Supplier Selection by Multi-Attribute Combinatorial Bidding

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Abstract

Supplier selection is a challenging decision that has strategic importance for organizations. Cost is no longer the sole factor in the selection of suppliers, and the complexity of this issue arises from the interplay of several situation-specific criteria (such as total cost, CO_{2e} emissions, development time, lead time) as well as the combinatorial nature of this problem. This paper proposes an approach based on combinatorial optimization (integer linear programming) combined with multi-criteria value analysis to establish priorities and trade-offs among the defined criteria for combinatorial bidding. The approach was employed in a real-world decision, the selection of a supplier for a cosmetics packaging set for a new product line. The obtained solution is compared against standard multi-criteria optimization (without a combinatorial auction formulation) and also against single criterion optimization. The paper also reports on the challenges and advantages of applying the framework in the case study.

1. Introduction

In modern supply chain management, the vendor selection process is a strategic activity that requires a comprehensive evaluation of various criteria. Vendor selection is determined not only by cost but according to the potential for other important benefits to the company. For instance, innovations in supply chain management include the consideration of social and environmental concerns. The intersection between sustainable and strategic factors for businesses plays a vital role in the long-term resilience of a supply chain [1]. However, combining these criteria to select and evaluate suppliers is complex and often controversial because of the contradictions among several factors, as well as the combinatorial nature of this problem.

There is a growing demand for supplier selection processes that consider both quantitative and qualitative attributes, and more efforts should be directed to combine these factors in a rational and systematic way [22]. Besides the traditional attributes that focus on cost, quality and service level, an additional requirement has been gaining attention in recent years, which is the incorporation of sustainability attributes in the selection of suppliers, driven by greater consumer awareness about the environment and new regulations stipulated for companies [22,13].

The paper has two potential contributions. First, it suggests a framework for employing combinatorial optimization combined with multi-attribute value analysis to support the choice of providers in competitive bidding. Second, it describes a real world case study, where the approach was employed to support the selection of suppliers for a large Brazilian cosmetics company. Given the dearth of real world applications in this field, its results might be of interest to researchers and practitioners interested in multi-criteria combinatorial bidding.

This paper has the following structure: section 2 presents a brief literature review on multi-criteria and optimization based supplier selection and combinatorial bidding. Section 3 describes the proposed model-based multi-criteria bidding methodology, elaborating on the multi-criteria decision analysis and detailing the optimization model. Section 4 shows the methodology steps applied to the case, while section 5 discuss the results. Section 6 lists the conclusions and suggests topics for further research.

2. Literature review

The supplier selection process usually deals with large quantities of information. Furthermore, many areas within an organization are affected by the final decision, which justifies the use of multi-criteria methods for the problem analysis. Supplier selection problems are complex because of the large number of criteria that can be considered important for decision.

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Since 1960, authors in the academic community and in industry have sought ways to combine these criteria. There are more than 50 important factors involved with sourcing decisions, but the most common are price, delivery performance, product quality and production capacity [24].

Ho, Xu and Dey [14] and Sonmez [22] investigated some decision-making methods in the literature on supplier selection problems and verified that the most frequent integrated approach is a combination using AHP and goal programming. The combination of optimization tools with a multi-criteria method is beneficial because it enables the comparison of the relative importance of the criteria weights while considering resources and system constraints.

An important feature of the supplier selection problem is whether there is an interaction between a proposed set of items (combinatorial bidding, i.e., if it is possible to combine a package in which the acquisition of two or more items from one vendor is more advantageous than an individual purchase). A combinatorial auction is beneficial when there are complementarities (economies of scope) between items, which may differ according to the supplier. In the classic example of transportation bidding, the cost of a particular route to a carrier depends on the carrier flow in the other routes, which encourages a more economical package of routes over single routes [8].

Economies of scope prevail when the cost of a particular product or service is not only dependent on its volume (economy of scale) but also on the existence of another product or service that makes the whole set more advantageous [7]. Hohner et al. [15] applied combinatorial auctions to strategic purchases in a large North American food company, Mars, Inc. These authors show the benefits of this approach when exists synergy between sets of supplies, and is useful when a company needs to buy small volumes of similar material that would not create economies of scale alone. Auctions can be overlain, and suppliers can offer packages for only one item or for two or complete sets, allowing each supplier to choose items according to its interest.

While the conventional combinatorial auctions usually focus only on cost, there has been growing demand for mechanisms to consider several attributes in the decision [4], which is consistent since the complexity of decisions increased over the years. Thereby, conducting a supplier selection process as merely a price competition may have harmful results, as other important factors are hidden in decision making. However, multi-criteria auctions are more difficult to analyze than conventional auctions, due to various attributes and parameter settings needed to build a comprehensive model [4,9]. In this context,

decision support methodologies are crucial to solving the problem, especially when there are three or more attributes involved in the decision [4,23].

Bichler and Kalagnanam [5] found that the multi-criteria techniques MAUT and AHP are commonly used for decision support in this kind of problems, to elicit the buyer's preferences and structure the prioritization of the attributes. In general, models use a weighted linear value function, with attributes that are preferentially independent (i.e., the performance of a criterion does not interfere with the performance of other criteria). It is known that scale ratios for preferences are undefined because there is no zero point to anchor the values [2]. For that is important to define the value functions and value ranges according to the specific case needs, obtained from a consensual decision about priorities for each behavior, which is a soft decision. In contrast to combinatorial auctions with only one criterion, the results of a multi-criteria combinatorial auction are much more sensitive to the preferences of the decision-making group and to the environment [4]. The multi-criteria process also gives the supplier more freedom in formulating the proposals, encouraging competition between suppliers for their strengths and allowing an assessment focused on value added [5].

Although essential to support sourcing decisions, academic literature on multi-criteria combinatorial auctions is still scarce [6], and focuses on theoretical models and software applications, especially e-commerce. The combinatorial problem involving multiple items, multiple suppliers and multiple attributes for a decision has not been widely reported in the literature and is known as puzzle problem [18]. Furthermore, there is a scarcity of real-world applications reported in the literature.

3. A model-based method for multi-criteria bidding

The proposed method to solve the supplier selection problem is a multi-criteria value analysis approach combined with combinatorial optimization. The process flow used to guide such decisions is shown in Figure 1 and is based on the methods of Franco and Montibeller [10] and Belton and Stewart [3]. The process presents three steps to analyzing the problem. The first step is to structure the problem, based on consensus among the group of interest. The second step is the creation of the solution model itself to identify all of the important parameters for the decision and to construct the mathematical model. The third step evaluates the results to determine the best solution in terms of the established preferences.
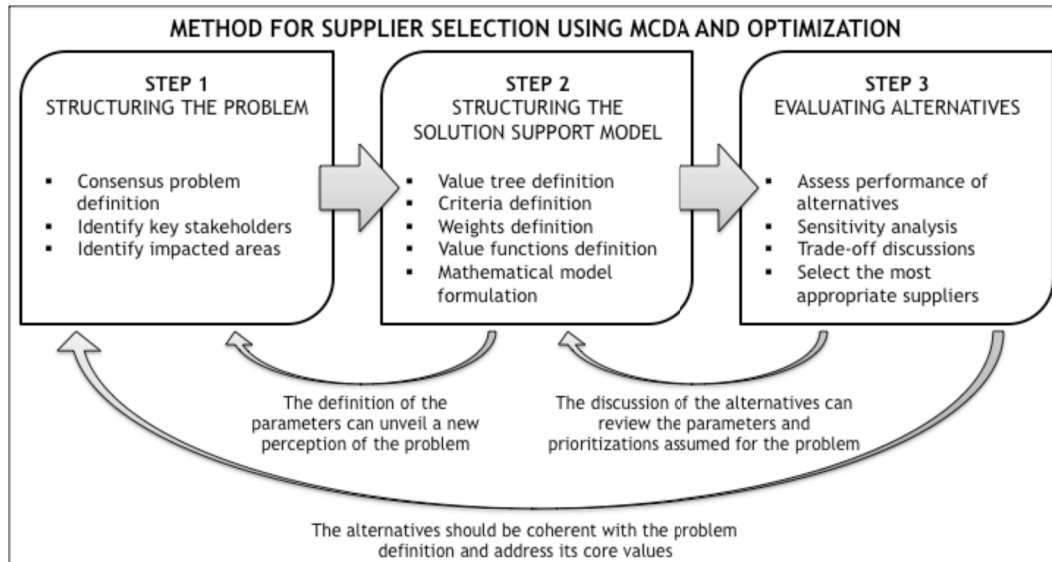
Figure 1. Scheme for the applied method, blending multi-criteria decision analysis (MCDA) and optimization

We now discuss the main definitions used in the multi-criteria evaluation and the formulation of the mathematical model.

3.1. Multi-Criteria Decision Analysis

We adopt a multi-attribute value analysis, a transparent and effective tool designed to assist decision makers in difficult and complex environments. It allows the aggregation of multiple dimensions of desired benefits into a single parameter set by the value preferences [21].

The objective function to be optimized assesses the offers from each vendor based on all of the criteria specified for the case, and should be maximized to obtain the best overall result for the problem and meet the customer's priorities. Each criterion has a designated weight, which indicates its relative value to the decision, and a value function that converts the index to a common unit of measurement; allowing comparison among criteria. Our approach aims to avoid the restrictions of some of the models described in the literature, such as the difficulty of implementing the model in a business environment, the measurement of subjective criteria or the time required to reach a decision [20].

The criteria should be established according to the company goals for the decision in question. These criteria are identified through the consensus of an interdisciplinary group from the organization that represents the various areas interested in the decision. It is important that all of the participants are knowledgeable about the process and have a global business perspective to select effective criteria for the decision. A facilitator is used in most cases to mediate the discussion constructively and to limit the number of criteria so that the model is able to produce a meaningful analysis and does not lose sight of the critical goals.

Thus, for the criteria to be effective, it is necessary to clearly identify the goals of the decision. The value tree is an effective tool to guide this analysis and is used in many multi-criteria decision models, particularly in multi-attribute value theory (MAVT) and multi-attribute utility theory (MAUT) [17]. Franco and Montibeller [10] explain in detail the two classical approaches for structuring a value tree: top-down (in line with value-focused thinking) and bottom-up (following the alternative-focused thinking).

The prioritization of the decision objectives is made by the assignment of weights for each criterion. A higher weight does not mean that a criterion is the most important in the decision. To avoid this mistake, the maximum and minimum parameters of each criterion should be explained in the trade-off process, as argued by Montibeller and Franco [19], to describe the swing-weights procedure. This procedure is one of the most effective methods to be applied in groups and evaluates the relative improvement of each criterion compared to the others, assigning the weights [12].

Each established criterion must have a value function, which represents customer preferences regarding the consequences and trade-offs of each choice in the decision. In a value function, there are no right or wrong values. Rather, the decision-maker's attitude toward the preferences must be represented

through value judgments about the possible consequences of alternatives [16].

The value functions may be established using two traditional methods: bisection, which can be applied only to quantitative criteria, or direct rating, which can be applied to both quantitative and qualitative criteria. In the bisection method, decision makers identify average points in the value function according to the desired amplitude levels for each criterion. In the direct rating method, the group evaluates the relevant levels of each criterion [12,19].

Although the value functions generated are non-linear, they can be modeled in the optimization model as piecewise linear functions supported by binary variables. Thus, they can be applied to integer linear programming models.

3.2. The optimization model

The existence of systemic properties requires the use of optimization modeling to analyze the supplier selection problem. These properties are widely found in single criterion classical optimization, such as the minimization of total costs [20]. For this problem, a traditional combinatorial optimization model was adapted for aggregating multiple criteria. This model can be applied for small problems, with quick computational time for resolution (less than 30 seconds). In this section the theoretical model is showed, and it will be applied on the case on section 4, where the indexes, parameters and variables stated on Tables 1 to 3 will be defined according to the case.

The multi-criteria optimization model is based on a prioritization among the criteria and can be seen in the overall value function shown in Equation 1, which acts as the objective function in the optimization. Each criterion has an assigned value multiplied by its weight value. When all of the criteria are preferentially independent they can be aggregated by a simple weighted sum [17].

Table 1. Description of indexes used in the mathematical model

Index	Description	Interval	Set
I	Item	$i = 1,2,...,I$	$i \in N$
J	Supplier	$j = 1,2,...,J$	$j \in N$
P	Package	$p = 1,2,...,P$	$p \in N$
K	Criterion	$k = 1,2,...K$	$k \in N$
M	Breakpoint of value function for criterion k	$m = 1,2,...,n$	$m \in N$

Table 2. Description of decision variables used in the mathematical model

Variable	Description	Unit
x_{ij}	Allocation of supplier j for item i (binary: 1 if it's chosen, 0 if it's not)	-
$x_j^{\ p}$	Allocation of supplier j for package p (binary: 1 if it's chosen, 0 if it's not)	-
y_j	Auxiliary variable to count suppliers (binary: 1 if it's chosen, 0 if it's not)	-
z_{km}	Auxiliary variable to locate the point m in the value function curve	-
z_{kmi}	Auxiliary variable to locate the point m in the value function curve, for item i	-
a_{km}	Auxiliary variable to identify the point z_{km} in determined place of the value function curve (binary: 1 if it's exists, 0 if it's not)	-
a_{kmi}	Auxiliary variable to identify the point z_{kmi} in determined place of the value function curve for item i (binary: 1 if it's exists, 0 if it's not)	-

Table 3. Description of parameters and constants used in the mathematical model

Parameter	Description	Unit
V	Total value of project purchasing contract	%
w_k	Weight attributed to criterion k (the sum of all criteria weights must be 1)	%
v_k	Value attributed to criterion k	%
$l_{ij}^{\ p}$	Designation of item i by supplier j, according to package p (binary: 1 if it's chosen, 0 if it's not)	-
k_{ij}	Value of criterion k for item i, supplied by j	(depends of the criterion)
$k_j^{\ p}$	Value of criterion k for package p, supplied by j	(depends of the criterion)
$f(k_m)$	Value of value function for criterion k in point k_m, for the set	%
$f(k_{mi})$	Value of value function for criterion k in point k_{mi}, for item i	%
k_m	Value of criterion k in breakpoint m, for the set	(depends of the criterion)
k_{mi}	Value of criterion k in breakpoint m, for item i	(depends of the criterion)
M	Any number large enough, compared to the items total	-
N	Maximum number of suppliers permitted	-
I	Set of items considered in the model: $I = \{1,2,...,I\}$	-
J	Set of suppliers considered in the model: $J = \{1,2,...,J\}$	-
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	(Cont.)	
P	Set of packages considered in the model: $P = \{1,2,...,P\}$	
K	Total value of criterion K for the chosen set	(depends of the criterion)
K_i	Total value of criterion K for chosen item i	(depends of the criterion)
n	Total number of breakpoints of value function	-

Equation 1 is subjected to a series of constraints that have been organized into two groups: overall constraints and criteria constraints. The overall constraints are directly subjected to the main equation, and the criteria constraints that are subject to each criterion equation are shown below. The overall constraints are shown in Equations 2 to 7.

$$Max\ V = \sum_{k \in K} w_k \cdot v_k \qquad (1)$$

Where $0 \leq v_k \leq 1,\ \forall\ k$ and $\sum_k w_k = 1$.

$$\sum_{j \in J} \sum_{p \in P} a_{ij}^p \cdot x_j^p = 1,\ \forall\ i \in I \qquad (2)$$

$$\sum_{j \in J} y_j \leq N \qquad (3)$$

$$\sum_{i \in I} x_{ij} \leq M \cdot y_j, \forall\ j \in J \qquad (4)$$

$$x_j^p \in \{0,1\} \qquad (5)$$

$$y_j \in \{0,1\} \qquad (6)$$

$$x_{ij} \in \{0,1\} \qquad (7)$$

All of the criteria are connected in the model through their specific value functions, as seen in the objective function (Equation 1), and each criterion has a performance equation that can be determined by Equation 8 or 9, depending on how the criterion is accounted for in the set of items. In the first case, Equation 8, the criterion is cumulative (i.e., has a summation characteristic); to analyze the set in this case, we add up the values of each participant item. An example of such a criterion is cost because to get two different items we must pay the price of both, accruing both costs. Equation 9 is for a non-cumulative criterion that has a bottleneck characteristic: the worst performance of the set will affect all of the others. For example, the elapsed time for an activity is enough to

analyze the most critical case because the remaining time occurred in parallel with the higher range.

$$K = \sum_{j \in J} \sum_{p \in P} k_j^p \cdot x_j^p \qquad (8)$$

$$K = \max\left[\sum_{j \in J} \sum_{p \in P} k_j^p \cdot x_{ij}\right], \forall\ i \in I \qquad (9)$$

The criteria constraints address the linearization of their value functions, whose formulations are shown below. For the linearization of the cumulative criteria (Equation 8), the set of items is analyzed globally (i.e., through an analysis of the sum of the whole and not separately for each item), as shown in Equations 10 to 16.

$$v_k = z_{k1} \cdot f(k_1) + z_{k2} \cdot f(k_2) + ... + z_{kn} \cdot f(k_n) \quad (10)$$

$$z_{k1} \leq a_{k1},\ z_{k2} \leq a_{k1} + a_{k2}, \cdots,$$
$$z_{kn-1} \leq a_{kn-2} + a_{kn-1},\ z_{kn} \leq a_{kn-1} \qquad (11)$$

$$a_{k1} + a_{k2} + ... + a_{kn-1} = 1 \qquad (12)$$

$$z_{k1} + z_{k2} + ... + z_{kn} = 1 \qquad (13)$$

$$z_{km} \geq 0 \qquad (14)$$

$$a_{km} \in \{0,1\} \qquad (15)$$

$$K = z_{k1} \cdot k_1 + z_{k2} \cdot k_2 + ... + z_{kn} \cdot k_n = \sum_{j \in J} \sum_{p \in P} k_j^p \cdot x_j^p \quad (16)$$

For the linearization of the non-cumulative criteria (Equation 9), the set of items is considered as the maximum of values, and the linearizing equations are applied to each item, as indicated by Equations 17 to 25. Equations 17 and 18 show two ways of measuring value for this type of criterion. In Equation 17, the lowest value function of all of the items, or the one with the worst performance, is chosen as the total value. In the Equation 18, the resulting function is an average value of each item and applies to items that are independent; the worst performance of one does not affect the other, thus, they are modeled as an average.

$$v_k = \min\left[z_{k1i} \cdot f(k_{1i}) + z_{k2i} \cdot f(k_{2i}) + ... + z_{kni} \cdot f(k_{ni})\right];\ \forall\ i \in I \qquad (17)$$

$$v_k = \sum_{i \in I}\left[z_{k1i} \cdot f(k_{1i}) + z_{k2i} \cdot f(k_{2i}) + \ldots + z_{kni} \cdot f(k_{ni})\right]\Big/ \sum_{i \in I} i$$
$$(18)$$

The following equations compose the remaining restrictions for the value function linearization of non-cumulative criteria.

$$z_{k1i} \leq a_{k1i}, \; z_{k2i} \leq a_{k1i} + a_{k2i}, \cdots,$$
$$z_{kni-1} \leq a_{kni-2} + a_{kni-1}, \; z_{kni} \leq a_{kni-1}; \forall \, i \in I \;(19)$$

$$a_{k1i} + a_{k2i} + \ldots + a_{kni-1} = 1; \; \forall \, i \in I \qquad (20)$$

$$z_{k1i} + z_{k2i} + \ldots + z_{kni} = 1; \; \forall \, i \in I \qquad (21)$$

$$K_i = z_{k1i} \cdot k_{1i} + z_{k2i} \cdot k_{2i} + \ldots + z_{kni} \cdot k_{ni}; \; \forall \, i \in I \quad (22)$$

$$z_{kmi} \geq 0 \qquad (23)$$

$$a_{kmi} \in \{0,1\} \qquad (24)$$

$$K = \max[K_i] = \max\left[\sum_{j \in J}\sum_{p \in P} k_j^p \cdot x_{ij}\right], \forall \, i \in I \qquad (25)$$

The results for the application of this model are presented in the following sections of this paper.

4. Application in the cosmetics industry

The case study focuses on packaging supplier selection for cosmetics companies based on a real-life problem of a large Brazilian company. The competitiveness of this market makes product innovation, both in packaging and in formulations, essential to motivating purchases by consumers. Thus, even successful products are constantly renewed, and their life cycle is short. Packaging undergoes the most frequent updates, with a shelf life of approximately three years.

4.1. Applying the method

4.1.1. Step 1: Structuring the problem. The problem addressed is the selection of suppliers for eight kinds of plastic packaging in a new line of cosmetics. Five suppliers with the ability to manufacture all of the items were invited to participate in the bidding for an overall set evaluation; they were not examined based on individual items. For this evaluation, the project would release all of the items at the same time.

All of the pre-selected providers were considered incumbent, and each item was assigned to a single vendor. Each supplier was encouraged to submit two types of proposal: one proposal for an individual item and one proposal for a set of items to create a package with some advantage over the purchase of individual items. Each supplier thus decided which items are part of the package and uses his or her awareness of synergies to propose efficient packages. The auction setting collects all the proposals in the same deadline, selecting the winner suppliers from the information received at this moment. An improvement in the bid proposal is not allowed in the process.

Compared to the company's regular bidding process (Request For Quotation – RFQ – with open cost structure, for all items), the major differences experienced by the suppliers were the creation of a package bid and the multiple criteria included for winner selection. Some suppliers needed assistance to understand the items package concept, but once this was clear the complexity to fill the RFQ has not

Table 4. Classification of chosen criteria

Criterion	Reason to be a criterion	Independency between items	Accounting characteristic	Set accounting
Cost	Represent the main driver in the purchasing department and is indispensable in a bidding process	Independent	The total cost is the sum of each chosen item cost	Sum
GHG Emissions	Company is committed to reducing its GHG emissions by 33% in 5 years, including those accounted for by its extended supply chain	Independent	The total emission is the sum of each chosen item emission	Sum
Development Time	Time to market is critical in the cosmetics business	Dependent	The total development time is the time for the longest chosen item	Bottleneck (maximum)
Lead Time	Implies stock reduction and responsive supply chain	Independent	The total lead time is the average of all chosen items lead times	Bottleneck (average)

increased. The explicitness of the criteria used to assess the suppliers brought greater reliability to the process, sharing with the supplier the characteristics that are priorities to each item, and also providing them a rational feedback about winning or losing the bid.

The client group involved in the decision is cross-functional, including representatives of the procurement, packaging development, logistics planning, manufacturing, engineering and product marketing areas of the company. It was assumed that all the participants are empowered with knowledge of their areas and the business process and have autonomy for consensual decision making within the group, even though the final decision must be submitted for approval by a higher management level.

4.1.2. Step 2: Structuring the solution support model. The cross-functional group mentioned above identified the essential criteria in the decision process according to the company's fundamental objectives. Initially, members of the group brainstormed all of the desirable characteristics of the packaging of the new products aligned with business goals. The second step was to select which of the resulting list of features were relevant to the evaluation of suppliers. At this stage, it is important to distinguish the features that are mandatory for all suppliers (e.g., contract signing) and which are not criteria because they do not differentiate among the alternative suppliers.

The characteristics chosen by consensus through group discussions form the value tree, with the company's desired objectives for the project. These chosen values will be the criteria specific for this project, showed in Table 4. The criteria can be classified into three categories that determine how they are treated in the mathematical model.

It is important to clarify that development time only affects the time before a product is launched in the market. It considers the manufacture of tools, the acquisition of machinery for assembly or finishing operations, the hiring of labor or outsourced services and the time needed for the required tests so that the product will be ready for large-scale production. In contrast, lead time is the time promised by the supplier for a production batch delivery and includes planning, all stages of manufacturing, and delivery terms if applicable.

The next step is to assess the performance of the company goals through the criteria. That is one of the biggest challenges of the method application, together with obtaining a consensus from the decision group about the behavior of each criterion depending on its value (definition of value functions). Long discussions among the group are normal, but the presence of a facilitator will help to maintain the focus in this phase, and stimulate rational arguments.

The swing-weights procedure was carried out by the cross-functional decision group, to indicate the company's priorities. The criteria tree shown in Figure 2 illustrates the swing-weight bars (on the right), in which total cost is the top priority, followed by GHG emissions (equivalent to 75% of cost improvement), and development time and lead time, which make up 60% and 30% of the cost improvement, respectively, compared to the first improvement. At the end of the process, these priorities are normalized to total 100%, and the final weights of each criterion are shown on the left side of Figure 2.
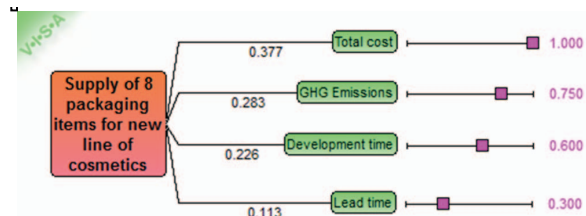

Figure 2. Weights attribution for criteria prioritization

The value functions were determined by the direct rating method, and are presented in Figure 3. The cost function has two breakpoints, at US$ 5.0 million/year and US$ 5.4 million/year, which represent a challenge for the project and its budget. Any cost greater than US$ 5.4 million/year loses more value than to that point. The GHG emissions function is linear because the company sees the increase in emissions as directly proportional to the decrease in value. The development time function has one breakpoint (at 212 days), which is the minimum time for the release of the entire project. The lead time has a breakpoint at 49 days, which is the maximum fixed planning horizon of the company. A longer period would lose more value because the item would require safety stock.

The mathematical model was presented in the previous section and was applied to all of the parameters defined so far. The computer software V.I.S.A (http://www.visadecisions.com) supported the application of multi-criteria decision analysis (MCDA). This approach is not essential (one can build an MCDA model using MS-Excel), but it facilitates problem visualization and the modeling of weights and values.
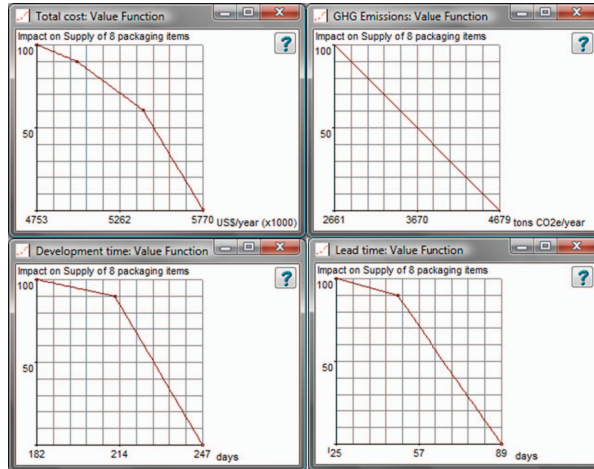
Figure 3. **Value functions for the four criteria**

4.1.3. Step 3: Evaluating alternatives. The data provided in the suppliers' proposals during the project bidding are the alternatives to the supplier selection problem. In the bidding announcement, the suppliers are informed of all of the criteria by which they will be evaluated, so they know the most important parameters for the decision in advance. This practice also clarifies the selection of the winners, which provides feedback for the losing suppliers so they can recognize and improve their weak points.

All of the relevant information about the project is provided in the RFQ, including technical specifications and quality requirements, the minimum production capabilities required, lot sizes, safety stock requirements, the expected shelf life of each item and planned volume variations. Each supplier can submit a proposal for each individual item and two proposals for combinations of items (packages) that promote any benefit derived from the package. Each vendor presented ten proposals. The results and discussion of this analysis are presented in the following section.

5. Results and discussion

The software used to model all of the cases in this paper was the Premium Solver Platform within MS-Excel, in a notebook computer. This tool was chosen to allow the storage of data and modeling in an accessible interface. Due to problem size (five suppliers, eight items), all of the scenario results were obtained within a few seconds, which indicates that this tool can be used as a decision support system in real-time, facilitated decision analysis [11].

The supplier selection problem was also analyzed using single criterion optimization for each established criterion and multi-criteria optimization without a combinatorial auction. Thus, it was possible to compare the results and assess the relative success of multi-criteria combinatorial optimization. The graph in Figure 4 illustrates the supremacy of the multi-criteria combinatorial decision over others optimizations based on the weights defined for the problem.

The graph shows that the multi-criteria combinatorial optimization solution (the gray, solid line) best balances all of the factors and benefits of the weighted solution, as illustrated in the right-most column, titled "Multi-criteria weighting". The multi-criteria solution attains a value of 88%, whereas the other solutions are between 52% and 67% for the single criterion solutions and a maximum optimal of 83% for the multi-criteria non-combinatorial. This result illustrates that single criteria optimization, which looks at single aspect of a decision, penalizes other factors that are important to the company.

To assess the use of combinatorial bidding in a multi-criteria optimization, we evaluated the outcome of each optimization by varying the number of selected suppliers, limiting N (number of suppliers) in Equation 3[1] to between 1 and 5. All of the other parameters and variables were maintained. The result is shown in Figure 5.
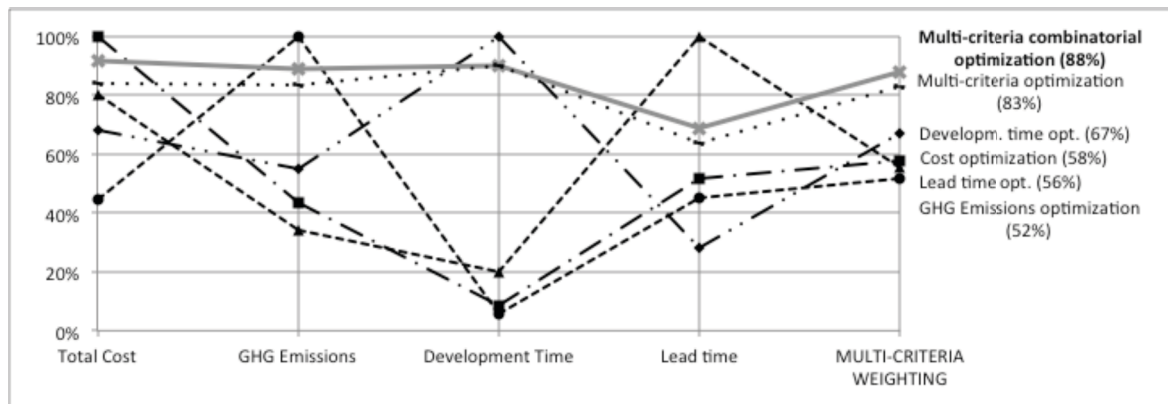

Figure 4. Graph for the decision value comparing the multi-criteria solution with the non-combinatorial and the single criterion ones

When we restrict the number of winning suppliers, the combinatorial multi-criteria solution remains superior to the non-combinatorial solution at all points. This result was expected because the combinatorial optimization offers benefits in cost and GHG emissions when all other parameters remain the same. Thus, it is possible to deduce from the graph that at least one combinatorial package was chosen at all points in the combinatorial multi-criteria solution; otherwise, there would be a combined single point for both cases at some point on the curve, which does not occur. With 5 suppliers, which is the optimal solution for both cases, the economic gain for the combinatorial solution is US\$ 120,000 per year. It also generates an annual saving of 113.3 tons CO2e.
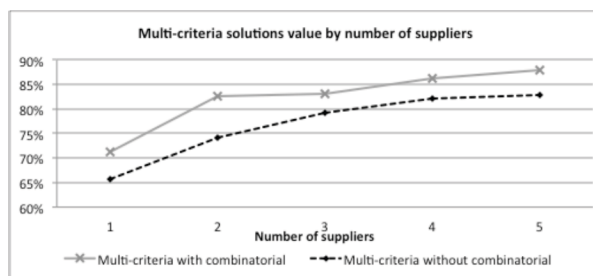

Figure 5. Comparison of both solutions, varying the number of selected suppliers

Observing the graph in Figure 5 it is apparent that the selection of five suppliers, the optimal solution, does not present a great advantage over the choice of four suppliers, for both cases. This is an important consideration, since the use of a smaller base of suppliers means having lower management complexity and therefore lower cost, which should be balanced with the risk of shortages due to the dependence on fewer partners.

Although not presented here, a sensitivity analysis could be performed by changing the criteria weights and evaluating the models by modifying their relative value to verify the validity of the initial solution compared to a hypothetical scenario with another set of priorities for the same criteria.

6. Conclusions

This paper proposed a solution to the supplier selection problem using combinatorial multi-criteria optimization based on multi-attribute value function assessment. The framework allows the selection of criteria, whether quantitative or qualitative, identified according to the organization's priorities and may be customized for many different applications. The case presented was conducted for a Brazilian cosmetics company that must account for sustainability in its decisions and consider economic, social and environmental factors. However, the framework is generalizable to other sectors and industries because different institutional environments face complex decisions and must consider multiple criteria.

The paper's main contribution is in its use of combinatorial optimization integrated with multi-criteria value analysis applied to a real decision, which is rather underexplored in the academic literature. The results show that there are many benefits in this approach, such as in a line of products in which the items have similar features, and that several synergies should be exploited to obtain gains for both buyers and vendors. For the company, the main lesson learned is how a purchasing decision can change considering multiple criteria, especially considering a long term scenario.

Despite the model developed being practical and easy to interface in the routine of the company, one of the obstacles encountered was the creation of the model at the same time of the packaging development. The time taken to discuss and come to consensus on values, attributes, weights and value functions for the first time in a group was demanding, a fact that led to develop the model in a mixed approach, initially by facilitate mode (group with the help of a facilitator) and at the end by specialist mode (decisions taken by the buyer specialist). The method was used not used by the company in a continuous way due to that reason, but some alternatives are being discussed to implement a faster multiple criteria decision support system in purchasing.

Further research on the problem of supplier selection may encourage the development of a real-time decision support system for corporate purchasing based on MCDA. Templates could be built for the situations companies usually face (sets of criteria, weights and value functions) to reduce the time spent in group discussions.

7. References

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