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It is entitled:
Applying Multiple Query Optimization in Mobile Databases

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# Applying Multiple Query Optimization in Mobile Databases 

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#### Abstract

Mobile computing is a fast growing research and commercial area. An important application of mobile networks is data dissemination over limited bandwidth channels. There are different modes of data dissemination: push-based, pull-based, or a combination of both. In push-based, the data is broadcast in the form of broadcast disks. In pull-based, a mobile unit sends an uplink query to a central server, the server processes the data and sends the answer on a downlink channel. If the number of uplink queries is large, a lot of channel bandwidth is expended in sending the answers on the downlink channels. In this study, we apply multiquery optimization to batches of pull requests in mobile databases. Materialized views are created that can be used to answer several queries at once. The materialized views are then broadcast on a push-pull channel dedicated for this purpose (answers to multiple pull queries). Each mobile unit receives a short message from the server that contains information about when and for how long to tune to the channel to retrieve the requested information. We compare multiple query processing for pull requests (MQPR) with a basic pull request method (PR) in which each query is handled separately. Appropriate algorithms and formulae are given to calculate the bandwidth usage and the wait time for the mobiles sending the requests. A performance study is conducted by simulating different query loads over a testbed schema. The studies indicate a significant savings in the channel bandwidth usage and also a significant reduction in the wait time in MQPR compared to PR.


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## Chapter 1 Mobile Databases

Wireless technology is a fast growing research and commercial area. Wireless devices such as cellular phones, personal digital assistants (PDAs) or laptop computers have become popular because they are convenient and economical. Currently there are around 200 million users of some form of wireless networks; it is anticipated that there will be around 1 billion subscribers in the next five years [UC 1999]. Mobile device users need a way to communicate with other (probably larger and more powerful) systems in order to use remote data or services. However, there is no need to establish wired communication links. Often, a wireless network can be installed in places where it is not possible to install a wireline. Users of mobile devices have immediate access to services regardless of their location. Application areas include electronic mail, field audit, public safety, stock trading, airline activities, weather information, bill paying, warehouses, healthcare and the transportation industry [MA 2000]. Most of these applications need access to databases, digital libraries, online services, and location-dependent information, which is provided by mobile databases.

### 1.1 Mobile Databases

The main aim of mobile databases is to provide information to a mobile user. The term 'mobile database' does not necessarily mean that the database is mobile. Sistla and others [SWCD 1997] propose a centralized and distributed mobile database architecture where some data is present at the central server and other data is present at mobile nodes. In a distributed architecture there is a possibility of nodes being disconnected [IB 1993, B 1999], and thus may not be available to answer a request at all times. In order to focus on query optimization issues, we assume a purely centralized architecture in our work.

In a centralized wireless architecture the whole geographical area is divided into hexagonal cells where each cell mimics a circle (shown in Figure 1.1). At the center of each cell
is a Base Station (BS) that communicates with the Mobile Stations (MS) in its cell area through a wireless link. The BS is also referred to as server and the MS as mobile units, devices, or simply mobiles. BSs serving an area are connected by a backbone wired network through a Mobile Switching Center (MSC). The MSCs are connected to the Public Switched Telephone Network (PSTN). As a mobile station moves, the calls are relayed from one cell to another. When an MS moves from one cell to another the radio link with the old BS has to be broken and a new radio link has to be established with a new BS.


Figure 1.1: Centralized Wireless Network Architecture

In a centralized mobile database the database resides in the central server (or BS). There are two ways the server can provide data for a mobile user: pull-based and push-based. In a pullbased method the user sends a request for data on an uplink channel and the server processes the request and sends the data to the client on a downlink channel. An uplink channel is a channel on which a mobile can send its query to the server. The downlink channel or pull channel is the channel on which an answer to a query is sent to an individual mobile. Other mobiles cannot
access the downlink information. Uplink channel bandwidth is used to send queries and downlink channel bandwidth is required to send the answers to the queries. In the push-based method the server broadcasts the data on a broadcast channel and the mobiles tune to that particular channel to retrieve the information [AAFZ 1995]. A broadcast channel or push channel is a channel on which the server broadcasts information that all the mobiles can access. In this mode there is a wait for the data but there is a reduction in the channel bandwidth that is used since the data need not be sent to each client separately. In a hybrid model, the push-only model is augmented with a pull-based approach [AFZ 1997] by using an uplink channel to allow clients to send explicit requests for data to the server. This model accommodates that queries whose answers cannot be obtained from the broadcast information.

There are many limitations in using wireless devices such as frequent disconnections, limited power, limited screen size, security and authentication requirements, and limited channel bandwidth [IB 1993, AAFZ 1995]. In this thesis we address the issue of limited channel bandwidth.

In a wireless mobile network the servers have relatively high bandwidth broadcast capability while the clients can communicate only over a lower bandwidth link. Such an environment is called an asymmetric communication environment; a new architecture for this environment called broadcast disks has been proposed by Acharya and others [AAFZ 1995]. In this approach the server continuously and repeatedly broadcasts data to the clients. The broadcast channel becomes a disk from which clients can retrieve data as it goes by. The broadcast is created by assigning data items to different disks of varying sizes and speeds and then multiplexing the disks on the broadcast channel. Items stored on faster disks are broadcast more often than items on slower disks. Figure 1.2 shows a simple broadcast program in which disk pages $A, B, C, D$, and $E$ are continuously broadcast. The mobiles that need this information tune to the channel and retrieve the required information. Lee and others give channel allocation methods for the broadcast data [LHL 1999].


Figure 1.2: A Broadcast Disk
Unlike the push architecture, in a pull model the answers for each query are sent separately. When the queries are processed in groups there may be many common expressions among the queries. A multiple query optimizer processes a group of queries together and executes the common operations once. This technique is very useful for query workloads that contain identical queries (queries are repeated), subsumption queries (the answer for a query is a subset of the answer for another query), and overlapping queries (the two answers share some data). In the case of disjoint queries (all the queries are all distinct), this technique behaves as an ordinary query processor.

In this thesis, we apply the techniques of MQP for the queries sent as pull requests by a group of users and refer to the approach as multiple query processing for pull requests (MQPR). In the naïve approach for answering pull requests (PR), a user sends a request for data on an uplink channel, and the server processes all these requests independently and sends the answers separately on downlink channels; and this data is accessible only by a mobile that has sent a request. In MQPR, we group queries that are sent to the server in a given time window. If the queries in a particular group have some commonalities, we construct a query graph for these queries and obtain a common answer for them, called a materialized view. The answers for the queries in the group can be obtained from this materialized view. Instead of using the pull channel, which is a downlink channel, we use a push-pull channel, which is a broadcast channel for views that answer a group of pull requests. The individual mobiles are sent small packets
stating when and for how long to tune to the channel and retrieve the answers. In this method we use a broadcast channel instead of a downlink channel, and we eliminate redundancy in query answers. We study the savings in wait time and bandwidth that are obtained using this approach. In the next section, we motivate of our work with an example.

### 1.2 A Motivating Example

To illustrate the problem addressed in this thesis, consider a mobile database where data pages $A$, $B, C, D, E$ and $F$ are continuously broadcast over a push channel. Figure 1.3 gives the representation of the push data, while its channel bandwidth usage is given in Figure 1.4(a). The channel is allocated for the push data and the disks $A-F$ continuously occupy this channel. The channel bandwidth usage for broadcasting the disks once is equal to the sum of the sizes of all the disks. It is assumed that there is no repetition of data in the disks. A time window occurs in which 5 mobiles cannot retrieve the answers to their queries (Q1-Q5) from the push data and thus they send pull requests to the server.


Figure 1.3: Relationships between Queries and Broadcast Data

It is assumed that the answers to Q1 and Q5 are identical, the answer to Q2 overlaps that of Q1, the answer to Q2 subsumes that of Q3 (i.e., answers for Q3 can be obtained by doing a simple select or join operation on the result of Q2), and the answer to Q4 does not have any data in common with any other query answers. The representation of these queries is given in Figure 1.3. The universe of discourse is a mobile database, and the ellipses represent subsets of the database.

The server processes the queries and sends the answers on a downlink channel in the order the requests are sent. The channel bandwidth usage for the pull requests on a downlink channel is shown in Figure 1.4(b).

(a) Bandwidth Usage for Push Data

(b) Bandwidth Usage for Pull Requests on a Downlink Channel

(c) Bandwidth Usage for Pull Requests on a Push-Pull (Broadcast) Channel

Figure 1.4: Bandwidth Usage for Different Channels

In our approach we apply MQP, which helps us in determining the commonalities between queries in the given time window. The resulting broadcast data is shown in Figure 1.4(c).

Since Q1 and Q5 are identical, the results are broadcast just once. The overlapping portion of Q1 and Q2 is broadcast just once. The result of Q3 is obtained from the result of Q2. Q4 is broadcast. Channel bandwidth and wait time savings are expected to result from using MQP to form a broadcast program.

### 1.3 General Research Objective

This thesis applies techniques from multiple query optimization to obtain bandwidth and wait time reduction for answering pull requests in a mobile database environment.

### 1.4 Specific Research Objective

The specific research objectives are as follows:
a) To determine what techniques are proposed for processing pull requests and devise improvements.
b) To measure the effectiveness of algorithms for processing pull requests.
c) To design experiments to determine performance characteristics.
d) To analyze the results to determine conditions under which savings occur.

Our approach to meeting these objectives is given in next section.

### 1.5 Research Methodology

The research methodology used to achieve our research objectives is given here.
a) In order to define a new method for handling pull requests, we investigate the use of multiple query processing techniques to process a set of requests.
b) In order to determine the effectiveness of algorithms for processing pull requests, we
i. propose characteristics such as bandwidth and wait time,
ii. develop metrics to calculate the bandwidth and wait time, and
iii. select a testbed to conduct the experiments.
c) To conduct experiments, we develop a software system for implementing MQP techniques that computes bandwidth and wait time.
d) To analyze the results, we examine them in the form of graphs and draw conclusions about the savings in bandwidth and also see how the wait time varies under different conditions.

### 1.6 Expected Contributions

By accomplishing our research objectives we expect to make the following contributions:
a) an improved method to handle the pull-requests using MQP techniques,
b) cost metrics to calculate the bandwidth and wait time and also define a testbed appropriate for MQP scenarios,
c) a software system that implements MQP and also computes bandwidth and wait time in both the methods, and
d) identification of the key factors that affect bandwidth and wait time.

### 1.7 Overview of the Thesis

In Chapter 2, we discuss multiple query processing, a multigraph MQP technique, and details of our software implementation. In Chapter 3, we present the overview of our complete system, the testbed, design of experiments, and the results and their analysis. In Chapter 4, we present contributions of the thesis and discuss future work.

## Chapter 2 Multiple Query Processing

Multiple Query Processing (MQP) optimizes a set of queries together by executing the common operations once in order to save query execution time and evaluation cost. It has been shown by Sellis that MQP typically offers substantial improvement to the performance of a system [S 1998]. Exhaustive algorithms have been proposed for doing MQP [SG 1990]. These are impractical and explore an exponential search space. Roy and others propose heuristic algorithms that are practical and provide significant benefits in the optimization of queries [RSSB 2000]. We adopt a heuristic approach here.
a) SELECT customer.name, customer.custkey, orders.orderkey, orders.orderdate, orders.totalprice
FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND lineitem.quantity = '24';
b) SELECT customer.name, customer.custkey, orders.orderkey, orders.orderdate, orders.totalprice
FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND lineitem.quantity = '24'
AND orders.orderstatus = 'shipping';

Figure 2.1: Two Sample Queries
We illustrate MQP with an example. Figure 2.1 contains two queries that retrieve information from an order processing database. The first query retrieves customer and order information for a particular quantity of items ordered. The second query retrieves customer and order information for a particular quantity of items ordered and whose order status is shipping. The answer for the second query is a subset of the results of first query. The results of the first query enable fast computation of the second. These requests can be optimized using MQP by first
finding the Customers whose lineitem quantity is 24 and then using this to find the Orders with the additional constraint that the order status is shipping.

Section 2.1 discusses how MQP techniques can be applied to mobile databases. Section 2.2 gives an overview of MQP techniques from the literature and we select one for our work; Section 2.3 details the multigraph processing algorithms we use here. Section 2.4 illustrates an approach with an example, while Section 2.5 gives the details of software implementation.

### 2.1 Applying MQP in Mobile Databases

In the mobile database architecture that we adopt, there are hundreds of mobile users served by a central server (BS) that receives requests on an uplink channel and sends answers on the downlink channel. In a naïve approach, when a server receives an uplink request, it processes the request and sends a request for data to the data source, and obtains the answer from it. This answer is sent to the mobile on a downlink channel. Figure 2.2 shows the information flow between a mobile, a server, and a data source in the naïve method (called PR here for pull request). The vertical lines represent the information source, server, and a mobile and the horizontal lines show the data flow. The numbers on the lines show the sequence of the flow. A dotted horizontal line shows that the link is an air link. The continuous horizontal lines show that the link is a wireline. The direction of an arrow represents the direction of the flow of data.

Significant channel bandwidth can be expended in sending query answers on the downlink channel. If there is an overlap in the results of the queries meant for different mobiles, channel bandwidth is wasted by sending the same data on the downlink more than once. Response time and channel traffic are both increased. To address these issues, we propose a multiple query processing method for pull requests (called MQPR) to batch the queries and perform common sub-expression processing.


Figure 2.2: Information Flow in PR

The major tasks in MQP are common operation/sub-expression identification and global plan construction. In our approach, MQPR, we develop a view from the results of various queries and broadcast this view on a channel we call the push-pull channel. The push-pull channel broadcasts information on a broadcast channel instead of sending answers on a downlink channel. On a downlink channel, mobiles are sent a small packet containing information about when to tune to the push-pull channel and which part of the view they need to download to find the results to their queries. When there are subsumptions between queries, i.e., when the answer for a query is to be obtained by doing a select operation on the answer being broadcast, we can sort the answer before it is broadcast so that the mobile does not have to do a select. It can just tune at the time given in the packet and get its answer. This is referred as filtering of data [IVB1 1994]. This method reduces the bandwidth that would previously be wasted by transmitting almost the same information for different users separately. Figure 2.3 shows the information flow between the mobile, the server, the data source, and the broadcast channel for MQPR.


Figure 2.3: Information Flow in MQPR

In Figure 2.3, the mobile initially sends an uplink query. The server collects all the requests it obtains in a given time window, applies an MQP technique, and evaluates the common expressions. It sends the requests for data to the information source and obtains the results. The mobile is sent a small packet containing information regarding when and for how long to tune to the channel to get the information. Later these results are broadcast on a broadcast channel. The mobile then tunes to the channel according to the information in the packet and obtains the information.

In the next section, various MQP techniques from the literature are reviewed and one is selected for our work.

### 2.2 MQP Techniques

The problem of identifying common subexpressions is NP-hard [J 1985, RH 1980, SKL 1989]. Jarke indicates that multi-relation subexpressions can only be addressed heuristically since determining subexpressions has an exponential search space [J 1985]. There are two main heuristic approaches, one using AND/OR graphs and the other using multigraphs. In an AND/OR graph, the AND nodes represent the relations and the OR nodes represent the operators [RC 1988]. Roy and others have demonstrated that multiquery optimization using heuristics with AND/OR graphs is practical and provides significant benefits [RSSB 2000]. Their method detects subsumption by comparing each pair of operator nodes from distinct queries. There may be many AND/OR graphs corresponding to a particular set of queries; thus the performance of an algorithm depends on the chosen representation. AND/OR graphs are also procedural (i.e., they specify an evaluation order) so that some potential optimization choices are not considered. Both of these disadvantages do not occur in the multigraph approach.

We use a multigraph technique for processing subsumptions [CE 1994, CD 1998]. The multigraph approach can be used to identify common subexpressions for identical, subsumption and overlap cases. A multigraph is a non-procedural representation of multiple queries and the representation of the multiple queries is unique. We choose a multigraph for MQP in our work as it represents queries in a unique way compared to operator graphs and also because of the ease of detection of common subexpressions. The detection of common subexpressions using a multigraph is the same as the detection of common edges, i.e., edges connecting to the same node(s) on the graph. The time and space complexities for processing common subexpressions using a multigraph are much lower than those that use an operator graph [CE 1994]. In our work we consider the identical, subsumption, and disjoint cases. We describe the multigraph technique in the following sections.

### 2.3 Multigraph Processing

A definition and common sub-expression processing algorithms for multigraphs are given here [CE 1994]. A multigraph $\mathrm{G}(\mathrm{R}, \mathrm{SE}, \mathrm{JE})$ is defined as follows:

1. A node, $r \in R$, of the multigraph represents a relation or an intermediate result derived from relational operation(s).
2. A selection edge, $\mathrm{se}_{\mathrm{i}} \in \mathrm{SE}$, loops on a node and represents a select operation on the relation. A selection edge is labeled with a query ID and selection condition(s).
3. A join edge, $\mathrm{je}_{\mathrm{i}} \in \mathrm{JE}$, between two relations represents the join operation. A join edge is labeled with a query ID and a join condition(s).

R is a relation, SE is a select edge, JE is a join edge and QL is the query list.
A multigraph is constructed for a given set of queries. Each relation is represented as a node. A select edge is represented as a loop on a relation. A join edge is represented by a line connecting two relations. When a relational operation is evaluated, the node(s) and edge(s) related to the operation are contracted into a new node. At the end of a sequence of operations, the single remaining node represents the final result. The main idea of MQP is to execute the common operations only once. Two conditions may have nothing in common, be identical, or the result of one condition subsumes the result of other condition; we refer to these relationships as the commonality between two conditions. Chen's subsumption processing algorithm is shown in Figure 2.4. A group of edges and type of commonality among the edges in the multigraph are selected for processing based on the heuristics given in Figure 2.5. The type of commonality among the edges is identified and the common operations are performed by calling appropriate procedures.

```
Algorithm comsubproc
Input: A multigraph G(R, SE, JE)
Output: A multigraph \(\mathrm{G}^{\prime}\left(\mathrm{R}^{\prime}, \mathrm{SE}, \mathrm{JE}\right.\) ') with no related conditions on same type of edges (SE or
JE)
Method:
While common operations exist
    Use the heuristics to select a group of edges/operations for processing;
    Identify the type of edges/operations (select or join) and the type of commonality
    (identical, subsumption or overlap) among the operations;
    Perform the common operation for this group of operations and contract the
        node for this operation;
```

Figure 2.4: Common Subexpression Processing Algorithm

## Edge Processing Heuristics

1. Select a group of edges $\operatorname{SE}(\mathbf{u}, \mathbf{u})$ such that they have identical select conditions and $\mathrm{QL}\{\operatorname{JE}(\mathrm{u}, \mathrm{v})\} \subseteq \operatorname{QL}\{\operatorname{SE}(\mathrm{u}, \mathrm{u})\}$;
2. Select a group of edges $\operatorname{SE}(\mathbf{u}, \mathbf{u})$ such that they have identical select conditions and $\operatorname{QL}\{\operatorname{JE}(\mathrm{u}, \mathrm{v})\} \not \subset \mathrm{QL}\{\mathrm{SE}(\mathrm{u}, \mathrm{u})\} ;$
3. Select a group of edges $\operatorname{SE}(\mathrm{u}, \mathrm{u})$ such that they have subsumed select conditions and $\mathrm{QL}\{\operatorname{JE}(\mathrm{u}, \mathrm{v})\} \subseteq \operatorname{QL}\{\operatorname{SE}(\mathrm{u}, \mathrm{u})\}$;
4. Select a group of edges $\operatorname{JE}(\mathrm{u}, \mathrm{v})$ such that they have identical join conditions and $\mathrm{QL}\{\mathrm{JE}(\mathrm{u}, \mathrm{v})\} \subseteq \mathrm{QL}\{\operatorname{SE}(\mathrm{u}, \mathrm{u})\}$ where $\mathrm{w} \neq \mathrm{u}$ or $\mathrm{w} \neq \mathrm{v}$.
5. Select a group of edges $\operatorname{JE}(u, v)$ such that they have subsumed join conditions and $\operatorname{QL}\{\operatorname{JE}(\mathrm{u}, \mathrm{v})\} \subseteq \operatorname{QL}\{\operatorname{SE}(\mathrm{u}, \mathrm{u})\}$ where $\mathrm{w} \neq \mathrm{u}$ or $\mathrm{w} \neq \mathrm{v}$.

Figure 2.5: Heuristics for Edge Processing
All six algorithms for processing the edges found with the heuristics are discussed by
Chen [CE 1994]. The algorithms are for processing (1) identical selects, (2) subsumption selects, (3) overlap selects, (4) identical joins, (5) subsumption joins, and (6) overlap joins. In our work, algorithms (1), (2), (4) and (5) are implemented. Figure 2.6 describes the algorithm for processing identical select conditions. This algorithm is illustrative of other identical and subsumption processing algorithms. In the algorithm for processing identical selects all the select operations with identical common subexpressions are performed just once and a contracted node is created by removing all the identical edges representing the same common subexpressions. The algorithm in Figure 2.4 continues to find the common subexpressions and processes them until no
further common conditions exist. The output of this algorithm is a multigraph with all the common conditions removed; it is a materialized view that can answer all of the queries represented in the input multigraph.

```
Algorithm S_identical
Input: A multigraph
    G(R, SE, JE), QL(sc, Ri) // list of query Ids and their associated
    identical select operation }\mp@subsup{\sigma}{\textrm{sc}}{}(\textrm{Ri})
Output: A contracted multigraph G'(R', SE', JE')
Method:
Perform }\mp@subsup{\sigma}{\textrm{sc}}{}(\textrm{Ri})\mathrm{ and create a contracted node n for this operation;
Delete edges representing the }\mp@subsup{\sigma}{\textrm{sc}}{}(\textrm{Ri})\mathrm{ operation;
For all selection edges se(Ri,Ri) with query ID in QL(sc,Ri) do
Move se(Ri,Ri) to se(n, n);
For all edges (t,Ri) where t }=\mathrm{ Ri do
If query ID of edge (t,Ri) is in the QL(sc,Ri), change original links from (t,Ri) to
(t,n);
```

If there are no remaining edges on node Ri, delete node Ri;

Figure 2.6: Algorithm for Identical Select Processing
In the following section we give an illustrative example of the multigraph MQP technique.

### 2.4 Illustrative Example

Consider the example Queries A and B shown in Figure 2.7. These queries are from an orderprocessing database. The base relations underlying these queries are given in Figure 2.8. The construction of the multigraph for these queries and the transformations are given here. First the base relations for these queries are identified as Orders, Customers and Lineitem. In the following figures $O$ stands for Orders relation, $C$ for Customer relation and $L$ for the Lineitem relation. Figure 2.9 shows the representation of Query A in the form of a query graph. The nodes are the relations, the edge joining two nodes is a join edge and the dotted edge looping to a relation is a select condition. Figure 2.10 shows the query graph representation for Query B.

Figure 2.11 shows the multigraph for Queries A and B.
A) SELECT customer.name, customer.custkey, orders.orderkey, orders.orderdate, orders.totalprice
FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND lineitem.quantity = '24';
B) SELECT customer.name, customer.custkey, orders.orderkey, orders.orderdate, orders.totalprice
FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND lineitem.quantity = '24'
AND orders.orderstatus $=$ 'shipping';

Figure 2.7: An Example

| Part (partkey, name, mfgr, brand, type, size, container, retailprice, comment) |
| :---: |
| Supplier (suppkey, name, address, nationkey, phone, acctbal, comment) |
| Partsupp (partkey, suppkey, availqty, supplycost, comment) |
| Customer (custkey, name, address, nationkey, phone, acctbal, mktsegment, comment) |
| Nation (nationkey, name, regionkey, comment) |
| LineItem (orderkey, partkey, suppkey, linenumber, quantity, extendedprice, discount, tax, returnflag, linestatus, shipdate, commitdate, receiptdate, shipinstinct, shipmode, comment) |
| Region (regionkey, name, comment) |
| Orders (orderkey, custkey, orderstatus, totalprice, orderdate, orderpriority, clerk, shippriority comment) |

Figure 2.8: Base Relations


Figure 2.9: Initial Query Graph for Query A


Figure 2.10: Initial Query Graph for Query B


Figure 2.11: Initial Multigraph for Queries A and B


Figure 2.12: Transformation Steps in Processing Multigraph for Queries A and B

Figure 2.12 gives the transformation steps in the processing of the multigraph. In Figure 2.12(a) the select condition 'L.quantity $=24$ ' is processed and the node L is replaced by the reduced node L'. In Figure 2.12(b) the join condition 'c.custkey=o.custkey' is processed and the node O is replaced by O '. The condition ' O .orderkey=L.orderkey' is processed and the node O ' is replaced by node O". The view in Figure 2.12(c) is now broadcast. The broadcast is sorted on orderstatus so that the mobiles that have sent Query B can get the data without doing a select operation. A small packet is sent to the mobiles that have sent Query B when they need to tune in so that they get the required information (i.e., they can tune when the information for 'orderstatus $=$ shipping' $^{\prime}$ is being broadcast). For any select conditions on other attributes that cannot be reduced for different queries, the mobile is sent information in order to perform a select.

### 2.5 Implementation

The multiquery processing algorithm is coded in C++. The algorithm takes a manually created multigraph as input. Creating a multigraph from a given query set could be automated, but we focus on the actual MQP coding rather than pre-processing.

(a) A Multigraph Node

(b) A Node in the Selection List

(c) A Node in the Join List

(d) A Node in Join Pair List

Figure 2.13: Data Structure for Representing a Multigraph

Figure 2.13(a) gives the data structure for our implementation of a multigraph. The data structure is the same as that given by Chen [CE 1994] with small modifications. All the relations
have a unique relation identifier, REL_ID. The select and join conditions are given select condition numbers and join condition numbers, respectively. We introduce a variable REL_SIZE to store the size of the node and a next pointer to facilitate traversal of the multigraph. Consider a relation with REL_ID R1. MAX_SEL is the number of select conditions on node R1, and MAX_JOIN is the number of relations that $R 1$ is joined with. There are two pointers, one to the selection list and other to the join list. Figure 2.13(b) shows a node in a selection list. Each node in the selection list stores the query number and the selection condition number along with a pointer to the next node in the selection list. Figure 2.13(c) shows a node in the join list. Each node in the join list has the REL_ID of the node to which the node $R 1$ is joined with. If node RI is joined with relation $R 2$, REL_ID is $R 2$ for this join node. The Num_of_ Joins in the join node is the number of joins between node $R 1$ and node $R 2$. There is a pointer to the join pair list and a pointer to the next node in the join list. Join pair list has all the join pairs between the nodes $R 1$ and $R 2$. Figure 2.13(c) shows a node in the join tuple list. Each node in a join tuple list is a join tuple that is a query number and a join condition number.


Figure 2.14: Multigraph for Queries A and B


Figure 2.15: Data Structure Representation of Multigraph for Queries A and B

In Figure 2.14, we repeat the multigraph of Figure 2.11 for Queries A and B given in Section 2.4. We give selection condition numbers and join condition numbers for the selection and join conditions. The data structure representing the multigraph is given in Figure 2.15. There are three nodes representing the three relations in the multigraph. Each node has a selection list representing the select conditions on that relation and a join list representing the joins it has with other relations.

A multigraph such as that shown in Figure 2.15 is given as input to the MQPR algorithm. The algorithm uses heuristics (Figure 2.5) to select a group of edges for processing. The type of edge and commonality among the operations is identified and the common operations are performed and a contracted node is obtained. The multigraph is modified (i.e., nodes are constructed and edges are eliminated) when an operation is carried out. This processing of common edges is done until no further common operations exist. The size of the result to be broadcast for the given set of queries is computed from the final multigraph obtained after the transformations. This is used in determining the channel bandwidth and the wait time; the cost metrics for these calculations are discussed in the next chapter.

## Chapter 3 Performance Study

In this chapter the performance of algorithms to process pull requests are compared. Since the algorithms determine the content of a push-pull broadcast channel, two relevant performance criteria are bandwidth usage and wait time. We design simulations with different query workloads to investigate the impact of the algorithms in different scenarios for these two metrics. Section 3.1 details the complete system. Section 3.2 gives the performance model. Section 3.3 gives the testbed we use in our experiments. Section 3.4 outlines the design of experiments and Section 3.5 gives the results and their analysis.

### 3.1 Overview of the System

There are two phases for the creation of a push-pull broadcast program. One is to determine the content (answers to queries) and the other is to assign the broadcast elements to the available push-pull channels.

In the PR method the elements to be broadcast are the answers to all the queries that the server receives. In MQPR the queries are grouped based on subsumptions and the elements to be broadcast are the answers of each of these groups. Figure 3.1(a) shows Phase 1 in MQPR. The MQP algorithms are given in Chapter 2. The query workload is a form of an unreduced multigraph. Figure 3.1(b) shows Phase 2, the channel assignment algorithm. This is the same for both PR and MQPR; only the input values change.

(a) Phase 1 in MQPR


Figure 3.1: Phases in MQPR

In PR the ordering of the elements is the order in which the server received the queries. In MQPR we define two ways of ordering the elements: $M Q P R_{1}$ and $M_{Q P R}^{2}$. In $M Q P R_{1}$, the elements are sorted in the order of their size and the smaller elements are given higher priority for broadcast than the larger elements. $\mathrm{MQPR}_{2}$ is an improved method for broadcasting the elements. In this the elements to be broadcast are sorted in increasing value of the ratio of the size of the element to the count of the number of users waiting for the element $\left(\mathrm{S}_{\mathrm{i}} / \mathrm{Sc}_{\mathrm{i}}\right)$. The element with smallest $\mathrm{S}_{\mathrm{i}} / \mathrm{Sc}_{\mathrm{i}}$ value has the highest priority to be broadcast. In our study, significant lowering of the average wait time is obtained using this method.

The other input for the channel assignment algorithm is the heuristics of assignment. These heuristics give us the strategy to assign elements to the channels. The heuristic we use here takes one element at a time from the ordered list and assigns it to the channel with the least sum of sizes of all the previously assigned elements.

Our channel assignment algorithm gives the broadcast program for each of the channels. The results within a broadcast element are ordered so that the subsumed queries can avoid doing
selects. Each mobile is sent a packet stating when to tune to get its answer. In the next section we give the channel assignment algorithm.

### 3.2 Performance Model

This section introduces terms and variables used for describing our performance model, followed by an algorithm for calculating wait time.

The broadcast is done in the form of buckets that are the smallest logical unit of broadcast. All buckets are the same size. Bucket size is given in bytes.

The parameters we use in this chapter and their explanations are as follows:

- C : channel bandwidth usage in bytes,
- $\mathrm{N}:$ number of requests,
- b: the size of the that needs to be broadcast for a query set in bytes,
- L: bucket length in bytes,
- B: the number of buckets to be broadcast ( $B=b / L$ ),
- t : time to broadcast a single bucket,
- T : time to broadcast the given data $(\mathrm{T}=\mathrm{B} * \mathrm{t})$,
- n : number of elements to be broadcast,
- nc: number of channels available for broadcast,
- $\mathrm{S}_{\mathrm{i}}$ : size of the $\mathrm{i}^{\text {th }}$ element that is being broadcast,
- $\mathrm{Sc}_{\mathrm{i}}$ : count of the number of users who are waiting for the $\mathrm{i}^{\text {th }}$ element, and
- $\mathrm{w}_{\mathrm{i}}$ : wait time for a mobile waiting on the $\mathrm{i}^{\text {th }}$ element.

The total channel bandwidth is the sum of the sizes of all the elements that are broadcast. Access time is the time a mobile spends determining where in the broadcast the answer for its query can be obtained. In our system, access time is not computed because the mobile is sent a packet detailing when the mobile can tune in to retrieve the desired information. Only tuning time
is considered. Tuning time is measured in terms of the number of buckets. The wait time is the time elapsed between the times a request is sent to the time the mobile starts downloading the data. The algorithm for calculating the wait time is given in Figure 3.2. It takes as input the number of elements to be broadcast, the element sizes in a given order, and the number of available channels. It gives as output the wait time for each mobile waiting on these elements. The algorithm combines our channel assignment heuristic with the computation of individual wait times.

```
Assign_and_find
input: n, //number of elements that need to be broadcast
    int size[] //array having sizes of elements to be broadcast in the order
    float waittime[] //array to hold the calculated wait_times
    nc //number of channels available for broadcast
output: waittime[] //the wait times are returned
BEGIN
    initialize the size of each channel to zero //channel size is the sum of elements scheduled to
                    be broadcast on that channel
    for each of the elements in the array
        compute the channel with smallest size
        add this element to that particular channel, compute the new channel size
        add this element in the element list of that channel
    wait time for an element in the element list of a channel=
    (sum of sizes of elements before it in the element list* t/L)
return waittime[]
END
```

Figure 3.2: Channel Assignment and Wait Time Calculation Algorithm

After obtaining the wait times for each mobile from the algorithm in Figure 3.2, the average wait time is calculated as follows:

$$
\text { Average wait time }=\left(\sum_{i=1}^{N} w_{i}\right) / \mathrm{N}
$$

The next section gives the testbed for our experiments.

### 3.3 Testbed

This thesis extends the TPC-H-SPJ query set [T 1999] to include additional queries with subsumption relationships. A detailed explanation of how the queries are derived and the subsumption relationships between the queries is described in Appendix A. TPC-H-SPJ contains select-project-join (SPJ) subset of the TPC-H benchmark's query set [TB 1999]. The TPC-H-SPJ query set modifies the TPC-H queries to discard the aggregation, ORDERBY and GROUPBY functions, but retains the original schema. TPC-H benchmark is a decision support benchmark that consists of a suite of business-oriented ad-hoc queries with broad industry-wide relevance. We choose this benchmark as our basis since it does not represent the activity of any particular business segment, but rather an industry that manages, sells, or distributes products worldwide.


Figure 3.3: Relative Query Sizes

Our testbed extends the TPC-H-SPJ query set to a total of fifteen queries. The queries are either the same or modifications of queries in TPC-H-SPJ benchmark. We introduce subsumption
relationships in order to apply an MQP technique. A summary of the relative sizes of the queries appears in Figure 3.3. The subsumption relationships are given as a Venn diagram in Figure 3.4.


Figure 3.4: Subsumption Relationships between Queries

### 3.4 Design of Experiments

We design simulations to study the bandwidth savings between PR and MQPR, and study the effect on wait time for the $\mathrm{PR}, \mathrm{MQPR}_{1}$ and $\mathrm{MQPR}_{2}$ algorithms for different kinds of workloads at the server. Some examples of loads that a server may have are light loads (small queries), heavy loads (large queries) and balanced loads (all types of queries). For each experiment we vary the number of user requests from one to fifty in increments of size 1 . For a given number of requests, we randomly select the queries from the testbed query set.

In the first experiment, we use a balanced load, i.e., all the queries have equal probability of being selected. In the second one, termed Skewed Load1, the first $k$ queries have higher probability of occurring than the rest. By increasing the probability of subsumption relationships between queries, we can study how this impacts bandwidth. The third load, termed Skewed Load2, k small queries (queries for which the results are small) have higher probability of occurring than other queries. This query set helps us to determine how performance
characteristics vary with a light load. In the fourth one, termed Skewed Load3, k large queries (queries for which the results are large) have higher probability of occurring and this load depicts the system performance with a heavier load. In the fifth one, termed Skewed Load4, k randomly selected queries have higher probability than the rest. This query set is used to study the system behavior for a skewed load with no particular pattern. The sixth experiment is conducted with distinct queries, called Distinct Query Load that consists of queries with no subsumption relationships between them, although some queries are repeated according to random selection. Figure 3.5 summarizes all the query loads.

| Type of Load | Load Description | Rationale |
| :--- | :--- | :--- |
| Balanced | All queries have equal probability | General workload |
| Skewed Load1 | First k queries have higher probability | Increased subsumptions |
| Skewed Load2 | k small queries have higher <br> probability | Lighter load |
| Skewed Load3 | k large queries have higher <br> probability | Heavier load |
| Skewed Load4 | k random queries have higher <br> probability | Random skewed load |
| Distinct Query Load | All the queries are disjoint queries | No subsumptions |

Figure 3.5: Query Loads

For our study, we double the likelihood of a query occurring in a workload if it has a higher probability according to a particular strategy for that workload. We let $\mathrm{k}=7$ for our study since it represents approximately half of the queries in the testbed. In Figure 3.6 we give the fixed parameters that are constant for all the experiments. These parameters are identical to those used in other performance studies in the literature [IVB 1994]. For each query set we vary the number of channels available for broadcast as shown in Figure 3.7. The value 10 is chosen for PR, but a smaller number of channels for MQPR is sufficient to give comparable or better performance.

| Variable | Description | Value |
| :--- | :--- | :--- |
| C | Channel bandwidth | 10 Kbps |
| L | Bucket length in bytes | 128 bytes |
| t | Time to broadcast a single bucket | 0.1 sec |

Figure 3.6: Fixed System Parameters

| Method | Number of Channels |
| :--- | :--- |
| PR | 10 |
| MQPR $_{1}$ | 3,5 |
| MQPR $_{2}$ | 3,5 |

Figure 3.7: Number of Channels for Broadcast
Each experiment has a given number of requests (from 1 to 50 ), and we group the selected queries so that queries with subsumptions fall in a particular group. We construct a multigraph for each of these groups and use it as an input to our MQP algorithm given in Section 3.1. The MQP algorithm computes the size of elements that needs to be broadcast. The elements are ordered for the different methods (PR, MQPR ${ }_{1}, \mathrm{MQPR}_{2}$ ), and passed to the channel assignment algorithm for computing broadcast plan. The bandwidth is computed for PR and MQPR methods. The bandwidth is same for $\mathrm{MQPR}_{1}$ and $\mathrm{MQPR}_{2}$, they only differ in the ordering of the elements in the broadcast. Wait times are computed for $\mathrm{PR}, \mathrm{MQPR}_{1}$ and $\mathrm{MQPR}_{2}$ techniques.

### 3.5 Results

For different numbers of user requests, for each of the channel number variations and push-pull broadcast (program creation algorithm), the bandwidth usage and the wait times are calculated. Thus 600 simulations are conducted for bandwidth and 1,500 simulations are conducted for wait time. The complete results are tabulated in Appendix B. Representative examples are given in this section. We plot three graphs for each type of load. The first one is the bandwidth usage graph, the second graph gives the average wait time with 3 channels, and the third graph gives the
average wait time with 5 channels. The complete graphs are given in Appendix C. We introduce an example of each of these graphs and analyze the results in the following subsections.

### 3.5.1 Bandwidth Usage

Figure 3.8 shows the bandwidth usage graph for a balanced load. The queries are randomly selected for a given number of user requests (from 1 to 50 ). On the x -axis is the number of user requests and on the $y$-axis is the bandwidth usage in bytes. The bandwidth plot for MQPR levels out after a certain number (in this case 39) of user requests. The bandwidth plot for PR continues to increase. The bandwidth usage in MQPR is always less than or equal to that of PR. In some points the plot in PR increases but the plot of MQPR decreases (e.g., where $x=18$ and $x=19$ ). The reason for this is the savings due to identical queries (at $\mathrm{x}=19$ there are more repetitions of certain queries than at $\mathrm{x}=18$ ). Figure 3.9 gives the percentage savings of MQPR compared to PR. On the x -axis is the number of user requests and on the y -axis is the percentage savings in MQPR compared to PR. The findings from the graph are tabulated in Figure 3.10. We observe that significant savings result from MQPR for any type of load. The savings are lowest for queries with smaller sizes (Skewed Load2) and highest for the load with large queries (Skewed Load3).

We have also computed the bandwidth usage in the Disjoint Query Load and similar results are observed. The percentage savings are $47.35 \%$ for up to 30 requests and $59.86 \%$ for up to 50 requests. The results of this experiment depict the savings in the channel bandwidth usage when there are no subsumptions (but there are identical queries); we cannot compare the results directly to those of the other workloads since the Distinct Query Load has only 8 possible queries to choose from and the repetitions of individual queries is higher.


Figure 3.8: Bandwidth Usage


Figure 3.9: Percentage Savings in MQPR Compared to PR

| Type of Load | Up to 30 <br> requests | Up to 50 <br> requests |
| :--- | :--- | :--- |
| Balanced | $39.44 \%$ | $51.21 \%$ |
| Skewed Load1 | $40.39 \%$ | $50.95 \%$ |
| Skewed Load2 | $30.83 \%$ | $42.63 \%$ |
| Skewed Load3 | $41.12 \%$ | $55.21 \%$ |
| Skewed Load4 | $43.23 \%$ | $53.74 \%$ |

Figure 3.10: Percentage Savings of Bandwidth in MQPR Compared to PR

### 3.5.2 Wait Time

For calculating wait times, we assume that 10 channels are available for broadcast in PR. In $M_{Q P R}^{1} 1$ and $M Q P R_{2}$, we vary the number of channels as 3 and 5. In Figure 3.11 the number of user requests is on the x -axis and the average wait time in seconds is on the y -axis. The wait times are reduced using MQPR ${ }_{1}$ although just 5 channels are used for broadcast compared to 10 for PR. It is further observed that the wait times with $\mathrm{MQPR}_{2}$ are slightly lower than $\mathrm{MQPR}_{1}$. The average wait times in PR and $\mathrm{MQPR}_{1}$ are almost the same up to 17 user requests. After that the wait time calculated for $\mathrm{MQPR}_{1}$ remains almost the same but the wait time calculated for PR increases rapidly. Figure 3.12 gives the plot of average wait time. This is similar to Figure 3.11, but only 3 channels are used for broadcast in $\mathrm{MQPR}_{1}$ and $\mathrm{MQPR}_{2}$. Even when using just 3 channels for broadcast the average wait times are either below PR or comparable. Wait times increase up to a certain maximum value (in this case 250 seconds) for 12 requests and after that they remain almost constant using either MQPR technique.


Figure 3.11: Average Wait Time with 5 Channels


Figure 3.12: Average Wait Time with 3 Channels
Figure 3.13 compares the average wait times for the mobiles for up to 30 requests (which we refer to as Case 1). We do the comparison for varying loads, broadcast methods, and
broadcast channels. The same comparisons for up to 50 requests (which we refer to as Case 2) are given in Figure 3.14.

| Type of Load | Avg Wait <br> Time in PR <br> with 10 <br> Channels | Avg Wait <br> Time in <br> MQPR $_{1}$ with <br> 5 Channels | Avg Wait <br> Time in <br> MQPR $_{2}$ with <br> 5 Channels | Avg Wait <br> Time in <br> MQPR $_{1}$ with <br> $\mathbf{3}^{\text {Channels }}$ | Avg Wait <br> Time in <br> MQPR $_{2}$ with 3 3 <br> Channels |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Balanced | 64.61 | 4.98 | 4.17 | 68.19 | 61.89 |
| Skewed Load1 | 81.92 | 4.36 | 3.34 | 51.08 | 43.64 |
| Skewed Load2 | 31.50 | 3.10 | 2.77 | 36.97 | 31.49 |
| Skewed Load3 | 105.82 | 6.53 | 5.38 | 78.62 | 66.85 |
| Skewed Load4 | 98.57 | 3.93 | 3.21 | 58.21 | 48.27 |

Figure 3.13: Average Wait Times for up to 30 Requests

| Type of Load | Avg Wait <br> Time in PR <br> with 10 <br> Channels | Avg Wait <br> Time in <br> MQPR $_{1}$ with <br> 5 Channels | Avg Wait <br> Time in <br> MQPR $_{2}$ with <br> $\mathbf{5}^{\text {Channels }}$ | Avg Wait <br> Time in <br> MQPR $_{\mathbf{1}}$ with <br> $\mathbf{3}^{\text {Channels }}$ | Avg Wait <br> Time in <br> MQPRR with $^{\text {3 Channels }}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Balanced | 208.11 | 7.51 | 6.91 | 86.05 | 81.31 |
| Skewed Load1 | 221.04 | 7.19 | 5.37 | 76.59 | 64.28 |
| Skewed Load2 | 121.35 | 4.80 | 4.35 | 52.10 | 47.48 |
| Skewed Load3 | 346.53 | 9.15 | 8.33 | 118.77 | 110.36 |
| Skewed Load4 | 262.28 | 5.91 | 4.09 | 78.55 | 63.54 |

Figure 3.14: Average Wait Times for up to 50 Requests

The wait times for PR are high even though it uses 10 channels. These wait times are comparable to the wait times in MQPR approaches with just 3 channels in Case 1, but they are much higher in Case 2. The wait times for $\mathrm{MQPR}_{2}$ are uniformly lower than wait times calculated for $\mathrm{MQPR}_{1}$. The large impact of a small increase in the number of channels using MQPR techniques is evident from the relatively lower wait times. The average wait times computed by MQPR approaches remain almost the same in Case 1 and Case 2, but the average wait times
computed by PR are higher for large number of requests than smaller number of user requests. In Case 1 the wait times calculated for Balanced and Skewed Load2 using MQPR ${ }_{1}$ are slightly higher than those calculated using PR, but this is still reasonable since we are using just 3 channels in $\mathrm{MQPR}_{1}$ compared to 10 in PR. $\mathrm{MQPR}_{2}$ overcomes this and the wait times in these cases are almost the same as those in PR. In other words, MQPR $_{2}$ always outperforms MQPR $_{1}$ and PR in our study for average wait time calculations. Figure 3.15 gives the percentage reduction of the average wait time for $\mathrm{MQPR}_{2}$ compared to PR. It can be seen that reasonably good reduction in wait time is obtained by using $\mathrm{MQPR}_{2}$.

| Type of Load | Up to 30 <br> Requests with <br> 3 Channels | Up to 30 <br> Requests with <br> 5 Channels | Up to 50 <br> Requests with <br> 3 Channels | Up to 50 <br> Requests with <br> 5 Channels |
| :--- | :--- | :--- | :---: | :---: |
| Balanced | $4.21 \%$ | $93.55 \%$ | $60.93 \%$ | $96.68 \%$ |
| Skewed Load1 | $46.73 \%$ | $95.92 \%$ |  | $70.92 \%$ |
| Skewed Load2 | $0.03 \%$ | $91.21 \%$ | $60.87 \%$ | $97.57 \%$ |
| Skewed Load3 | $36.83 \%$ | $94.92 \%$ | $68.15 \%$ | $97.60 \%$ |
| Skewed Load4 | $51.03 \%$ | $96.74 \%$ |  | $75.77 \%$ |

Figure 3.15: Percentage Reduction in Average Wait Time using MQPR ${ }_{2}$ Compared to PR
We have seen that MQPR techniques outperform PR for both bandwidth and wait time calculations. $\mathrm{MQPR}_{2}$ performs well in wait time calculations. We have seen that by grouping the queries and adopting the push-based broadcast mode for pull queries, we obtain reduction in the bandwidth usage. Wait times computed by MQPR approaches are lowered in case of higher number of requests and they are comparable to the wait times computed by PR approach in case of lower number of queries.

## Chapter 4 Conclusions

We present the contributions of our thesis, followed by a discussion of open research questions.

### 4.1 Contributions

We give below the contributions of the thesis.

- We investigate MQP techniques, adopt a multigraph technique, and use it to propose an improved method for handling pull requests that creates multiquery materialized views to be broadcast.
- We give cost metrics to calculate bandwidth usage and wait time for materialized views that are broadcast to answer queries.
- We extend the TPC-H-SPJ query set with additional subsumption queries to test MQP scenarios.
- We develop a software system to implement MQP using multigraphs.
- We conduct a performance study for different types of loads and see how the bandwidth and wait time are affected by the type of method used and the nature of loads.

In our performance study, we observe that by grouping queries and creating materialized views to broadcast, we obtain reductions in bandwidth usage and also lowering of wait times. MQPR techniques outperform PR for both bandwidth and wait time calculations in our study; $\mathrm{MQPR}_{2}$ performs better than $\mathrm{MQPR}_{1}$ in all cases. More savings in bandwidth usage are obtained for heavy load compared to the lighter load, the reason for this being more bandwidth is used for large size queries and by taking care of repetitions and subsumptions a lot of savings are obtained.

### 4.2 Future Work

In this section, we outline future problems that can be investigated in the framework presented here.

In our work we do not consider the case of queries with overlap. A performance evaluation including this type of query relationship can be done using our framework. The study can be done using the AND/OR operator graphs and applying MQP techniques suggested by Roy and others [RSSB 2000] instead of muligraphs to compare their relative merits.

Determining optimal time window is an important issue and algorithms can be developed to determine the length of time window for which the server needs to wait. In our work we assume a centralized architecture but a distributed architecture is a more realistic one and work can be done on how different requests can be handled in this environment.

Algorithms can be developed to study the frequency of pull requests and determine what to broadcast on the push channels to reduce the number of pull requests. Dynamic broadcasting techniques can be developed and broadcast disks can be reorganized so that the number of pull requests remains low.

Subieta proposes a method based on stored queries in a cache, which is a <query, response> pair [SR 1987]. When a collection of stored queries is available, responses to some queries may be obtained locally. In the mobile environment algorithms can be developed for updating the views stored in the cache so that the cache can answer future queries. This may reduce the number of pull requests that a mobile needs to send to a remote server.

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## Appendix A Testbed

This appendix gives the base relations and the database schema that are used in the experiments.

## A. 1 Base Relations

The testbed schema shown in Figure A. 1 has eight relations [TB 1999]. The base relations along with the attributes of each where the primary keys are in bold and the foreign keys are in italics.

| Part (partkey, name, mfgr, brand, type, size, container, retailprice, comment) |
| :--- |
| Supplier (suppkey, name, address, nationkey, phone, acctbal, comment) |
| Partsupp (partkey, suppkey, availqty, supplycost, comment) |
| Customer (custkey, name, address, nationkey, phone, acctbal, mktsegment, comment) |
| Nation (nationkey, name, regionkey, comment) |
| LineItem (orderkey, partkey, suppkey, linenumber, quantity, extendedprice, discount, tax, <br> returnflag, linestatus, shipdate, commitdate, receiptdate, shipinstinct, shipmode, <br> comment) |
| Region (regionkey, name, comment) |
| Orders (orderkey, custkey, orderstatus, totalprice, orderdate, orderpriority, clerk, shippriority <br> comment) |

Figure A.1: Testbed Schema

In the next section, the adaptation of TPC-H-SPJ benchmark and the definitions of the selected queries that are used as materialized views in this thesis are given.

## A. 2 Views

We present the definitions of fifteen queries we selected and modified from the TPC-H-SPJ benchmark. The fifteen queries that are used as materialized views are presented in SQL. We give below the queries we considered in our test bed. The explanation of how these queries are derived from the TPC-H-SPJ [T 1999] query set is given in Figure A.2.

| Testbed | TPC-H-SPJ |
| :---: | :--- |
| Q1 | Same as Q2 |
| Q2 | Select clause added to Q2 |
| Q3 | Select clause removed from Q3 |
| Q4 | Same as Q3 |
| Q5 | Same as Q4 |
| Q6 | Select clause added to Q4 |
| Q7 | Select clauses added to Q4 |
| Q8 | Same as Q5 |
| Q9 | Select clause added to Q5 |
| Q10 | Select clause removed from Q6 |
| Q11 | Same as Q6 |
| Q12 | Same as Q7 |
| Q13 | Select clause added to Q7 |
| Q14 | Select clause removed from Q8 |
| Q15 | Same as Q8 |

Figure A.2: Testbed Derivation from TPC-H-SPJ Query Set

Q1) CREATE VIEW extended_price AS
SELECT part.type, lineitem.extendedprice
FROM lineitem, part
WHERE lineitem.partkey = part.partkey
AND lineitem.shipdate = '1995-09-01'

Q2) CREATE VIEW extended_price1 AS
SELECT part.type, lineitem.extendedprice
FROM lineitem, part
WHERE lineitem.partkey = part.partkey
AND lineitem.shipdate = '1995-09-01'
AND part.retailprice > 50,000

Q3) CREATE VIEW parts_supplier_relationship AS
SELECT part.brand, part.type, part.size
FROM PartSupp, part
WHERE PartSupp.partkey = part.partkey
AND part.brand = 'brandno45'

Q4) CREATE VIEW parts_supplier_relationship1 AS
SELECT part.brand, part.type, part.size
FROM PartSupp, part
WHERE PartSupp.partkey = part.partkey
AND part.brand = 'brandno45'
AND part.size = '45'

Q5) CREATE VIEW important_stock_identification AS

SELECT PartSupp.partkey, PartSupp.supplycost, PartSupp.availqty
FROM PartSupp, supplier, nation
WHERE PartSupp.suppkey = supplier.suppkey
AND supplier.nationkey $=$ nation.nationkey
AND nation.name = 'germany'

Q6) CREATE VIEW important_stock_identification1 AS
SELECT PartSupp.partkey, PartSupp.supplycost, PartSupp.availqty
FROM PartSupp, supplier, nation
WHERE PartSupp.suppkey = supplier.suppkey
AND supplier.nationkey $=$ nation.nationkey
AND nation.name $=$ 'germany'
AND supplycost > 12,000

Q7) CREATE VIEW important_stock_identification2 AS
SELECT PartSupp.partkey, PartSupp.supplycost, PartSupp.availqty
FROM PartSupp, supplier, nation
WHERE PartSupp.suppkey = supplier.suppkey
AND supplier.nationkey $=$ nation.nationkey
AND nation.name $=$ 'germany'
AND supplycost > 12,000
AND availqty > 10

Q8) CREATE VIEW large_volume_customer AS
SELECT customer.name, customer.custkey, orders.orderkey, orders.orderdate, orders.totalprice
FROM customer, orders, lineitem WHERE orders.custkey = customer.custkey

AND lineitem.orderkey $=$ orders.orderkey
AND lineitem.quantity $=$ ' 24 '

Q9) CREATE VIEW large_volume_customer1 AS
SELECT customer.name, customer.custkey, orders.orderkey, orders.orderdate, orders.totalprice FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND lineitem.quantity $=$ ' 24 '
AND orders.orderstatus $=$ 'shipping'

Q10) CREATE VIEW shipping_priority AS
SELECT lineitem.orderkey, orders.orderdate, orders.shippriority
FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND orders.orderdate $=$ '1995-03-15'

AND lineitem.shipdate $=$ '1995-03-15'

Q11) CREATE VIEW shipping_priority1 AS
SELECT lineitem.orderkey, orders.orderdate, orders.shippriority
FROM customer, orders, lineitem
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND orders.orderdate $=$ '1995-03-15'
AND lineitem.shipdate $=$ '1995-03-15'
AND customer.mktsegment = 'building'

Q12) CREATE VIEW returned_item_reporting AS
SELECT customer.custkey, customer.name, customer.acctbal, nation.name, customer.address, customer.phone
FROM customer, orders, lineitem, nation
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND customer.nationkey $=$ nation.nationkey
AND orders. orderdate $=$ '1994-10-01'

Q13) CREATE VIEW returned_item_reporting1 AS
SELECT customer.custkey, customer.name, customer.acctbal, nation.name, customer.address, customer.phone
FROM customer, orders, lineitem, nation
WHERE orders.custkey = customer.custkey
AND lineitem.orderkey = orders.orderkey
AND customer.nationkey $=$ nation.nationkey
AND orders.orderdate $=$ '1994-10-01'
AND nation.name='canada'

Q14) CREATE VIEW cost_supplier AS
SELECT supplier.acctbal, supplier.name, nation.name, part.partkey, part.mfgr supplier.address, supplier.phone, supplier.comment
FROM part, supplier, PartSupp, nation, region
WHERE PartSupp.partkey = part.partkey
AND PartSupp.suppkey = supplier.suppkey
AND supplier.nationkey $=$ nation.nationkey
AND nation.regionkey $=$ region.regionkey
AND part.size=' 15 '

Q15) CREATE VIEW cost_supplier1 AS
SELECT supplier.acctbal, supplier.name, nation.name, part.partkey, part.mfgr supplier.address, supplier.phone, supplier.comment
FROM part, supplier, PartSupp, nation, region
WHERE PartSupp.partkey = part.partkey
AND PartSupp.suppkey = supplier.suppkey

AND supplier.nationkey = nation.nationkey
AND nation.regionkey $=$ region.regionkey
AND part.size=' ${ }^{\prime}{ }^{\prime}$
AND region.name $=$ ' europe'

Figure A. 3 gives the subsumptions introduced in the testbed query set.

| Query | Subsumes |
| :---: | :---: |
| 1 | 2 |
| 3 | 4 |
| 5 | 6,7 |
| 6 | 7 |
| 8 | 9 |
| 10 | 11 |
| 12 | 13 |
| 14 | 15 |

Figure A.3: Subsumption Relationships in the Testbed
Figure A. 4 gives the sizes of the queries.

| Query | Size in Bytes |
| :---: | ---: |
| Q1 | 105,035 |
| Q2 | 94,532 |
| Q3 | $1,440,000$ |
| Q4 | 28,800 |
| Q5 | $1,024,000$ |
| Q6 | 512,000 |
| Q7 | 460,800 |
| Q8 | $5,201,053$ |
| Q9 | 520,106 |
| Q10 | 18,000 |
| Q11 | 12,000 |
| Q12 | 244,122 |
| Q13 | 9,765 |
| Q14 | $1,312,000$ |
| Q15 | 262,400 |

Figure A.4: Query Sizes
The ordering of the queries computed in decreasing order of their final sizes is as follows:
Q8, Q3, Q14, Q5, Q9, Q6, Q7, Q15, Q12, Q1, Q2, Q4, Q10, Q11, Q13.
The distinct query load considered includes the following queries:
Q1, Q3, Q5, Q6, Q8, Q10, Q12, Q14.
These queries do not have any subsumption relationships between them.

## Appendix B Results from the Experiments

The results of all the experiments are given in the following tables for different kinds of loads. Table B. 1 gives the results for the Balanced Load. Table B.2, Table B.3, Table B. 4 and Table B. 5 give the results for Skewed Load1, Skewed Load2, Skewed Load3 and Skewed Load4, respectively. Table B. 6 gives the results for the Distinct Query Load. The description of these loads is given in Chapter 3. The following are the notations used in the tables.

- N

Number of queries

- (1-15) Frequencies of queries 1-15
- $\quad \mathrm{BW}(\mathrm{N}) \quad$ Total bandwidth used in PR (in bytes)
- BW(M) Total bandwidth used in MQPR (in bytes)
- Wait(N) 10
- Wait(M)5
- Wait(NM)5
- Wait(M)3 Average wait time in $\mathrm{MQPR}_{1}$ using 3 channels (in secs)
- Wait(NM)3 Average wait time in MQPR 2 using 3 channels (in secs)

| N | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | BW(N) | BW(M) | Wait(N)10 | Wait(M)5 | it(NM)5 | Wait(M)3 | (NM)3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 10250 | 10250 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 2126 | 2126 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 2272 | 2272 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 10368 | 10368 | 0 | 0 | 0 | 2.34375 | 2.34375 |
| 5 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 14964 | 10964 | 0 | 0 | 0 | 3.75 | 3.75 |
| 6 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 58637 | 51703 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | 0 | 0 | 2 | 1 | 23065 | 10390 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 0 | 1 | 3 | 0 | 0 | 0 | 94745 | 46234 | 0 | 0 | 0 | 3.51562 | 3.51562 |
| 9 | 0 | 1 | 2 | 1 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 39020 | 19945 | 0 | 0 | 0 | 88.1911 | 71.7793 |
| 10 | 1 | 1 | 1 | 1 | 2 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 115950 | 62753 | 0 | 0 | 0 | 57.4117 | 57.4117 |
| 11 | 3 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 0 | 1 | 0 | 0 | 15173 | 12287 | 0.69354 | 0 | 0 | 1.38707 | 1.38707 |
| 12 | 0 | 0 | 3 | 0 | 1 | 0 | 0 | 2 | 1 | 0 | 0 | 1 | 1 | 2 | 1 | 151613 | 72040 | 16.5291 | 0 | 0 | 247.68 | 263.573 |
| 13 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 3 | 0 | 2 | 1 | 1 | 1 | 1 | 79519 | 61109 | 2.02915 | 2.88462 | 2.88462 | 63.587 | 63.587 |
| 14 | 1 | 0 | 0 | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 0 | 0 | 0 | 1 | 3 | 81623 | 60069 | 9.4774 | 2.00893 | 2.00893 | 22.1691 | 22.1691 |
| 15 | 1 | 2 | 0 | 4 | 0 | 2 | 1 | 1 | 0 | 0 | 2 | 0 | 0 | 1 | 1 | 67918 | 56022 | 8.125 | 0.625 | 0.625 | 10.3456 | 10.3456 |
| 16 | 2 | 0 | 0 | 2 | 1 | 1 | 1 | 0 | 1 | 3 | 0 | 0 | 2 | 1 | 2 | 36678 | 23575 | 7.75854 | 4.06714 | 2.30933 | 22.8677 | 13.4146 |
| 17 | 2 | 1 | 3 | 3 | 0 | 1 | 2 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 54428 | 22400 | 19.7195 | 5.41199 | 1.27597 | 82.8028 | 28.0223 |
| 18 | 2 | 0 | 2 | 1 | 1 | 1 | 0 | 1 | 1 | 3 | 1 | 2 | 1 | 1 | 1 | 97769 | 73001 | 33.4213 | 11.4614 | 10.6801 | 132.918 | 130.659 |
| 19 | 0 | 4 | 1 | 2 | 2 | 0 | 1 | 2 | 0 | 1 | 1 | 2 | 2 | 0 | 1 | 121771 | 64719 | 31.3426 | 9.99441 | 9.99441 | 65.5741 | 65.5741 |
| 20 | 0 | 0 | 2 | 0 | 0 | 2 | 3 | 0 | 2 | 1 | 1 | 1 | 5 | 2 | 1 | 74499 | 31611 | 127.847 | 1.40625 | 1.40625 | 70.0143 | 70.0143 |
| 21 | 1 | 3 | 2 | 1 | 0 | 1 | 2 | 2 | 3 | 1 | 3 | 0 | 1 | 1 | 0 | 141165 | 67170 | 87.6975 | 4.43806 | 2.48605 | 110.537 | 42.0474 |
| 22 | 0 | 2 | 1 | 0 | 1 | 5 | 3 | 2 | 1 | 1 | 1 | 0 | 2 | 1 | 2 | 151593 | 71088 | 171.349 | 2.26438 | 2.26438 | 118.527 | 118.527 |
| 23 | 0 | 2 | 1 | 2 | 1 | 3 | 0 | 3 | 1 | 3 | 0 | 1 | 3 | 2 | 1 | 184248 | 72919 | 110.327 | 14.6783 | 14.6783 | 178.531 | 178.531 |
| 24 | 1 | 2 | 2 | 0 | 2 | 0 | 1 | 1 | 1 | 2 | 1 | 1 | 3 | 5 | 2 | 146955 | 73001 | 149.447 | 8.01009 | 8.01009 | 109.423 | 103.757 |
| 25 | 2 | 1 | 3 | 2 | 1 | 2 | 0 | 2 | 2 | 1 | 3 | 2 | 1 | 2 | 1 | 168835 | 73001 | 101.002 | 15.9419 | 14.8169 | 179.929 | 176.675 |
| 26 | 0 | 1 | 3 | 3 | 0 | 4 | 5 | 0 | 2 | 2 | 2 | 2 | 1 | 0 | 1 | 83699 | 24149 | 164.218 | 18.1248 | 3.92224 | 91.3291 | 50.8845 |
| 27 | 4 | 1 | 2 | 2 | 0 | 3 | 1 | 3 | 2 | 2 | 1 | 1 | 2 | 2 | 1 | 197582 | 69001 | 190.453 | 17.2794 | 16.7585 | 116.134 | 112.11 |
| 28 | 2 | 0 | 3 | 0 | 3 | 1 | 5 | 1 | 1 | 1 | 1 | 2 | 2 | 4 | 2 | 175389 | 73001 | 362.804 | 7.36802 | 7.36802 | 100.686 | 100.686 |
| 29 | 1 | 2 | 2 | 3 | 1 | 1 | 2 | 2 | 1 | 5 | 2 | 3 | 3 | 0 | 1 | 138893 | 64801 | 104.006 | 10.9134 | 10.9134 | 67.3478 | 67.3478 |
| 30 | 5 | 1 | 2 | 2 | 1 | 1 | 4 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 91353 | 36431 | 240.168 | 12.8161 | 12.8161 | 98.8313 | 98.8313 |
| 31 | 3 | 3 | 1 | 3 | 2 | 0 | 2 | 4 | 2 | 3 | 2 | 3 | 0 | 1 | 2 | 231142 | 73001 | 198.813 | 17.6968 | 17.2432 | 191.925 | 188.42 |
| 32 | 2 | 3 | 2 | 1 | 0 | 2 | 0 | 3 | 4 | 2 | 0 | 1 | 3 | 6 | 3 | 242802 | 69001 | 494.005 | 19.2687 | 19.2687 | 132.414 | 132.414 |
| 33 | 1 | 3 | 4 | 2 | 2 | 2 | 3 | 0 | 0 | 3 | 4 | 3 | 0 | 5 | 1 | 143104 | 32368 | 292.784 | 2.55682 | 2.55682 | 52.5791 | 52.5791 |
| 34 | 2 | 3 | 0 | 2 | 2 | 2 | 4 | 2 | 3 | 4 | 2 | 2 | 3 | 2 | 1 | 163506 | 61976 | 206.831 | 4.54963 | 4.54963 | 44.7177 | 44.7177 |
| 35 | 2 | 3 | 2 | 5 | 1 | 4 | 0 | 2 | 2 | 2 | 2 | 3 | 1 | 2 | 4 | 175841 | 73001 | 256.835 | 12.1906 | 11.7888 | 147.256 | 144.151 |
| 36 | 1 | 3 | 2 | 2 | 5 | 2 | 2 | 2 | 2 | 2 | 5 | 0 | 2 | 1 | 5 | 191981 | 71170 | 367.64 | 2.41016 | 2.41016 | 103.105 | 103.105 |
| 37 | 4 | 0 | 4 | 3 | 0 | 3 | 4 | 4 | 4 | 2 | 3 | 2 | 1 | 1 | 2 | 272946 | 69001 | 353.333 | 20.4029 | 18.8826 | 134.923 | 119.755 |
| 38 | 3 | 2 | 2 | 1 | 1 | 6 | 3 | 5 | 3 | 1 | 3 | 2 | 3 | 0 | 3 | 295435 | 64801 | 322.457 | 18.3857 | 18.3857 | 83.8798 | 83.8798 |
| 39 | 2 | 4 | 4 | 6 | 4 | 1 | 3 | 1 | 0 | 1 | 5 | 1 | 2 | 3 | 2 | 175897 | 73001 | 424.955 | 5.70984 | 3.90695 | 83.181 | 69.25 |
| 40 | 4 | 3 | 3 | 1 | 5 | 1 | 7 | 3 | 1 | 1 | 3 | 5 | 1 | 1 | 1 | 256970 | 73001 | 396.25 | 9.61211 | 8.90898 | 109.152 | 103.718 |
| 41 | 2 | 2 | 2 | 3 | 2 | 2 | 2 | 2 | 3 | 4 | 4 | 4 | 2 | 6 | 1 | 223218 | 73001 | 480.186 | 11.7221 | 11.7221 | 138.602 | 138.602 |
| 42 | 2 | 4 | 4 | 2 | 2 | 6 | 3 | 3 | 4 | 2 | 1 | 2 | 2 | 0 | 5 | 253590 | 64801 | 421.653 | 15.6854 | 15.6854 | 86.9218 | 86.2522 |
| 43 | 3 | 2 | 2 | 4 | 3 | 4 | 2 | 1 | 3 | 2 | 3 | 5 | 2 | 5 | 2 | 192963 | 73001 | 569.974 | 9.59557 | 9.59557 | 118.641 | 118.641 |
| 44 | 2 | 1 | 2 | 3 | 0 | 3 | 4 | 3 | 5 | 2 | 5 | 2 | 4 | 3 | 5 | 240040 | 69001 | 587.302 | 16.5178 | 16.5178 | 114.114 | 114.114 |
| 45 | 3 | 3 | 2 | 2 | 4 | 4 | 2 | 3 | 3 | 2 | 5 | 2 | 4 | 2 | 4 | 250486 | 73001 | 438.759 | 12.1911 | 12.1911 | 139.561 | 139.561 |
| 46 | 2 | 5 | 3 | 1 | 2 | 2 | 3 | 4 | 2 | 8 | 0 | 2 | 5 | 3 | 4 | 289039 | 73001 | 443.91 | 11.9261 | 11.9261 | 137.394 | 137.394 |
| 47 | 1 | 6 | 4 | 1 | 4 | 3 | 6 | 3 | 2 | 2 | 2 | 2 | 4 | 2 | 5 | 281441 | 73001 | 704.185 | 10.2256 | 10.2256 | 123.003 | 123.003 |
| 48 | 2 | 4 | 3 | 3 | 2 | 3 | 4 | 1 | 2 | 4 | 6 | 6 | 2 | 1 | 5 | 163400 | 73001 | 363.499 | 6.88647 | 6.88647 | 87.613 | 87.613 |
| 49 | 3 | 5 | 3 | 5 | 2 | 8 | 3 | 2 | 3 | 1 | 1 | 5 | 3 | 3 | 2 | 238135 | 73001 | 536.404 | 10.6692 | 9.80828 | 126.31 | 119.657 |
| 50 | 6 | 2 | 3 | 1 | 5 | 2 | 4 | 3 | 1 | 1 | 4 | 2 | 5 | 3 | 8 | 280600 | 73001 | 607.704 | 7.68969 | 7.68969 | 101.529 | 101.529 |

Table B.1: Balanced Load

| N | 1 | 2 | 3 |  | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | BW(N) | BW(M) | Wait(N)10 | Wait(M)5 | Wait(NM) | ait(M) | (NM)3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 10250 | 10250 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 12070 | 12070 | 0 | 0 | 0 | 0 |  |
| 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 8139 | 8139 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 15466 | 15466 | 0 | 0 | 0 | 1.90723 | 1.90723 |
| 5 | 1 | 1 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13559 | 4820 | 0 | 0 | 0 | 0 | 0 |
| 6 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 8926 | 5186 | 0 | 0 | 0 | 4.6875 | 4.6875 |
| 7 | 0 | 0 | 0 | 1 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 18427 | 10351 | 0 | 0 | 0 | 3.26953 | 3.26953 |
| 8 | 1 | 0 | 1 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 35384 | 27827 | 0 | 0 | 0 | 44.3547 | 44.3547 |
| 9 | 1 | 3 | 1 | 2 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 22876 | 20211 | 0 | 0 | 0 | 4.6875 | 1.5625 |
| 10 | 3 | 2 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 56263 | 52920 | 0 | 0 | 0 | 2.93203 | 2.93203 |
| 11 | 1 | 1 | 2 | 0 | 1 | 1 | 2 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 45400 | 22118 | 1.27841 | 0 | 0 | 20.0334 | 20.0334 |
| 12 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 3 | 0 | 0 | 3 | 1 | 0 | 1 | 1 | 151463 | 67734 | 1.5625 | 2.34375 | 2.34375 | 123.349 | 123.349 |
| 13 | 1 | 1 | 1 | 2 | 2 | 1 | 0 | 1 | 0 | 0 | 0 | 3 | 0 | 0 | 1 | 81663 | 64661 | 9.14255 | 6.3122 | 6.3122 | 78.7182 | 78.7182 |
| 14 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 2 | 1 | 1 | 2 | 1 | 0 | 0 | 3 | 106678 | 53694 | 3.95089 | 6.83036 | 6.83036 | 60.2584 | 59.0996 |
| 15 | 2 | 2 | 0 | 2 | 1 | 4 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 75901 | 51728 | 29.2588 | 0 | 0 | 14.4706 | 14.4706 |
| 16 | 2 | 0 | 1 | 6 | 1 | 1 | 2 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 76057 | 62611 | 10.3206 | 0 | 0 | 47.8207 | 32.4347 |
| 17 | 1 | 3 | 0 | 1 | 3 | 4 | 0 | 1 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | 94408 | 60098 | 23.0928 | 1.00023 | 1.00023 | 12.2546 | 10.9083 |
| 18 | 1 | 1 | 3 | 0 | 1 | 1 | 3 | 1 | 2 | 0 | 1 | 2 | 1 | 1 | 0 | 121103 | 72954 | 85.3466 | 15.2389 | 14.1973 | 173.846 | 161.772 |
| 19 | 1 | 5 | 1 | 2 | 1 | 3 | 3 | 0 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 47341 | 20211 | 24.9491 | 0 | 0 | 2.22039 | 2.22039 |
| 20 | 1 | 1 | 6 | 1 | 3 | 0 | 2 | 0 | 0 | 1 | 0 | 2 | 1 | 2 | 0 | 125015 | 32368 | 184.77 | 4.92188 | 1.40625 | 78.4736 | 51.3082 |
| 21 | 1 | 3 | 2 | 4 | 1 | 2 | 1 | 0 | 1 | 0 | 2 | 1 | 0 | 3 | 0 | 82944 | 36384 | 82.8372 | 24.7846 | 5.24684 | 162.096 | 86.6645 |
| 22 | 0 | 4 | 2 | 5 | 1 | 2 | 1 | 2 | 0 | 0 | 1 | 1 | 1 | 2 | 0 | 150022 | 72872 | 104.923 | 9.69688 | 7.56619 | 142.682 | 113.19 |
| 23 | 4 | 3 | 1 | 1 | 0 | 3 | 1 | 1 | 0 | 3 | 3 | 0 | 0 | 3 | 0 | 104659 | 67094 | 43.1365 | 0.61141 | 0.61141 | 26.3611 | 26.3611 |
| 24 | 1 | 5 | 4 | 1 | 1 | 2 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 3 | 2 | 146898 | 72954 | 203.434 | 5.37223 | 5.37223 | 92.1154 | 92.1154 |
| 25 | 2 | 2 | 4 | 1 | 4 | 2 | 0 | 5 | 1 | 0 | 1 | 2 | 0 | 0 | 1 | 301530 | 64754 | 340.868 | 21.5691 | 21.5691 | 109.663 | 112.57 |
| 26 | 5 | 3 | 2 | 0 | 1 | 1 | 2 | 3 | 0 | 3 | 2 | 2 | 1 | 0 | 1 | 176468 | 64801 | 70.9195 | 10.55 | 10.55 | 53.1125 | 53.1125 |
| 27 | 3 | 3 | 1 | 1 | 4 | 1 | 5 | 2 | 0 | 0 | 1 | 4 | 0 | 1 | 1 | 171441 | 72954 | 250.166 | 6.77286 | 6.77286 | 83.6318 | 83.6318 |
| 28 | 2 | 2 | 3 | 1 | 7 | 1 | 5 | 1 | 1 | 2 | 0 | 1 | 1 | 0 | 1 | 164104 | 64801 | 369.149 | 7.87026 | 7.87026 | 81.4939 | 53.694 |
| 29 | 5 | 1 | 5 | 3 | 5 | 2 | 2 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 131484 | 32368 | 202.608 | 3.87931 | 0.96983 | 62.6359 | 37.7296 |
| 30 | 2 | 2 | 4 | 3 | 4 | 5 | 1 | 1 | 2 | 1 | 2 | 0 | 2 | 0 | 1 | 155683 | 62970 | 415.895 | 3.18633 | 1.66055 | 45.3517 | 37.2654 |
| 31 | 4 | 4 | 0 | 2 | 2 | 6 | 1 | 1 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 125815 | 61976 | 196.159 | 3.53831 | 3.53831 | 35.652 | 35.652 |
| 32 | 3 | 6 | 3 | 1 | 1 | 3 | 4 | 2 | 2 | 2 | 1 | 2 | 1 | 1 | 0 | 179176 | 73001 | 293.011 | 12.0151 | 4.32214 | 131.678 | 63.0524 |
| 33 | 3 | 3 | 5 | 3 | 1 | 3 | 3 | 4 | 2 | 1 | 1 | 1 | 0 | 0 | 3 | 271353 | 64801 | 280.854 | 18.3288 | 15.3459 | 104.75 | 101.037 |
| 34 | 2 | 3 | 2 | 3 | 5 | 3 | 6 | 2 | 1 | 0 | 3 | 1 | 2 | 0 | 1 | 190352 | 64754 | 270.803 | 8.61914 | 8.61914 | 81.0271 | 57.9952 |
| 35 | 1 | 2 | 5 | 3 | 2 | 2 | 7 | 1 | 2 | 3 | 1 | 3 | 2 | 1 | 0 | 173822 | 73001 | 314.042 | 10.2479 | 7.43538 | 120.134 | 98.4018 |
| 36 | 1 | 2 | 6 | 3 | 2 | 3 | 5 | 3 | 3 | 0 | 1 | 2 | 2 | 1 | 2 | 268973 | 72954 | 472.466 | 16.0202 | 14.4577 | 192.018 | 173.908 |
| 37 | 1 | 0 | 4 | 4 | 4 | 3 | 3 | 2 | 3 | 2 | 1 | 1 | 3 | 2 | 4 | 226188 | 73001 | 459.314 | 14.1295 | 13.3694 | 168.353 | 160.199 |
| 38 | 2 | 3 | 3 | 6 | 0 | 2 | 5 | 1 | , | 1 | 3 | 3 | 2 | 3 | 3 | 152849 | 69001 | 287.708 | 7.64947 | 6.53927 | 82.5104 | 73.9319 |
| 39 | 2 | 2 | 2 | 6 | 6 | 3 | 5 | 1 | 3 | 1 | 1 | 1 | 1 | 3 | 2 | 194859 | 73001 | 521.165 | 11.3009 | 10.2192 | 138.184 | 125.859 |
| 40 | 2 | 7 | 4 | 4 | 4 | 2 | 3 | 4 | 0 | 2 | 2 | 1 | 2 | 3 | 0 | 299322 | 73001 | 537.778 | 11.0184 | 9.26055 | 128.869 | 115.286 |
| 41 | 6 | 4 | 3 | 5 | 3 | 3 | 5 | 1 | 1 | 2 | 1 | 1 | 4 | 0 | 2 | 148136 | 64801 | 364.938 | 6.74676 | 6.74676 | 70.6014 | 58.9923 |
| 42 | 1 | 5 | 4 | 2 | 5 | 5 | 5 | 2 | 2 | 3 | 2 | 2 | 2 | 1 | 1 | 234232 | 73001 | 443.209 | 9.82403 | 8.48475 | 113.705 | 103.357 |
| 43 | 8 | 1 | 7 | 5 | 1 | 3 | 6 | 0 | 2 | 4 | 2 | 1 | 2 | 1 | 0 | 149964 | 36431 | 344.655 | 23.2271 | 2.56241 | 141.492 | 48.9261 |
| 44 | 3 | 2 | 5 | 4 | 5 | 4 | 3 | 1 | 5 | 2 | 2 | 2 | 2 | 3 | 1 | 226074 | 73001 | 517.418 | 14.0662 | 12.4682 | 161.315 | 148.967 |
| 45 | 4 | 9 | 4 | 3 | 1 | 4 | 4 | 0 | 2 | 6 | 1 | 2 | 3 | 0 | 2 | 111211 | 28231 | 275.961 | 15.5772 | 5.83455 | 76.4925 | 48.8853 |
| 46 | 4 | 6 | 4 | 6 | 3 | 3 | 2 | 3 | 2 | 1 | 2 | 1 | 5 | 2 | 2 | 254506 | 73001 | 419.285 | 11.9765 | 10.1422 | 139.527 | 125.354 |
| 47 | 3 | 5 | 8 | 5 | 6 | 5 | 2 | 2 | 4 | 1 | 1 | 2 | 1 | 1 | 1 | 286424 | 73001 | 824.916 | 14.3652 | 11.074 | 164.057 | 134.436 |
| 48 | 3 | 3 | 5 | 4 | 4 | 2 | 7 | 2 | 3 | 4 | 2 | 2 | 1 | 4 | 2 | 270224 | 73001 | 547.081 | 11.1845 | 10.3056 | 134.624 | 125.196 |
| 49 | 2 | 4 | 0 | 6 | 5 | 3 | 10 | 1 | 0 | 6 | 2 | 5 | 1 | 2 | 2 | 169822 | 61976 | 556.671 | 1.60714 | 1.60714 | 20.8651 | 20.8651 |
| 50 | 9 | 5 | 5 | 3 | 3 | 6 | 4 | 1 | 2 | 2 | 1 | 3 | 1 | 3 | 2 | 220185 | 73001 | 667.343 | 7.17352 | 6.32977 | 91.2211 | 84.7014 |

Table B.2: Skewed Load1

| N | 1 | 2 |  | 4 | 5 | 67 | 78 | 8 | 9 | 10 | 11 | 12 | 13 |  | 15 | BW(N) | BW(M) | Wait(N)10 | Wait(M) | Wait(NM) | ait(M) | t(NM)3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 40633 | 40633 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 8140 | 8140 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 10132 | 10132 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 17361 | 17361 | 0 | 0 | 0 | 3.51562 | 3.51562 |
| 5 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 2 | 0 | 1 | 0 | 0 | 0 | 7009 | 6868 | 0 | 0 | 0 | 2.8125 | 2.8125 |
| 6 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 22904 | 11466 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 1 | 0 | 24514 | 20297 | 0 | 0 | 0 | 2.00893 | 2.00893 |
| 8 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 1 | 1 | 1 | 0 | 1 | 16408 | 12097 | 0 | 0 | 0 | 3.51562 | 3.51562 |
| 9 | 2 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 3 | 0 | 0 | 2 | 0 | 26760 | 14764 | 0 | 0 | 0 | 2.08333 | 2.08333 |
| 10 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 3 | 0 | 0 | 21497 | 20287 | 0 | 0 | 0 | 3.57539 | 2.93203 |
| 11 | 0 | 0 | 1 | 1 | 2 | 1 | 0 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 2 | 41826 | 27411 | 1.27841 | 2.55682 | 2.55682 | 90.5658 | 58.4462 |
| 12 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 2 | 2 | 1 | 3 | 0 | 0 | 15088 | 10931 | 1.27148 | 0 | 0 | 9.18197 | 9.18197 |
| 13 | 2 | 2 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 2 | 0 | 0 | 1 | 0 | 70617 | 60069 | 3.53365 | 2.16346 | 2.16346 | 16.5186 | 16.5186 |
| 14 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 6 | 0 | 1 | 2 | 1 | 106014 | 51873 | 2.55385 | 0 | 0 | 2.97405 | 2.97405 |
| 15 | 1 | 3 | 0 | 0 | 1 | 0 | 1 | 2 | 1 | 2 | 1 | 0 | 3 | 0 | 0 | 100569 | 49670 | 3.71328 | 0 | 0 | 3.82969 | 3.82969 |
| 16 | 1 | 2 | 1 | 3 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 2 | 0 | 1 | 1 | 87127 | 72954 | 15.4426 | 12.6011 | 11.4292 | 160.281 | 146.698 |
| 17 | 2 | 2 | 1 | 4 | 4 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 60838 | 26041 | 20.6159 | 0 | 0 | 85.0561 | 35.3537 |
| 18 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 2 | 2 | 0 | 0 | 86587 | 62704 | 26.2717 | 0.52083 | 0.52083 | 34.9521 | 26.8762 |
| 19 | 1 | 2 | 3 | 2 | 2 | 0 | 0 | 1 | 1 | 3 | 1 | 2 | 1 | 0 | 0 | 101600 | 62751 | 33.0287 | 1.48026 | 1.48026 | 43.1505 | 32.4142 |
| 20 | 3 | 2 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 5 | 1 | 2 | 2 | 1 | 1 | 36077 | 27968 | 15.7484 | 1.40625 | 1.40625 | 27.981 | 27.981 |
| 21 | 4 | 0 | 2 | 1 | 0 | 0 | 2 | 1 | 2 | 2 | 2 | 2 | 2 | 0 | 1 | 88452 | 60401 | 42.0879 | 13.7316 | 13.7316 | 67.0252 | 73.501 |
| 22 | 1 | 1 | 0 | 4 | 0 | 0 | 1 | 1 | 1 | 2 | 1 | 4 | 2 | 0 | 4 | 67112 | 49376 | 67.8048 | 2.68466 | 2.68466 | 30.2727 | 27.0767 |
| 23 | 2 | 1 | 1 | 2 | 1 | 1 | 3 | 0 | 3 | 3 | 1 | 2 | 1 | 2 | 0 | 73976 | 36431 | 71.5207 | 11.9261 | 8.96977 | 91.0916 | 81.1057 |
| 24 | 1 | 2 | 1 | 1 | 1 | 1 | 3 | 0 | 3 | 1 | 3 | 4 | 2 | 0 | 1 | 59015 | 28231 | 102.4 | 9.7679 | 7.42415 | 68.8319 | 59.7767 |
| 25 | 2 | 2 | 2 | 3 | 1 | 1 | 1 | 1 | 1 | 4 | 0 | 1 | 3 | 2 | 1 | 111838 | 73001 | 92.3398 | 9.37719 | 8.25219 | 114.804 | 106.111 |
| 26 | 2 | 1 | 3 | 1 | 0 | 3 | 0 | 1 | 1 | 3 | 2 | 3 | 4 | 1 | 1 | 111987 | 69001 | 50.8438 | 8.47566 | 7.39393 | 69.1273 | 60.7688 |
| 27 | 3 | 2 | 0 | 4 | 0 | 0 | 1 | 0 | 3 | 4 | 3 | 3 | 2 | 0 | 2 | 31446 | 12806 | 34.6393 | 3.02083 | 2.39583 | 30.0637 | 20.973 |
| 28 | 5 | 1 | 1 | 3 | 2 | 1 | 1 | 1 | 0 | 0 | 3 | 3 | 5 | 1 | 1 | 99683 | 72954 | 68.2684 | 4.26995 | 3.60031 | 63.3526 | 55.5911 |
| 29 | 3 | 2 | 0 | 2 | 4 | 1 | 1 | 1 | 0 | 2 | 4 | 2 | 2 | 3 | 2 | 124095 | 61976 | 173.331 | 3.20043 | 3.20043 | 27.8043 | 27.8043 |
| 30 | 3 | 1 | 0 | 1 | 0 | 4 | 0 | 3 | 3 | 1 | 5 | 3 | 3 | 2 | 1 | 182624 | 57976 | 118.372 | 5.90625 | 5.90625 | 54.9749 | 54.9749 |
| 31 | 4 | 3 | 1 | 2 | 0 | 2 | 0 | 2 | 2 | 1 | 3 | 6 | 4 | 0 | 1 | 128811 | 60801 | 90.7042 | 11.9491 | 11.9491 | 52.4707 | 59.0509 |
| 32 | 2 | 1 | 0 | 2 | 2 | 0 | 2 | 2 | 1 | 0 | 6 | 4 | 4 | 0 | 6 | 132155 | 53729 | 137.466 | 3.28125 | 3.28125 | 35.5788 | 34.4069 |
| 33 | 3 | 2 | 0 | 3 | 1 | 3 | 2 | 3 | 2 | 1 | 4 | 4 | 4 | 0 | 1 | 172339 | 53776 | 115.691 | 5.96591 | 3.83523 | 50.0384 | 41.0143 |
| 34 | 5 | 3 | 4 | 1 | 1 | 2 | 0 | 1 | 1 | 3 | 4 | 5 | 3 | 1 | 0 | 133051 | 73001 | 129.242 | 6.89499 | 5.24058 | 79.5874 | 66.8037 |
| 35 | 4 | 2 | 0 | 1 | 5 | 2 | 4 | 2 | 2 | 2 | 2 | 4 | 0 | 1 | 4 | 183325 | 61976 | 481.18 | 4.58036 | 4.58036 | 43.805 | 43.805 |
| 36 | 7 | 5 | 2 | 0 | 0 | 1 | 2 | 1 | 2 | 5 | 4 | 3 | 2 | 0 | 2 | 102948 | 60801 | 131.814 | 7.61947 | 7.61947 | 36.4702 | 36.4702 |
| 37 | 2 | 1 | 2 | 2 | 1 | 2 | 1 | 3 | 2 | 4 | 3 | 3 | 5 | 1 | 5 | 202402 | 73001 | 206.516 | 12.6093 | 12.6093 | 145.454 | 150.967 |
| 38 | 6 | 3 | , | 5 | 2 | 3 | 0 | 1 | 0 | 2 | 4 | 4 | 4 | 0 | 3 | 102887 | 64801 | 132.32 | 4.37983 | 4.37983 | 47.7859 | 47.7859 |
| 39 | 3 | 3 | 1 | 3 | 1 | 3 | 0 | 2 | , | 0 | 5 | 3 | 7 | 7 | 0 | 200406 | 72954 | 365.232 | 7.27374 | 7.27374 | 97.5107 | 97.5107 |
| 40 | 1 | 3 | 1 | 4 | 5 | 1 | 2 | 1 | 2 | 3 | 7 | 5 | 5 | 0 | 0 | 126141 | 62751 | 162.698 | 1.05469 | 1.05469 | 27.3738 | 27.3738 |
| 41 | 3 | 4 | 1 | 5 | 1 | 4 | 3 | 2 | 2 | 5 | 2 | 2 | 2 | 3 | 2 | 181691 | 73001 | 259.242 | 10.0636 | 9.72066 | 120.082 | 117.432 |
| 42 | 2 | 4 | 0 | 4 | 4 | 1 | 2 | 4 | 4 | 3 | 5 | 6 | 2 | 0 | 1 | 242017 | 53776 | 365.611 | 6.62946 | 4.62054 | 55.897 | 47.3886 |
| 43 | 3 | 5 | 0 | 3 | 4 | 1 | 3 | 2 | 1 | 3 | 3 | 3 | 8 | 2 | 2 | 170594 | 61976 | 171.368 | 2.87791 | 2.87791 | 29.704 | 29.704 |
| 44 | 4 | 5 | 3 | 4 | 1 | 1 | 1 | 1 | 2 | 6 | 3 | 5 | 3 | 2 | 3 | 143524 | 73001 | 222.891 | 7.83212 | 6.55371 | 96.1298 | 92.1148 |
| 45 | 1 | 2 | 0 | 2 | 1 | 0 | 0 | 3 | 8 | 6 | 5 | 8 | 4 | 1 | 4 | 200479 | 61976 | 231.014 | 7.0625 | 3.9375 | 63.4256 | 31.172 |
| 46 | 2 | 6 | 2 | 3 | 1 | 3 | 2 | 3 | 1 | 2 | 4 | 6 | 5 | 2 | 4 | 223591 | 73001 | 259.931 | 8.66406 | 8.66406 | 104.056 | 104.056 |
| 47 | 2 | 6 | 1 | 3 | 4 | 4 | 2 | 2 | 1 | 4 | 8 | 6 | 0 | 2 | 2 | 195882 | 73001 | 314.737 | 6.43459 | 6.43459 | 78.1687 | 78.1687 |
| 48 | 4 | 3 | 1 | 3 | 6 | 1 | 2 | 3 | 5 | 3 | 4 | 7 | 2 | 1 | 3 | 249539 | 73001 | 589.072 | 14.8483 | 14.8483 | 161.045 | 161.045 |
| 49 | 3 | 4 | 0 | 2 | 1 | 3 | 3 | 3 | 2 | 5 | 6 | 2 | 3 | 3 | 9 | 221201 | 61976 | 342.642 | 5.7398 | 4.30485 | 45.6414 | 39.564 |
| 50 | 4 | 2 | 3 | 2 | 4 | 2 | 2 | 2 | 4 | 5 | 8 | 5 | 3 | 2 | 2 | 219497 | 73001 | 413.133 | 11.2533 | 10.972 | 125.574 | 123.401 |

Table B.3: Skewed Load2

| N | 1 | 2 | 3 | 45 | 5 |  | 7 | 8 |  | 10 | 11 | 12 | 13 | 14 | 15 | BW(N) | BW(M) | Wait(N)10 | Wait(M) | Wait(NM) | Wait(M)3 | t(NM)3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 738 | 738 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 15313 | 15313 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 40998 | 40998 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 29640 | 29640 | 0 | 0 | 0 | 3.51562 | 3.51562 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 62840 | 60790 | 0 | 0 | 0 | 38.1441 | 38.1441 |
| 6 | 1 | 0 | 0 | 1 | 0 | 2 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 13202 | 9202 | 0 | 0 | 0 | 6.875 | 6.875 |
| 7 | 1 | 0 | 0 | 0 | 2 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 71797 | 59797 | 0 | 0 | 0 | 13.0619 | 13.0619 |
| 8 | 1 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 45396 | 19146 | 0 | 0 | 0 | 1.90723 | 1.90723 |
| 9 | 0 | 0 | 1 | 0 | 2 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 50700 | 31500 | 0 | 0 | 0 | 22.2328 | 22.2328 |
| 10 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 3 | 1 | 134129 | 58883 | 0 | 0 | 0 | 0 | 0 |
| 11 | 1 | 2 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 0 | 0 | 1 | 1 | 1 | 1 | 32307 | 21041 | 0.69354 | 0 | 0 | 49.5962 | 49.5962 |
| 12 | 0 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 0 | 1 | 1 | 41364 | 36349 | 2.65625 | 14.6526 | 8.49818 | 109.179 | 80.3004 |
| 13 | 1 | 2 | 3 | 0 | 2 | 1 | 0 | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 1 | 106857 | 62753 | 23.9864 | 0 | 0 | 66.2443 | 66.2443 |
| 14 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 1 | 0 | 3 | 2 | 0 | 2 | 1 | 90542 | 60884 | 16.3015 | 0 | 0 | 29.2547 | 29.2547 |
| 15 | 0 | 1 | 0 | 2 | 0 | 6 | 0 | 1 | 1 | 1 | 0 | 2 | 0 | 1 | 0 | 84090 | 57894 | 42.0816 | 3.9375 | 3.9375 | 43.1029 | 38.2279 |
| 16 | 2 | 1 | 0 | 1 | 1 | 2 | 1 | 1 | 3 | 2 | 1 | 0 | 0 | 1 | 0 | 85652 | 60069 | 19.5021 | 3.51562 | 0.87891 | 25.4365 | 14.2693 |
| 17 | 0 | 2 | 1 | 1 | 4 | 1 | 2 | 0 | 1 | 2 | 0 | 1 | 0 | 1 | 1 | 74703 | 36349 | 70.8566 | 10.343 | 10.343 | 103.133 | 85.7561 |
| 18 | 0 | 1 | 0 | 1 | 1 | 2 | 0 | 2 | 3 | 2 | 1 | 0 | 1 | 4 | 0 | 151871 | 60063 | 77.6451 | 5.60156 | 5.17773 | 30.4015 | 29.9327 |
| 19 | 0 | 0 | 1 | 3 | 6 | 0 | 1 | 1 | 3 | 0 | 1 | 0 | 2 | 0 | 1 | 118644 | 62103 | 80.1277 | 1.60609 | 1.60609 | 47.9422 | 47.1392 |
| 20 | 1 | 1 | 1 | 1 | 3 | 1 | 1 | 3 | 0 | 2 | 0 | 1 | 1 | 4 | 0 | 209798 | 73001 | 145.78 | 13.715 | 13.715 | 161.109 | 161.109 |
| 21 | 1 | 1 | 1 | 0 | 0 | 5 | 0 | 4 | 0 | 3 | 0 | 0 | 3 | 2 | 1 | 218542 | 67170 | 80.6384 | 3.04185 | 2.12277 | 85.3765 | 38.8664 |
| 22 | 1 | 0 | 3 | 1 | 2 | 2 | 4 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 211626 | 73001 | 205.018 | 17.4766 | 6.28675 | 191.531 | 91.7126 |
| 23 | 2 | 0 | 2 | 1 | 0 | 2 | 5 | 1 | 4 | 1 | 0 | 1 | 0 | 2 | 2 | 133900 | 69001 | 170.283 | 19.6731 | 19.6731 | 133.441 | 128.55 |
| 24 | 3 | 1 | 3 | 0 | 1 | 2 | 3 | 1 | 1 | 0 | 1 | 1 | 1 | 3 | 3 | 147424 | 72954 | 224.881 | 8.01009 | 8.01009 | 114.146 | 114.146 |
| 25 | 1 | 1 | 5 | 0 | 0 | 1 | 3 | 1 | 2 | 0 | 2 | 2 | 3 | 3 | 1 | 158399 | 68954 | 270.368 | 11.722 | 11.347 | 101.898 | 97.552 |
| 26 | 0 | 2 | 1 | 0 | 2 | 1 | 3 | 2 | 4 | 0 | 2 | 1 | 0 | 6 | 2 | 208741 | 72872 | 492.598 | 17.4036 | 5.00397 | 219.002 | 97.1892 |
| 27 | 1 | 1 | 2 | 2 | 1 | 2 | 2 | 3 | 3 | 2 | 0 | 1 | 2 | 5 | 0 | 235389 | 73001 | 346.019 | 20.3186 | 20.3186 | 226.958 | 226.958 |
| 28 | 0 | 1 | 1 | 1 | 4 | 1 | 5 | 4 | 2 | 1 | 2 | 2 | 0 | 3 | 1 | 273815 | 72919 | 448.52 | 16.8301 | 16.8301 | 203.638 | 197.611 |
| 29 | 1 | 2 | 2 | 1 | 1 | 2 | 1 | 2 | 2 | 2 | 0 | 3 | 1 | 5 | 4 | 199544 | 73001 | 264.827 | 12.7732 | 12.7732 | 159.42 | 157.076 |
| 30 | 4 | 1 | 2 | 2 | 1 | 3 | 2 | 3 | 2 | 2 | 2 | 2 | 1 | 1 | 2 | 202906 | 73001 | 191.905 | 15.5514 | 15.0827 | 172.125 | 168.503 |
| 31 | 2 | 2 | 2 | 0 | 4 | 0 | 1 | 4 | 3 | 3 | 2 | 1 | 2 | 4 | 1 | 281660 | 73001 | 378.981 | 19.4366 | 19.4366 | 211.628 | 211.628 |
| 32 | 0 | 0 | 5 | 0 | 5 | 3 | 3 | 2 | 3 | 3 | 2 | 2 | 0 | 3 | 1 | 249730 | 72181 | 546.219 | 2.19727 | 2.19727 | 156.558 | 151.037 |
| 33 | 1 | 1 | 2 | 5 | 2 | 6 | 3 | 4 | 3 | 0 | 1 | 0 | 1 | 2 | 2 | 275477 | 71123 | 568.844 | 3.60689 | 2.91335 | 192.401 | 185.793 |
| 34 | 2 | 2 | 4 | 3 | 5 | 3 | 1 | 4 | 3 | 1 | 1 | 1 | 0 | 3 | 1 | 314057 | 73001 | 493.475 | 19.7896 | 18.5488 | 220.243 | 202.106 |
| 35 | 0 | 1 | 1 | 3 | 2 | 8 | 2 | 2 | 1 | 2 | 1 | 4 | 2 | 4 | 2 | 206449 | 72919 | 387.543 | 7.93741 | 7.93741 | 109.055 | 105.841 |
| 36 | 2 | 0 | 1 | 2 | 3 | 3 | 5 | 4 | 3 | 3 | 3 | 2 | 1 | 1 | 3 | 263057 | 73001 | 350.054 | 17.1277 | 17.1277 | 187.598 | 187.598 |
| 37 | 1 | 3 | 2 | 0 | 5 | 6 | 4 | 4 | 1 | 0 | 2 | 1 | 2 | 5 | 1 | 326079 | 72954 | 728.272 | 11.5958 | 11.5958 | 136.792 | 132.991 |
| 38 | 2 | 3 | 2 | 2 | 1 | 3 | 5 | 4 | 3 | 2 | 2 | 3 | 2 | 2 | 2 | 270472 | 73001 | 553.531 | 16.5963 | 16.5963 | 182.003 | 182.003 |
| 39 | 1 | 2 | 2 | 2 | 3 | 2 | 6 | 6 | 5 | 1 | 2 | 1 | 2 | 3 | 1 | 378151 | 73001 | 825.464 | 24.587 | 24.587 | 261.551 | 261.551 |
| 40 | 4 | 2 | 7 | 2 | 2 | 6 | 2 | 1 | 5 | 2 | 2 | 0 | , | 3 | 1 | 225454 | 71170 | 605.587 | 3.82588 | 2.87227 | 142.921 | 134.421 |
| 41 | 1 | 4 | 4 | 1 | 1 | 5 | 3 | 5 | 7 | 1 | 1 | 0 | 3 | 2 | 3 | 346522 | 71170 | 745.664 | 5.04621 | 5.04621 | 249.776 | 249.776 |
| 42 | 4 | 4 | 3 | 0 | 4 | 2 | 5 | 2 | 1 | 2 | 1 | 1 | 3 | 6 | 4 | 255527 | 73001 | 592.537 | 6.86579 | 6.86579 | 94.991 | 94.991 |
| 43 | 4 | 2 | 1 | 4 | 6 | 3 | 2 | 4 | 5 | 1 | 2 | 1 | 0 | 6 | 2 | 334794 | 73001 | 656.969 | 18.8102 | 18.8102 | 211.426 | 203.685 |
| 44 | 0 | 1 | 2 | 1 | 3 | 3 | 4 | 5 | 2 | 4 | 1 | 3 | 4 | 6 | 5 | 363589 | 72919 | 1007.43 | 12.7082 | 12.7082 | 164.173 | 164.173 |
| 45 | 3 | 3 | 5 | 2 | 4 | 3 | 6 | 4 | 4 | 3 | 0 | 4 | 1 | 2 | 1 | 336440 | 73001 | 751.672 | 16.7757 | 15.5257 | 183.923 | 174.264 |
| 46 | 0 | 2 | 3 | 0 | 7 | 8 | 2 | 5 | 2 | 2 | 1 | 5 | 2 | 4 | 3 | 398933 | 72919 | 918.115 | 12.1556 | 12.1556 | 152.753 | 152.753 |
| 47 | 1 | 3 | 3 | 3 | 6 | 6 | 6 | 4 | 2 | 0 | 0 | 4 | 1 | 3 | 5 | 350425 | 72861 | 911.861 | 10.4756 | 10.4756 | 140.442 | 140.442 |
| 48 | 3 | 8 | 2 | 2 | 3 | 2 | 2 | 5 | 2 | 4 | 0 | 4 | 4 | 3 | 4 | 329259 | 73001 | 635.476 | 13.1388 | 13.1388 | 148.628 | 148.628 |
| 49 | 5 | 3 | 7 | 1 | 2 | 4 | 3 | 4 | 4 | 3 | 0 | 1 | 3 | 4 | 5 | 360687 | 73001 | 994.711 | 15.6932 | 15.6932 | 181.701 | 181.701 |
| 50 | 1 | 2 | 8 | 4 | 1 | 5 | 2 | 11 | 1 | 3 | 3 | 1 | 3 | 4 | 1 | 625315 | 73001 | 1499.62 | 23.0691 | 21.1003 | 251.604 | 247.27 |

Table B.4: Skewed Load3

|  | 12 |  |  | 011 | 1112 | 1213 |  |  | BW（N） |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 10 | 00000 | 0 | 0 | 0 | 0 | 0 | 0 | 820 | 820 | 0 | 0 | 0 | 0 |  |
|  | 1 | 00000 |  | 0 | 1 | 10 | 0 | 0 | 2645 | 645 |  |  |  | 0 |  |
|  |  | 000 |  | 0 | 0 | 00 | 0 |  | 17363 | 17363 |  |  |  | 0 |  |
|  |  | 01110 |  | 0 | 0 | 01 | 10 |  | 230 |  |  |  |  | 562 |  |
| 5 | 00 | 10201 |  | 0 | 0 | 0 | 0 | 0 | 71483 | 5988 |  |  |  | 38.1441 | 38.1441 |
| 6 | 0 | 12001 | 10 | 0 | 0 | 01 | 10 | 0 | 56009 | 55559 |  |  | 0 | ． 87 | 6.875 |
|  | 1 | 21010 | 00 | 0 | 0 | 0 | 0 | 0 | 28377 | 16164 |  |  |  | 13.0619 | 3.0619 |
| 8 |  | 30200 | 00 | 1 | 0 | 01 | 10 |  | 50060 | 19466 |  |  | ， | 1.90723 | 1.90723 |
| 9 |  | 01 | 00 | 0 | 11 | 1 |  | 0 | 2171 | 17296 |  | 67 | 67 | 22.2328 | 22.2328 |
| 10 |  | 2001 |  | 2 | 0 | 03 | 30 | 0 | 2719 | 5466 | 0 | 0 | 0 |  |  |
| 11 | 21 | 01 |  | 0 | 01 | 11 |  | 2 | 137 | 4091 | 9354 | 87 | 87 | 5962 |  |
| 12 |  | 02122 |  | 0 | 0 | 0 |  |  | 寿100 | 929 | 3.7 |  |  | 109.179 |  |
| 13 |  | 21211 |  | 1 | 01 | 11 | 10 | 0 | 54153 | 26181 | 3.98618 | 519 | 173 | ． 24 | 66.2443 |
| 14 |  | 2111 | 12 | 1 | 01 | 10 | 0 | 2 | 94053 | 64801 | 23.115 | 20.5974 | 20.5974 | 29.25 | 29.2547 |
| 15 | 0 | 31 | 2 | 0 |  | 00 | 02 | 0 | 9554 | 34395 | 62.18 | 1.87 | 1.25 | 43.1029 | 38.2279 |
| 16 |  | 2000 | 03 | 0 | 10 | 02 | 2 |  | 45263 | 21953 | 15.387 | 2.60229 | 64868 | 25.4365 | 14.2693 |
| 17 |  | 3202 | 0 | 0 | 1 | 11 | 10 | 0 | 89751 | 58704 | 18.9691 | 0.55147 | 0.55147 | 103.133 | 85.756 |
| 18 | 00 | － 41232 | 01 | 1 | 0 | 00 | 02 | 0 | 10531 | 3703 | 15 |  |  | 30.401 | 29.9327 |
| 19 | 40 | 22120 | 01 | 2 | 1 | 1 |  |  | 6095 | 643 | 42.238 | 18.7558 | 37788 | 47．942 | 俍 |
| 20 | 0 | 4131 | 10 | 0 | 0 | 02 | 20 | 2 | 13334 | 2841 | 305 | 2.375 | 2.3759 | 61. |  |
| 21 | 30 | 50110 | 10 | 3 | 1 | 12 | 21 | 1 | 1263 | 300 | 34.173 | 7.255 | 5．2468 | 85.376 | 8.866 |
|  | 22 | 41131 | － | 0 | 0 | 02 | 2 | 0 | 2185 | 103 | 219.15 | 1.3870 | ． 387 | 191.5 | 1.7 |
| 23 | 22 | 4104 | 00 | 0 |  |  | 03 |  | 11354 | 8321 | 171 |  | 1.63043 | 133.44 |  |
| 24 |  | 02 |  | 3 | 12 | 21 | 12 |  | 15518 |  | 10.68 | 15.434 | 15.4342 | 114.146 |  |
| 25 | 12 | 3313 | 0 | 2 | 1 | 11 | 12 | 2 | 13151 | 73001 | 131.06 | 6.6 | 5.53234 | 101.898 | 97.55 |
|  | 21 | 8 | 31 | 1 | 0 | 01 | 11 |  | 26717 | 1170 | 495.47 | 4.8042 | ．750 | 219.00 | 7．1892 |
|  |  | 2136 | 00 | 1 | 2 | 22 | 1 | 4 | 10491 | 228 | ． 37 | 1.562 | 1.562 | 226.95 | 崖． |
|  |  | 3422 | 30 | 1 | 2 | 23 | 31 | 1 | 20662 | 300 | 90． | 12.30 | 9．79646 | 203.638 | 崖．611 |
|  |  | 7441 | 11 | 0 | 01 | 11 | 11 | 1 | 18593 | 286 | 412.8 | 5.659 | 5．659 | 99．4 | 57.076 |
|  | 51 | 6222 | 10 | 0 |  | 01 | 13 | 3 | 1852 | 112 | 450.06 | 2.346 | ． 83 | 172.1 | 68.503 |
|  | 3 | 3351 | 11 | 0 | 21 | 11 | 13 | 3 | 1790 | 2954 | 298 | 7.1086 | 7.10862 | 211. |  |
| 32 | 3 | 5333 | 20 | 2 |  | 12 | 1 |  |  | 300 | 543.65 | 8．64 | 6.00757 | 156.55 |  |
|  | 20 | 2357 | 01 | 1 | 2 | 21 | 11 |  | 1349 | 643 | 5．6 | 13.28 | ． 399 | 92.4 |  |
|  |  | 5 3141 |  | 1 |  | 12 | 21 |  | 1092 | 643 | 0．6 | 20.13 | 240 | 220.2 |  |
|  |  | 4330 | 21 | 36 | 2 | 24 | 40 | 0 | 1740 | 27 | 213.5 | 1.205 | ． 205 | 09.05 |  |
|  |  | 46214 | 14 | 2 | 2 | 21 | 13 | 0 | 1759 | 300 | 328.39 | 15.303 | 2.568 | 187.59 | 87.5 |
|  | 32 | 6482 | 01 | 1 | 11 | 12 | 22 | 1 | 1840 | 643 | 571.77 | 23.318 | 3.3 | 136.7 | 32.9 |
|  | 32 | 44 | 34 | 2 |  | 22 | 2 | 0 | 269 | 3001 | 497.46 | 18.07 | 6.2 | 182.0 | 82.00 |
| 39 |  | 75 | 23 | 1 | 0 | 01 | 11 | 1 | 399 | 1170 | 500.20 | 4.150 | 2.194 | 261.55 | 261.55 |
| 40 | 22 | 6255 | 14 | 3 | 3 | 31 | 12 | 2 | 2259 | 300 | 10.4 | 3.06 | 1.66 | 142 | 134.42 |
|  | 23 | 63 | 14 | 0 | 0 | 03 | 3 | 3 |  | 1123 | 562.4 | 2.8179 | 2.25972 | 249.77 | 49．77 |
|  | 83 | 51254 | 1 | 1 | 40 | 02 | 2 | 2 | 189 | 7178 | 394.27 | 2.0943 | 1.7310 | 4．99 |  |
|  | 44 | 4104354 | 12 | 2 | 0 | 01 | 1 | 1 | 行 | 崖 | 603.47 | 3.4649 | 1.3359 | 211.426 | 3．68 |
|  |  | 95663 | 11 | 1 | 0 | 02 | 2 | 1 | 23798 | 2970 | 704.599 | 3.0665 | 0.81259 | 164.173 | 4．1 |
| 45 | 52 | 92746 | 10 | 0 | 12 | 21 | 12 | 3 | 27214 | 2954 | 805.914 | 4.1151 | 2.86519 | 183.923 | 74 |
| 46 | 71 | 66223 | 33 | 2 | 22 | 23 | 32 | 2 | 2733 | 73001 | 574.955 | 14.3718 | 11.926 | 152.75 | 52.75 |
| 47 | 54 | 85 | 00 | 2 | 23 | 31 | 12 | 2 | 1950 | 32368 | 541.091 | 3.8896 | 1.19681 | 140.4 | 0.4 |
| 48 | 34 | 6525 | 11 | 2 | 4 | 43 | 3 | 2 | 1889 | 64801 | 489.296 | 6.641 | 4.00505 | 148.628 | 48.62 |
|  | 51 | 5657 | 21 | 0 | 2 | 22 | 2 | 2 | 269 | 29 | 659.5 | 7.128 | ． 78 | 181.7 | 81 |
|  | 47 | 814 | 11 | 16 | 64 | 41 | 10 | 4 | 221581 | 64801 | 721.118 | 5.813 | 5.81359 | 251.6 |  |

Table B．5：Skewed Load4

| N 1 |  | 2 |  |  |  | 78 |  | 10 |  | 12 |  |  |  | (N) | BW(M) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 0 | 0 | 0 | 00 | 0 | 00 | 1 | 0 | 0 | 0 | 0 | 0 | 140 | 140 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 10 | 0 | 00 | 0 | 0 | 0 | 0 | 0 | 0 | 19250 | 19250 | 0 | 0 | 0 | 0 | 0 |
| 3 |  | 0 | 0 | 0 | 01 | 0 | 10 | 0 | 0 | 0 | 0 | 1 | 0 | 54883 | 54883 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 00 | 0 | 10 | 1 | 0 | 0 | 0 | 2 | 0 | 61273 | 51023 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 10 | 1 | 0 | 20 | 0 | 00 | 1 | 0 | 0 | 0 | 0 | 0 | 28211 | 20211 | 0 | 0 | 0 | 2.8125 | 2.8125 |
| 6 | 0 | 0 | 1 | 0 | 11 | 0 |  | 1 | 0 | 1 | 0 | 0 | 0 | 65931 | 61931 | 0 | 0 | 0 | 34.1305 | 34.1305 |
| 7 | 1 | 10 | 0 | 0 | 11 | 0 | 3 | 0 | 0 | 0 | 0 | 1 | 0 | 144970 | 59703 | 0 | 0 | 0 | 35.168 | 35.168 |
| 8 | 1 | 10 | 1 | 0 | 2 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 0 | 50477 | 32227 | 0 | 0 | 0 | 44.3547 | 44.3547 |
| 9 | 1 | 10 | 1 | 0 | 21 | 0 | 0 | 2 | 0 | 1 | 0 | 0 | 0 | 74892 | 62751 | 0 | 1.5625 | . 5625 | 34.9963 | 34.9963 |
| 10 | 1 | 10 | 2 | 0 | 10 | 0 | 2 | 1 | 0 | 0 | 0 | 3 | 0 | 143477 | 71094 | 0 | 2.8125 | 2.8125 | 180.63 | 187.43 |
| 11 | 1 | 1 | 1 | 0 | 12 | 0 | 10 | 0 | 0 | 3 | 0 | 2 | 0 | 94925 | 72861 | 7.45987 | 7.45987 | 7.45987 | 104.985 | 104.985 |
| 12 | 1 | 10 | 1 | 0 | 12 | 0 | 20 | 1 | 0 | 4 | 0 | 0 | 0 | 117106 | 62751 | 8.01009 | 2.34375 | 2.34375 | 42.1406 | 42.1406 |
| 13 | 1 | 10 | 3 | 0 | 21 | 0 | 0 | 1 | 0 | 2 | 0 | 2 | 0 | 119658 | 73001 | 31.505 | 9.55739 | 8.47566 | 122.502 | 114.144 |
| 14 |  | 0 | 3 | 0 | 11 | 0 | 0 | 3 | 0 | 1 | 0 | 1 | 0 | 220861 | 72181 | 17.6407 | 4.01786 | 1.00446 | 270.445 | 114.648 |
| 15 |  | 4 | 5 |  | 11 | 0 | 20 | 0 | 0 | 1 | 0 | 1 | 0 | 164956 | 72861 | 56.4793 | 10.9411 | 10.9411 | 175.711 | 141.264 |
| 16 |  | 4 | 1 | 0 | 20 | 0 | 0 | 4 | 0 | 1 | 0 | 3 | 0 | 104385 | 73001 | 19.7805 | 6.00757 | 6.00757 | 79.9427 | 79.9427 |
| 17 |  | 2 | 1 | 0 | 32 | 0 | 20 | 3 | 0 | 1 | 0 | 3 | 0 | 159236 | 73001 | 70.4132 | 10.4812 | 10.4812 | 125.608 | 121.472 |
| 18 | 1 | 10 | 1 | 0 | 14 | 0 | 20 | 1 | 0 | 3 | 0 | 5 | 0 | 174449 | 73001 | 167.951 | 9.89887 | 9.89887 | 127.747 | 127.747 |
| 19 |  | 2 | 3 | 0 | 13 | 0 | 30 | 4 | 0 | 2 | 0 | 1 | 0 | 19191 | 73 | 75.0229 | 15.177 | 13.6968 | 165.929 | 154.491 |
| 20 |  | 2 | 2 | 0 | 23 | 0 | 60 | 5 | 0 | 0 | 0 | 0 | 0 | 296643 | 60844 | 285.437 | 0 | 0 | 26.0238 | 26.0238 |
| 21 |  | 2 | 3 | 0 | 4 | 0 | 20 | 3 | 0 | 1 | 0 | 5 | 0 | 194236 | 73001 | 165.16 | 9.82403 | 9.82403 | 127.661 | 125.653 |
| 22 |  | 3 | 4 | 0 | 53 | 0 | 40 | 0 | 0 | 2 | 0 | 1 | 0 | 276059 | 72 | 342.879 | 14.9197 | 3.72994 | 183.861 | 85.9598 |
| 23 |  | 5 | 4 | 0 | 22 | 0 | 20 | 2 | 0 | 5 | 0 | 1 | 0 | 174436 | 73001 | 119.025 | 9.58118 | 7.74694 | 109.97 | 95.7969 |
| 24 |  | 6 | 1 | 0 | 23 | 0 | 50 | 2 | 0 | 2 | 0 | 3 | 0 | 282185 | 7300 | 210.695 | 17.6815 | 6.3488 | 190.73 | 86.8397 |
| 25 |  | 4 | 1 | 0 | 03 | 0 | 0 | 3 | 0 | 7 | 0 | 3 | - | 233587 | 69001 | 155.908 | 13.6919 | 5.53234 | 85.4133 | 58.6123 |
| 26 |  | 2 | 4 | 0 | 43 | 0 | 20 | 5 | 0 | 4 | 0 | 2 | 0 | 200739 | 7300 | 234.864 | 8.47566 | 7.39393 | 102.06 | 93.7014 |
| 27 |  | 10 | 5 | 0 | 45 | 0 | 30 | 5 | 0 | 1 | 0 | 3 | 0 | 264330 | 73 | 471.854 | 11.7218 | 10.6801 | 139.575 | 127.359 |
| 28 |  | 2 | 4 | 0 | 52 | 0 | 40 | 4 | 0 | 4 | 0 | 3 | 0 | 296115 | 73001 | 470.161 | 13.7316 | 13.2294 | 155.848 | 151.967 |
| 29 |  | 4 | 5 | 0 | 33 |  | 20 | 4 | 0 | 3 | 0 | 5 | 0 | 234332 | 73001 | 723.025 | 8.08378 | 8.08378 | 106.083 | 106.083 |
| 30 |  | 2 | 3 | 0 | 64 | 0 | 30 | 3 | 0 | 5 | 0 | 4 | 0 | 272248 | 73001 | 663.888 | 9.61211 | 9.61211 | 116.107 | 116.107 |
| 31 |  | 4 | 4 | 0 | 23 | 0 | 50 | 5 | 0 | 6 | 0 | 2 | 0 | 312094 | 7300 | 412.296 | 15.0498 | 14.1425 | 163.472 | 156.461 |
| 32 |  | 8 | 5 | 0 | 44 | 0 | 0 | 5 | 0 | 3 | 0 | 2 | 0 | 372 | 730 | 267.632 | 4.7616 | 3.44324 | 63.8838 | 53.6968 |
| 33 |  | 1 | 6 | 0 | 29 | 0 | 50 | 3 | 0 | 2 | 0 | 5 | 0 | 378973 | 73001 | 719.829 | 14.9899 | 14.5638 | 175.14 | 168.012 |
| 34 |  | 4 | 3 | 0 | 73 | 0 | 0 | 2 | 0 | 7 | 0 | 4 | 0 | 322196 | 73001 | 531.547 | 10.8948 | 10.8948 | 126.39 | 126.39 |
| 35 |  | 5 | 5 |  | 24 | 0 | 30 | 3 | 0 | 11 | 0 | 2 | 0 | 256 | 730 | 481.744 | 9.04252 | 7.83717 | 104.122 | 94.8085 |
| 36 |  | 5 | 2 | 0 | 75 | 0 | 40 | 8 | 0 | 3 | 0 | 2 | 0 | 292482 | 7300 | 569.799 | 9.89887 | 9.89887 | 110.293 | 110.293 |
| 37 |  | 4 | 2 | 0 | 46 | 0 | 80 | 2 | 0 | 5 | 0 | 6 | 0 | 478165 | 730 | 1064.54 | 18.5025 | 7.47614 | 203.43 | 102.348 |
| 38 |  | 4 | 6 | 0 | 54 | 0 | 20 | 7 | 0 | 6 | 0 | 4 | 0 | 261476 | 73001 | 553.676 | 6.53927 | 5.79914 | 84.9275 | 79.2084 |
| 39 | 10 | 0 | 6 | 0 | 63 | 0 | 4 | 2 | 0 | 4 | 0 | 4 | 0 | 347148 | 73001 | 940.039 | 10.5797 | 9.85857 | 124.497 | 118.924 |
| 40 |  | 3 | 11 | 0 | 21 | 0 | 50 | 3 | 0 | 8 | 0 | 7 | 0 | 436807 | 73001 | 1203.19 | 14.1245 | 11.312 | 169.621 | 156.371 |
| 41 | 4 | 4 | 4 | 0 | 17 | 0 | 20 | 7 | 0 | 8 | 0 | 8 |  | 263790 | 73001 | 723.64 | 5.37481 | 5.37481 | 77.0726 | 77.0726 |
| 42 |  | 9 | 3 | 0 | 65 | 0 | 70 | 4 | 0 | 4 | 0 | 4 | 0 | 442759 | 73001 | 926.225 | 14.6809 | 14.6809 | 160.798 | 160.798 |
| 43 | 8 | 80 | 4 | 0 | 85 | 0 | 30 | 7 | 0 | 3 | 0 | 5 |  | 315420 | 73001 | 646.362 | 7.03316 | 7.03316 | 88.3296 | 85.3863 |
| 44 |  | 6 | 9 |  | 65 | 0 | 50 | 3 | 0 | 3 | 0 | 7 | 0 | 455233 | 73001 | 1186.65 | 12.2013 | 11.5621 | 148.088 | 143.149 |
| 45 | 9 | 90 | 7 | 0 | 36 | 0 | 40 | 6 | - | 3 | 0 | 7 | 0 | 374983 | 73001 | 921.185 | 9.4816 | 9.4816 | 117.606 | 117.606 |
| 46 | 6 | 60 | 7 | 0 | 102 | 0 | 40 | 5 | 0 | 6 | 0 | 6 | 0 | 407852 | 73001 | 1227.57 | 9.27548 | 8.96977 | 114.182 | 111.82 |
| 47 |  | 40 | 8 | 0 | 75 | 0 | 30 | 10 | 0 | 5 | 0 | 5 | 0 | 353374 | 73001 | 909.192 | 7.6314 | 6.73379 | 96.7445 | 89.8087 |
| 48 | 6 | 60 | 4 | 0 | 87 | 0 | 50 | 5 | 0 | 8 | 0 | 5 | 0 | 412300 | 73001 | 1152.54 | 9.71965 | 9.71965 | 113.634 | 113.634 |
| 49 |  | 20 | 10 | 0 | 77 | 0 | 70 | - 4 | 0 | 6 | 0 | 6 | 0 | 556079 | 73001 | 1891.64 | 14.5926 | 13.4446 | 169.283 | 160.413 |
| 50 | 8 | 80 | 8 | 0 | 63 | 0 | 60 | 6 | 0 | 4 | 0 | 9 | 0 | 501086 | 73001 | 1143.27 | 12.097 | 12.097 | 145.505 | 145.505 |

Table B.6: Distinct Query Load

## Appendix C Graphs

This appendix gives the graphs plotted for different loads. Section C. 1 shows the graphs for a Balanced Load and Sections C. 2 to C. 5 give graphs for Skewed Loads 1-5. Section C. 6 gives the graphs for a Distinct Query Load. There are three graphs plotted for each type of load: bandwidth usage, average wait time with 5 channels, and average wait time with 3 channels. Discussion of these graphs appears in Chapter 3.

## C. 1 Balanced Load



Figure C.1: Bandwidth Usage for Balanced Load


Figure C.2: Average Wait Time with 5 Channels for Balanced Load


Figure C.3: Average Wait Time with 3 Channels for Balanced Load

## C. 2 Skewed Load1



Figure C.4: Bandwidth Usage for Skewed Load1


Figure C.5: Average Wait Time with 5 Channels for Skewed Load1


Figure C.6: Average Wait Time with 3 Channels for Skewed Load1

