When Common Knowledge Becomes Common Doubt – Modeling IT-Induced Ambiguities about the Strategic Situation as Reasons for Flash Crashes

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Abstract

Flash crashes, perceived as sharp drops in market prices that rebound shortly after, have turned the public eye towards the vulnerability of IT-based stock trading. In this paper we explain flash crashes as the result of actions made by rational agents. We argue that information technology, which has long been associated with competitive advantages, may cause ambiguities with respect to the game form that give rise to a Hypergame. We employ Hypergame Theory to demonstrate that a market crash constitutes an equilibrium state if players misperceive the true game. Once the ambiguity is resolved, prices readjust to the appropriate level, creating the characteristic flash crash effect. We also discuss endogenous and exogenous mechanisms that may alleviate the threat of a flash crash and present possible options for future research.

1. Introduction

On May 6, 2010, the effects of the "Flash Crash" rippled through U.S.-based equity markets and beyond. Indices dropped by several percentage points, just to rebound almost as quickly [1]. In the immediate aftermath, this has sparked several hypotheses explaining why the crash occurred. Easley et al. [2] highlight some of the most prominent: a quickly debunked "fat-finger trade", which is a trader entering the wrong number for a transaction; technical reporting difficulties at the NYSE; currency movements between dollar and yen; and predatory quote stuffing. Several of these hypotheses assume irrational behavior on behalf of the traders. In this paper we show that flash crashes can be explained as a result of rational behavior resulting from IT-induced ambiguities about the strategic situation.

While at first it may appear strange to believe that the simultaneous selling frenzy of many traders marks a mutual best-response equilibrium, we argue that ambiguity in the strategic environment changes the nature of the game such that these flash crashes are bestresponse equilibria. Our key assumption is that in recent years information technology has eroded the competitive advantage dominant traders held over the rest. As the reaction time in financial markets dropped to a few milliseconds, the strategic environment changed. It turned from a situation that can be described as sequential game with a leader informing the actions of other traders towards a purely simultaneous game where all traders literally trade at the same time. We investigate situations where traders do not recognize this shift in the strategic situation. This ambiguity violates the classic common knowledge assumption that all traders know the accurate strategic situation they are acting in. As a consequence of this ambiguity, flash crashes can be explained as best-response equilibria. All traders play the dominant strategy in their subjective (sequential) game – what they perceive to be the true game - while in reality the game is simultaneous. Misperceptions by many traders at the same time may then result in a flash crash.

Thus, our approach can explain flash crashes as a result of ambiguities in the strategic situation of the traders. Furthermore, as flash crashes require the misperception of many traders at the same time, we can show why such events occur only very rarely.

The remainder of this paper is structured as follows. In Section 2 we will review related work. This includes research on flash crashes, the erosion of IT-induced competitive advantages, and hypergame theory which we will use later in the paper. In Section 3 we first further motivate our assumption that the steady improvement of information technology in electronic markets may create ambiguities in the underlying strategic game. We then theoretically analyze the hypergame caused by these ambiguities. This allows us to establish conditions under which flash crashes may occur. Section 4 discusses implications of the model, as well as mechanisms to detect flash crashes in advance. Section 5 concludes with a summary and an outlook on future research.

2. Related Work

The flash crash on May 6, 2010, caused extensive investigations by the U.S. Securities and Exchange Commission [1]. They identified liquidity issues as a result of large sell orders as a possible reason for this crash. Easley et al. [2] provide further evidence that liquidity problems were developing in the days before the event. Kirilenko et al. [3] argue that the response of high frequency traders to selling pressure further amplified market volatility. Crashes in financial markets have also been attributed to herd behavior. Lux [4] proposes a cyclic model that explains bubbles and crashes as the result of such behavior. Barlevy and Veronesi [5] attribute herd behavior to rational, but uninformed traders. We argue that flash crashes can occur even if traders are rational and equally well informed about market conditions. We suggest, however, that traders may be unaware of ambiguities in the underlying game form.

Consider, for instance, Easley et al. [6], who highlight that while technology has enabled some traders to be faster than others during the last two centuries, speed is no longer a defining characteristic under a high frequency trading paradigm. This relates to the notion by Carr [7] that questions sustainability of ITinduced competitive advantages. Such advantages provided by information technology have been extensively discussed in information systems research (e.g. [8-10]). As we consider a market that is highly information sensitive - the financial market - such a competitive advantage would materialize as the ability to react faster to new information. However, it is beyond doubt that the emergence of the Internet and the technologies surrounding it, from smartphones to social networks, have fundamentally changed the way we gain access to and process new information-we can find realtime news on anything at any time basically anywhere [11, 12]. Aldridge [13] provides further evidence that the influence of information technology is not only limited to the general public, but extends to the financial sector, as well. The facts high frequency traders are limited to a few communication protocols and are often fast but quite simple algorithms [14] further suggest a homogenization of software capabilities. This is amplified by the replacement of human factors, which used to provide a large variance depending on individual capabilities. Therefore, we argue that it has become more and more difficult to create competitive advantages in such information-sensitive mechanisms, since everybody uses similar information channels and processes this information with similar software employing similar algorithms.

Unarguably, those agents who were best able to collect and process new information have always been

at an advantage in financial markets by reacting first and making the best deals. The role of information arrival, i.e. whether agents receive news simultaneously or sequentially, has been researched for financial markets by Copeland and Friedman [15]. More recently, Groth [16] analyzes the relationship between news-related liquidity shocks and automated trading engines. Xu and Zhang [17] investigate how social media has changed the information environment in financial markets. Both studies show that information technology has fundamentally altered information dissemination.

In the following sections we will make the argument that the eroding competitive advantage causes games to turn from sequential to simultaneous, as the first mover advantage that could be exploited over the centuries is all but disappearing. We analyze what happens when agents are not aware of this game form ambiguity using Hypergame Theory, which was first proposed by Bennett [18, 19]. It was originally used to model conflicts where each party has different perceptions of the game being played. More recently, Sasaki and Kijima [20] have compared hypergames with Bayesian games. While they show that most concepts from hypergame theory can be captured by a Bayesian game, they do point out that hypergames exhibit a persuasive simplicity. This often leads to them being more intuitive. In the following model we exploit this intuitiveness to illustrate the effects of game misperception.

3. Modeling the Strategic Effect of Information Technology

The central assumption in this paper is that in some instances information technology has changed the game that is played in financial markets from sequential to simultaneous. However, we argue that agents may not always be aware of this – thereby we explicitly relax the assumption that the type of game is common knowledge. Instead, some players continue to play a subjective sequential game and choose their strategies according to this game.

Before we can analyze the impact of those misperceptions, we first need to explain why some games are sequential and others are simultaneous.

3.1. When Technology Causes Ambiguities

Consider a simplified scenario of shareholders of a company trading at a stock exchange. The company is publishing its quarterly financial report, thus supplying shareholders with new information. Without losing the general implications of our research later in the paper we make the following assumptions:

- There are homogenous expectations among the shareholders which were built during the weeks preceding the event.
- These expectations are fully reflected in the portfolio of the shareholders.
- The newly released information is substantially worse than expected, inducing portfolio adjustments.
- Agents make a binary decision between selling and keeping their entire stock in the company.
- Shares are equally divided among agents.

Now consider how new information is received and processed. In the past news were disseminated by, for instance, television, radio, telephone, or word of mouth and processing used to be executed by human workers. Today the central instrument for information dissemination is the Internet and processing has often been replaced by computer algorithms. The "fastest" player is the one who best collects and processes new information. This player holds an advantage over its competitors, which we refer to as the informational edge. This is the time between the reaction of the information leader and the fastest follower. The reaction of the leader must be observable by the follower to affect the decision of the follower, for instance, by being reflected in a change of the market price. Yet, in many cases this threshold used to be sufficiently low that the leader holds a first-mover advantage.

The rise of the Internet and the cosmos of information technology surrounding it have undoubtedly changed the way we acquire information and the speed at which we do this. This certainty has even spread beyond academia into popular culture. This culminates, for instance, in a famous essay by Carr [21] where he describes the Internet as "the conduit for most of the information that flows through my eyes and ears and into my mind." As for financial markets, technology has improved information dissemination, the quality of financial analysis, and the speed at which market participants communicate [22].

This last statement emphasizes the influence information technology exerts on game form determination. Information technology increases the speed of information dissemination and processing – similar to the argument made by Aldridge [22]. Since it does so for most, perhaps even all traders, it also substantially decreases standard deviations and, thereby, the informational edge a potential leader may hold over its followers. While information technology also accelerates market technologies, there is no guarantee that this happens to the same degree as with player technologies. Hence, this *acceleration effect* may remove the first-mover advantage and turn a game that used to be sequential into a simultaneous one.

Also, consider the influence of information technology on the variance of player technologies independent from acceleration. We have previously discussed the large variety of channels used for the acquisition of information before the Internet. Combined with the differing cognitive capabilities of involved human workers resulted even on a normalized level in a much higher variance of response times when compared to today. Nowadays, the Internet is the ultimate information channel and certain (automated) software packages dominate information processing. Therefore, we further argue that the dominance of the Internet and specific software packages causes a homogenization of response times across players, diminishing the informational edge and increasing the likelihood of simultaneous games.

Hence, information technology may change a game from sequential to simultaneous through the combined effect of acceleration and homogenization. The common knowledge assumption would require that players are aware of this change and always know what type of game they play. However, the common knowledge assumption is difficult to argue for in the first place, since it is more grounded in a desire for mathematical tractability of games rather than an approximation of Information reality. acquisition, information technology, and information systems have long been associated with competitive advantage - in the context of information-sensitive businesses the informational edge. It is easy to argue that this mindset may still prevail even when companies are no longer creating edges but merely keeping pace. The central problem is that from a leader's perspective the sequential and simultaneous games are indistinguishable. Thus, players may believe to be leaders in a sequential game simply because that is how it has always been like - an argument that relates to game theoretic focal points [23]. In this case, players assume the type of game that seems natural, i.e. the sequential game. Nevertheless, even if players are aware of the ambiguity, they may simply misperceive the game.

In the next subsection we briefly discuss the sequential and simultaneous games if correctly perceived before we use hypergame theory to investigate the influence of misperceptions and how they can explain market crashes.

3.2. Sequential and Simultaneous Games

If the informational edge exceeds the market specific threshold value, the sequential game occurring in the aftermath of information dissemination and processing is essentially a Stackelberg game with a leader and follower(s). Without loss of generality we set N, the number of players, equal to 2 for the time being. Thus,





there are only two players that own stock in the company (or that care about the report) and both know what selling or keeping their shares entails for the outcome. Figure 1a illustrates this game with payoffs expressing a preference structure.

While the payoffs are not set in stone, we argue that they make sense for the example introduced above. If both players keep their shares, they basically ignore the information from the disastrous financial report (the event, Θ) and it is reasonable to expect that the value of their shares will decrease in the future as the company struggles compared to the price before publication of the report. If player 1 sells, it can get out while prices are still reasonably high, thus utilizing the informational edge. The same applies to player 2 if player 1 did not sell. However, if player 2 sells after player 1 did so, the market has processed the action of player 1 and prices have dropped substantially.

There is a clear subgame perfect Nash equilibrium within the game, which can be easily traced by backward induction. For player 2 it is always better to play the opposite strategy of player 1 (red markings) since A > B in the left information set and B > C in the right one. Player 1, anticipating this, sells its shares (blue markings). In this particular example player 1 does not even need to anticipate, since player 2's action does not change the outcome for player 1, it is A in either case.

The opposite case occurs if the informational edge fails to exceed the threshold value. The resulting simultaneous game is depicted in Figure 1b. The only change in outcome compared to the sequential game that can be reasonably argued is for the event of both players selling. In the simultaneous game there is an immediate oversupply and the associated price drop. Essentially the leader from the sequential game cannot monetize its informational advantage, such that both players sell at the follower's price. Since shares are equally divided among shareholders, the game is also symmetric.

The changes in strategies are more fundamental. The resulting simultaneous game is similar to a Game of Chicken [24] in that there are two Nash equilibria in pure strategies when players choose opposing strategies (blue background). This is illustrated by the best responses in Figure 1b (blue and red as the best response for players 1 and 2, respectively). Additionally, there is an equilibrium in mixed strategies, where the payoffs determine the probabilities to sell or to keep.

The implication of the latter equilibrium is that, absent knowledge of the others' strategic decisions, players are likely to mix the strategies they act on. Thus, given a large N the probability that all (most players) sell, thereby inducing a crash, is significantly reduced. However, this is all under the assumption that players are aware of the change in the type of the game.

3.3. The Hypergame for Two Players

Consider, again, Figure 1a. The leader, in this case player 1, decides upon its action given the information of the event Θ . The follower can observe this action and select its own action given the information of Θ and the action by player 1. In the simultaneous game all players react exclusively to Θ , as no other actions are observable. However, reacting exclusively to Θ is the leader's mark in the sequential game. Hence, if there is uncertainty about the game form, players may misperceive the simultaneous game and instead believe they are leaders in a sequential game. To model this misperception we use hypergame theory [18, 19]. Sasaki and Kijima [20] describe a hypergame as a collection of subjective games where "each agent believes that it is common knowledge among all the agents (who she thinks participate in the game) that the game they play is her own subjective game". More precisely, they define a hypergame $H = (I, (G^i)_{i \in I})$ with a finite set of agents I and each agent's subjective game $G^i = (I^i, A^i, u^i)$, where:

- *Iⁱ* is the finite set of agents perceived by agent *i*, assuming *Iⁱ* ⊆ *I*.
- $A^i = \times_{j \in I^i} A^i_j$ with A^i_j as the finite set of agent j's actions perceived by agent *i*.
- $\pi^{i} = (\pi_{j}^{i})_{j \in I^{i}}$ with $u_{j}^{i}: A^{i} \to \mathbb{R}$ as agent *j*'s payoff as perceived by agent *i*.

Furthermore, they define $\alpha^* = (\alpha_i^*, \alpha_{-i}^*) \times_{i \in I} A_i^i$ as a hyper Nash equilibrium of H iff $\forall i \in I$, $\alpha_i^* \in N_i(G^i)$, where $N_i(G^i)$ is the set of Nash actions of player i in its subjective game. An action $\alpha_i^* \in A_i^i$ is player i's Nash action in G^i iff there exists $\alpha_{j\neq i}^i \in A_{j\neq i}^i$ such that $(\alpha_i^*, \alpha_{j\neq i}^i) \in N(G^i)$ with $N(G^i)$ as the set of Nash equilibria of subjective game G^i . Thus, a hyper Nash equilibrium is defined as an outcome, where every player chooses an action that may result in a Nash equilibrium in the subjective game of that particular player. The set of hyper Nash equilibria of hypergame H is called HN(H).

Adopting the definition by Sasaki and Kijima [20] we derive the following hypergame when both players misperceive the simultaneous game as a sequential game:

with

and

$$H = (I, (G^{1}, G^{2}))$$

h $I = N = \{1, 2\}, \qquad G^{1} = (I, A^{1}, u^{1}),$
 $G^{2} = (I, A^{2}, u^{2})$
d $A_{i}^{i} = \{K, S\}, \qquad A_{j\neq i}^{i} = \{KK, KS, SK, SS\}$
 $\forall i \in I$

From the preferences we derive the following payoffs:

 $\pi_i^i (\alpha_i = K) = B \qquad \pi_i^i (\alpha_i = S) = A$ $\pi_j^i (\alpha_i^i = KK) = B \qquad \pi_j^i (\alpha_i^j = KS | \alpha_i^i = K) = B$

$$\pi_j^i(\alpha_j^i = KS \mid \alpha_i^i = S) = C$$

$$\pi_j^i(\alpha_j^i = SK \mid \alpha_i^i = K) = A$$

$$\pi_j^i(\alpha_j^i = SK \mid \alpha_i^i = S) = B$$

$$\pi_j^i(\alpha_j^i = SS \mid \alpha_i^i = K) = A$$

$$\pi_j^i(\alpha_j^i = SS \mid \alpha_i^i = S) = C$$

$$A > B > C$$

Figure 2 provides an illustration of the subjective game G^i .

Evidently, both subjective games are identical to the sequential game in Figure 1a with the only difference being that each player sees itself as leader in its own subjective game. Hence, the best responses and Nash equilibria reflect Figure 1a, as well. Note particularly that $\pi_i^i (\alpha_i = S) > \pi_i^i (\alpha_i = K) \forall \alpha_{j\neq i}^i \in A_{j\neq i}^i$. No matter what the other player does, player *i* always prefers selling within its subjective game – selling is the dominant strategy and, thereby, the only Nash strategy of player *i* in the subjective game. Thus,

$$\begin{aligned} \pi_i^i(\alpha_i = S) > \pi_i^i(\alpha_i = K) \; \forall \; \alpha_{j \neq i}^i \in \; \mathbf{A}_{j \neq i}^i \\ \Rightarrow \; N_i(G^i) = \{S\} \; \; \forall i \in I. \end{aligned}$$

If both players misperceive the simultaneous game as sequential, they consider themselves to be the leader in their respective subjective game. From the definition above we derive a unique hyper Nash equilibrium as

$$N_i(G^i) = \{S\} \quad \forall i \in I \quad \Rightarrow \quad HN(H) = \{(S,S)\}.$$

The implications of this result are illustrated in Figure 3a. If both players perceive a sequential game and themselves as leader, the true game must be simultaneous. Otherwise, one player would go first and the other player would realize that it is the follower. However, the collective misperception causes the worst possible outcome in the simultaneous game, since all players are selling and prices crash. Notice that in the objective game itself this outcome does not constitute a Nash equilibrium, as either player would improve its position by deviating. Even when both players deviate, the outcome would still be preferable. However, in the game as each player perceives it, selling is the dominant



Figure 2. Subjective Game Gⁱ



Figure 3. Hyper Nash Equilibria with Both Players and Only One Player Misperceiving

strategy. Thus, we conclude the central insight of our model:

If players misperceive the simultaneous game due to game form ambiguity, each player believes to be the leader in a sequential game. Playing the leader's Nash strategy in the perceived sequential game induces a disastrous outcome in the underlying simultaneous game.

Such an outcome can, for instance, be a flash crash. Players react to news, but do not consider the simultaneous reaction of other players, thereby inducing an unwarranted price decline.

However, so far we have assumed that all players are misperceiving the game. Figure 3b shows that implications become less clear once this assumption is dropped. In this particular case player 1 misperceives the game and plays the sequential Nash strategy while player 2 correctly perceives the simultaneous nature of the game. From Figure 1b we can see that both selling and keeping constitute Nash strategies for player 2. Hence, as evident in Figure 3b, there are two Hyper Nash Equilibria in the game. Intuitively, this seems to soften the negative impact of player 1's misperception. We will further analyze the implications of only a subset of N misperceiving the game in the next subsection.

3.4. Misperceptions in Games with Many Players

In the model above we analyzed a two player case and showed the detrimental effects of game misperception caused by game form ambiguity. However, in most financial markets two player games are arguably rare. If there are many players and only a small subset misperceives the game, this may already lead to suboptimal outcomes. To calculate this we, again, need to consider the sequential game and the simultaneous game to estimate the effect of misperceptions. For this we redefine our set of players as

$$\mathbf{P} = \{p_1, \dots, p_n, \dots, p_N\}$$

where each player is defined by the tuple $p_i = (a_i, t_i) \forall 1 \le i \le N$. a_i is the action of player *i* and is defined as above as $a_i \in \{S, K\}$, i.e. either selling or keeping. $t_i \in \{1, ..., t_{max}\}$ is the position of player *i* in the game. A strictly sequential game is a game where all players have different positions, which implies that a player can observe the actions of all players with a smaller *t*-value. A strictly simultaneous game is a game where all players have the same *t*-value, i.e. no player moves first.

We further define M_{τ} as a subset of *P* with

$$A_{\tau} = \{ p_m \in P \mid s_m = S \land t_m \le \tau \}.$$

Thus, M_{τ} contains those players, who sold before or at position τ . We can now generalize the payoff functions from the two-player game above as:

$$E(\pi_j | s_j = K) = \nu,$$

$$E(\pi_j | s_j = S \land t_j = \tau) = w_0 - w_1 |M_\tau|$$

$$w_0 - w_1 > \nu > w_0 - w_1 N.$$

The argument is similar to the one we made earlier in the paper. The payoff for keeping represents the longterm value of the stock, which is expected to be less than before the negative new information. If only a few people are selling, the price has not yet incorporated this negative information. Therefore, they get a payoff above v. However, if many players are selling, prices drop eventually below v. Note that the inequality above reflects the preference structure A > B > C from the two-player game. In a strictly sequential game every player knows its position and the actions of all players before. Hence, every player can calculate $E(\pi_j | s_j = S \land t_j = \tau)$. If the resulting value exceeds v, it is a strictly dominant strategy for this player to sell, since payoffs do not depend on the actions of the following players. Similarly, if the value is less than v, the dominant strategy is keeping. We assume that all players weakly prefer selling if the value is equal to v. It is important to note that it is always dominant for the player in the first position to sell, since $w_0 - w_1 > v$. Hence, the subgame perfect Nash Equilibrium is for the first t^* players to sell, where $w_0 - w_1t^* \ge v > w_0 - w_1(t^* + 1)$, and for all other players to keep.

In a strictly simultaneous game $t_i = 1 \forall 1 \le i \le N$, i.e. all players are at the same position. We can calculate the Nash Equilibrium in mixed strategies by setting the expected payoff under each strategy equal, resulting in

$$v = w_0 - w_1 E(|M|).$$

E(|M|) is the expected number of selling agents, which depends on the probability $q_j = \Pr(s_j = S)$. Due to the strict symmetry in payoff functions, we know that $q_j = q \quad \forall \quad 1 \le j \le N$. Thus, we can calculate the probability of agents selling from

$$v = w_0 - w_1 \sum_{i=1}^{N} \left[i \binom{N-1}{i-1} q^{i-1} (1-q)^{N-i} \right]$$

From this we get $q^* = \frac{w_0 - (v + w_1)}{w_1 (N-1)}$ as the probability to sell in the mixed strategy Nash equilibrium. Naturally, the expected payoff for any player is v, the aggregate payoff is Nv.

However, consider the case when player p_1 misperceives the game, such that it always plays $s_1 = S$. Its expected payoff is still v, since all other players mix their strategies according to the Nash equilibrium. Yet, the expected payoff to all other players changes, since p_1 does not mix. More precisely the expected payoff is

$$\begin{split} E(\pi_{j\neq 1}) &= (1-q^*)v + q^* \left[w_0 - w_1 \left[2 \right. \right. \\ &+ \sum_{i=1}^{N-2} \left[i \left({N-2 \atop i} \right) (q^*)^i (1 \\ &- q^*)^{N-i-2} \right] \right] \end{split}$$

While it can be shown that this is always less than v, we omit doing so for brevity. Since the payoff does not change for the single misperceiving player, this implies that the aggregate payoffs of all players decrease. Hence, we can conclude that in a symmetric game where

every agent plays the mixed Nash strategy, a single agent misperceiving the game does not decrease its own payoff, but the payoff of every other agent.

3.5. Numerical Examples

First, consider again the two-player case discussed above. We can satisfy the preference structure in Figure 3 by setting A = 2, B = 1, C = 0. This also satisfies the condition from the previous subsection with v = $1, w_0 = 4$, and $w_1 = 2$. The resulting simultaneous game is illustrated in Figure 4.

Plugging the values into the equation above yields $q^* = 0.5$ as the probability to sell in the mixed strategy Nash equilibrium. Hence, the expected payoff in the simultaneous game would be 1 for either player. If player 1 misperceives the game, its payoff is still 1, while player 2's payoff decreases to 0.5, such that the aggregate payoff is 1.5. If both players misperceive the aggregate payoff is zero. This supports the suggestion from Figure 3, that aggregate payoff decreases in the number of misperceiving players.

To further investigate this we construct a threeplayer game with $v = 3, w_0 = 6$, and $w_1 = 2$, which is illustrated in Figure 5. In this case players mix with a probability to sell of $q^* = 0.25$. If there is no misperception, the expected payoff is naturally 3 for every player. Assuming that player 1 misperceives and plays *S*, its payoff is still 3. The payoffs of players 2 and 3 are $E(\pi_{j\neq 1}) = 2.625$. Thus, the aggregate payoff is 8.5, but the loss is borne by the players who are not misperceiving.

However, considering the case of two players misperceiving shows that playing the subjective sequential game has detrimental effects on the misperceiving players, as well. If players 1 and 2 are misperceiving, payoffs can be seen in the bottom row of the right panel of Figure 5. Since player 3 is still mixing with $q^* = 0.25$, its payoff is 2.25, while the payoff of each misperceiving player is 1.5. The decrease in expected payoff is much more pronounced for players who misperceive than for the one who is not. Aggregate payoff is, thus, equal to 5.25. Obviously, if all players are misperceiving, aggregate and each player's



Figure 4. Numerical Example of Two-Player Game



----- Information Set

Figure 5. Numerical Example of Three-Player Game

individual payoff are equal to zero. This leads us to the following conjectures:

- There is a tipping point in the number of misperceiving players, after which the players who misperceive suffer more than those who are not.
- The number of misperceiving players has a nonlinear and increasing influence on the overall welfare loss.

We will consider these conjectures in more detail in future research. However, the examples presented in this section show that even if only a small subset of agents misperceive the game that is objectively played, this has a detrimental impact on the aggregate performance.

4. Implications and Discussion

The model introduced in the previous sections provides a cautionary tale against ignoring game theoretic implications of the continuing adoption of information technology. It argues that competitive advantages in information sensitive mechanisms are more difficult to create than ever and illustrates how this may lead to ambiguities concerning the underlying game form. It also illustrates the possibly detrimental effects of failing to adapt strategic behavior to these ambiguities. In this section we will discuss several paths for future research concerning implications and extensions of our model.

The emergence of algorithmic and high-frequency trading has fundamentally changed stock trading. While such automated traders have been associated with beneficial effect like higher increased market efficiency and decreased intraday volatility [25, 26] during generally rising stock prices, their impact during financial turmoil is less clear. Particularly the flash crash on May 6, 2010, has caused several investigations into the role of high-frequency traders [1, 2, 6]. Aside from crashes it is worthy to investigate the general effect of game misperception on volatility. Our model suggests that even if only a small fraction of agents misperceive the game, there is an overreaction to unexpected news.

Nevertheless, our model provides an intuitive explanation for flash crashes, i.e. sudden drops in market prices which quickly rebound once the game form ambiguity is resolved. Further empirical research is necessary to investigate the explanatory effect of our model regarding particular flash crash on May 6, as well as the general effect of game misperceptions on market prices. A central criterion is to identify markets with price functions where the price decreases in the number of simultaneously made offers. For this it must be possible to make quasi-simultaneous offers in the first place.

We also need to emphasize the importance of the "event", i.e. the news, in our model. The impact of unanticipated new information could most recently be observed during another, albeit smaller, flash crash on April 23, 2013. After the Twitter account of the Associated Press had been hacked, a tweet reporting explosions at the White House and an injury of Barack Obama caused the Dow Jones industrial average to drop by 143 points [27]. This event illustrates the vulnerability of financial markets, particularly when algorithmic traders play simple trigger strategies, which relate to the sequential game in our model. Once new information is detected, shares are sold without regard to simultaneous actions of other trading agents and aggregated effects on the market.

Our model also provides implications on how trading algorithms can be improved to decrease the probability of flash crashes in possibly simultaneous markets. The central idea is to detect whether the realization of an event substantially differs from expectations concerning the event. One such warning system is the VPIN metric proposed by Easley et al. [2], which measures toxicity directly from the order flow. The VPIN metric is the ratio of average unbalanced volume to total trading volume. While Easley et al. [2] use it to detect flash crashes based on liquidity evaporation, the VPIN metric can also be used to measure expectations within a particular market. Once a news event occurs that contradicts these expectations, traders should act more cautious. A similar effect can be achieved by using sentiment analysis. Trading algorithms may already use sentiment analysis to detect news that trigger a particular strategy. This can be expanded to employ sentiment analysis to detect the general expectations concerning the market. Once an event contradicting these expectations occurs, the algorithm may still trigger a particular strategy. However, this strategy should reflect the probability of game form ambiguities.

In our future research we will investigate the empirical support for our model in appropriate markets. We will also research the influence of warning mechanisms as discussed above. Furthermore, we will explore the use of hypergames as a general methodology to assess and evaluate mechanisms. This does not only relate to misperceptions, but also misconstructions of mechanisms and can apply to financial contexts, as well as other areas of research, such as demand-side management of energy consumption or social media applications.

5. Conclusion

Flash crashes of prices and sudden peaks in demand or supply have been observed in various IT-supported mechanisms, such as financial. In this paper we have presented a model that can explain these events as a result of the actions of rational agents when agents are unaware of an ambiguity with respect to the underlying game form.

We have first analyzed how the game form is determined by introducing the concept of the informational edge as a competitive advantage that enables information leaders to act faster than other agents. We subsequently analyze the differences in payoffs between the resulting sequential and simultaneous games. In this context we argue why information technology has eroded the competitive advantage and turned some games from sequential to simultaneous. This results in an ambiguity concerning the game form, since agents cannot distinguish between their participation in a simultaneous game and being the leader in a sequential game.

We model the impact of this ambiguity using hypergame theory, first as an intuitive two-player scenario and later for an arbitrary large number of players. We show that agents playing their rationally dominant Nash strategy – but misperceiving the true objective game – have a detrimental impact on market outcomes. In fact, we demonstrate that when a large number of players misperceive the game, an unwarranted sudden drop in market prices constitutes an equilibrium outcome. As the drop exaggerates the negative implications of the news, prices adjust upwards once the ambiguity is resolved, creating a characteristic flash crash effect. Hence, we explain flash crashes as the result of actions by rational agents.

Having described potential sources for flash crashes, we address corresponding warning mechanisms that may detect their occurrence. Once detected, a strategic adaption alleviates ambiguities and, thus, contributes to defusing the threat of a flash crash. On the one hand we illustrate how a measurement for order toxicity proposed by Easley et al. [2] to detect liquidity-based flash crashes may be adapted to estimate market expectations endogenously, i.e. from the order flow. On the other hand we suggest sentiment analysis to detect those expectations exogenously by analyzing the news flow. In either case, news event that substantially deviate from these expectations should be treated cautiously in simultaneous markets.

In our future research we will explore applications of the model beyond crashes in financial markets. For instance, we observe similar effects when considering demand side management in energy markets. Gottwalt et al. [28] describe sudden peaks in demand when agents react simultaneously to price signals whose actual purpose it is to balance demand.

Beside this, the work presented in this paper also opens a number of other avenues for future research. For instance, laboratory experiments can be used to investigate the predictive effect of order flow toxicity and news sentiment, as well as a combination of these methods. These experiments, as well as analysis of real world data, can furthermore be used to establish empirical support for the model introduced in this paper.

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