Robust Optimization Based Economic Dispatch for Managing System Ramp Requirement

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Abstract

The increasing penetration of renewable generation poses a challenge to the power system operator's task of balancing demand with generation due to the increased inter-temporal variability and uncertainty from renewables. Recently major system operators have been testing approaches to managing inter-temporal ramping requirement. In this paper we propose a robust optimization based economic dispatch model for ensuring adequate system ramp capability. The proposed model is critically assessed with the ramp product which is currently under consideration by several system operators. We conduct theoretical assessment based on a proposed lack-of-ramp probability (LORP) index and numerical assessment using Monte Carlo simulations. It is shown that compared with the recently proposed industry model, the proposed robust formulation of ramp requirement yields more smoothed generation cost variation and is capable of ensuring lower lack of ramp probability.

1. Introduction

The increasing penetration of renewable resources such as wind and solar poses a challenge to the goal of the Independent System Operators (ISOs) to manage the power system with a reliable and cost effective approach. Due to the limited control over the output of renewable resources as well as associated forecast errors the ISOs will have to deal with an increasing amount of uncertainty and variability in the system [1].

One significant issue is the temporary price spikes experienced by many ISOs in the real time electricity market due to shortages attributed to a lack of system ramp capability [2]. The main causes of these shortages include variability of load, scheduled interchanges and non-controllable generation resources (primarily wind) as well as uncertainty associated with short term forecasts. Due to the physical limitations on ramp rates generators are unable to respond effectively to these price spikes. The current practices to deal with ramp shortages include increasing reserve margins, starting fast-start units (such as gas turbines) and out of market dispatch methods that involve operator action. However, these approaches are usually high cost or create some market distortion. It is important for ISOs to have *additional flexibility* for dispatchable generation resources through the market clearing process. The Security Constrained Economic Dispatch (SCED) decision needs to be robust to the uncertainties so that the critical system power balance requirement is not violated.

In recent years, robust optimization has attracted significant interest as a framework for optimization under uncertainty, led by the work [3–7]. The approach has several attractive modeling and computational advantages. First, it uses a deterministic set-based method to model parameter uncertainty. This method requires only moderate amount of information, such as the support and moments, of the underlying uncertainty. At the same time, it provides the flexibility to incorporate more detailed information. There is also a deep connection between uncertainty sets and risk theory [8]. The work in [9] uses this approach to devise a bidding strategy for a virtual power plant consisting of wind generation and energy storage. Second, the robust optimization approach yields a solution that immunizes against all realizations of uncertainty data within the uncertainty sets, rather than a finite number of sample scenarios. Such robustness is consistent with the reliability requirement of power systems operation, given that the cost associated with constraint violations is very high. Third, for a wide class of problems, the robust optimization models have similar computational complexity as the deterministic counterparts. This computational tractability makes robust optimization a practical approach for many real-world applications. The interested reader can refer to [10] for a comprehensive treatment of the topic.

Applications of robust optimization in power system scheduling have been actively pursued in the

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past few years. In [11], a two-stage robust optimization model and a two-level cutting plane algorithm are proposed for the security constrained unit commitment problem with net load uncertainty. Computational experiments show improved economic efficiency as well as significant reduction in cost volatility in comparison to the reserve adjustment approach widely used in the current industry. The work of [12, 13, 14] proposes similar robust optimization models with different uncertainty sets. Exact and efficient solution methods are also explored in these works. In this paper we propose a robust optimization based economic dispatch model which gives dispatch decisions that are robust to uncertainties in the system net load.

An alternate approach is currently being investigated by Midwest ISO (MISO) which involves a modification to the SCED formulation to include additional *ramp capability* constraints [15]. The proposed economic dispatch with *ramp product* aims to cover forecast variability in net load as well as uncertainty, which is calculated based on a statistical analysis of historical data available to the system operator. California ISO (CAISO) is also investigating a *flexible ramping product* in order to create additional flexibility in the dispatch so that the occurrences of ramp shortage and temporary price spikes are greatly reduced [16]. However, even with the ramp capability modification there is a significant probability of shortage events due to lack of system ramp capability.

The main contributions of our paper are as follows.

1) We present a robust optimization based economic dispatch model for ensuring a reliable dispatch solution for the power system.
2) We propose a reliability index for system ramp capability based on a probabilistic risk measure.
3) We illustrate the proposed robust model on a small test system for the real time dispatch. Further, we compare the robust model to the dispatch with ramp product model using a Monte Carlo simulation to evaluate their reliability.

This paper is organized as follows. Section 2 describes the proposed robust economic dispatch formulation. Section 3 reviews the economic dispatch with ramp product model. Section 4 presents the proposed reliability index for measuring the adequacy of system ramp capability. Section 5 presents a numerical on a small test system to illustrate and compare the dispatch models. Section 6 discusses the conclusions and proposed future work.

2. Robust Economic Dispatch Formulation

In this section we present a robust optimization based economic dispatch formulation for the real time market. The aim of the security constrained economic dispatch (SCED) process is to find the least cost generation dispatch in order to satisfy the system power balance constraint while at the same time meeting other constraints such as generator power output and ramping limits. In the real time market (RTM) time frame ISOs use the SCED to allocate required power generation to participating dispatchable generators in order to maintain system wide power balance. Additionally some ISOs also procure regulation reserve and contingency reserves through SCED by means of co-optimization with energy. As the penetration of renewables in electricity grids increases the impact of their uncertain and variable output will increase and the need for robustness of dispatch will become more important.

For simplicity regulation reserves and contingency reserves are omitted from this presentation. Regulation reserves are used in the frequency regulation time scale rather than the economic dispatch time scale. Contingency reserves are used in case of reportable disturbances and not for handling normal power system operations.

The system *net load* is defined as follows:

Net Load = Total Load – Renewable Generation + Scheduled Interchanges (i.e., Exports – Imports)

Notation

$C_i^g()$	Cost function of generator *i*
$P_i^g(t)$	Dispatched output of generator *i* at time *t*
N_g	Total number of dispatched generators
P_i^{max}	Maximum output of generator *i*
P_i^{min}	Minimum output of generator *i*
$\hat{P}^l(t)$	System net load forecast at time *t*
$\tilde{P}^l(t)$	System net load uncertain variable
RCU_i	Cleared ramp up capability of resource *i*
RCD_i	Cleared ramp down capability of resource *i*
RCU_s	System wide ramp up requirement
RCD_s	System wide ramp down requirement
R_i	One interval (5 min.) ramp rate of resource *i*
$F(t)$	Vector of branch flows at the time *t*
F^{max}	Vector of branch transmission constraints
U	Uncertainty set for net load

The robust economic dispatch formulation is as follows.

Objective

The objective is to minimize total generation cost over current and next time interval.

$$\min_{P_i^g(t),P_i^g(t+1)} \sum_{i=1}^{N_g} C_i^g \left(P_i^g(t) + P_i^g(t+1)\right) \tag{1}$$

s.t.

Worst-case Power Balance Constraint

This constraint is included so that the dispatch solution in the next time interval will be feasible under even the worst case realization of net load. The net load in the next time interval is assumed to be an uncertain variable which belongs to a given deterministic uncertainty set *U*. The uncertainty in net load arises from its components viz., system load, renewable generation (such as wind, solar etc.) and scheduled interchanges (i.e., imports and exports).

$$\sum_{i=1}^{N_g} P_i^g(t+1) \geq \max_{\tilde{P}^l(t+1)\in U} \tilde{P}^l(t+1) \tag{2}$$

System Power Balance Constraint

The current interval net load forecast is assumed to be accurate, and if there are any deviations they can be handled by the frequency regulation control.

$$\sum_{i=1}^{N_g} P_i^g(t) = \hat{P}^l(t) \tag{3}$$

Generator Active Power Output Limits

The scheduled output power for each generator must remain within its active power output limits.

$$P_i^g(t) \leq P_i^{max}\ \forall\, i, t \tag{4}$$

$$P_i^g(t) \geq P_i^{min}\ \forall\, i, t \tag{5}$$

Single Interval Ramp Limits

The change in power output is limited by the ramping ability of each generator in the given time period.

$$P_i^g(t) - P_i^g(t-1) \leq R_i\ \forall\, i \tag{6}$$

$$P_i^g(t-1) - P_i^g(t) \leq R_i\ \forall\, i \tag{7}$$

$$P_i^g(t+1) - P_i^g(t) \leq R_i\ \forall\, i \tag{8}$$

$$P_i^g(t) - P_i^g(t+1) \leq R_i\ \forall\, i \tag{9}$$

Branch Power Flow Constraints

The transmission line capacity constraints must be satisfied for all the branches in the transmission network.

$$-F^{max} \leq F(t) \leq F^{max}\ \forall\, t \tag{10}$$

Uncertainty Set for Net Load

The deterministic uncertainty set defines the range of the uncertain future net load variable.

$$U := \tilde{P}^l(t+1) \in \left[\hat{P}^l(t+1) - \Delta P^l(t+1), \hat{P}^l(t+1) + \Delta P^l(t+1)\right] \tag{11}$$

where $\Delta P^l(t+1)$ is the maximum deviation of net load from the point forecast value $\hat{P}^l(t+1)$.

The real time market bidding and clearing for the robust economic dispatch model will work as follows. At each time step *t* the generating resources will submit their bids for the current and the next time interval, namely t and $t+1$, similar to a look-ahead economic dispatch model [17]. The system operator will perform a uniform price auction and the cost will be minimized while at the same time ensuring that all the constraints are satisfied. At each time step the current dispatch solution will be binding, whereas the future interval dispatch result will be advisory and can be modified in the subsequent interval.

In the general case the robust optimization formulation presented above can be extended to include more than one future time steps. That is we can include the uncertain net load variables $\tilde{P}^l(t+1), \tilde{P}^l(t+2), \tilde{P}^l(t+3), \ldots$ where each of these variables can be assumed to belong to a deterministic uncertainty set $U_1, U_2, U_3, \ldots$ each of which can be defined similar to (11). Accordingly, the objective function can be modified and additional constraints added to account for these additional variables. In this paper a two period model is considered since it is comparable to the proposals being considered by the aforementioned ISOs. The general case will be the subject of our future work where we will investigate aspects such as accuracy of dispatch, efficiency of dispatch in terms of generation costs and computational burden.

3. Dispatch with Ramp Product Formulation

In this section we review the formulation of the economic dispatch with ramp product which comprises of the following additional constraints which are to be added to the current SCED formulation [18, 19]:

1) Ten minute Ramp Capability for each dispatchable resource
2) System (or Zonal) Ramp Capability requirement

The system ramp capability requirement would allow dispatchable generators to respond to any forecast variations in *net load* as well as uncertainty. The uncertainty in net load can be calculated based on a statistical analysis of its components and then combining them. The statistical characterization of individual components of net load may be obtained from historical data [15].

We present a simplified version of the formulation of the dispatch with ramp product for the real time market. The actual formulation includes regulation reserve and contingency reserves, which are omitted here for simplifying the exposition. The dispatch scheme is posed as an optimization problem with the aim of obtaining the least cost dispatch solution to maintain the system power balance as well as meet generator power output and ramp constraints.

Objective

$$\min_{P_i^g(t)} \sum_{i=1}^{N_g} C_i^g\left(P_i^g(t)\right) \tag{12}$$

s.t.

System Power Balance Constraint

$$\sum_{i=1}^{N_g} P_i^g(t) = \hat{P}^l(t) \tag{13}$$

Generator Active Power Output Limits

$$P_i^g(t) + RCU_i(t) \le P_i^{max}\ \forall\, i \tag{14}$$

$$P_i^g(t) - RCD_i(t) \ge P_i^{min}\ \forall\, i \tag{15}$$

Single Interval Ramp Limits

$$P_i^g(t) - P_i^g(t-1) \le R_i\ \forall\, i \tag{16}$$

$$P_i^g(t-1) - P_i^g(t) \le R_i\ \forall\, i \tag{17}$$

Branch Power Flow Constraints

The transmission line capacity constraints must be satisfied for all the branches in the transmission network.

$$-F^{max} \le F(t) \le F^{max}\ \forall\, t \tag{18}$$

Ten Minute Ramp Capability Limits

$$RCU_i(t) \le 2R_i\ \forall\, i \tag{19}$$

$$RCD_i(t) \le 2R_i\ \forall\, i \tag{20}$$

System Ramp Capability Requirements

$$\sum_i RCU_i(t) \ge RCU_s(t) \tag{21}$$

$$\sum_i RCD_i(t) \ge RCD_s(t) \tag{22}$$

Where

$$RCU_s(t) = \hat{P}^l(t+2) - \hat{P}^l(t) + u \tag{23}$$

$$RCD_s(t) = -\left\{\hat{P}^l(t+2) - \hat{P}^l(t)\right\} + u \tag{24}$$

where *u* is the system net load uncertainty.

(12)-(18) comprise the conventional SCED formulation, whereas, (19)-(22) are the modifications for the ramp capability. The objective of SCED (12) is the sum of dispatch costs of all generators which are committed by the unit commitment (UC). (13) is the power balance constraint where it is assumed that the current net load forecast $\hat{P}^l(t)$ is accurate. Any small deviations are handled in the frequency regulation time frame. Violation of this constraint carries with it a very high cost and so ISOs would like to avoid such events.

Unlike the robust model in this case the generators will submit their bids only for the current time interval, namely $P_i^g(t)$. The system operator will run the optimization with the ramp capability constraints and thereby obtain the dispatch allocation for each generator. The LMPs will be based on the Lagrangian multipliers associated with the system power balance constraint (13). Thus the operation of the dispatch with ramp product model will be similar to the conventional single interval economic dispatch which is presently used in the ISO electricity markets.

The possibility of allowing resources to bid availability prices for ramp capability also exists, in which case the objective function would be:

$$\min_{P_i^g(t)} \sum_{i=1}^{N_g} C_i^g\left(P_i^g(t)\right) + \sum_{i=1}^{N_g} C_i^{rcu}\left(RCU_i(t)\right) + \sum_{i=1}^{N_g} C_i^{rcd}\left(RCD_i(t)\right) \tag{25}$$

where $C_i^{rcu}(t)$ and $C_i^{rcd}(t)$ are the availability offer prices for resource i ramp capability up and down respectively. The availability offers may lead to a change in the dispatch allocation as well as the LMPs [18].

4. Ramp Capability Reliability Index

In the probabilistic determination of contingency reserves the loss of load probability (LOLP) is used as a reliability index [20]. It is the probability that the generation resources combined with reserves will not be able to meet the demand. Analogous to this concept we propose a risk index for the system ramp capability being insufficient to meet the change in net load, due to a lack of available rampable capacity from dispatched generators. We call this the *lack of ramp probability* (LORP) and define it as follows:

$$LORP^{up} = Pr\left[\sum_{i=1}^{N_g}\left\{P_i^g(t) + \min\left(2R_i, P_i^{max} - P_i^g(t)\right)\right\} < \tilde{P}^l(t+2)\right] \tag{26}$$

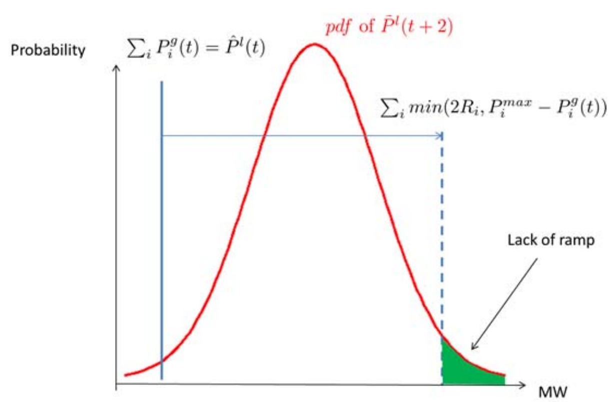

Figure 1. Lack of ramp probability

Figure 1 illustrates the concept, where the shaded area under the curve represents the probability that the system power balance will be violated in the future (second interval ahead from current) due to insufficient available system ramp capability.

In this example we have assumed that the 10 minute ahead net load $\tilde{P}^l(t+2)$ is a normally distributed random variable with known mean (equal to the point forecast of net load) and known standard deviation (estimated from historical data). Similar to (26) for the ramp up case we can also define the lack of ramp probability for the ramp down case.

$$LORP^{down} = Pr\left[\sum_{i=1}^{N_g}\left\{P_i^g(t) - \min\left(2R_i, P_i^g(t) - P_i^{min}\right)\right\} > \tilde{P}^l(t+2)\right] \quad (27)$$

Next we investigate the link between the ramp capability requirement and the lack of ramp index in the ramp up case. The link for the ramp down case can be derived similarly.

Based on (14) and (19) the cleared ramp capability of each resource i obeys the following constraints.

$$RCU_i(t) \le \min\left(2R_i, P_i^{max} - P_i^g(t)\right) \quad (28)$$

The probability that the cleared ramp capability from all resources is inadequate to meet system requirement is given by

$$Pr\left[\sum_{i=1}^{N_g} RCU_i(t) < RCU_s(t)\right] = Pr\left[\sum_{i=1}^{N_g} RCU_i(t) < \hat{P}^l(t+2) - \hat{P}^l(t) + u\right] = Pr\left[\hat{P}^l(t) + \sum_{i=1}^{N_g} RCU_i(t) < \hat{P}^l(t+2) + u\right] \quad (29)$$

We now make the following assumptions.

1. The current interval net load forecast is accurate and any deviations are handled in the frequency regulation time scale, thus $\sum_{i=1}^{N_g} P_i^g(t) = \hat{P}^l(t)$.
2. The cleared ramp up capability from each resource is at its maximum, thus $$RCU_i(t) = \min\left(2R_i, P_i^{max} - P_i^g(t)\right) \forall i$$
3. We can write $\hat{P}^l(t+2) + u = \tilde{P}^l(t+2)$ which is an uncertain variable.

Thus from (29) we have

$$Pr\left[\sum_{i=1}^{N_g} RCU_i(t) < RCU_s(t)\right] = Pr\left[\sum_{i=1}^{N_g} P_i^g(t) + \sum_{i=1}^{N_g} \min\left(2R_i, P_i^{max}(t) - P_i^g\right) < \tilde{P}^l(t+2)\right] = LORP^{up} \quad (30)$$

In the more general case, from the derivation (28)-(30) without Assumption 2 we know that

$$Pr\left[\sum_{i=1}^{N_g} RCU_i(t) < RCU_s(t)\right] \ge LORP^{up} \quad (31)$$

because $\sum_{i=1}^{N_g} RCU_i(t) < RCU_s(t)$ implies $\sum_{i=1}^{N_g}\left\{P_i^g(t) + \min\left(2R_i, P_i^{max} - P_i^g(t)\right)\right\} < \tilde{P}^l(t+2)$.

Thus LORP gives a bound on the probability that a ramp shortage event will occur in the dispatch model with ramp capability.

Also if $Pr\left[\sum_{i=1}^{N_g} RCU_i(t) < RCU_s(t)\right] \le \epsilon$, then we can guarantee $LORP^{up} \le \epsilon$, but not the other way around.

LORP can be used to calculate the probability of ramp shortage event occurring under the current SCED formulation. LORP can also be used to obtain the reliability in case we have an empirical probability distribution of net load.

5. Numerical

We compare the current single interval economic dispatch to the economic dispatch with ramp product and also to the robust economic dispatch by using a simple test system. For simplicity we neglect the branch power flow constraints (10) and (18) in our examples. Table 1 shows the generator characteristics for 3 conventional (dispatchable) generators.

Table 1. Generator characteristics

Generator	G1	G2	G3
Minimum Output (MW)	10	10	10
Maximum Output (MW)	130	130	100
Ramp Rate (MW/min)	4	1	1
Offer Price ($/MWh)	30	31	36
Initial Output (MW)	100	10	10

Table 2 shows the net load forecasts, which are used for calculating the ramp capability requirements in each interval Tn. Table 3 shows the required ramp capability up and ramp capability down requirements which are based on the change in forecast net load ΔNL and the uncertainty. Assuming a normal distribution of net load, taking the maximum uncertainty as ±3σ around the mean value should cover 99.73% of uncertainty cases as per the theory of the *3 sigma method.*

Table 2. Net load forecasts

Forecast	T1	T2	T3	T4	T5	T6
@T1	136	149	164			
@T2		151	163	173		
@T3			160	174	177	
@T4				171	175	179

Table 3. Ramp capability requirements

Interval	T1	T2	T3	T4
ΔNL(MW)	28	22	17	8
3σ Uncertainty (MW)	8	8	8	8
RCU_s (MW)	36	30	25	16
RCD_s (MW)	20	14	9	0

Since the system net load is generally increasing in this example we will focus on the ramp up capability. The total system ramp capability up requirement RCU_s in each time interval is the sum of the change in forecast net load ΔNL and the uncertainty.

In what follows, we first show a detailed comparison of the three models, in terms of generators output, total dispatch cost, LMPs and $LORP^{up}$, when the net load is fixed at the forecast value. The results are shown in Tables 4-8. Then, we generate 1000 scenarios of net load using Monte Carlo simulation and compare the average dispatch costs, volatility of the cost, and average $LORP^{up}$ of the models.

Table 4. Conventional economic dispatch results

Interval	T1	T2	T3	T4
Net Load (MW)	136	151	160	171
G1 (MW)	116	130	130	130
G2 (MW)	10	11	16	21
G3 (MW)	10	10	14	19
Total Output (MW)	136	151	160	**170**
LMP ($/MWh)	30	31	36	**3500**
$LORP^{up}$	0.0122	0.7735	0.1304	≈ 0

From Table 4 we can see that with conventional economic dispatch, in interval T4 the total generation is insufficient to meet the net load. The lack of system ramp capability results in a violation of the power balance constraint. To avoid this constraint violation the system operator will have to take some action such as sending a turn-on signal to some fast start generating unit to bridge the power gap. This shortage results in a temporary price spike in the real time market. In MISO the price associated with system power balance constraint violation is assumed to be equal to the Value of Lost Load (VOLL), which is $3500/MWh [18].

Table 5. Results of dispatch with ramp product

Interval	T1	T2	T3	T4
Net Load (MW)	136	151	160	171
G1 (MW)	114	120	125	130
G2 (MW)	12	17	22	27
G3 (MW)	10	14	13	14
Total Output (MW)	136	151	160	171
LMP ($/MWh)	31	36	36	36
$LORP^{up}$	0.0013	0.0013	0.0013	≈ 0

However, in the economic dispatch with ramp product, the dispatch solution is adjusted to avoid the shortage event. The inclusion of ramp capability constraints may lead to higher locational marginal prices (LMPs) in other non-shortage intervals. For instance we see from Table 5 (using u = 3σ = 8 MW) that in the interval T1 due to the different dispatch the LMP changes from $30/MWh to $31/MWh.

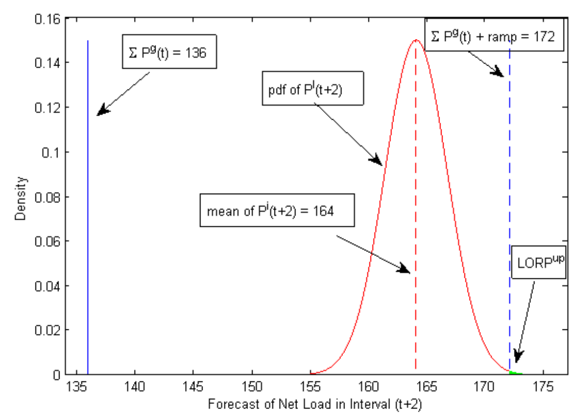

Figure 2. Lack of ramp probability for T1

We evaluate the lack of ramp probability index for the interval T1. As shown in Figure 2 the total generation in interval T1 is 136 MW, and the total available two interval ramp capability is 36 MW. The net load is assumed to be a normally distributed random variable with mean assumed equal to the point forecast value in interval T3, namely mean = 164 MW and the standard deviation σ = 8/3 MW. Since the system can't ramp up to greater than 172 MW the shaded area under the pdf of the net load represents the lack of ramp probability. Thus for the interval T1 the $LORP^{up} = 0.0013$.

Next we consider the robust economic dispatch model. In order to define the uncertainty set we choose the maximum deviation $\Delta P^l = 4$ MW for each interval. Table 6 shows the dispatch results for the robust model.

Table 6. Robust economic dispatch results

Interval	T1	T2	T3	T4
Net Load (MW)	136	151	160	171
G1 (MW)	116	124	123	130
G2 (MW)	10	15	20	25
G3 (MW)	10	12	17	16
Total Output (MW)	136	151	160	171
LMP ($/MWh)	30	30	30	36
$LORP^{up}$	0.0122	0.0669	≈ 0	≈ 0

Table 7 shows the generation (offer) costs for each economic dispatch approach on a 5 minute interval basis. For the conventional economic dispatch the generation cost in interval T4 is high because the committed generators G1-G3 are not able to satisfy the net load. Therefore, in this interval the system operator has to dispatch a fast start unit to ensure that the system power balance constraint is not violated. The cost associated with this generator is assumed to be the VOLL.

As shown in Table 7 the total generation cost associated with the robust approach (Table 6), is higher than that of the dispatch with ramp product approach (Table 5), due to the conservative nature of the robust approach. However, this approach avoids the shortage situation that we encounter in the conventional dispatch approach (Table 4).

Table 7. Generation cost comparison

Interval	Conventional ($)	Ramp product ($)	Robust ($)
T1	345.83	346	345.83
T2	383.42	385.92	384.75
T3	408.33	408.33	410.17
T4	727.92	436.75	437.58
Total	**1865.5**	**1577**	**1578.33**

In Table 8 the total generation cost for the 4 intervals and the total $LORP^{up}$ is shown for different levels of uncertainty in net load, for both the dispatch with ramp product approach and the robust dispatch approach. From Table 8 we see that the robust dispatch solutions have higher reliability (i.e., lower aggregate $LORP^{up}$) for all four cases and slightly higher generation costs for u = 8 MW and u = 4 MW. In the dispatch with ramp product cases with uncertainty u = 2 MW and u = 1 MW we find that a shortage event occurs in interval T4, which requires the system operator to dispatch a fast-start unit and therefore incur high cost.

Table 8. Generation cost and reliability comparison of dispatch methods

Dispatch w/ ramp product			Robust dispatch		
u	ΣGenCost ($)	$\Sigma LORP^{up}$	ΔP^l	ΣGenCost ($)	$\Sigma LORP^{up}$
8	1577	0.0039	8	1581	0.0026
4	1575.33	0.1460	4	1578.33	0.0792
2	1862.5	0.3694	2	1576.33	0.2403
1	1863.25	0.4966	1	1576.33	0.2403

In the general case when network constraints are considered, in the deterministic dispatch model the parameter u will be a system-wide load uncertainty parameter, whereas in the robust model the parameter ΔP^l would likely have to be defined for each node which has an uncertain net power injection [11]. For a more direct comparison between the two methods we consider the first two rows of Table 8. We see that with the robust model the generation costs are only slightly higher, but we get significant improvement in the reliability level as measured by $\Sigma LORP^{up}$. The system operator can adjust the choice of ΔP^l keeping in mind this tradeoff.

Next we use Monte Carlo simulation to assess the performance of the robust approach relative to the conventional economic dispatch and the economic dispatch with ramp product. To generate the net load scenarios each net load forecast in Table 2 is assumed to be a random variable. In each case the net load forecast is chosen at random from a truncated Gaussian distribution with the mean values indicated in Table 2, the standard deviation σ = 8/3, and maximum deviation ±8 MW. Thus 1000 scenarios are generated for a 20 minute real time dispatch time frame, and thus with 4 consecutive dispatch intervals in each scenario we simulate a total of 4000 intervals.

In the conventional economic dispatch, shortages occur in 983 intervals, in the economic dispatch with ramp product (taking u = 8 MW) they occur in 540 intervals and in the robust economic dispatch (taking $\Delta P^l = 4$ MW) shortages occur in 42 intervals. Further we calculate the mean and the standard deviation of the total generation cost of 4 intervals for the scenarios. In case of the dispatch with ramp product the mean generation cost = \$2,208.33 and standard deviation = \$717.25, whereas for the robust dispatch approach the mean generation cost = \$1600.92 and standard deviation = \$238.75. The mean lack of ramp probability on a single interval basis across all scenarios (i.e., mean $LORP^{up}$) for dispatch with ramp capability is 0.1890, whereas for the robust dispatch mean $LORP^{up}$ is 0.0512.

Thus with robust dispatch on average the total generation cost is expected to be lower since there is lower probability of a shortage event occurring. Additionally, the variance of generation cost in the robust dispatch approach is lower than that in the dispatch with ramp product approach. Finally, the robust dispatch solutions yield a lower mean lack of ramp probability compared to the dispatch with ramp capability solutions, indicating that the robust model is more reliable than the dispatch with ramp product.

6. Conclusions

In this paper we propose and evaluate a robust optimization based approach to managing system ramping requirement in real-time economic dispatch. The robust model is compared with a recently proposed industry model for ramp products. In order to assess the performance of different dispatch models targeted at managing the increasing system-wide ramping requirements, we propose Lack of Ramp Probability (LORP) as a performance index. This index measures the probability of insufficient system ramp capability resulting in system power imbalance.

The tradeoff of reliability and dispatch cost for both the ramp product approach as well as the robust approach is shown through a numerical on a simple test system. Additionally, the generation costs and reliability of the dispatch with ramp product and the robust economic dispatch model are evaluated using Monte Carlo simulation. It is shown that our proposed robust model yields a higher reliability of dispatch as well as lower mean and lower variability of generation cost relative to the dispatch with ramp product for the same level of uncertainty in net load.

Based on the finding from this paper, future work will include simulations on a larger system using realistic data from power systems. Another avenue of research is to construct a proper market mechanism that enables the implementation of the robust dispatch with guaranteed system ramping capability.

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8. References

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