z_2} g(z)$ corresponds to the left hand limit, and $\lim^{'}_{z \to z_2} g(z) < z_2$. Hence, from the decreasing nature of $g(z) - z$ we know that there is a unique solution in the range $[-1, \frac{1-w}{w})$. Since $1 > w > \frac{1}{2}$ then $[-1, \frac{1-w}{w}) \subset [-1, 1]$ we need to show that there is no solution in the range $(\frac{1-w}{w}, 1]$. In the range $(\frac{1-w}{w}, 1]$ the function $g(z)$ is not necessarily continuous. There exists a discontinuity if $2e^{\frac{\lambda_d r}{1-w-wz}} - 1 = 0$ and $z \in (\frac{1-w}{w}, 1]$. Let's assume that there is a discontinuity. In that case, the function $g(z)$ will decrease values from -1 to $-\infty$, then to the right of the discontinuity at $2e^{\frac{\lambda_d r}{1-w-wz}} - 1 = 0$ the function decreases from ∞ to $\frac{1}{2e^{\frac{\lambda_d r}{1-2w}}-1}$. Since $w > \frac{1}{2}$ and $2e^{\frac{\lambda_d r}{1-w-wz}} - 1 = 0$ for some $z \in (\frac{1-w}{w}, 1]$ $1 > 2e^{\frac{\lambda_d r}{1-2w}} - 1 > 0$ we can say that $\frac{1}{2e^{\frac{\lambda_d r}{1-2w}}-1} > 1$. Hence, there is no point in the range in $[-1, 1]$ which intersects with this function. In the case, when there is no discontinuity it is straightforward to show that there is no solution of $g(z) = z$ as the function $g(z)$ will only take negative values less than -1. Also, when $w = 1$ the individuals welfare is fixed to zero all the time, hence there is a symmetry in the proportion of good and bad quality individuals, which leads to a unique solution $\bar{Q}(\lambda_b, \lambda_d, r, w) = 0$.

Lemma 1. Good and bad quality individuals attain positive and negative welfare values respectively.

Proof: The proof of theorem 1, already contains the proof for this lemma as we show that R_1 and R_{-1} attain positive and negative welfare values respectively.

Lemma 2. The average quality $\bar{Q}(\lambda_b, \lambda_d, r, w)$ and the average welfare $\bar{X}(\lambda_b, \lambda_d, r, w)$ of an individual a). Decrease as the level of collectivism, w is increased., b). Decrease as the rate of natural deaths, λ_d increases., c). Decrease as the the death boundary,$-r$ decreases.

Proof: We already know that the solution for $\bar{Q}(\lambda_b, \lambda_d, r, w)$ requires solving a transcendental equation (1), which means that we do not have a closed form analytical expression for it. It can be shown that the expression for $\bar{X}(\lambda_b, \lambda_d, r, w)$ expressed in terms of the $\bar{Q}(\lambda_b, \lambda_d, r, w)$ is $(r + \frac{1}{\lambda_d}).\bar{Q}(\lambda_b, \lambda_d, r, w)$. From Theorem 1, we know that for every set of parameters there does exist a solution $\bar{Q}(\lambda_b, \lambda_d, r, w)$. For part a), as the level of collectivism is increased let us assume that the average quality $\bar{Q}(\lambda_b, \lambda_d, r, w)$ increases. However, if there is an increase in both the collectivism and the average quality, the expression $g(\bar{Q}(\lambda_b, \lambda_d, r, w)) = \frac{e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}}{2-e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}}$ decreases which contradicts the increase in $\bar{Q}(\lambda_b, \lambda_d, r, w)$. Hence, $\bar{Q}(\lambda_b, \lambda_d, r, w)$ has to decrease with an increase in collectivism. And from the expression of $\bar{X}(\lambda_b, \lambda_d, r, w)$ expressed in terms of $\bar{Q}(\lambda_b, \lambda_d, r, w)$ it is straightforward that the average welfare also decreases with an increase in the level of collectivism. For part b), again as the rate of natural deaths increases assume that $\bar{Q}(\lambda_b, \lambda_d, r, w)$ increases. However the decrease in $\frac{e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}}{2-e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}}$ will contradict the assumption. With an increase in λ_d the first term in the expression of $\bar{X}(\lambda_b, \lambda_d, r, w)$ which inversely related to λ_d has to decrease, this combined with the decrease in $\bar{Q}(\lambda_b, \lambda_d, r, w)$ leads to a decrease in the average welfare. For part c), we arrive at the expression of the derivative of average quality $\bar{Q}(\lambda_b, \lambda_d, r, w)$ w.r.t. r, $-\frac{\lambda_d(d)(d+1)(1-w-wd)}{(1-w-wd)^2+\lambda_d rw(d+1)}$ which is negative. Hence, we know that the average quality indeed decreases with an increase in r. For average welfare we give an intuitive explanation first, increasing r decreases the average quality as a result of which the growth of a good quality individual slows down and the decay of a bad quality individual becomes faster. As a result the average welfare levels attained by a good and bad quality individual are lower. Moreover, increase in r increases the proportion of the bad quality individuals which further has a negative effect on the average welfare. To prove this formally we will show that the average welfare of both good and bad quality individuals decreases and the proportion of the bad quality individuals increases. Since the average welfare value of a bad quality individual is always lower than that of a good quality individual this is sufficient to show the result. The average welfare of good quality individuals is given as $\frac{1}{\lambda_1} = \frac{1-w+w\bar{Q}(\lambda_b,\lambda_d,r,w)}{\lambda_d}$. This can be derived as follows, the distribution of the welfare conditional on the fact that individuals are of good quality $f_{\lambda_b,\lambda_d,r,w}(x|Q=+1)$ can be shown to be an exponential distribution with parameter λ_1 exactly on the same lines as we derived the joint densities $f^1_{\lambda_b,\lambda_d,r,w}(x)$ in Theorem 1. Since $\bar{Q}(\lambda_b, \lambda_d, r, w)$ decreases as a function of r, the average welfare of a good quality individual also decreases as a function of r. Similarly we need to arrive at the distribution $f_{\lambda_b,\lambda_d,r,w}(x|Q=-1)$,which turns out to be $f_{\lambda_b,\lambda_d,r,w}(x|Q=-1) = \frac{\lambda_2}{1-e^{-\lambda_2 r}}e^{\lambda_2 x}$, $x<0$. The average welfare value of bad quality individual can be arrived at using this distribution and it turns out to be, $-\frac{1}{\lambda_2} + \frac{re^{-\lambda_2 r}}{1-e^{-\lambda_2 r}}$. As r is increased, $\bar{Q}(\lambda_b, \lambda_d, r, w)$ decreases and thus λ_2 decreases as well. The partial derivative of average welfare of bad quality individual $-\frac{1}{\lambda_2} + \frac{re^{-\lambda_2 r}}{1-e^{-\lambda_2 r}}$ w.r.t. r is given as $\frac{e^{\lambda_2 r}-\lambda_2 re^{\lambda_2 r}-1}{(e^{\lambda_2 r}-1)^2}$ and this expression turns out to be negative for $(\lambda_2, r) \in \mathbb{R}^2_+$. Also, it can be shown that the partial derivative of $-\frac{1}{\lambda_2} + \frac{re^{-\lambda_2 r}}{1-e^{-\lambda_2 r}}$ w.r.t λ_2 is given as $(\frac{1}{\lambda_2})^2 - \frac{r^2e^{\lambda_2 r}}{(e^{\lambda_2 r}-1)^2}$ and this expression turns out to be positive. Hence, from the sign of these partial derivatives we can easily see the result.

Theorem 2. a) Total population $Pop(\lambda_b, \lambda_d, r, w)$ increases as the rate of birth λ_b increases. b) $Pop(\lambda_b, \lambda_d, r, w)$ increases as the level of collectivism w increases. c) $Pop(\lambda_b, \lambda_d, r, w)$ increases as the death boundary$-r$ decreases. d) If $w < \frac{1}{2}$ then $Pop(\lambda_b, \lambda_d, r, w)$ increases as the rate of natural deaths λ_d decreases.

Proof: In order to compute the total population in the steady state, we need to have the rate of birth equals the rate of death which is formally stated as follows,

$$(\lambda_d + f_{\lambda_b,\lambda_d,r,w}(-r).(1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w))).Pop(\lambda_b,\lambda_d,r,w) = \lambda_b$$

$$Pop(\lambda_b, \lambda_d, r, w) = \frac{\lambda_b}{\lambda_d.(1 + \bar{Q}(\lambda_b, \lambda_d, r, w))}$$

For part a), $\bar{Q}(\lambda_b, \lambda_d, r, w)$ does not depend on the rate of births and it is clear that the result holds since the population is directly proportional to λ_b. For part b) as well it can be seen that the only term in the expression which depends on w is $\bar{Q}(\lambda_b, \lambda_d, r, w)$ which will decrease as w is increased (Lemma 2). Therefore, it is clear that the population has to increase with level of collectivism. For part c), again we can see that the only term in the expression which depends on the death boundary $-r$ is $\bar{Q}(\lambda_b, \lambda_d, r, w)$. We know that as the death boundary decreases $\bar{Q}(\lambda_b, \lambda_d, r, w)$ decreases as well (Lemma 2), thereby leading to an increase in the population. In part d), as the rate at which natural deaths occur decreases, the rate of deaths due to achieving poor welfare levels or hitting the death boundary can increase. However, if the level of dependence on the society is low then the decrease in the rate of natural deaths dominates, as a result the total population increases such that the mass of deaths equals mass of birth. We now show this formally. Let us take the derivative of the term in the denominator w.r.t λ_d,

$$(1 + \bar{Q}(\lambda_b, \lambda_d, r, w)) + \frac{d\bar{Q}(\lambda_b, \lambda_d, r, w)}{d\lambda_d}$$

$$(1+\bar{Q}(\lambda_b, \lambda_d, r, w))(\frac{(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))^2 + \lambda_d rw\bar{Q}(\lambda_b, \lambda_d, r, w) - \lambda_d r\bar{Q}(\lambda_b, \lambda_d, r, w)(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))}{(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))^2 + \lambda_d rw\bar{Q}(\lambda_b, \lambda_d, r, w)}$$

$$(1+\bar{Q}(\lambda_b, \lambda_d, r, w))(\frac{(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w) - \lambda_d r\bar{Q}(\lambda_b, \lambda_d, r, w)) + \lambda_d rw\bar{Q}(\lambda_b, \lambda_d, r, w)}{(1 - w - w.\bar{Q}(\lambda_b, \lambda_d, r, w))^2 + \lambda_d rw\bar{Q}(\lambda_b, \lambda_d, r, w)})$$

If we can show that $(1-w-w.\bar{Q}(\lambda_b, \lambda_d, r, w)-\lambda_d r\bar{Q}(\lambda_b, \lambda_d, r, w)) > 0$ then the above expression will be positive. We know from lemma 2 that $\bar{Q}(\lambda_b, \lambda_d, r, 0) \geq \bar{Q}(\lambda_b, \lambda_d, r, w)$, $\forall w \in [0, 1]$. This leads to $\bar{Q}(\lambda_b, \lambda_d, r, 0) < \frac{1-w}{w+\lambda_d r}$ which is a sufficient for the above derivative to be positive. It can be checked that this condition is satisfied if $w < \frac{1}{2}$.

Theorem 3: a) Cumulative welfare $CF(\lambda_b, \lambda_d, r, w)$ decreases as the rate of birth λ_b decreases. b) $CF(\lambda_b, \lambda_d, r, w)$ decreases as the rate of natural deaths λ_d increases. c) If $\lambda_d r \leq \epsilon < \frac{1}{2}$ & $w < \frac{1}{2} - \epsilon$ with $\epsilon > 0$, then $CF(\lambda_b, \lambda_d, r, w)$ decreases as the death boundary $-r$ decreases. d) $CF(\lambda_b, \lambda_d, r, w)$ decreases as the level of collectivism w increases.

Proof: For part a), we know that $CF(\lambda_b, \lambda_d, r, w) = \bar{X}(\lambda_b, \lambda_d, r, w)Pop(\lambda_b, \lambda_d, r, w)$. Also, since the average welfare of an individual is independent of λ_b we only need to consider the effect on total population which we already know from Theorem 2. For part b), let us simplify the expression of cumulative welfare, $CF(\lambda_b, \lambda_d, r, w) = (r + \frac{1}{\lambda_d}).\frac{\lambda_b}{\lambda_d}.\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)}{(1+\bar{Q}(\lambda_b,\lambda_d,r,w))}$. From this expression we can see that as λ_d increases the term $(r + \frac{1}{\lambda_d}).\frac{\lambda_b}{\lambda_d}$ will definitely decrease. In fact the other term will also decrease, as can be seen from the derivative of the second term w.r.t. λ_d , $\frac{1}{(1+\bar{Q}(\lambda_b,\lambda_d,r,w))^2}\frac{d\bar{Q}(\lambda_b,\lambda_d,r,w)}{d\lambda_d}$ and this combined with Lemma 2. For part d), we can see that only $\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)}{(1+\bar{Q}(\lambda_b,\lambda_d,r,w))}$ depends on the weight w and its derivative w.r.t. w is $\frac{1}{(1+\bar{Q}(\lambda_b,\lambda_d,r,w))^2}\frac{d\bar{Q}(\lambda_b,\lambda_d,r,w)}{dw}$. This expression of the derivative and Lemma 2, lead us to the result. For part c), as the death boundary decreases, the total population in the society increases whereas the average welfare of an individual decreases, leading to opposing effects. Therefore, if the $\lambda_d r$ is sufficiently low then the proportion of the population with bad quality is sufficiently low as well. Also, if the level of collectivism, w is low then then the rate at which the welfare of bad quality individuals decays with time is high, hence the effect of decreasing the death boundary on the average welfare is high. Under these conditions the decrease in average welfare dominates the increase in population. We next show this formally. The derivative of cumulative welfare w.r.t. r is given as,

$$\Big(\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)}{\bar{Q}(\lambda_b,\lambda_d,r,w)+1}\Big).\Big(\frac{(1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)^2-(\lambda_d r+1)(1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)))}{(1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)^2+\lambda_d rwd(d+1)}\Big)$$

$$\Big(\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)}{\bar{Q}(\lambda_b,\lambda_d,r,w)+1}\Big).\Big(\frac{(w.(\bar{Q}(\lambda_b,\lambda_d,r,w)).(-(1-w.(1+\bar{Q}(\lambda_b,\lambda_d,r,w))+\lambda_d r+\lambda_d r\bar{Q}(\lambda_b,\lambda_d,r,w))-\lambda_d r}{(1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)^2+\lambda_d rwd(d+1)}\Big)$$

If $-(1-w.(1+\bar{Q}(\lambda_b,\lambda_d,r,w))+\lambda_d r+\lambda_d r\bar{Q}(\lambda_b,\lambda_d,r,w)<0$ then the above derivative is negative. Note $\bar{Q}(\lambda_b,\lambda_d,r,0)<\frac{1-w-\lambda_d r}{w+\lambda_d r}$ is sufficient for this condition to hold and it leads to the following condition, $w<\frac{1}{2}-\epsilon$ and $\lambda_d r\leq\epsilon<\frac{1}{2}$ where $\epsilon>0$. This proves part c.

Theorem 4. a) Average life time $\bar{T}(\lambda_b,\lambda_d,r,w)$ decreases with an increase in rate of natural deaths λ_d. b) If $\lambda_d r>\theta^*=\ln(1+\frac{\sqrt{2}}{2})$[1], then $\bar{T}(\lambda_b,\lambda_d,r,w)$ increases with an increase in level of collectivism w else, it first decreases and then increases with an increase in level of collectivism w. c), If $\lambda_d r>\theta^*$, then $\bar{T}(\lambda_b,\lambda_d,r,w)$ increases with a decrease in death boundary $-r$ else, it first decreases and then increases with a decrease in death boundary $-r$.

Proof: The expression for the average life-time of an individual $\bar{T}(\lambda_b,\lambda_d,r,w)$ involves the computation of the average life-time of good quality individuals and bad quality individuals separately and then combining the two using the conditional probabilities. Hence, $\bar{T}(\lambda_b,\lambda_d,r,w)=\frac{1}{\lambda_d}+(\frac{1}{\lambda_d})\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)-\bar{Q}(\lambda_b,\lambda_d,r,w)^2}{\bar{Q}(\lambda_b,\lambda_d,r,w)+1}$. The derivative of $\bar{T}(\lambda_b,\lambda_d,r,w)$ w.r.t w can be expressed as $\frac{1}{\lambda_d}\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)^2+2\bar{Q}(\lambda_b,\lambda_d,r,w)-1}{(\bar{Q}(\lambda_b,\lambda_d,r,w)+1)^2}\frac{d\bar{Q}(\lambda_b,\lambda_d,r,w)}{dw}$. If $\bar{Q}(\lambda_b,\lambda_d,r,0)<\sqrt{2}-1$ then the above derivative is positive. This leads to the condition $\lambda_d r>\ln(1+\frac{\sqrt{2}}{2})$. However, if $\lambda_d r<\ln(1+\frac{\sqrt{2}}{2})$ then $\bar{Q}(\lambda_b,\lambda_d,r,0)>\sqrt{2}-1$ and as a result the derivative is negative. However, $\bar{Q}(\lambda_b,\lambda_d,r,w)$ will decrease with increase in w and it can be observed that at $w=1$, $\bar{Q}(\lambda_b,\lambda_d,r,w)$ will be zero, this is due to the fact that the individuals completely depend on the society and the rate of growth is zero for all individuals. Hence, for some $w=w^*$ the $\bar{Q}(\lambda_b,\lambda_d,r,w^*)=\sqrt{2}-1$ where the life-time will take the minimum value. Therefore, we know that in the region $w>w^*$, the life-time will increase. This explains part b). For part c), a similar explanation can be given. The expression for the derivative changes to $\frac{1}{\lambda_d}\frac{\bar{Q}(\lambda_b,\lambda_d,r,w)^2+2\bar{Q}(\lambda_b,\lambda_d,r,w)-1}{(\bar{Q}(\lambda_b,\lambda_d,r,w)+1)^2}\frac{d\bar{Q}(\lambda_b,\lambda_d,r,w)}{dr}$ and the rest of the explanation follows from above and Lemma 2. For part a), we will first show that the average life-time of both a good and bad quality individual decrease. Then, we will show that the proportion of the bad quality individuals increase. Since the average life-time of a bad quality individual is always lesser than that of a good quality individual, this will lead to a

[1] θ^* is a fixed constant which in general will depend on $P(Q=1)$, and when $P(Q=1)=\frac{1}{2}$ it is $\ln(1+\frac{\sqrt{2}}{2})$.

decrease in the average life-time unconditional on the quality of the individual. First of all the average life-time of a good quality individual is $\frac{1}{\lambda_d}$ and it decreases with λ_d. Next, the average life-time of an individual with bad quality is arrived at by computing the expectation of $\min\{T^{'}, T_2(\lambda_b, \lambda_d, r, w) = \frac{r}{1-w(1+\bar{Q}(\lambda_b,\lambda_d,r,w))}, \}$ where $T^{'}$ is an exponential random variable with mean $\frac{1}{\lambda_d}$. The life-time of a bad quality individual is $\frac{1}{\lambda_d}.(1-e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}})$, the derivative of this expression is $-\frac{1}{\lambda_d^2}.(1-e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}} - \frac{\lambda_d r}{1-w-w\bar{Q}(\lambda_b,\lambda_d,r,w)}e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}}) + \frac{r.w}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}\frac{d\bar{Q}(\lambda_b,\lambda_d,r,w)}{d\lambda_d}$. The term $(1-e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}} - \frac{\lambda_d r}{1-w-w\bar{Q}(\lambda_b,\lambda_d,r,w)}e^{-\frac{\lambda_d r}{1-w-w.\bar{Q}(\lambda_b,\lambda_d,r,w)}})$ has to be positive since $(x+1)e^{-x} < 1$. Hence, we can see that the derivative is negative which implies the result.

Theorem 5. The average inequality $Var_X(\lambda_b, \lambda_d, r, w)$ is always more in an individualistic society $w = 0$ as compared to a collectivistic society $w = 1$. Also if the person only dies a natural death, i.e. $r \to \infty$, then a) $\lim_{r\to\infty} Var_X(\lambda_b, \lambda_d, r, w)$ decreases with an increase in level of collectivism w and b) $\lim_{r\to\infty} Var_X(\lambda_b, \lambda_d, r, w)$ decreases with an increase in rate of natural deaths λ_d.

Proof: $Var_X(\lambda_b, \lambda_d, r, w = 1) = 0$ since all the individuals have the same welfare value of zero. So, we need to show that $Var_X(\lambda_b, \lambda_d, r, w = 0) > 0$. The expression for variance is,

$$Var_X(\lambda_b, \lambda_d, r, w = 0) = (\frac{1}{\lambda_d})^2 \frac{(8e^{2\lambda_d r} + e^{\lambda_d r}(-2(\lambda_d r)^2 + 4\lambda_d r - 8) - 3\lambda_d r + (\lambda_d r)^2 + 1)}{(2e^{\lambda_d r} - 1)^2}$$

It can be shown that the expression in the numerator of the above expression is indeed positive. To do so we show that at any point $(\lambda_d, r) \in \mathbb{R}^2_+$the partial derivative w.r.t to either λ_d or r is positive and also that $Var_X(\lambda_b, \lambda_d = 0, r = 0, w = 0) > 0$ which helps us establish the result.

For part a), the case when an individual only dies a natural death there is a symmetry in the proportion of individuals with good and bad quality. Hence, the average quality of an individual is zero. Therefore, the rate of decay for an individual with bad quality is $1 - w$ and the same is the rate of growth for an individual with good quality. Hence, increasing w slows the rate of decay and growth, thereby allowing individuals to neither take too low or too high welfare values, which leads to a lower average disparity. Formally if $r \to \infty$, $\bar{Q}(\lambda_b, \lambda_d, r, w) = 0$, this leads to the density distribution given as, $f^1_{\lambda_b,\lambda_d,r,w}(x) = \frac{\lambda_d}{1-w}e^{-\frac{\lambda_d}{1-w}x}$ and $f^{-1}_{\lambda_b,\lambda_d,r,w}(x) = \frac{\lambda_d}{1-w}e^{\frac{\lambda_d}{1-w}x}$. This leads to the expression of the variance given as, $(\frac{1-w}{\lambda_d})^2$ and therefore, part a) and b) follow directly from this.

References

[1] G. Hofstede, "Culture and organizations," *International Studies of Management & Organization*, pp. 15–41, 1980.

[2] G. E. Lenski, *Ecological-evolutionary theory: Principles and applications*. Paradigm Publishers Boulder, CO, 2005.

[3] D. S. Landes, "The wealth and poverty of nations," *WORLD AND I*, vol. 13, pp. 258–263, 1998.

[4] D. Landes, "Culture makes almost all the difference," *Culture matters: how values shape human progress*, pp. 2–13, 2000.

[5] Y. Gorodnichenko and G. Roland, "Which dimensions of culture matter for long-run growth?" *The American Economic Review*, vol. 101, no. 3, pp. 492–498, 2011.

[6] A. Greif, "Cultural beliefs and the organization of society: A historical and theoretical reflection on collectivist and individualist societies," *Journal of political economy*, pp. 912–950, 1994.

[7] H. C. Triandis, K. Leung, M. J. Villareal, and F. I. Clack, "Allocentric versus idiocentric tendencies: Convergent and discriminant validation," *Journal of Research in personality*, vol. 19, no. 4, pp. 395–415, 1985.

[8] Y. Gorodnichenko and G. Roland, "Culture, institutions and the wealth of nations," National Bureau of Economic Research, Tech. Rep., 2010.

[9] T. M. Singelis, H. C. Triandis, D. P. Bhawuk, and M. J. Gelfand, "Horizontal and vertical dimensions of individualism and collectivism: A theoretical and measurement refinement," *Cross-cultural research*, vol. 29, no. 3, pp. 240–275, 1995.

[10] W. B. Group, *World Development Indicators 2012*. World Bank Publications, 2012.

[11] W. Kopczuk, E. Saez, and J. Song, "Earnings inequality and mobility in the united states: evidence from social security data since 1937," *The Quarterly Journal of Economics*, vol. 125, no. 1, pp. 91–128, 2010.

[12] R. G. Wilkinson, "Socioeconomic determinants of health. health inequalities: relative or absolute material standards?" *BMJ: British Medical Journal*, vol. 314, no. 7080, p. 591, 1997.

[13] D. Acemoglu, S. Johnson, and J. A. Robinson, "Reversal of fortune: Geography and institutions in the making of the modern world income distribution," *Quarterly journal of economics*, pp. 1231–1294, 2002.

[14] D. Acemoglu, J. A. Robinson, and D. Woren, *Why nations fail: the origins of power, prosperity, and poverty*. SciELO Chile, 2012, vol. 4.

[15] J. M. Diamond and D. Ordunio, *Guns, germs, and steel*. National Geographic, 2005.

[16] S. P. Brown and R. A. Johnstone, "Cooperation in the dark: signalling and collective action in quorum-sensing bacteria," *Proceedings of the Royal Society of London. Series B: Biological Sciences*, vol. 268, no. 1470, pp. 961–965, 2001.

[17] H. C. Triandis, *Individualism & collectivism.* Westview Press, 1995.

[18] A. Razin and E. Sadka, *Population economics*. MIT Press, 1995.