

Accelerating Combinatorial Clock Auctions using Bid Ranges

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Abstract

Auctions are important tools for resource allocation and price negotiations, while combinatorial auctions are perceived to achieve higher efficiency when allocating multiple items. In the recent decade, many auction designs are proposed and proven to be efficient, incentive compatible, and tractable. However most of the results hinge on quasi-linear preference bidders with ultimate patience, which is not quite realistic. In reality human bidders cannot engage in hundreds of auction rounds evaluating thousands of package combinations simultaneously. They either withdraw early or bid only on a limited subset of valuable packages. In this paper, we introduce bid ranges, with an additional sealed-bid phase before a Combinatorial Clock auction for information elicitation. With range information, the auction can start at higher prices with fewer rounds, and bidders are informed with the most relevant packages. Our design reduces the complexity both for the bidders and auctioneer, and is verified with computational simulations.

1. Introduction

Auctions are important tools for resource allocation and price negotiations. With the help of inter-connected computers, electronic auctions are capable of allocation much faster than multi-lateral negotiations. However human participation is still critical for the final decisions, as many attributes cannot be fully specified electronically and need to be traded off carefully. Since human bidders cannot iterate through auction rounds as fast as computers do, one way to improve the efficiency of auctions is to reduce auction rounds. Over the years, auction theory has perfected in single-unit negotiations, thus in this paper we focus on the more general and realistic multi-item cases.

Combinatorial auctions are perceived to achieve higher efficiency when allocating multiple items. The increase in efficiency comes from the possibility to express sub- or super-additive valuations among items, which is impossible for traditional single-unit auctions. One classic example is a purchase of return tickets: An outbound ticket is only useful if the inbound ticket is also secured. If a bidder has bidden for both the return tickets but only wins a single one, he encounters the *Exposure Problem*, i.e. winning an incomplete package at prices above valuations [15].

Enabling package bidding solves the exposure problem, at the cost of increased complexity. In this case the bidders need to evaluate an exponential amount of combinations for optimal bidding, while the auctioneer needs to solve an NP-hard winner determination problem (WDP) for efficient allocation [21]. Between the bidders and the auctioneer, an exponential amount of information needs to be communicated for finding the efficient allocation [25], and the auction may require excessive rounds before converging to the final allocation [37].

The bid elicitation problem and the WDP have been studied extensively over the years [26, 36]. Most designs fall into two categories, the open Ascending Auctions which are extensions of the popular English auction, and the sealed-bid Vickrey auction [5]. In practice, ascending auctions are favored over sealed-bid auctions for better price discovery and transparency, which may enhance allocative efficiency and auction revenues [13].

The other two issues from package bidding, the *Communication Complexity* and *Excessive Rounds*, receive relatively less attention. One remedy is proxy bidding [3], which essentially converts the auction into a sealed-bid format. In many other studies, the problem is simply assumed away with Quasi-Linear Preference bidders, who is only interested about packages and bid prices, and have no time preference.

It is a crucial mistake to assume that bidders have unlimited patience. In practice, excessive auction rounds do dampen the allocative efficiency. Shachat et

al. [38] points out that bidders exit early as a response to “the tediousness of the English auction”, while Isaac et al. [17] unveils the rationale behind jump-biddings as bidder impatience and strategic manipulation. Once bidders deviate from the equilibrium bidding strategies, nice theoretical properties, such as full efficiency and incentive compatibility, are no longer guaranteed. Thus accelerating the auction process is highly relevant for actually achieving these theoretical virtues in practice, especially with internet auctions such as *eBay* or *uShip*, where the stake is lower than spectrum auctions and bidders are more impatient.

One observation from actual auctions is that real competition occurs mostly in the final rounds of the auction. The early rounds just drive up the prices without changing demand or supply. Skipping these early rounds reduces the information communicated and the tediousness of the auction.

In this paper, we introduce bid ranges to address the issue, based on the highly successful Combinatorial Clock (CC) auction format [32, 14]. Our design adds an additional sealed-bid phase before the clock auction, in which bidders submit binding ranges indicating their interested bid intervals. The auctioneer learns the demand and derives the non-zero item prices from the lower bounds of the submitted ranges. These prices serve as the starting price of the second phase clock auction. In the clock phase, bidders are required to bid within the range, so to prevent strategic abuse of the signaling opportunity with the ranges. The winners are exempt from the lower bounds to prevent “winner’s curse” and to eliminate the incentive of bid-shading.

This paper is structured as follows: Section 2 compares a selection of combinatorial auction designs, and section 3 introduces the bid ranges to a CC auction setup. We then evaluate the effect of bid ranges through computational simulations in section 4, before concluding briefly with a research outlook in section 5.

2. Related Work

The primary goal of auction design is *Allocative Efficiency*, which is to maximize the welfare of all agents [10], or to award items to the bidder with the highest valuations. Efficiency is strictly distinguished from auction revenues. Another common objective is *Incentive Compatibility* where truth revelation of valuations is optimal for the bidders [7].

The benchmark for combinatorial auctions is the Generalized Vickrey Auction. Vickrey auction is theoretically remarkable for being the only direct revelation mechanism that is efficient and *Strategy Proof*, with general valuations and quasi-linear preference agents [28]. However, Vickrey auction suffers serious drawbacks, such as low revenue and

vulnerable to collusive bidding [5, 35]. The revenue drawback is more severe for procurement auctions since arbitrarily large payments to the bidders can be induced [11].

Combinatorial auctions gain popularity with the rise of high-stake auctions like the Federal Communications Commission (FCC) Spectrum auction. The FCC, however, has been using the non-package Simultaneous Ascending Auction (SAA) for years, due to concerns over complexity and the *Threshold Problem*, where small bidders try to “free ride” on each other and fail to outbid a large package bid [12]. Recently, the FCC is reported to choose a new design called Hierarchical Package Bidding (HPB) for the upcoming auctions [16]. In HPB, packages are predefined and structured in hierarchies. The tree structure of packaging allows HPB to solve the allocation problem in linear complexity, which is impossible if bidders can freely bid on any combinations of packages.

The relationship between various *true* combinatorial auctions (CA) is depicted in Figure 1 (based on [31]). Some designs transfer the WDP to the bidder side, so that the auctioneer does not solve the complex allocation problem (AUSM [6], PAUSE [20]). In most designs the computation burden of allocation is bore by the auctioneer, and they can be classified with respect to the pricing schemes.

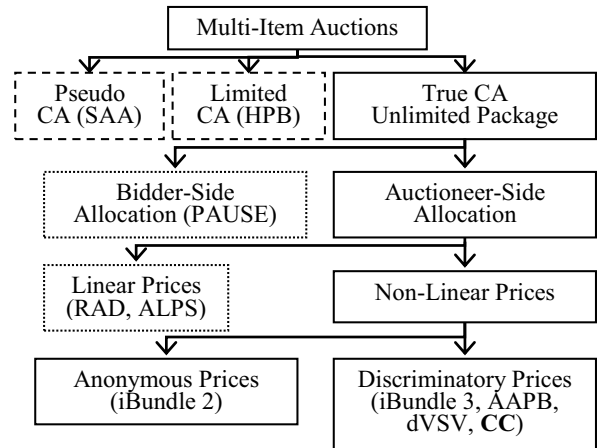


Figure 1. Classification of Combinatorial Auctions

Prices are linear if the price of a package equals the sum of the item prices within the package; Prices are anonymous if every bidder pays the same price for the same package. Non-anonymous prices are also called discriminatory prices. Although linear and anonymous prices simplify the auction design, there are valuations such that only non-linear and discriminatory prices can support the efficient allocation [10].

Examples of linear and anonymous pricing are the Resource Allocation Design (RAD) [19] and ALPS [8]. The linear ask prices are calculated from the dual problem of the efficient allocation problem, and need to be approximated when the dual prices are non-linear. Inefficiency may occur when such linear dual prices do not exist.

With non-linear and non-anonymous prices, several ascending auction designs are proven to be efficient: The iBundle auction [27, 29], the Ascending Auctions with Package Bidding (AAPB) [3, 4], and the dVSV auction [39]. These are extensions to the English auction where WDP is solved in each round, with losing bidders trying to outbid the provisional winners. Besides the complexity in solving WDP, solving it in each round requires multiple tie-breakers. In iBundle and AAPB auctions jump-bidding is still possible, while dVSV is essentially a clock auction where prices are set by the auctioneer.

An alternative to the English ascending auctions is the Combinatorial Clock auction (CC) [32]. CC maintains a price clock for each item, and the prices tick up in each auction round if the item is demanded by more than one bidder. CC stops when all item demands are equal to or below supply. WDP is only needed in under-supply situations and the winners are determined with all previous bids. Although price communication is linear in each round, it should be emphasized that CC is a non-linear discriminatory price auction, since the final allocation is calculated from all previous bids, whose price need not equal to the final ask prices.

Experiments [9, 37] showed that CC achieves high efficiency with less auction rounds, and the feature that provisional winners are not announced enhances CC's robustness against tacit collusion and strategic gaming [18]. With demand masking valuations CC might be inefficient, but this can be fixed by enforcing *PowerSet* bidding and a *Partial Revelation Pricing* rule [9], or introducing an additional sealed-bid round after the clock phase [2, 14].

With the jump-bidding gone in CC, impatient bidders are more likely to exit early. Thus, it is more beneficial to accelerate CC than the English ascending auctions. In this paper, we introduce bid ranges, which provide non-zero starting prices for the CC auction. Our design starts with an additional sealed-bid phase, where bidders indicate a lower bound and an upper bound for each bid. The starting prices are derived from the submitted bounds, and bidders are required to bid between the bounds throughout the clock auction, with winners being exempt from the lower bounds. We showed that this contributes to reduced auction rounds without reducing efficiency. In practice human bidders do not bid on all packages with value [37], thus, if the

auctioneer partially provides the bound information to the bidders, the bidders can learn about the demand and focus on the most relevant packages. We believe that the reduced complexity makes combinatorial auctions more accessible for common applications such as finding carriers for transportation requests.

The efficiency of the auction often requires truth revelation of the bidder valuations. Adopting Vickrey pricing is an instant solution for incentive compatibility [30]. However, Vickrey prices at times lie outside the *core* of the efficient outcome, i.e. they are too low to separate the winning bidders from the losing ones. The solution is either to enforce gross substitutes valuations for bidders so that Vickrey prices stay in the core [3, 39], to provide one-time discount on the final prices so that bidders get the core allocation but pays the Vickrey prices [1, 24, 30], or to forgo Vickrey pricing and identify an optimal price in the core that minimizes the distortion of deviating from Vickrey prices [14]. Since these payment rules are well studied and trivial to switch in CC, we leave the payment part of the auction open, as there is no general best option and the choices largely depend on the specific applications.

3. Bid Ranges and the Combinatorial Clock Auction

Our design builds upon the CC+ auction in [9]. A pre-auction sealed-bid round is added for learning the bid ranges and setting the starting prices, and then the bid ranges are enforced during the auction through activity rules. In the following subsections, we first introduce CC and CC+ auction in 3.1, and then the bid ranges in 3.2.

3.1 The Combinatorial Clock Auction

The combinatorial clock (CC) auction is a generalization of the single-unit Japanese auction [33], while CC+ is an improvement over the CC on efficiency and incentive compatibility [9].

For notation we assume a total of n bidders, each bidder i bids $b_i(S)$ for the subset S of the m items. In CC each item j has a price clock P_j . The auction starts with zero prices and all bidders demand all items with positive valuations. During the auction, the price P_j increases by an increment of ϵ , if more than one bidder demands the item j . Bidder i withdraws the bid $b_i(S)$ on package S , when the price-sum for the package is too high. The price-sum at which the bid withdraws is recorded as the bid price $b_i(S) = \sum_{j \in S} P_j$. When all prices stop increasing, i.e. there is no *over-demand*, the auctioneer solves for the provisional winner based on the recorded bid prices.

If a demanded item j does not belong to the final allocation, the auction enters the *under-supply* stage. Under-supply happens when the price P_j is too low to be provisional winning. Thus the auctioneer needs to increase P_j further until either item j becomes part of the allocation or the bidder withdraws the bid. The final allocation assigns package X_i to bidder i at price $P_i(S)$ solved from WDP.

Since the winner determination problem is solved with all the bids in the end, the efficiency of CC hinges on the bidding strategies. One common assumption is straight forward bidding, where bidders only bid on packages with highest payoff ($u_i(S) = v_i(S) - b_i(S)$). This can be inefficient for CC, when bidder valuations are demand-masking [9].

An example clarifies the auction process and the problem with demand-masking valuations. Supposed there are two bidders with two items A, B. The bidder valuations are listed in table 1.

Table 1. An example of the CC Auction

Bidder	A	B	AB
1	70€	170€	-
2	10€	-	160€

Suppose P_A increases first. Bidder two withdraws the bid on A at 10€, and then bidder one withdraws at 70€. Since there is only one bid AB left covering A, P_A stops increasing at 70€.

Then P_B increases. At 90€, bidder two withdraws the bid on AB as the price-sum is 160€. Since there is no over-demand, the auctioneer solves for allocation and the winner is bidder two with {AB, 160€}. However, bidder one demands B without getting it, so item B is undersupplied. Thus P_B will continue to increase beyond 150€. The allocation assigns bidder one with {B, 150€} and bidder two with {A, 10€}.

However, if bidder two bids straight-forwardly, he will not demand A as a single item at all, because when P_B is below 150€, bidding A alone yields less payoff than bidding AB, i.e. A is “demand-masked” by AB [9]. The final allocation assigns AB to bidder two, which is inefficient (lower total valuations).

The remedy is to require bidders to demand all valuable packages (*PowerSet* bidding strategy [9]), or to provide an additional round, in which bidder two learns the market demand and bid on A instead [2, 14].

To promote incentive compatibility, [9] further applies a *Full Revelation* price update rule and Vickrey pricing, under which truthful bidding with the *PowerSet* strategy is an ex-post equilibrium.

3.2 Bid Ranges and the Activity Rules

The efficiency of CC with *PowerSet* bidding and CC+ has been theoretically proven in [9], in which the starting prices need not be zero. The goal is thus to increase the starting prices as much as possible, so as to skip the early auction rounds.

Our design features a pre-auction sealed-bid round for bidders to submit the bid ranges, two methods for setting the starting prices, and the activity rule that enforces consistency during the auction.

At the beginning of the auction bidders are required to submit ranges $[L_i(S), U_i(S)]$ on every potential bid $b_i(S)$. The ranges have a predefined length and are binding during future rounds: bidders can only bid within the ranges, they cannot overbid the ranges, or bid on new packages that are not covered by any range. Therefore by backward induction, bidders have the incentive to partially reveal their true valuations.

Common critiques on identifying all valuations are the associated complexity in bid valuation. However, as [25] shows, for L items, “a price must be revealed for each of the $2^L - 1$ bundles” for efficient allocation, such complexity cannot be avoided if full efficiency is desired for auction design.

From the submitted ranges the auctioneer calculates the starting prices of the auction. Intuitively, prices can be set to the infimum of all lower bounds, which is to find price that best matches the lower bounds. The goal is to minimize the distortions $\delta_i(S)$ on package S , if the price-sum must be lower than the lower bound $L_i(S)$:

$$\begin{aligned}
& \min_{P_j, \delta_i(S)} \{ \max \delta_i(S) \}, \forall i, j, S \\
& \text{s.t. } \sum_{j \in S} P_j + \delta_i(S) = L_i(S), \forall i, S \\
& P_j \geq 0, \delta_i(S) \geq 0, \forall i, j, S
\end{aligned} \tag{I}$$

A simpler alternative is to raise the price from zero uniformly, until certain lower bound is broken:

$$\begin{aligned}
& \max \{P_j\} \\
& \text{s.t. } \sum_{j \in S} P_j \leq L_i(S), \forall i, S \\
& P_j \geq 0, \forall j
\end{aligned} \tag{U}$$

The two methods for setting the starting prices are evaluated in section 4. Once prices are set, the normal clock auction can start at these higher prices. Our clock auction adopts a normal price update rule (or *Partial Revelation* in [9]), with a modification in bidding. In CC+ [9] prices increase for *all* over-demanded items in a round, which may be inefficient, since for a package of k items the effective price increase would be k -times the price increment. Hence, we only increase the price for *one* over-demanded item in each round.

Facing the changing prices, the only action a bidder can take is to withdraw the bid at certain prices within the pre-specified range. The price-sum at which the bid is withdrawn becomes the effective bid price.

To prevent strategic gaming, activity rules are enforced during the auction to ensure bid consistency:

1. Bid $b_i(S)$ cannot be withdrawn at price-sums below the lower bound $L_i(S)$, except in cases that $b_i(S)$ is the provisional winning bid.
2. Bid $b_i(S)$ is automatically withdrawn when the price-sum exceeds the upper bound $U_i(S)$.
3. Withdrawn bids cannot re-enter the auction in future auction rounds.

The rule on the lower bound practically enforces the *PowerSet* bidding rule since every desired package must be covered by certain range, which effectively places a binding bid higher than the lower bound. Winning bids are exempt to avoid “winners’ curse” and the incentive of bid-shading.

The strict upper bounds discourage bid-shading as well, since bid-shading strictly reduces the probability of winning. The bounds also guarantee auction termination. When all the bids are withdrawn (either by the bidders or by the upper bounds), the Winner Determination Problem (WDP) is solved for the final allocation. In undersupplied situations the auction continues with more rounds.

The pseudo-code of the auction is as follows. The main process is based on the CC+ [9], with the addition of bid ranges, and the modified *Partial Revelation* price update rule.

Input: Package bids $b_i(S)$, Range $[L_i(S), U_i(S)]$
Result: Allocation X and prices $P_i(S)$
Initialization:
 for $j = 1$ to m **do**
 Calculate the Starting Prices P_j from $L_i(S)$
 for $i = 1$ to n **do** $X_i \leftarrow \emptyset$
Repeat
 $over-demand \leftarrow FALSE$
 $under-supply \leftarrow FALSE$
 for $j = 1$ to m **do**
 for $i = 1$ to n **do**
 if $P_i(S)$ satisfies the bid range **then**
 Bidder i update bids $b_i(S)$
 else
 Enforce the Bid Ranges
 end
 if ≥ 2 bidders $i \neq i'$ demand item j **then**
 $P_j \leftarrow P_j + \epsilon$

$over-demand \leftarrow TRUE$
end
 if item j is not part of any bid $b_i(S)$ **then**
 $under-supply \leftarrow TRUE$
 end
 if $over-demand = TRUE$ **then** go to the next auction round
 else if $under-supply = TRUE$ **then**
 for $j = 1$ to m **do**
 Assign X based on all previous bids
 Calculate $P_i(S)$ for X
 if there are demanded items not in X **then**
 foreach demanded item j not in X **do**
 $P_j \leftarrow P_j + \epsilon$
 Enforce the Bid Ranges
 else
 X is the final allocation,
 auction ends
 end
 else exit repeat, auction ends.
until stop

Using the example in table 1, assume a range of 50€ and both bidders place their true valuation in the middle of the range, the lower bounds are $\{A, 45\text{€}\}$, $\{B, 145\text{€}\}$ for bidder one, and $\{A, 0\text{€}\}$, $\{AB, 135\text{€}\}$ for bidder two. The infima are $P_A = 0\text{€}$, $P_B = 135\text{€}$. At $P_A = 10\text{€}$ bidder two withdraws bid A. Then at $P_A = 25\text{€}$ bidder two withdraws bid AB ($P_A + P_B = 160\text{€}$). Since bidder one still demands A or B without getting any, P_B starts to increase from 135€. At $P_B = 150\text{€}$ the provisional winning bid AB is replaced by $\{A, 10\text{€}\}$ from bidder two and $\{B, 150\text{€}\}$ from bidder one. Now there is neither over-demand nor under-supply, the auction stops with the same result as in the previous example. Starting the auction with uniform prices is the same as zero starting prices for this example.

Having a range rather than a specific value in the sealed-bid phase protects bidders’ privacy, and leaves them flexibility in bidding. They can bid aggressively by setting the true valuations as the lower bounds, or bid conservatively with the true valuations as the upper bounds. Bidders with fuzzy valuations can benefit from such flexibility through price discovery in later rounds, a feature that sealed-bid or proxy auctions lack. The strict upper bound also protects over-ambitious bidder from bidding above their true valuations, which can be highly useful for risk management. Moreover, if a bid on a package is strictly dominated by another bid on the same package (i.e. the upper bound is lower than

the lower bound of the other bid), then the bidder would be notified beforehand. This helps the bidders to focus on the most relevant packages and forgo those packages of low values. The sealed-bid phase is one-shot only. It is impossible to enter new ranges later, so as to prevent strategic abuse of this opportunity.

4. Computational Simulations

The goal of the simulations is to find out:

1. The impact of different price increments and bid ranges on auction rounds and efficiency.
2. How increased starting prices affect auction efficiency, revenue, and auction rounds.

The CC auction with bid ranges is programmed in C# and tested with the Combinatorial Auction Test Suite (CATS) [22].

CATS contains distributions stemmed from real world applications and previous auction studies (*L*). The valuation distributions are mostly normal (*-NPV*) or uniform (*-UPV*), and the uniform distributions are generated with fixed bounds (*Fixed Random*) or bounds that scale to the number of items (*Linearly Random*). The valuations for *Matching*, *Paths* and *Regions* are determined by problem specific common values plus random deviations. The detailed setups are as follows [23]:

- *Arbitrary*: arbitrary distribution for both bids and valuations.
- *Matching*: bids with complementarities in time slot matching, e.g. airport take-off and landing rights.
- *Paths*: bids as paths in space, e.g. rail, bandwidth, and spectrum auctions.
- *Regions*: bids with complementarity in two-dimensional adjacency, e.g. real-estate auctions.
- *Scheduling*: The job shop scheduling problem. The bids are multiple jobs with different deadlines competing for one resource, and the valuations are determined by the latency and deadline of the job.
- *L1*: Uniform bids, Fixed Random valuations.
- *L2*: Uniform bids, Linearly Random valuations.
- *L3*: Constant bids, Fixed Random valuations.
- *L4*: Decay distribution for bids ($\alpha = 0.55$), Linearly Random valuations.
- *L5*: Normal bids and valuations.

- *L6*: Exponential distribution for bids ($q = 5$), Linearly Random valuations.
- *L7*: Binomial distribution for bids ($p = 0.2$), Linearly Random valuations.
- *L8*: Constant bids, quadratic valuation distribution.
- *Default-hard*: Mixed distributions to maximize the predicted runtime of WDP.

We test our auction design with all distributions except for *Default-hard* and *L8*, since *Default-hard* are designed for algorithm performance testing, while the *L8* instances have constant valuations. For each distribution, 10 random instances are generated with 10 items and 100 bids.

To find out the effect of price increments and bid ranges, we test each instance with two increments and two ranges. The small increment is determined by $1/100$ of the difference between the highest valuation and the lowest valuation, while the large increment is $1/5$ of that difference, i.e. with the large increment, the price increase on any item is at most 5 times. For the bid ranges, the small one has an interval of 10% of the highest valuation, while the large one is 50% of the highest valuation, i.e. $|U_i(S) - L_i(S)| = r \cdot \max \{v_i(S)\}$, $\forall i, S$, $r_{small} = 0.1$, $r_{large} = 0.5$.

In reality these parameters would be determined from experience and clear digits must be used to prevent code bidding [14]. In addition, we compare two methods for calculating the starting prices: infimum of the lower bounds (I) versus uniform prices (U). These yield 10 setups for each distribution.

All instances are benchmarked to the CC auction. We evaluate our design by *Efficiency* and *Auction Rounds*. Efficiency is calculated by the valuation sum of the winning packages compared to that of the valuation-maximizing package. For higher accuracy, we do not round-off values within one unit of the price increment. The average results are reported in table 2. In the test, all bidders withdraw their bids when the price-sum of the package exceeds the true valuations, and they all follow a conservative strategy by setting the lowest possible range, i.e. the truth valuation as the upper bound. We have also tested the situations where ranges are set randomly, the results are similar, so they are not reported. For clarity, we report relative changes for auction rounds.

Table 2. Simulation on CATS instances with true valuation as upper bound

Large price increment: 1/5 Valuation Difference. (efficiency / auction rounds)										
Distribution	Efficiency	Large Range 50%		Small Range 10%		Rounds	Large Range 50%		Small Range 10%	
	CC	Infimum	Uniform	Infimum	Uniform	CC	Infimum	Uniform	Infimum	Uniform
Arbitrary	96.0%	92.6%	96.0%	92.2%	96.0%	29.3	-10%	0%	-18%	0%
Arbitrary-NPV	97.7%	96.4%	97.7%	95.1%	97.7%	25.5	-9%	0%	-16%	0%

Arbitrary-UPV	96.7%	96.1%	96.7%	93.9%	96.7%	27.6	-9%	0%	-19%	0%
Matching ^[1]	99.6%	98.6%	99.6%	99.0%	99.6%	41.8	-9%	0%	-15%	0%
Paths	99.8%	99.1%	99.8%	99.6%	99.8%	51.9	-16%	0%	-15%	0%
Regions	94.9%	91.8%	94.9%	94.2%	94.9%	30.4	-13%	0%	-18%	0%
Regions-NPV	93.4%	94.1%	93.4%	94.2%	93.4%	27.9	-10%	0%	-18%	0%
Regions-UPV	96.1%	95.9%	96.1%	93.8%	96.1%	28.9	-13%	0%	-23%	0%
Scheduling	99.2%	99.7%	99.2%	99.5%	99.2%	31.9	-3%	0%	-6%	0%
L1 ^[2]	100.0%	100.0%	100.0%	100.0%	100.0%	212	-25%	-40%	-46%	-73%
L2	98.8%	98.6%	98.8%	99.5%	98.8%	29.2	-3%	0%	-8%	0%
L3	99.9%	96.2%	99.9%	95.7%	99.9%	43.6	-3%	0%	-4%	0%
L4	97.3%	97.9%	97.3%	96.3%	97.3%	35.0	-10%	0%	-19%	0%
L5 ^[2]	100.0%	100.0%	100.0%	99.4%	100.0%	54.3	-19%	-19%	-35%	-38%
L6	97.1%	95.7%	97.1%	97.8%	97.1%	30.5	-8%	0%	-17%	0%
L7	98.0%	97.7%	98.0%	96.6%	98.0%	42.2	-12%	0%	-23%	0%
Small price increment: 1/100 Valuation Difference. (efficiency / auction rounds)										
Distribution	Efficiency	Large Range 50%		Small Range 10%		Rounds	Large Range 50%		Small Range 10%	
	CC	Infimum	Uniform	Infimum	Uniform	CC	Infimum	Uniform	Infimum	Uniform
Arbitrary	100.0%	100.0%	100.0%	100.0%	100.0%	377.7	-18%	-11%	-35%	-24%
Arbitrary-NPV	99.9%	99.9%	99.9%	99.9%	99.9%	321.5	-15%	-11%	-30%	-23%
Arbitrary-UPV	99.9%	99.8%	99.9%	99.8%	99.9%	356.1	-21%	-12%	-35%	-24%
Matching ^[1]	100.0%	100.0%	100.0%	100.0%	100.0%	640.5	-12%	-13%	-20%	-24%
Paths	100.0%	100.0%	100.0%	100.0%	100.0%	783.9	-21%	-8%	-23%	-14%
Regions	100.0%	100.0%	100.0%	99.9%	100.0%	379.5	-15%	-6%	-33%	-15%
Regions-NPV	100.0%	100.0%	100.0%	100.0%	100.0%	347.7	-16%	-9%	-30%	-19%
Regions-UPV	99.9%	99.9%	99.9%	99.9%	99.9%	358.0	-18%	-11%	-34%	-24%
Scheduling	99.9%	100.0%	99.9%	100.0%	99.9%	355.3	-6%	-5%	-11%	-10%
L1 ^[2]	100.0%	100.0%	100.0%	100.0%	100.0%	4106.7	-34%	-43%	-57%	-76%
L2	100.0%	100.0%	100.0%	100.0%	100.0%	331.3	-10%	-3%	-22%	-10%
L3	100.0%	100.0%	100.0%	100.0%	100.0%	659.1	-1%	0%	-3%	-1%
L4	99.8%	99.9%	99.8%	99.8%	99.8%	437.4	-16%	-21%	-34%	-41%
L5 ^[2]	100.0%	100.0%	100.0%	100.0%	100.0%	952.6	-21%	-29%	-36%	-49%
L6	100.0%	100.0%	100.0%	99.9%	100.0%	348.2	-16%	-15%	-32%	-32%
L7	99.9%	100.0%	100.0%	99.9%	100.0%	569.6	-19%	-24%	-35%	-45%
^[1] Matching has 12 items due to distribution requirement. ^[2] L1, L5 has 5 items and 20 bids due to the complexity in generating test cases.										

The simulation result shows that the CC auction achieves high efficiencies across all distributions, even with very large price increments. Smaller increments improve auction efficiency at the cost of more auction rounds.

Introducing bid ranges and setting non-zero starting prices do not affect auction efficiency in all settings. In the worst case, the efficiency loss is less than one price increment (20% for large increment and 1% for small increment), which in some studies is regarded as full efficiency [27].

Even with conservative bid ranges, the reduction in auction rounds is significant, except for distribution *L2* and *L3*. The effect is most noticeable using small bid ranges and infimum starting prices. Intuitively smaller bid ranges lead to higher starting prices. Surprisingly,

even a range as large as 50% of the highest valuation can still reduce auction rounds effectively.

There are also two caveats from the simulation: Firstly large price increment combined with uniform starting price has almost no effect. This is because the lower bounds of the low bids are so close to zero that the uniform price is actually zero. Secondly although infimum price is more effective, it also requires more time to solve the more complex linear program.

To find out the revenue implications from the bid ranges and non-zero starting prices, we test the CATS instances with increasing starting prices that block certain low bids before the auction. For each instance, we first obtain the uniform starting price, then raise the price by a fixed portion of the price increment. The increase is 30%, 60%, 90% of the large increment, and 200%, 400% and 600% of the small increment. The

increase is not proportion to the uniform starting prices because they are zero in many cases. Since no specific payment rule is chosen, we calculate auction revenue from the sum of the winning bid prices.

Each instance is tested with two increments, with a fixed bid range of 50%. We did not test the small bid range or the infimum method, since small ranges give very limited room for raising the starting prices, while

the infimum price is highly instance-dependent. All instances are benchmarked to the CC auction, and for auction rounds and revenue we report the relative changes.

From the listed results in table 3 we see that increasing the starting prices modestly has little impact on efficiency, strong effect on reducing auction rounds, and mixed effect on auction revenue.

Table 3. Simulation on CATS instances with increased starting prices

Large price increment: 1/5 Valuation Difference. (efficiency / auction rounds)									
Distribution \ Increased Price	Efficiency			Auction Rounds			Revenue		
	30%	60%	90%	30%	60%	90%	30%	60%	90%
Arbitrary	90%	97%	100%	-34%	-68%	-94%	+1%	0%	-89%
Arbitrary-NPV	97%	87%	100%	-35%	-74%	-94%	0%	-20%	-95%
Arbitrary-UPV	96%	93%	99%	-32%	-70%	-95%	-1%	-5%	-95%
Matching ^[1]	99%	99%	99%	-9%	-19%	-29%	+1%	-1%	+1%
Paths	99%	98%	99%	-10%	-19%	-29%	-1%	+2%	-2%
Regions	93%	94%	99%	-36%	-73%	-94%	-2%	-15%	-95%
Regions-NPV	92%	94%	98%	-35%	-73%	-92%	0%	-26%	-92%
Regions-UPV	95%	93%	96%	-33%	-63%	-93%	+2%	-4%	-89%
Scheduling	99%	100%	100%	-50%	-96%	-97%	-1%	-91%	-100%
L1 ^[2]	100%	100%	98%	-13%	-25%	-38%	-5%	-5%	-8%
L2	100%	100%	100%	-35%	-97%	-97%	0%	-100%	-100%
L3	100%	100%	100%	-16%	-29%	-44%	+2%	+1%	+1%
L4	98%	99%	100%	-25%	-52%	-85%	+3%	-9%	-68%
L5 ^[2]	100%	100%	100%	-7%	-14%	-20%	0%	-3%	0%
L6	98%	100%	100%	-30%	-90%	-97%	0%	-67%	-100%
L7	97%	97%	100%	-19%	-38%	-54%	+3%	+4%	+6%
Small price increment: 1/100 Valuation Difference. (efficiency / auction rounds)									
Distribution \ Increased Price	Efficiency			Auction Rounds			Revenue		
	200%	400%	600%	200%	400%	600%	200%	400%	600%
Arbitrary	100%	100%	99%	-26%	-51%	-69%	0%	0%	-13%
Arbitrary-NPV	100%	100%	98%	-26%	-50%	-69%	0%	0%	-4%
Arbitrary-UPV	100%	99%	94%	-27%	-50%	-69%	0%	-1%	-11%
Matching ^[1]	100%	100%	100%	-27%	-53%	-76%	0%	-2%	-11%
Paths	100%	100%	98%	-15%	-30%	-45%	0%	0%	-4%
Regions	100%	100%	98%	-21%	-41%	-60%	0%	+1%	0%
Regions-NPV	100%	100%	99%	-25%	-49%	-71%	+1%	+1%	-3%
Regions-UPV	100%	100%	100%	-22%	-43%	-62%	0%	+1%	-2%
Scheduling	100%	100%	100%	-24%	-47%	-67%	0%	0%	0%
L1 ^[2]	100%	98%	98%	-85%	-98%	-98%	-16%	-19%	-48%
L2	100%	100%	100%	-21%	-42%	-62%	0%	0%	0%
L3	100%	100%	100%	-7%	-14%	-20%	0%	0%	0%
L4	100%	100%	100%	-41%	-80%	-95%	0%	0%	-65%
L5 ^[2]	100%	97%	100%	-54%	-93%	-97%	-8%	-28%	-44%
L6	100%	100%	100%	-32%	-60%	-82%	0%	0%	-48%
L7	100%	100%	100%	-47%	-88%	-100%	0%	-12%	-91%
[1] Matching has 12 items due to distribution requirement.									
[2] L1, L5 has 5 items and 20 bids due to the complexity in generating test cases.									

Compared to the results in table 2, increasing the starting prices slightly can effectively reduce auction rounds even with uniform price and large increments. Continuing to increase the starting price would further reduce auction rounds. Although efficiency is not greatly affected by the high starting prices, auction revenue drops quickly due to reduced competition. Thus, unlike in the single-unit auctions where optimally setting the reserve price can increase auction revenue [34], using starting prices to increase revenue is a more challenging issue for combinatorial auctions, because the excluded low bids might be part of the winning package (as in our example).

For managerial implications, if the allocation requires lots of human interactions, then CC with large price increment is sufficiently good. Targeting full efficiency requires sufficiently small price increments and a high number of auction rounds. The auction rounds can be effectively reduced by setting the starting prices as the infimum of the lower bounds, or slightly higher than the highest uniform price below the lower bounds. Using smaller bid ranges has only a modest impact.

5. Conclusion and Research Outlook

In this paper we present a novel idea of introducing bid ranges for accelerating combinatorial auctions. We apply the bid ranges on one of the most successful design, the Combinatorial Clock (CC) auction, and show that bid ranges will not dampen the allocative efficiency of CC. Even with very large bid ranges and large price increments, the combined approach is able to maintain high efficiency with significantly reduced auction rounds in simulations with CATS [22] instances. We believe that the reduction in auction rounds contributes to the reduction in communication and decision complexity in combinatorial auctions, making it more accessible as a mean of multi-dimensional internet negotiation. Stronger starting prices derived from bid ranges also reveal incompetent bids, making it easier to solve WDP and allowing bidders to focus on the most relevant packages.

We are currently expanding the simulation to more setups to quantify the impact of price increment setting, bid range setting, different starting prices and more complex bidding strategies. Further sensitivity analyses are also planned to optimally set the bid ranges and starting prices, so that auction rounds can be reduced as much as possible without dampening efficiency or auction revenue.

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