# **Automatic Generation Control and its Implementation in Real Time**

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Abstract—In power systems, the control mechanism responsible for maintaining the system frequency to the nominal value and the real power interchange between balancing authority areas to the scheduled values is referred to as automatic generation control (AGC). The purpose of this paper is to present a systematic way to determine, in real time, the power allocated to each generator participating in AGC by taking into account the cost and quality of the AGC service provided. To this end, we formulate the economic dispatch process and gain insights into the economic characteristics of the generating units. We value the quality of AGC service by taking into consideration the ramping constraints of the generating units. The proposed methodology is illustrated in the WECC system and is compared with other allocation methods.

Keywords-Automatic Generation Control, Area Control Error, Optimal Power Flow, System Dynamics, Automatic Generation Control Allocation

### I. Introduction

In power system operations, there is a need to meet reliability criteria in an economic way. Power systems are divided into several balancing authority (BA) areas that are responsible for maintaining (i) load-interchange-generation balance within the BA area, and (ii) the interconnection frequency as close as possible to its nominal value at all times [1]. As a result, each BA implements several decisionmaking processes acting on different time scales that reflect the information on the system conditions. These include unit commitment (UC), economic dispatch (ED), automatic generation control (AGC) and primary generation control. The UC process determines the schedule of the hourly start-up and shutdown of units and, as a result, which generating units are used to supply the forecasted load (see, e.g., [2]). Next, the ED serves to allocate the total generation among the committed units, determined by the UC, so as to minimize the costs of serving the system load subject to physical constraints (see, e.g., [3]). The ED process is performed every hour to meet the day-ahead forecasted load and every 5-10 minutes to meet the minute-ahead forecasted load (see, e.g., [4]). However, more frequent adjustments to the output of generators are necessary due to generation outages, line outages, intermittent generation or just load demand fluctuations. In such a case, primary generation control reestablishes balance between load and generation. However, there remains an unavoidable frequency control error due to the action of the droop-based generator controllers responsible for primary generation control. The control mechanism that is responsible for returning the frequency to its nominal value is referred to as AGC. The AGC also maintains the correct value of interchange power between BA areas as scheduled. The AGC system updates the commands of generating units every 2-4 seconds. For the AGC mechanism there are several rules in the various independent system operators (ISOs).

Research in AGC systems spans in various areas. For instance, some papers focus on the determination of the area control error (ACE), which the AGC mechanism wishes to make zero [5]. The ACE is a function of the frequency, the real power interchange deviations from the desired values, and the frequency bias factor. A review of the current methods of calculating the frequency bias factor in frequency control is given in [6]. The authors propose a method of calculating the frequency bias factor on a more regular basis than the usual practice of once a year. The effects of the variability and intermittency of renewable generation on the AGC system are studied in [7], [8], [9], [10], [11]. On the other hand, there are papers that are focused on the consumer side, i.e., on the generators and how they should participate in AGC [12]. A thorough literature review of research on AGC is given in [13]; it includes an overview of AGC schemes, the types of power system models used, control techniques, load characteristics, incorporation of renewable resources and the AGC in the new market environment.

A critical aspect concerning the AGC system is the allocation of the total generation needed among the generators participating in AGC. This allocation is important, since it affects the cost and quality of the service offered. It needs to be simple, since it is conducted every 2-4 seconds, but also needs to meet certain criteria. The ISO wishes to maximize social welfare, therefore, the total cost for the AGC mechanism should be minimized. The most common type of AGC market is flat rate, because of its simplicity, however, in this case the response quality of the participating generating units are not taken into account [14, pp. 84-86]. The AGC payments, according to FERC Order No. 755 [15], include consideration for capacity set aside



to provide the regulation service and the energy that the resource injects into the system. These payments also cover the opportunity costs from foregone sales of electricity. A survey of the frequency control ancillary services in power systems from various parts of the world by focusing on the economic features is given in [16]. ISO New England (ISONE) makes payments for frequency regulation service to reflect the amount of work performed by a resource by taking into account the absolute amount of energy injected and withdrawn, which is referred to as a "mileage" payment. In ISONE, the fastest units are chosen among the ones cleared in the regulation market [17]. California ISO (CAISO), New York ISO (NYISO), Midwest ISO (MISO) and PJM pay a capacity payment to all resources that clear the frequency regulation market, and then net the amount of regulation up and regulation down provided by these resources. However it was found that this method does not acknowledge the greater amount of frequency regulation service being provided by faster-ramping units [15]. The deepening penetration of renewable resources with high variability intensify the need for fast responding units with high ramping rates participating in AGC, that need to be compensated accordingly.

In this paper, we present a systematic method of allocating the AGC signal among the generators by taking into consideration the ramping characteristics of each generator, as well as economic criteria defined by the ED process. We include in our modeling approach the ED process, represent the power system's dynamics and incorporate network and other physical constraints. We compare the proposed method with other two methods currently used in industry.

## II. POWER SYSTEM MODEL

This section provides a model for the ED process, the AGC and the electromechanical dynamics of a power system [18]. We consider a power system with N buses indexed by  $\mathscr{N} = \{1,\dots,N\}$  and L lines indexed by  $\mathscr{L} = \{\ell_1,\dots,\ell_L\}$ . We denote each line by the ordered pair  $\ell = (n,n'),\, n,n' \in \mathscr{N},$  with the real power flow  $f_\ell \geq 0$  whenever the flow is from n to n' and  $f_\ell < 0$  otherwise. The set of synchronous generating units is indexed by  $\mathscr{I} = \{1,2,\dots,I\}.$ 

## A. Economic Dispatch

The ED objective is to minimize the total cost, which is the sum of the costs of the individual units, subject to the essential constraint imposing that the sum of the generators' output must be equal to demand. In the ED process, other physical constraints may be included, such as voltage or real power flow constraints or in the power balance constraints the losses may be taken into consideration. The various available formulations of the ED are a result of what constraints as well as which power model (AC or DC) are used. For the ED that is implemented every 5 minutes,

common formulations are the ED with loss coefficients and DC optimal power flow (DCOPF), due to their simplicity and computational efficiency. Next, we formulate the ED with loss coefficients and subsequently the DCOPF.

1) ED with loss coefficients: We denote the total system load by  $P_{\text{load}}$ , the system losses by  $P_{\text{loss}}$ , the net interchange  $P_{\text{inter}}$ , the output of the  $i^{\text{th}}$  unit by  $P_{S_i}$  and the  $i^{\text{th}}$  generation cost function by  $\hat{c}_i(\cdot)$ . The mathematical formulation of the ED with loss coefficients is

$$\text{minimize } \sum_{i \in \mathscr{I}} \hat{c}_i(P_{S_i})$$

subject to 
$$\sum_{i \in \mathscr{I}} P_{S_i} = P_{\text{load}} + P_{\text{loss}} + P_{\text{inter}} \quad \longleftrightarrow \quad \sigma$$

$$P_{S_i} \geq P_{S_i}^m, \ \forall i \in \mathscr{I} \qquad \longleftrightarrow \quad \eta_i^m$$

$$P_{S_i} \leq P_{S_i}^M, \ \forall i \in \mathscr{I} \qquad \longleftrightarrow \quad \eta_i^M , \qquad (1)$$

where  $P^m_{S_i}(P^M_{S_i})$  are the lower (upper) permissible limits of the real power generation at bus i and  $\sigma, \eta^m_i$  and  $\eta^M_i$  are the Lagrangian multipliers or dual variables associated with the corresponding constraints of the problem. Define  $\eta^m = [\eta^m_1, \dots, \eta^m_I]^T$  and  $\eta^M = [\eta^M_1, \dots, \eta^M_I]^T$ . The calculation of the system losses makes the problem more complicated. We express the system losses as a function of the generators' output by using the so-called B-coefficients method [3, pp. 162-182]:

$$P_{\text{loss}} = \sum_{i \in \mathscr{I}} \sum_{j \in \mathscr{I}} P_{S_i} B_{ij} P_{S_j} , \qquad (2)$$

where  $B_{ij}$  are the loss coefficients considered to be constant under certain assumed conditions. More specifically, we have that

$$B_{ij} = \sum_{\ell \in \mathscr{L}} R_{\ell} \omega_{\ell}^{i} \omega_{\ell}^{j} , \qquad (3)$$

where  $R_{\ell}$  the line's  $\ell$  resistance and  $\omega_{\ell}^{i}$  the line  $\ell$  generalized generation distribution factor with respect to an injection/withdrawal at bus i [19].

2) DCOPF: We assume the network to be lossless. We denote the diagonal branch susceptance matrix by  $B_d \in \mathbb{R}^{L \times L}$  and the branch-to-node incidence matrix for the subset of nodes  $\mathscr{N}$  by  $A \in \mathbb{R}^{L \times N}$ . The corresponding nodal susceptance matrix is  $B \in \mathbb{R}^{N \times N}$ . Let  $P_{L_i}$  be the load at bus i,  $P_{I_i}$  be the interchange at bus i (positive if exporting, negative otherwise),  $f_l$  be the power flow through line  $\ell$ , and  $f_\ell^M$   $(f_\ell^m)$  be the limit of the real power flow on the same (opposite) direction of line  $\ell$ ; and define  $P_S = [P_{S_1}, \ldots, P_{S_N}]^T$ ,  $P_L = [P_{L_1}, \ldots, P_{L_N}]^T$ ,  $P_I = [P_{I_1}, \ldots, P_{I_N}]^T$ ,  $f = [f_{\ell_1}, \ldots, f_{\ell_L}]^T$ ,  $f^M = [f_{\ell_1}^M, \ldots, f_{\ell_L}^M]^T$  and  $f^m = [f_{\ell_1}^m, \ldots, f_{\ell_L}^m]^T$ . The optimiza-

tion problem describing the ED process is

minimize 
$$\sum_{i \in \mathscr{I}} \hat{c}_i(P_{S_i})$$
 subject to 
$$P_S - P_L - P_I = B \theta \qquad \longleftrightarrow \qquad \lambda$$
 
$$f = B_d A \theta \le f^M \qquad \longleftrightarrow \qquad \mu^M \qquad (4)$$
 
$$-f \le f^m \qquad \longleftrightarrow \qquad \mu^m$$
 
$$P_{S_i} \ge P_{S_i}^m, \ \forall i \in \mathscr{I} \qquad \longleftrightarrow \qquad \eta_i^m$$
 
$$P_{S_i} \le P_{S_i}^M, \ \forall i \in \mathscr{I} \qquad \longleftrightarrow \qquad \eta_i^M \ ,$$

where  $\theta_i$  is the voltage phase angle at bus i and  $\theta$  is the corresponding vector; and  $\lambda$ ,  $\mu^M$ ,  $\mu^m$ ,  $\eta_i^m$  and  $\eta_i^M$  are the dual variables associated with the corresponding constraints of the problem. The vector of dual variables associated with the power balance constraints is known as the vector of locational marginal prices (LMPs) and is denoted by  $\lambda = [\lambda_1, \lambda_2, \dots, \lambda_N]^T$ .

In both optimization problems in (1) and (4), the dual variables of the equality constraints may be interpreted as the cost needed to satisfy the constraint. As for an inequality constraint, if it is not binding the dual variable is zero. If the dual variable is not zero, i.e., if the constraint is binding, then the dual variable may be interpreted as the benefit associated with relieving the constraint. The ED, as defined by the solution of either (1) or (4), provides, usually every 5 minutes, the optimal output of each generator, which we denote by  $P_{ED_i} = P_{S_i}^{\star}$ .

#### B. Automatic Generation Control (AGC)

Consider an interconnected system with M BA areas indexed by  $\mathscr{A}=\{1,\ldots,M\}$ . For each  $m\in\mathscr{A}$ , we denote by  $\mathscr{A}_m\subset\mathscr{A}$  the set of BA areas that have ties with the BA area m. The actual power interchange out of BA area m to m' is  $P_{mm'}$  and is given by

$$P_{mm'} = \sum_{\substack{l \in \mathcal{B}_{mm'} \\ l' \in \mathcal{B}_{m'm}}} V_l V_{l'} \left( G_{ll'} cos(\theta_l - \theta_{l'}) + B_{ll'} sin(\theta_l - \theta_{l'}) \right),$$
(5)

where,  $V_l$  is the voltage magnitude of bus l,  $G_{ll'}+jB_{ll'}$  is the (l,l') entry of the network admittance matrix, and  $\mathscr{B}_{mm'}$  is the set of nodes in BA area m with tie lines to nodes in BA area m'. We denote by  $\mathscr{G}_m$  the set that indexes all the generators in BA area m. The actual frequency of BA area m is

$$f_m = \sum_{i \in \mathcal{A}_-} \gamma_i (f_{nom} + \frac{1}{2\pi} \frac{d\theta_i}{dt}) , \qquad (6)$$

with  $\gamma_i = \frac{H_i}{\sum_{i \in \mathscr{G}_m} H_i}$ , where  $H_i$  is the inertia constant for generator i. The ACE for BA area m, denoted by  $ACE_m$ , is given by

$$ACE_m = \sum_{m' \in \mathcal{A}_m} (P_{mm'}^{sch} - P_{mm'}) + b_m (f_m - f_{nom}) , (7)$$

where  $P_{mm'}^{sch}$  is the scheduled power interchange out of BA area m to m',  $b_m$  is the bias factor for BA area m, and  $f_{nom}$  is the system nominal frequency.

The objective of the AGC system is to make the ACE go to zero. We use a discrete time AGC model that follows from combining the models in [20, pp. 237] and [2, pp. 352-355]. We define a new state for the system,  $z_m(t)$ , which at the steady state is the total power generated in the BA area m. Let  $z_m[k] := z_m(kh)$ , where h takes values between 2 and 4 seconds and k is an integer. Then, the AGC system dynamic is given by:

$$z_{m}[k+1] = z_{m}[k] + h(-z_{m}[k] + \eta_{1}(ACE_{m}[k] - ACE_{m}[k-1]) + \frac{1}{\eta_{2}}ACE_{m}[k] + \sum_{i \in \mathcal{G}_{m}} P_{S_{i}}), \quad (8)$$

where  $\eta_1 = [0, 1]$ ,  $\eta_2 = [30, 200]$ . Each generator  $i \in \mathscr{G}_m$  participates in the AGC system with  $P_{C_i} = \phi_i(z_m)$  for  $i \in \mathscr{G}_m$ , where  $\phi_i(\cdot)$  is some function, to be defined later, determined by the so-called participation factors.

#### C. Electromechanical dynamics

We use a nine-state machine model as described in [18, pp. 140] for the representation of the machine dynamics. For the  $i^{\text{th}}$  synchronous machine, the nine states are: the field flux linkage  $E'_{q_i}$ , the damper winding flux linkage  $E'_{d_i}$ , the rotor electrical angular position  $\delta_i$ , the rotor electrical angular velocity  $\omega_i$ , the scaled field voltage  $E_{fd_i}$ , the stabilizer feedback variable  $R_{f_i}$ , the scaled output of the amplifier  $V_{R_i}$ , the scaled mechanical torque to the shaft  $T_{M_i}$ , the steam valve position  $P_{SV_i}$ . Then, for the  $i^{\text{th}}$  machine dynamics we have

$$T'_{do_i} \frac{dE'_{q_i}}{dt} = -E'_{q_i} - (X_{d_i} - X'_{d_i})I_{d_i} + E_{fd_i},$$
 (9)

$$T'_{qo_i} \frac{dE'_{d_i}}{dt} = -E'_{d_i} - (X_{q_i} - X'_{q_i})I_{q_i}, \tag{10}$$

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s, \tag{11}$$

$$\frac{2H_i}{\omega_s} \frac{d\omega_i}{dt} = T_{M_i} - E'_{d_i} I_{d_i} - E'_{q_i} I_{q_i} - (X_{q_i} - X'_{d_i})$$

$$I_{q_i} I_{d_i} - D_i(\omega_i - \omega_s), \tag{12}$$

$$T_{E_i} \frac{dE_{fd_i}}{dt} = -(K_{E_i} + 0.0039e^{1.555E_{fd_i}})E_{fd_i} + V_{R_i}, \tag{13}$$

$$T_{F_i} \frac{dR_{f_i}}{dt} = -R_{f_i} + \frac{K_{F_i}}{T_{F_i}} E_{fd_i},$$
 (14)

$$T_{A_i} \frac{dV_{R_i}}{dt} = -V_{R_i} + K_{A_i} R_{f_i} - \frac{K_{A_i} K_{F_i}}{T_{F_i}} E_{fd_i} + K_{A_i} (V_{ref_i} - V_i), \tag{1}$$

$$+K_{A_i}(V_{ref_i} - V_i), \tag{15}$$

$$T_{CH_i} \frac{dT_{M_i}}{dt} = -T_{M_i} + P_{SV_i}, \tag{16}$$

$$T_{SV_i} \frac{dP_{SV_i}}{dt} = -P_{SV_i} + P_{C_i} - \frac{1}{R_{D_i}} \left(\frac{\omega_i}{\omega_s} - 1\right). \quad (17)$$

The parameters in (9)-(17) describe the machine and their definitions may be found in [18].

In addition to the specified dynamics, we also have a set of algebraic equations. The  $i^{th}$  machine algebraic equations are

$$V_{i}e^{j\theta_{i}} + (R_{s_{i}} + jX'_{d_{i}})(I_{d_{i}} + jI_{q_{i}})e^{j(\delta_{i} - \frac{\pi}{2})} - [E'_{d_{i}} + (X'_{q_{i}} - X'_{d_{i}})I_{q_{i}} + jE'_{q_{i}}]e^{j(\delta_{i} - \frac{\pi}{2})} = 0 , \quad (18)$$

and the network equations

$$P_{S_i} - P_{L_i} = \sum_{k=1}^{n} V_i V_k \left( G_{ik} cos(\theta_i - \theta_k) + B_{ik} sin(\theta_i - \theta_k) \right) , \qquad (19)$$

$$Q_{S_i} - Q_{L_i} = \sum_{k=1}^{n} V_i V_k \left( G_{ik} sin(\theta_i - \theta_k) - B_{ik} cos(\theta_i - \theta_k) \right) , \qquad (20)$$

where

$$P_{S_{i}} = I_{d_{i}}V_{i}sin(\delta_{i} - \theta_{i}) + I_{q_{i}}V_{i}cos(\delta_{i} - \theta_{i}), (21)$$

$$Q_{S_{i}} = I_{d_{i}}V_{i}cos(\delta_{i} - \theta_{i}) - I_{q_{i}}V_{i}sin(\delta_{i} - \theta_{i}). (22)$$
III. AGC ALLOCATION

When a disturbance occurs, the system behaves as described in (9)-(22). The generators participating in AGC modify their output so that the generation meets the load at all time. Each generator i in area m participates in the AGC by a function  $\phi_i(z_m)$ . In particular, we have

$$P_{C_i} = P_{ED_i} + \xi_{m_i} (z_m - \sum_{j \in \mathscr{G}_m} P_{ED_j}) ,$$
 (23)

where  $\xi_{m_i}$  is the participation factor of generator i in the AGC system, with  $\sum_{i\in\mathscr{G}_m}\xi_{m_i}=1, \forall m\in\mathscr{A}$ . We propose a systematic way of determining the AGC participation factors  $\xi_{m_i}$  by using economic criteria (defined by the ED process) and by taking into account unit ramping characteristics.

We wish to specify the marginal cost for each generator of serving 1 MW of load. To this end, we use the mathematical formulations of the ED process and some basic optimization concepts [21]. For the ED with loss coefficients, as defined in (1), we denote the vector  $y = [P_{S_1}, \ldots, P_{S_I}, \sigma, \eta^{M^T}, \eta^{m^T}]^T$  and  $\eta_i^\star = \eta_i^{M^\star} - \eta_i^{m^\star}$ , for  $i = 1, \ldots, I$ ; the incremental cost as generator i changes its output by a small amount  $\Delta P_{S_i}$  is given by

$$\frac{\partial \hat{c}_i}{\partial P_{S_i}}\Big|_{y^*} = \sigma^* \Big( 1 - 2 \sum_{i \in \mathscr{I}} B_{ij} P_{S_j}^* \Big) - \eta_i^* = \rho_i. \tag{24}$$

For the DCOPF formulation, as defined in (4), we denote the vector  $x = [P_S^T, \theta^T, \lambda^T, \mu^{M^T}, \mu^{m^T}, \eta^{M^T}, \eta^{m^T}]^T$  and we have

$$\left. \frac{\partial \hat{c}_i}{\partial P_{S_i}} \right|_{x^*} = \lambda_i^* - \eta_i^* = \rho_i. \tag{25}$$

Now, we wish to take into account the ramping characteristics of the synchronous generating units. Each unit's i contribution to raise (lower) its output is constrained by its maximum (minimum) ramping capability  $\kappa_i^+$  ( $\kappa_i^-$ ) and the units upper (lower)  $P_{S_i}^M$  ( $P_{S_i}^m$ ) power limits. The convention we are using is that  $\kappa_i^-$  is a negative number. The units for the ramping rates are usually MW/min. We denote the binary variables  $\delta_m^+$ ,  $\delta_m^- \in \{0,1\}$  of BA area  $m \in \mathscr{A}$  to reflect if the total generation needed in the AGC system in BA area  $m, z_m$ , is positive or negative. More precisely,  $\delta_m^+ = 1$  and  $\delta_m^- = 0$  if  $z_m \geq 0$ , and  $\delta_m^+ = 0$  and  $\delta_m^- = 1$  if  $z_m < 0$ . For each BA area  $m \in \mathscr{A}$ , the allocation of the AGC signal among the generators is provided by the solution to the following optimization problem

$$\begin{aligned} & \text{minimize} & \sum_{i \in \mathcal{G}_m} \rho_i P_{C_i} - \delta_m^+ \zeta_m \sum_{i \in \mathcal{G}_m} \kappa_i^+ P_{C_i} + \\ & \delta_m^- \zeta_m \sum_{i \in \mathcal{G}_m} \kappa_i^- P_{C_i} \end{aligned}$$

subject to

$$\sum_{i \in \mathcal{G}_m} P_{C_i} = z_m$$

$$P_{C_i} \le P_{S_i}^M, \ \forall i \in \mathcal{G}_m$$

$$P_{C_i} \ge P_{S_i}^m, \ \forall i \in \mathcal{G}_m$$

$$f_\ell^m \le \sum_{i \in \mathcal{G}_m} \psi_\ell^i (P_{C_i} - P_{S_i}) \le f_\ell^M, \ \forall \ell \in \mathcal{L}_m \ ,$$
(26)

where  $\zeta_m$  is a parameter that weights the importance of using fast responsive units and is affected by the system characteristics. We denote by  $\psi_\ell^i$  the injection shift factor of line  $\ell$  with respect to an injection/withdrawal at bus i and by  $\mathcal{L}_m$  the set of lines in BA area m. The optimization problem in (26) determines  $P_{C_i}$  for  $i \in \mathcal{G}_m$ . Thus, we may determine the participation factors  $\xi_{m_i}$  for area m by  $\xi_{m_i} = \frac{P_{C_i}}{2}$ ,  $\forall i \in \mathcal{G}_m$ .

The value of the parameter  $\zeta_m$  is affected by the system characteristics. For example, systems with deep penetration of renewable resources have high values of  $\zeta_m$ . Some metrics to quantify the level of renewables in the system are the net load variations and the required ramping capability. A method of calculating the ramping requirements given some confidence level is given in [22]. The authors take into account the renewable based generation output and load forecast errors to determine the required ramping at a certain time. In a similar rationale, the ramping capability of the system may be calculated as shown in [23]. The ratio of required ramping to the available ramping capability of the system  $\varsigma_m$  is used as an input to determine the parameter  $\zeta_m$ . The ratio  $\varsigma_m$  provides a good metric of the net load variation, i.e., high values show large net variations and a deep integration of renewable resources. On the other had, low values demonstrate that the net load variations are low and that there are not a lot of renewable-based resources in the system. In addition, in order to insert a dollar value for each MW to the parameter  $\zeta_m$ , we use the average incremental costs of all generators  $\bar{\rho}_m = \frac{\sum_{i \in \mathscr{G}_m} \rho_i}{|\mathscr{G}_m|}$ , where  $|\mathscr{G}_m|$  the cardinality of the set  $\mathscr{G}_m$ . To have comparable values with the first term of the objective value we insert in the  $\zeta_m$  parameter the average ramping rates  $\bar{\kappa}_m^+ = \frac{\sum_{i \in \mathscr{G}_m} \kappa_i^+}{|\mathscr{G}_m|}$ . We define the parameter  $\zeta_m$  as  $\zeta_m = \zeta_m \frac{\bar{\rho}_m}{\bar{\kappa}_m^+}$ . An equivalent definition may be used for  $\bar{\kappa}_m^-$ .

We compare the results of the proposed method with other two AGC allocation methods: alternative method A1, where LMPs are used as economic signals to allocate the AGC command to each generator, and alternative method A2, where the ramping characteristics of each generator are taken into consideration, as discussed in [10]. In particular, by using method A1 the participation factors of each generator *i* are given by

$$\xi_{m_{i(A1)}} = \frac{1}{2} - \frac{\lambda_i}{2\sum_{j \in \mathscr{G}_m} \lambda_j},\tag{27}$$

where  $\lambda_i$  the LMP at bus i. Based on alternative method A2, the AGC participation of each generator i is

$$P_{C_{i_{(A2)}}} = \begin{cases} & \min\left(\frac{\kappa_{i}^{+}}{\sum_{j \in \mathcal{G}_{m}} \kappa_{i}^{+} j} z_{m}, P_{S_{i}}^{M} - P_{S_{i}}\right) & \text{if } z_{m} \geq 0 \\ & \max\left(\frac{\kappa_{i}^{-}}{\sum_{j \in \mathcal{G}_{m}} \kappa_{j}^{-}} z_{m}, P_{S_{i}}^{m} - P_{S_{i}}\right) & \text{if } z_{m} < 0 \end{cases}$$

$$(28)$$

Once  $P_{C_{i_{(A2)}}}$  are determined for  $i \in \mathcal{G}_m$ , the participation factors may be calculated accordingly.

The cost and quality of ACG service provided are different based on which allocation method is used. There are several similarities and differences between the three methods. The proposed method and alternative method A1 take into account economic signals, i.e., LMPs. However, the more appropriate economic signal is not the marginal cost of providing another MW at a bus i, i.e.,  $\lambda_i$ , but the marginal cost of modifying the output of a generator, which is affected by both the LMP at the bus where the generator is located and the output level of the generator. With alternative method A2 the fastest units are chosen. The proposed method values the necessity of fast ramping units with the use of the weighting parameter  $\zeta_m$  as described earlier in this section.

#### IV. NUMERICAL STUDIES

We illustrate the proposed methodology with the standard three-machine-nine-bus Western Electricity Coordination Council (WECC) power system, which is depicted in Fig. 1, it contains three synchronous generating units in buses 1,2 and 3, and load in buses 5, 6 and 8. The machine, network and load parameter values may be found in [18].

We consider one BA area for the WECC power system. As a result the ACE is only a function of the frequency deviation. We choose the frequency bias factor to be  $b=0.1\,$  MW/Hz. We formulate the ED process with the DCOPF, as

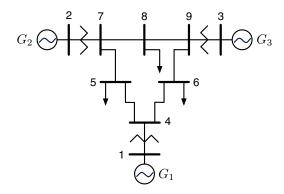


Figure 1: One-line diagram of the WECC three-machine nine-bus power system.

described in (4). The ED process is implemented every 5 minutes. The quantities in this section are expressed in per unit (p.u.) with respect to a 100 MVA base, unless stated otherwise. The load profile is as follows:  $P_{L_5}+jQ_{L_5}=1.25+j0.50, P_{L_6}+jQ_{L_6}=0.9+j0.30,$  and  $P_{L_8}+jQ_{L_8}=1.00+j0.35.$  The real power flow limits for all lines in the same (opposite) direction are 1 p.u. (-1 p.u.). The cost functions for the three generators are (units are in \$/MW):  $\hat{c}_1(P_{S_1})=0.025P_{S_1}^2+10P_{S_1}+100,\,\hat{c}_2(P_{S_2})=0.012P_{S_2}^2+20P_{S_2}+120$  and  $\hat{c}_3(P_{S_3})=0.010P_{S_3}^2+13P_{S_3}+150.$  The minimum (maximum) output in p.u. for each generator are:  $0\leq P_{S_1}\leq 1.2,\,0\leq P_{S_2}\leq 2$  and  $0\leq P_{S_3}\leq 1.5.$  The ramping characteristics for each unit in MW/min are:  $\kappa_1^+=3,\,\kappa_2^+=2,\,\kappa_3^+=1$  and  $\kappa_1^-=-3,\,\kappa_2^-=-2$  and  $\kappa_3^-=-1.$ 

In initial steady state, there is no congestion in the system, thus the uniform LMP for the system is 20.01 \$/MW. The synchronous generators in buses 1 and 3 are at their upper limits. The dual variables associated with the upper limits for the two generators are  $\eta_1^M=9.95$  \$/MW and  $\eta_3^M=6.98$  \$/MW. The timeframe of the simulations is described as follows: t=0s a disturbance occurs, t=60s the ED sends new signals to the generators and the AGC system is implemented every 2s. In the first case, we modify the load in bus 5 as follows  $P_{L_5}=1.7$  p.u.. In this case, the results

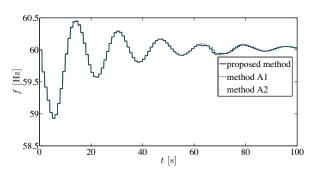


Figure 2: System frequency, with the three methods.

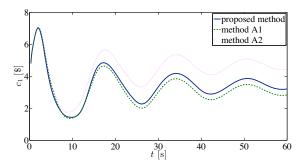


Figure 3: Cost associated with AGC service for generator 1, with the three methods.

of the updated ED process, show that congestion arises in the system and the LMPs at each node are  $\lambda_1=24.87$ ,  $\lambda_2=20.02$ ,  $\lambda_3=13.03$ ,  $\lambda_5=29.02$ ,  $\lambda_6=15.17$  and  $\lambda_8=22.85$  in \$/MW. We have 6 LMPs because in the DCOPF formulation buses  $1\equiv 4$ ,  $2\equiv 7$  and  $3\equiv 9$ , since they are connected by transformers.

The modification of the load causes a mismatch between generation and demand, and a deviation from the nominal frequency. We use three methods to allocate the AGC signal to restore the frequency to the nominal value: (i) our proposed method, (ii) alternative method A1 and (iii) alternative method A2. We compare the costs and the quality of AGC service for each method. For the WECC system we choose the value of  $\zeta$  to be 2 \$ min/MW<sup>2</sup>. The calculated values of the marginal cost in \$/MW for each generator are  $\rho_1 = 10.06$ ,  $\rho_2 = 20.02$  and  $\rho_3 = 13.03$ . We would expect that the participation factor for generator 1 would be the largest; however, since we also consider the network constraints, we end up with  $\xi_1 = 0.3710$ ,  $\xi_2 = 0.1653$ and  $\xi_3 = 0.4637$  at first. The participation factors after the updated ED process are:  $\xi_1=0.3220,\,\xi_2=0.3012$  and  $\xi_3=0.3768.$  Since, at first the LMPs are equal at all nodes, in method A1, we have that  $\xi_{i_{(A1)}} = \frac{1}{3}$ , for i = 1, 2, 3. Then, when the ED signal is updated and there is congestion in the system the participation factors become  $\xi_{1_{(A1)}} = 0.2853$ ,

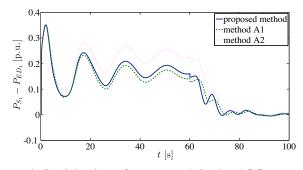


Figure 4: Participation of generator 1 in the AGC system, with the three methods.

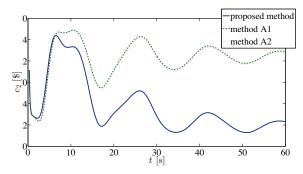


Figure 5: Cost associated with AGC service for generator 2, with the three methods.

 $\xi_{2_{(A1)}}=0.3272$  and  $\xi_{3_{(A1)}}=0.3876.$  For method A2, we have constant participation factors for the considered period of time, which are equal to  $\xi_{1_{(A2)}}=0.5,\;\xi_{2_{(A2)}}=\frac{1}{3}$  and  $\xi_{3_{(A2)}}=\frac{1}{6}.$ 

 $\xi_{3_{(A2)}}=rac{1}{6}.$  The system's frequency is depicted in Fig. 2. We notice that the AGC system serves its purpose, i.e., restores the frequency to its nominal value, with all three methods. The associated total cost for AGC service in \$ for each method are: c = 55.3738,  $c_{(A1)} = 55.3543$  and  $c_{(A2)} = 56.7635$  for the considered time period [0, 100] sec. The minimum cost is achieved by using method A1, as was expected, however in this case the quality of service (ramping characteristics) is not taken into account. In method A2, the cost is high but the fastest unit is mostly used to meet the AGC demands. In Fig. 3, we depict the cost for AGC service offered from generator 1 for all three methods. We only plot the cost until 60s, because after the new signals are sent from the ED, the participation of the units as well as the associated costs are small. Generator 1 has the highest ramp rate in the system. Thus, as we can see form the graph the cost associated with A2 is the highest. The lowest cost is observed with A1, since the participation of generators based on A1 is uniform and does not consider the ramp rates. The proposed method provides a balance between the two as shown in Fig. 3. A modification of the parameter  $\zeta$  gives more significance to

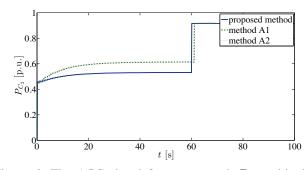


Figure 6: The AGC signal for generator 2  $P_{C_2}$ , with the three methods.

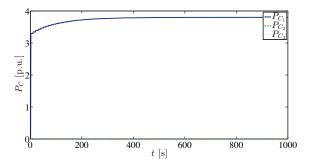


Figure 7: The AGC signal for all generators, with  $\varsigma = 0.1$ .

the cost or the quality of the AGC service. The participation of generator 1 in AGC is depicted in Fig. 4. We notice that after the new signal form the ED at  $t=60\mathrm{s}$ , the AGC signals of all methods are similar and have small values.

In Fig. 5, we depict the cost of AGC associated with generator 2. Both A1 and A2 assign a participation factor of  $\frac{1}{3}$ , thus the costs associated with A1 and A2 are identical. The proposed method utilizes generator 2 in a lower extent, since the marginal cost  $\rho_2$  is the highest and the ramp rate of the generator is 2 MW/min, which is in between the ramp rates of the other two generators. In Fig. 6, we depict the AGC signal to generator 2  $P_{C_2}$ . Method A1 uniformly allocates the AGC signal among the generators, until the ED signal is updated and the LMP at bus 2 becomes 20.02 \$/MW, which is higher than the LMP at bus 3, therefore the participation factor becomes  $\xi_{2_{(A1)}} = 0.3272 < \frac{1}{3}$  and the participation of generator 3 is greater, with  $\xi_{3_{(A1)}}=0.3876$ . The LMP at bus 1 is  $\lambda_1=24.87$  \$/MW, which is greater than the LMP at bus 2  $\lambda_2=20.02\$/\mathrm{MW}$ , thus  $\xi_{1_{(A1)}}=0.2853<\xi_{2_{(A1)}}.$  However, method A1 neglects the economic signals  $\eta_i^m$  and  $\eta_i^M$  associated with the lower and upper limit constraints for each generator i. Even if the LMP at bus 2 is smaller than that of bus 1, the associated benefit of relieving the constraint associated with the upper limit of generator 1 is  $\eta_1^M = 14.81$ \$/MW. Thus, the marginal cost of generator 1 is 10.06 \$/MW which is smaller than that of generator 2, which is 20.02 \$/MW. That is why the participation factor of our proposed

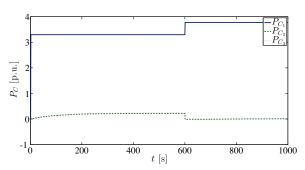


Figure 8: The AGC signal for all generators, with  $\varsigma = 0.5$ .

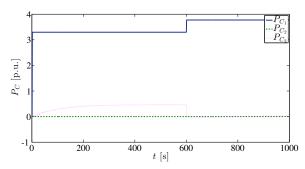


Figure 9: The AGC signal for all generators, with  $\varsigma = 0.8$ .

method for generator 2  $\xi_2=0.3012$  is smaller than that of method A1:  $\xi_{2_{(A1)}}=0.3272$  and  $\xi_1=0.3220>\xi_{1_{(A1)}}.$  Generator 2 has  $\kappa_2^+=2$ , therefore method A2 assigns a participation factor of  $\frac{1}{3}$  to generator 2.

We present another case by modifying the system, in order to demonstrate the capabilities of the proposed method, where the generators' cost functions are not overlapping and the system is not congested. We increase the line flow limits to 3 p.u. and the generators' limits to 5 p.u.. We now select non intersecting cost functions (units are in \$/MW): 
$$\begin{split} \hat{c}_1(P_{S_1}) &= 0.010 P_{S_1}^2 + 10 P_{S_1} + 100, \, \hat{c}_2(P_{S_2}) = 0.014 P_{S_2}^2 + \\ 15 P_{S_2} &+ 125 \; \text{ and } \; \hat{c}_3(P_{S_3}) \; = \; 0.025 P_{S_3}^2 + 20 P_{S_3} + 160. \end{split}$$
The ramping characteristics for each unit in MW/min are:  $\kappa_1^+ = 1, \; \kappa_2^+ = 2, \; \kappa_3^+ = 3 \; \text{and} \; \kappa_1^- = -1, \; \kappa_2^- = -2 \; \text{and}$  $\kappa_3^- = -3$ . Since the generators limits are much higher than the total load, only the least cost unit is dispatched. In this case, we have  $P_{S_1}=3.3$  p.u. and  $P_{S_2}=P_{S_3}=0$ . The system LMP is 10.06 \$/MW. A modification in the load occurs at time t=0s and we have  $P_{L_5}=1.7$  p.u.. In this case study we vary the parameter  $\zeta$  to illustrate the modifications in the AGC signal among the generators. The parameters used for the determination of  $\zeta$  are  $\bar{\rho} = 15.02$ \$/MW and  $\bar{\kappa}^+ = 2$  MW/min for this particular system. The reason  $\bar{\rho}$  is higher than the LMP is that generators 1 and 2 are at their lower limits. Then we modify the ratio  $\varsigma$ , i.e., we modify the variability of the net load. The values of  $\varsigma$ , for

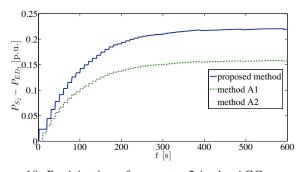


Figure 10: Participation of generator 2 in the AGC system, with the three methods.

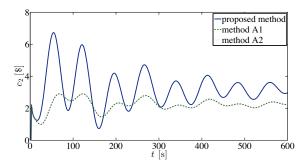


Figure 11: Cost associated with AGC service for generator 2, with the three methods.

which the AGC allocations are depicted in Figs 7-9, are 0.1, 0.5 and 0.8 respectively. We notice that as we increase the value of  $\varsigma$  the more expensive but faster ramping units are used in regulation. For small values of  $\varsigma$  only the cheapest generator, i.e., generator 1, participates in the AGC system, as seen in Fig. 7. Once, we increase the value of  $\varsigma$ , we notice that the other two more expensive generators participate in the AGC system, as is depicted in Fig. 8. When, the value of  $\varsigma$  exceeds a certain value, that is 0.8 in this particular system, only the fastest generator is used in the AGC system, as it may be seen in Fig. 9. We notice in all figures that once the ED sends the new signal, at t=60s, the entire load is met by generator 1 and the outputs of the other two generators are set to zero.

Now, we fix the value of  $\varsigma$  to 0.5 and compare the results of the proposed method with the two alternative methods. As it may be seen from Fig. 10, the two alternative methods assign equal participation of generator 2 in the AGC system equal to  $\frac{1}{3}$ . For  $\varsigma=0.5$ , the proposed method assigns a higher participation equal to 0.48, since the generator provides a good balance between the cost and the ramp rate. Generator 2 is more expensive than generator 1 but cheaper than generator 3. In addition, its ramp rate is 2 MW/min, which is in between the ramp rates of the other two generators. The cost associated with the AGC service offered by generator 2 is shown in Fig. 11. The cost is higher for the proposed method because the unit is utilized more with the proposed method than with the other two.

## V. CONCLUDING REMARKS

In this paper, we presented a systematic method of allocating the AGC signal among the generators by taking into consideration the quality of the AGC service as well as economic criteria. In our modeling approach, we include the ED process, we represent the power system's dynamics and incorporate network and other physical constraints. We use the information from the ED process to determine the marginal cost of increasing/decreasing a generator's output. We take into account the quality of service, i.e., how fast the generators respond, by including in the objective function

a parameter that quantifies the importance of using fast responsive units in AGC regulation. In the numerical studies, we compared the cost as well as the quality of AGC service among three different allocation methods and illustrated that the proposed methodology provides a good balance between cost and quality of AGC service offered. Furthermore, we modified the value of parameter  $\zeta_m$  and see its effect on the AGC allocation.

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