SPECTRAL CLUSTERING FOR BEAM-FREE SATELLITE COMMUNICATIONS

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ABSTRACT

This paper introduces the notion of beam-free satellite systems and it investigates different scheduling algorithms for this architecture. Attending to the current satellite gateway cloudification, this paper assumes that users from different beams can be scheduled over the same frame. Indeed, considering full frequency reuse among beams and on ground precoding, we show that whenever the scheduler is able to freely group users independently of their beam location, large attainable rates are obtained. In addition, we also consider that the gateway is able to select a number of simultaneous transmissions which leads to a substantial sum-rate increase. A scheduling scheme based on spectral clustering is proposed and it shows a higher performance compared to other state-of-the-art alternatives. In addition, our method is able to deal with different user terminal traffic classes. Based on the numerical results obtained considering a close-to-real multibeam satellite pattern, we point out that the current per-beam scheduling process is an inefficient network management for multibeam satellite systems using precoding.

***Index Terms*—** Multibeam satellite systems, precoding, spectral clustering, user scheduling.

1. INTRODUCTION

Current satellite network managers split the system resources in beams. These beams correspond to geographical areas of hundreds of kilometers created by one or multiple on board antennas. In this context, user terminals (UTs) are generally equipped with a global positioning system so that each UTs requests connectivity to a unique beam. Consequently, the gateway scheduling process is done on a per-beam basis (i.e. there are as many scheduler entities as beams).

The mentioned architecture is motivated by the disjoint frequency reuse among beams. That is, adjacent beams are served in different frequency bins in order to reduce the inter-beam interference. Therefore, there is no need of inter-beam scheduling process as the inter-beam signals have a very low power. However, in case full frequency reuse and precoding is employed, the mentioned per-beam network management shall be revisited.

In particular, it is known that given a set of on-board antennas N, the optimal number of simultaneous transmissions, B, might be lower than the *a priori* conceived number of beams by the satellite operator. Indeed, if a large population of UTs is located at the beam overlapping area, it might be more convenient to transform the two beams transmission to a single beam with a unique scheduling process.

This paper proposes the idea of beam-free satellite systems where we consider that there is no *a priori* relation between each UT and the payload architecture. Concretely, we envisage that over a certain refreshing time period, the number of simultaneous transmissions is updated. In this context, the GW scheduler shall be able to group UTs into a certain number of groups and embed its transmit data over the same frame, independently of their geographical location.

Scheduling in multibeam satellite system employing precoding has been investigated in the recent years [1–8]. Most of all satellite standards embed more than one UT information in the same codeword in order to obtain high channel coding gains. Due to that, the scheduling process consists of grouping users from the same beam to be served over the same frame. Attending the UT channel vector, the work in [1] proposes to opportunistically group users based on their fed back signal-to-interference-plus-noise ratio (SINR). The work in [2] clusters UTs which are geographically close. In [3] and [7] it is proposed to schedule UT having similar channel vectors over the same frame and schedule UTs in adjacent beams considering orthogonal channel vectors. A similar approach is done in [8] but only considering the geographical location of the UTs.

In [4, 5, 9] the authors consider user scheduling of UTs presenting the minimum Euclidean distance of their channel. This notion is extended in [6] by considering the k-means algorithm [10] and alternative similarity UT channel vector metrics.

In all the mentioned works, the GW is assumed to have a *fixed* number of simultaneous transmissions B. This is, UTs are geographically divided in beams so that groups can only be formed by UTs from the same beam. On the contrary, in this paper we consider the case where the GW is able to schedule users over the same frame from different beams and optimize the number of simultaneous transmissions.

In satellite communications one satellite covers thousands of UTs spread in a wide coverage area. Each UT is described by a set of different features; namely, channel vector, location, receiver sensitivity, traffic class, quality-of-service, user experience,... All of them can be encompassed in a user feature vector, which as any nonuniform data vector, contains a underlying structure due to the heterogeneity of the data. Graphs are used in this paper to represent these UT data, and its spectral domain results very useful to encode its structure and effectively cluster the UTs.

We propose a method based on spectral clustering [11,12] which is able to provide an adequate structure to perform the UTs scheduling over the multibeam coverage area. The numerical evaluations show that it is more convenient to simultaneously transmit a reduced number of beams rather than use all available payload radiofrequency beams. An additional case where the UTs have different traffic demands is also analysed and it is shown that the proposed method behaves well even for this scenario.

Spectral clustering is not only very simple to implement, but it is also very powerful in graph clustering in that it is guaranteed to reach global extreme points in principle. Indeed, this paper is not only the first that states the problem of user scheduling in a beam-free system, but also shows the potential of spectral clustering for effectively solving the non-convex user scheduling problem. We note that the right construction of the graph is key for a successful user selection in the multicast problem at hand.

The rest of the paper is organized as follows. Section II presents the system model of multibeam satellite systesm with precoding. Section III introduces the scheduling problem so as the proposed technique. Section IV presents the numerical results. Section V concludes.

Notation: Throughout this paper, the following notations are adopted. Boldface upper-case letters denote matrices and boldface lower-case letters refer to column vectors. $(.)^H$, $(.)^T$, $(.)^*$ denote a Hermitian transpose, transpose and conjugate matrices, respectively. $\mathbf{I}_N$ builds $N \times N$ identity matrix and $\mathbf{0}_{K\times N}$ refers to an all-zero matrix of size $K \times N$. If $\mathbf{X}$ is a $N \times N$ matrix. $[\mathbf{X}]_{ij}$ represents the (i-th, j-th) element of matrix $\mathbf{X}$. $\otimes$, $\circ$ and $||.||$ refer to the Kronecker product, the Hadamard product and the Frobenius norm, respectively. Vector $\mathbf{1}_N$ is a column vector with dimension N whose entries are equal to 1. vec $(\cdot)$ denotes the vectorization operator.

2. SYSTEM MODEL

We consider a forward link transmission of a single geostationary satellite system, consisting of one satellite payload equipped with an array fed reflector with N feed elements and K satellite UT. The UT set is denoted by $\mathcal{K} = \{1, 2, \ldots, K\}$. Adhering to the commercial satellite system scenario, we focus on the case where $K >> N$.

The forward link channels between the GW and the UTs are described by the channel matrix

$$\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K)^T \in \mathbb{C}^{K\times N}, \quad (1)$$

where $\mathbf{h}_k \in \mathbb{C}^{N\times 1}$ denotes the channel between the GW and the k-th UT. We adopt the line-of-sight channel model which is given by

$$[\mathbf{H}]_{k,n} = \frac{G_R a_{kn} e^{j\psi_{k,n}}}{4\pi \frac{d_k}{\lambda}\sqrt{K_B T_R B_W}} \;\; k = 1, \ldots, K; n = 1, \ldots, N. \quad (2)$$

d_k is the distance between the k-th UT and the satellite. λ is the carrier wavelength, K_B is the Boltzmann constant, B_W is the carrier bandwidth, G_R^2 is the UT receive antenna gain, and T_R is the receiver noise temperature. The term a_{kn} refers to the gain from the n-th feed to the k-th user. The time varying phase due to beam radiation pattern and the radio wave propagation is represented by $\psi_{k,n}$.

The phase value, $\psi_{k,n}$, presents different contributions. In particular,

$$\psi_{k,n} = \theta_{\text{RF},k} + \theta_{\text{LNB},k} + \theta_{\text{PL},n}, \quad (3)$$

where $\theta_{\text{RF},k} = \frac{2\pi}{\lambda} d_k$ is the phase rotation due to the radiofrequency signal propagation which depends on the UT distance to the satellite, $\theta_{\text{LNB},k}$ is the phase contribution of the receiver low noise block downconverters assumed to be Gaussian with zero mean and standard deviation of 0.24 degrees and $\theta_{\text{PL},n}$ which are the payload oscillator phase offsets which are assumed to be Gaussian with zero mean and standard deviation that is usually around 2 degrees.

We assume the availability of perfect channel state information at the GW. The GW performs user scheduling and serves the K users in B groups. The B simultaneous data frames can contain information from more than UT, leading to a multigroup multicast transmission. We denote as $\mathcal{B}_i$ for $i = 1, \ldots, B$ the set of groups. We assume that $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1,\ldots,B} \mathcal{B}_i = \mathcal{K}$.

The B data streams are precoded using the minimum mean square error multicast (MMSE-M) technique [9]. While having significantly lower computational complexity than other approaches [3], the sum-rate performance of the MMSE-M is generally good for diverse multibeam satellite systems. We adopt per-feed power allocation to ensure that none of the satellite high power amplifiers reach the saturation. Then, the precoded signal is given by

$$\mathbf{x} = \mathbf{W}\mathbf{s} = \gamma \left(\mathbf{G}^H\mathbf{G} + \frac{N}{P}\mathbf{I}_N \right)^{-1} \mathbf{G}^H\mathbf{s}, \quad (4)$$

where matrix $\mathbf{W} = (\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_B) \in \mathbb{C}^{N\times B}$ is the precoding matrix and $\mathbf{s} \in \mathbb{C}^{B\times 1}$ are the data symbols transmitted to the B UTs groups. The data symbols are assumed to have unit power, i.e., $\mathbb{E}\left(\mathbf{s}\mathbf{s}^H\right) = \mathbf{I}_B$. Matrix $\mathbf{G} = (\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_B)^T \in \mathbb{C}^{B\times N}$ is constructed by

$$\mathbf{g}_i = \frac{1}{|\mathcal{B}_i|} \sum_{k\in\mathcal{B}_i} \mathbf{h}_k. \quad (5)$$

Note that $\mathbf{g}_i$ is the vector that minimizes the average Euclidean distance of all UT channel vectors belonging to the same group:

$$\sum_{k\in\mathcal{B}_i} \|\mathbf{g}_i - \mathbf{h}_k\|^2. \quad (6)$$

In other words, $\mathbf{g}_i$ is the vector that represents the group $\mathcal{B}_i$ from the minimum Euclidean distance perspective.

The scalar γ is set such that the transmit power at each feed power amplifier is bellow P:

$$\gamma^2 = P / \max_n \left[\mathbf{W}\mathbf{W}^H\right]_{n,n}. \quad (7)$$

The K served UTs receive signal is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (8)$$

where $\mathbf{n} \in \mathbb{C}^{K\times 1}$ is the additive white Gaussian noise with zero mean and unit variance. Hence, the signal-to-interference-plus noise ratio (SINR) experience at each UT is

$$\text{SINR}_k = \frac{|\mathbf{h}_k^H\mathbf{w}_k|^2}{\sum_{j\neq k}^K |\mathbf{h}_k^H\mathbf{w}_j|^2 + 1}. \quad (9)$$

3. USER SCHEDULING

In general, typical satellite transmissions require that data from more than one UT is embedded into one frame. This means that the GW has to partition the set of users to be served $\mathcal{K}$ into B disjoint groups, which guarantees that all UTs will be served. Mixing the usual objective of sum rate maximization (see e.g. [3]) with the notion of user scheduling, we attempt to optimize the following problem

$$\underset{\{\mathcal{B}_i\}_{i=1}^B}{\text{maximize}} \quad \sum_{i=1}^{B} \min_{k\in\mathcal{B}_i} B_W \log_2(1 + \text{SINR}_k), \quad (10)$$

where it can be observed that the attainable rate of each of the groups is given by the achievable rate of the user with the lowest SINR of that group. This is due to the multicast transmission: all UTs belonging to the same group have to be able to decode the transmitted frame.

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Finding the optimal solution of the problem in (10) requires an exhaustive search as the SINR of each group can only be obtained after all users have been selected and the precoding matrix has been computed. We therefore apply an heuristic approach as it is described in the next subsection.

Once the UTs are grouped, each user group is served with the same frame using the precoding strategy described in the previous Section. This strategy guarantees that all K users are served simultaneously. Note that each group might contain a different number of users.

3.1. Spectral Clustering

Given a set of UT $\mathcal{K}$ and some notion of similarity $s_{ij} \geq 0$ between all pairs i, j of UTs, our intuitive goal is to divide the set of UTs, $\mathcal{K}$, into groups such that UTs in the same group are *similar* and UTs in different groups are dissimilar to each other in other to support the precoding operation.

The best way of representing the similarities of the different UTs is in form of the similarity graph were the vertices represent the UTs and the edge of each vertex is weighted by s_{ij}. In this context, the clustering problem is transformed in finding a partition of the graph such that edges between different groups have very low weight and edges within a group have high weight [11].

The graph is generally represented by its adjacency matrix $\mathbf{S}$ which can be constructed via a Gaussian kernel as follows

$$[\mathbf{S}]_{ij} = e^{\frac{-d_{ij}}{2\nu^2}}. \tag{11}$$

Here the parameter ν controls the width of the clusters. The election of this parameter influences the clustering operation severely. Due to that, different values of this parameters shall be tested for a given scenario.

The value of d_{ij} shall be the *distance* between the UTs. For the considered case, a relevant measurement of distance between UTs whenver precoding is going to be implemented is

$$d_{ij} = \|\mathbf{h}_i - \mathbf{h}_j\|^2. \tag{12}$$

This is, the Euclidean distance.

Alternatively, the GW might opt to consider additional clustering constraints apart from the similarity of the UTs channel vectors. For instance, if UTs require different services (e.g. real-time connectivity or broadcasting information), it might be convenient to group them in different sets. Assuming that different UTs might belong to different classes, we adopt the following distance measurement

$$d_{ij}^{\text{class}} = c_{ij}\|\mathbf{h}_i - \mathbf{h}_j\|^2, \tag{13}$$

where $c_{ij} = 1$ if UT i and j belong to the same class and $c_{ij} = \infty$ in case they do not belong to the same class. With this, we can construct a graph able to accommodate UTs bearing in mind the underlying upper layer traffic requirements.

The main tool for spectral clustering is the Laplacian matrix, which can be described as

$$\mathbf{L} = \mathbf{M} - \mathbf{S}, \tag{14}$$

where $\mathbf{M}$ is the degree matrix defined as a diagonal matrix whose entries are the UTs degrees $m_1, \ldots, m_K$ such that

$$m_i = \sum_{j=1}^{K} s_{ij}. \tag{15}$$

The Laplacian matrix has many properties which are beneficial for graph analysis. For instance, assuming and ideal similarity matrix (i.e. nodes which are not connected present a distance equal to zero and nodes which are connected present a distance equal to 1), the multiplicity of the eigenvalue 0 equals to the number of clusters.

In non-ideal similarity matrices spectral clustering has been widely employed in many clustering problems (e.g. image segmentation [13]). As a general statement, the B eigenvectors corresponding to the lowest B eigenvalues of the Laplacian matrix presents a well structured data for properly obtaining B clusters.

Data: $\mathbf{H}$, B, ν.
1 Construct the similarity graph, $\mathbf{S}$, as in (11) with ν. ;
2 Compute the Laplacian, $\mathbf{L}$, as in (14).;
3 Compute the first B eigenvectors (i.e. lowest eigenvalues), $\mathbf{v}_1, \ldots, \mathbf{v}_B$ of $\mathbf{L}$.;
4 Let $\mathbf{V} \in \mathbb{R}^{K \times B}$ the matrix containing the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_B$ as columns. ;
5 Let $\mathbf{z}_i \in \mathbb{R}^{1 \times B}$ the vector corresponding to the i-th row of $\mathbf{V}$ for $i = 1, \ldots, K$.;
6 Cluster the points $\{\mathbf{z}_i\}_{i=1}^{K}$ in B clusters using the k-means algorithm.;
Result: $\{\mathcal{B}_i\}_{i=1}^{B}$

Algorithm 1: Spectal Clustering

The proposed clustering method is described in Algorithm 1. The main idea of this technique is to consider the clustering over the data $\{\mathbf{z}_i\}_{i=1}^{K}$ extracted from the Laplacian matrix instead of considering $\{\mathbf{h}_i\}_{i=1}^{K}$. The vectors $\{\mathbf{z}_i\}_{i=1}^{K}$ are the rows of matrix $\mathbf{V}$ which is formed by the B eigenvectors associated to the lowest eigenvalues of matrix $\mathbf{L}$.

The clustering process is done via the k-means algorithm assuming the Euclidean distance between $\{\mathbf{z}_i\}_{i=1}^{K}$. This is, $l_{ij} = \|\mathbf{z}_i - \mathbf{z}_j\|^2$. The k-means algorithm randomly selects k (B) UTs and constructs groups based on their minimum Euclidean distance. At each iteration, the vector that represents the group is updated and the process ends where there are no changes in groups memberships.

Note that apart from the channel matrix, $\mathbf{H}$, the algorithm requires the number of clusters to be obtained B. This is a crucial election in general clustering algorithms. Heuristic approaches based on the eigenvalues of the Laplacian are discussed in [14]. In the numerical evaluation of the method, we consider different values of B and we observe its relation with the resulting sum-rate.

It is important to remark that the value of ν also plays a crucial role in the clustering process. With a low value of ν, the similarity matrix will present no connectivity between nodes and; alternatively, if ν becomes very high, full connectivity between nodes will appear. In this context, a carefully election of ν based on the d_{ij} values.

4. SIMULATION RESULTS

We now demonstrate the benefits of our proposed method in multibeam satellite systems. For our simulations, we adopt the geostationary satellite channel model described in Section II and the parameters depicted in Table 1. The values of a_{kn} have been obtained from a simulated satellite array fed reflector with $N = 245$ feed elements.

We compute the average sum-rate (i.e. the objective function of the optimization problem in (10)) over 1000 Monte Carlo runs for the values of $K = 490, 735, 980$ and 1225. In order to compute the similarity matrix, we tune ν to be $10^{5.4}$.

P	55 Watts
B_W	500 MHz
Frequency band	20 GHz (Ka band)
Number of feeds	245
G_R	42.2 dBi
Output back-off	5 dB

Table 1: System Parameters

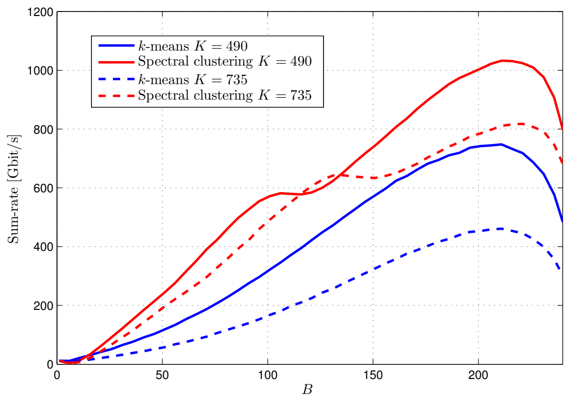

Fig. 1: Sum-rate analysis for different B values and $K = 490, 735$

Figures 1 and 2 show the sum-rate versus different B values for the proposed user scheduling technique based on spectral clustering and the k-means algorithm considering the UTs channel vectors. As it can be observed, in all cases our technique yields to sum-rate values higher than the k-means benchmark.

Remarkably, the largest attainable rates are obtained for $B < N = 245$, which differs to the current deployments where it is assumed a single-feed-per-beam/group multibeam architecture. In both k-means and the proposed spectral clustering, the efficient value of B is within the range of $211 \leq B \leq 221$.

Table 2 summarizes the sum-rate results and it compares them with the current precoding techniques in multibeam satellite systems. In particular, we consider the benchmark case where the GW can only group users that geographically belong to the same beam. As it can be observed, in the context of multibeam satellite systems employing precoding, substantially higher sum-rates are obtained if the GW is able to group users belonging to different geographical beams.

The larger K is assumed, the lower sum-rate values are obtained as the precoding gain becomes lower whenever the number of served users is increased. In addition, note that spectral clustering behaves better than the k-means approach. The larger K is considered, the larger gains between spectral and k-means are obtained. Intuitively, spectral clustering is able to properly collapse the critical features of the UTs in $\{\mathbf{z}_i\}_{i=1}^{K}$ rather than in $\{\mathbf{h}_i\}_{i=1}^{K}$. This effect is emphasized when large values of K are considered.

We now demonstrate our method for dealing with different traffic classes. Considering that the UTs can belong to $M = 2$ and 3 traffic types so that UT from different classes cannot be clustered in the same group. Table 3 summarizes the sum-rate results. It can be observed that again our proposed method offers a large sum-rate value compared to the pure k-means case apart from teh case where $K = 490$ and $M = 3$ where both schemes yield to similar sum-

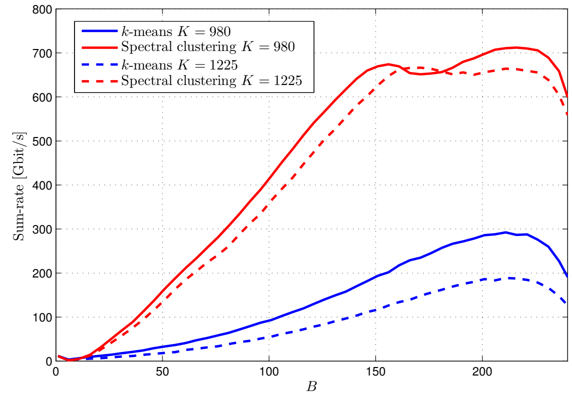

Fig. 2: Sum-rate analysis for different B values and $K = 980, 1225$

K	Benchmark	k-means	Spectral clustering
490	224	748	1033
735	146	461	817
980	125	287	712
1225	102	188	666

Table 2: Sum-rate comparison in Gbit/s

Setting/Technique	k-means	Spectral clustering
$K = 490$, $M = 3$	886	887
$K = 735$, $M = 2$	709	861
$K = 735$, $M = 3$	645	709

Table 3: Sum-rate comparison in Gbit/s for different traffic types.

rates. Remarkably, the sum-rate values when considering the different UTs classes do not decrease severely compared to the case where no classes are considered.

5. CONCLUSIONS

In this work, we considered the user scheduling problem for multibeam satellite systems with a cloud-based network manager system. We suggested an approach serving all K users in groups which can be formed by UTs belonging to different geographical beams. We examined two clustering techniques; namely, k-means and a technique based on spectral clustering. We demonstrated that our grouping based on spectral clustering outperforms the original k-means scheme. Moreover, our simulations revealed that transmitting $B < N$ simultaneous frames attains a larger sum-rate compared to current approaches that assume $B = N$. In addition, we consider the case where the UTs have different traffic needs and; thus, the scheduler can only group users with the same traffic demands. Therefore, we conclude that our proposed scheme is an efficient and flexible strategy for multibeam satellite systems using precoding.

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