# Bacteria inspired patterns grown with hyperbolic cellular automata

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#### Abstract

In this paper we give three examples of expending patterns defined by hyperbolic cellular automata whose growth seems to be very similar to the growth of colonies of bacteria.

## 1 introduction

A very intriguing phenomenon of diffusion is given by the growth of colonies of bacteria, see [1]. As explained by Professor Ben-Jacob, very surprising structures can be obtained by putting such colonies in very severe conditions, see Figures 1 and 13. This gives a striking power of adaptability of these colonies. These experimental data are comforted by the discovery of bacteria in almost every possible hard conditions as geysers, ocean fathoms, core of the earth and even atomic piles. In the introduction of [1], Professor Ben-Jacob says:

Eons before humans, bacteria inhabited a very different Earth. As the earliest life form they devised ways to counter the spontaneous course of increasing entropy and convert high-entropy, inorganic substances into low entropy, organic molecules...

To change environmental hazards, bacteria resort to a wide range of cooperative strategies...

They collectively glean information from the environment, communicate, distribute tasks, perform distributed information processing and learn from past experience.

In many cases, their growth on plates used by microbiologists to study them constitute figures with a more or less fractal symmetry. This may address to hyperbolic geometry. This is why we tried the other way: let us start from hyperbolic geometry, in fact from an appropriate tiling of the hyperbolic plane and try to simulate the observed growth.

In Section 2 we give the needed information for the reader about what to know about hyperbolic geometry in order to understand how our grid is obtained and to see how cellular automata are implemented in this context. In Section 3, we see how to proceed to the simulations indicated in the abstract. We conclude in Section 4 with ideas about possible continuations.



Figure 1 Examples of growth of bacteria colonies. Pictures by courtesy of Professor Ben-Jacob.

# 2 A triangular tiling of the hyperbolic plane

In this section, we provide the reader with the minimal material which allows the paper to be self-contained. Sub-section 2.1 is a very short introduction to hyperbolic geometry and to the Poincaré's disc model which is intensively used in the illustrations of the paper. Then, Sub-section 2.2 indicates which grid we consider for the simulations we propose. Then, in Sub-section 2.3 we introduce cellular automata adapted to this grid and the discussion we have there will lead to the various examples of simulation dealt with in Section 3.

## 2.1 Hyperbolic geometry

Hyperbolic geometry appeared in the first half of the 19<sup>th</sup> century, proving the independence of the parallel axiom of Euclidean geometry. Models were devised in the second half of the 19<sup>th</sup> century and we shall use here one of the most popular ones, Poincaré's disc. This model is represented by Figure 2.

Inside the open disc represented in the figure we have the points of the hyperbolic plane. Note that by definition, the points on the border of the disc do not belong to the hyperbolic plane. However, these points an important role in this geometry and are called **points at infinity**. Lines are trace of diameters or circles orthogonal to the border of the disc, e.g. the line m. In this model, two lines which meet in the open disc are called **secant** and two lines which meet at infinity, *i.e.* at a point at infinity are called **parallel**. In the figure, we can see a line s through the point A which cuts m. Now, we can see that two lines pass through A which are parallel to m: p and q. They touch m in the model at P and Q respectively which are points at infinity. At last, and not the least: the line n also passes through A without cutting m, neither inside the disc nor outside it. This line is called **non-secant**.



Figure 2 Poincaré's disc model.

## 2.2 The grid of our simulations

From a famous theorem established by Poincaré in the late 19<sup>th</sup> century, it is known that there are infinitely many tilings in the hyperbolic plane, each one generated by the reflection of a convex polygon P in its sides and, recursively, in the reflection of the images in their sides, provided that the number p of sides of P and the number q of copies of P which can be put around a point A and exactly covering a neighbourhood of A without overlapping satisfy the relation:  $\frac{1}{p} + \frac{1}{q} < \frac{1}{2}$ . The numbers p and q characterize the tiling which is denoted  $\{p, q\}$ and the condition says that the considered polygons live in the hyperbolic plane. Note that the three tilings of the Euclidean plane which can be defined up to similarities can be characterized by the relation obtained by replacing < with = in the above expression. We get, in this way,  $\{4, 4\}$  for the square,  $\{3, 6\}$  for the equilateral triangle and  $\{6, 3\}$  for the regular hexagon.

In the paper, we shall focus our attention on one of the simplest tilings which can be defined in this way in the hyperbolic plane:  $\{7,3\}$ . We call this tiling the **heptagrid** which is illustrated by Figure 3. On the right-hand side of the figure, we can see the tree which is in bijection with an angular sector, a basic

structure of the heptagrid, also see Figure 6. This property is the basis of a very efficient navigation tool to locate tiles in the heptagrid.

We refer the reader to [4, 5] for a detailed analyses and detailed explanations of these tools.

In the book, it is also indicated how to derive from the heptagrid an infinite family of tilings. We proceed to this point now.



Figure 3 The heptagrid. On the right-hand side: the key structure to explore the tiling.



**Figure 4** The subtilings of the heptagrid. On the left-hand side: generation 1; on the right-hand side, how generation 2, the **heptatrigrid**, is built from generation 1.

The idea is to split each tile of the heptagrid into seven triangles whose vertices are the centre of the tile and the end-points of its edges. Then, we split the triangles into four triangles by taking the mid-points of the previous triangles: the new triangles are defined by a vertex of the previous triangles and the mid-points of the edges of the previous triangle which meet at the vertex. This defines generation 1 and generation 2 of the process, see Figure 4.

can go on the process inductively: the generation n+1 is obtained from the generation n as generation 2 is obtained from generation 1. In [5], this process is defined up to infinity, giving the possibility to define coordinates for the points of the hyperbolic plane.

However, in this paper we shall focus on generation 2 only. Let us call it the **second triangular heptagrid**, **heptatrigrid** for short.



Figure 5 The heptatrigrid.

## 2.3 Cellular automata on the hyperbolic triangular grid

Now, we turn to the implementation of a cellular automaton in the heptatrigrid.

Remember that such an automaton consists in a finite automaton A attached to each 2-triangle of the heptatrigrid. A cell of the finite automaton consists of A and a 2-triangle which is the **support** of the cell. The neighbours of a cell c with T as its support have, as their supports, the 2-triangles which share a side with T.

Figure 6 indicates the basic elements of the location of a triangle. A coordinate is defined by four numbers in the format  $(\sigma, \nu, \tau, \pi)$ :  $\sigma$  is the number of the sector in which the triangle lies;  $\nu$  is the number of the heptagon of the sector in which the triangle lies;  $\tau$  is the number in [1..7] of the generation 1 triangle in which the triangle lies; and in this triangle,  $\pi$  is the number of the triangle itself. Later on,  $\tau$  will be called the **slice** and  $\pi$  will be called the **place**. Note that the central heptagon has the coordinate (0, 0) as it belongs to no sector.

The numbering of the generation 1 triangle, we say later **1-triangle**, is defined by the number of the side of the heptagon on which the 1-triangle is built, which explains why this numbers is in [1..7]. For the central heptagon,

side 1 is fixed once for all and the other sides are numbered by counter-clockwise turning around the tile. The side i touches the heading heptagon of the sector i, with i in [1..7]. For the other heptagons, side 1 is shared by the father of the heptagon in the tree. Note that we consider that the father of the root of the tree is the central cell.



Figure 6 From the heptagrid to the second triangular heptagrid, the heptatrigrid.

In each 1-triangle, we have four triangles of generation 2, we call them **2triangles**. The four 2-triangles of a 1-triangle are numbered from 0 to 3. First, we number the vertices of a 1-triangle T from 0 to 2: 2 is the centre of the heptagon, 0 and 1 are the vertices of the side of the heptagon defining T. Following the counter-clockwise orientation, 0 comes before 1. Now, the number of a vertex of T is the number of the 2-triangle which possesses this vertex. Accordingly, 3 is the number of the 2-triangle whose vertices are the mid-points of the edges of T. This numbering can be repeated for any further generation, see [5]. This numbering has interesting properties. The interested reader is referred to [5] for more information.

Now, to implement cellular automata, we have to compute the coordinates of the neighbours of a cell c from the coordinate of c itself. Let T be the 2-triangle which supports the cell. Number the neighbours of c from 0 to 3, c itself being neighbour 0. For the other numbers, the neighbour i of c is the 2-triangle which shares the side i of T. Let  $(\sigma, \nu, \tau, \pi)$  be the coordinate of T, the support of c. The coordinates of the neighbours of T are given by Table 1.

As can be seen in the table, each 2-triangle has at least one neighbour which is in the same heptagon and in the same slice of heptagon. Note that a 2-triangle with place 3 has all its neighbours in the same slice of the same heptagon. A 2-triangle with place 2 has all its neighbours in the same heptagon, but two neighbours are in the slices which are adjacent to its own one. This is indicated by the expressions  $\tau \ominus 1$  and  $\tau \oplus 1$ . As the number of a slice is in [1..7], subtracting 1 from 1 gives 7 and adding 1 to 7 gives 1. For  $\tau \in$ [2..7],  $\tau \ominus 1 = \tau - 1$ , and for  $\tau \in [1..6]$ ,  $\tau \oplus 1 = \tau + 1$ . For the 2-triangles with place 0 and 1, the computation of the coordinates of their neighbours is more complex. Indeed, in each case, one of the neighbours do not belong to the same heptagon H, but to a heptagon K neighbouring H. This changes the value of  $\nu$ and it may also change the values of  $\sigma$  and  $\tau$ . This is indicated in Table 1 by the expressions  $s(\sigma, \nu \tau), v(\tau, \nu)$  and  $t(\nu, \tau)$ .

The computation of these expressions can be found in [4, 5] but we repeat them for the self-containedness of the paper. The main point is the computation of  $v(\tau, \nu)$  as it may involve auxiliary functions, namely  $f(\nu)$ , the coordinate of the father of the tile  $\nu$  in its tree,  $\sigma(\nu)$ , the coordinate of the **preferred son** of  $\nu$  and  $st(\nu)$ , the **status** of  $\nu$ . As mentioned from [2, 3], the tree which we consider has two kinds of nodes: black and white ones with two and three sons respectively. The sons can be deduced from the node by the following rules  $B \to B_*W$  and  $W \to BW_*W$  in easy notations, where the star indicates the place of the preferred son. We assume here that the functions  $f(\nu)$  and  $\sigma(\nu)$ are known. Their computation is efficient but it requires notions which we have not mentioned here. We refer the reader to [3, 4, 5] for additional information.

neighbour	sector	number	slice	place	
0	$\sigma$	ν	au	0	
1	$\sigma$	u	au	3	
2	$s(\sigma, u, au)$	v( au, u)	t( u, au)	1	
3	$\sigma$	ν	$ au\ominus 1$	1	
0	σ	ν	au	1	
1	$s(\sigma, u au)$	v( au, u)	t( u, au)	0	
2	$\sigma$	u	au	3	
3	$\sigma$	ν	$\tau\oplus 1$	0	
0	$\sigma$	ν	au	2	
1	$\sigma$	u	$ au\ominus 1$	2	
2	$\sigma$	u	$ au\oplus 1$	2	
3	$\sigma$	ν	au	3	
0	$\sigma$	ν	au	3	
1	$\sigma$	u	au	0	
2	$\sigma$	u	au	1	
3	$\sigma$	ν	au	2	

**Table 1** Coordinates of the neighbours of a 2-triangle T in terms of the coordinates of T.

Clearly, the coordinate of a neighbour N of a tile T with coordinate  $\nu$  depends on the side  $\tau$  shared by T and N. Now, notice that this side numbered by  $\tau$  in T does not receive the same number in N and we shall say that N is the neighbour  $\tau$  of T. The correspondence between these numbers gives the

value of the function  $t(\nu, \tau)$  and, for completeness, we give it in Table 2. Note that the sides of the central cell are all numbered by 1 in its neighbours. For the other cells, the correspondence depends on the status of T and it may also depend on that of N. Side 7 is always the side shared by a neighbour which is on the same level of the tree, even when there is a change of tree by the change of sector. If T is black, its side 7 is numbered 2 on the other side. If T is white, the number of its side 7 in the other neighbour N depends on the status of N as indicated in the table.

From this table, we can indicate the values of  $v(\tau,\nu)$  which are given in Table 3. The basic point is that  $v(1,\nu)$  for the heptagon H defined by  $\nu$  is always  $f(\nu)$ , as its neighbour 1 is the father of H. Similarly, we have that  $v(4,\nu)$  is always  $\sigma(\nu)$  as neighbour 4 is the preferred son of H, regardless of the status of H. Note that in the case of a black heptagon H on the leftmost branch of the tree, two of its neighbours belong to the other tree on this side of the sector of H: neighbours 2 and 3. Neighbour 2 is still  $\nu - 1$  and, consequently, neighbour 3 is the rightmost son neighbour 2, hence it is  $\sigma(\nu-1)+1 = \sigma(\nu)-1$ . A symmetrical remark holds for a white H standing on the rightmost branch of the tree: neighbours 6 and 7 belong to another tree, the one which spans the other sector than that of H. Now, this time, neighbour 6 is numbered  $\nu+1$  and so, as neighbour 7 the father of neighbour 6, neighbour 7 is numbered  $f(\nu+1) = f(\nu)+1$ . At last, the root, which is a white node, belongs to both the left- and the rightmost branches of the tree. This is why it has a specific profile, different from both a standard white node and from a node on the rightmost branch of the tree.

**Table 2** Correspondence between the numbers of a side shared by two heptagons, H and K. Note that if H is white, the other number of side 1 may be 4 or 5 when K is white and that it is always 5 when K is black.

	black $H$		white $H$		
in	H in	K in $H$	in $K$		
	$1   3^{wK}$	$,4^{bK}$ 1	$4^{wK},$	5	
	2	6 2	7		
	3	7 3	1		
	4	1 4	1		
	5	1 5	1		
	6	2 6	2		
	7	2 7	$2^{wK}, 3^{b}$	οK	

It remains to indicate that in the case of a heptagon H which is on the leftor the rightmost branch, it is easy to define the number of the sector to which belongs the neighbours which do not belong to the tree of H. Indeed, let  $\sigma$  be the number of the sector in which H lies. If H is a black node, its neighbours 2 and 3 are in the sector  $\sigma \oplus 1$ . If H is a white node, its neighbours 6 and 7 are in the sector  $\sigma \oplus 1$ . Note that for the root of the sector  $\sigma$ , its neighbour 2 is in the sector  $\sigma \oplus 1$  and its neighbour 1 is the central cell which is outside all the sectors.

To conclude with this section, let us remember that cellular automaton have been implemented in several grids of the hyperbolic plane. The complexity classes of these cellular automata have been investigated leading to very surprising results. Several universal cellular automata also have been implemented in these grids. We refer the reader to [5, 6] for more information and more references.

**Table 3** The values of  $v(\tau, \nu)$ .

au	1	2	3	4	5	6	7	
black	f( u)	$f(\nu) - 1$	$\nu - 1$	$\sigma( u)$	$\sigma(\nu)$ +1	$\sigma(\nu)+2$	$\nu + 1$	
left	f( u)	$\nu - 1$	$\sigma(\nu) - 1$	$\sigma( u)$	$\sigma(\nu)$ +1	$\sigma(\nu)+2$	$\nu + 1$	
white	f( u)	$\nu - 1$	$\sigma(\nu) - 1$	$\sigma( u)$	$\sigma(\nu) + 1$	$\sigma(\nu)+2$	$\nu + 1$	
$\operatorname{right}$	f( u)	$\nu - 1$	$\sigma(\nu) - 1$	$\sigma( u)$	$\sigma(\nu) + 1$	$\nu + 1$	$f(\nu)+1$	
root	0	1	$\sigma(\nu) - 1$	$\sigma( u)$	$\sigma(\nu) + 1$	$\nu + 1$	1	

NOTE: left denotes the leftmost branch of the tree, right denotes its rightmost one.

# 3 The simulations

Now, we have the tools to implement cellular automata in the 2-triangles. In Sub-section 3.1, we shall look at the implementation of a cellular automata which propagates the tree structure of the heptagrid. The result, illustrated by Figure 7 convinced us that we could try to simulate colonies of bacteria. We propose three of them which are examined in Sub-section 3.2 and which differ by the number of states of the cellular automaton which is used for the simulation.

## 3.1 First simulation: the propagation of the tree structure

The tree structure of the heptagrid can be implemented by cellular automata on this grid: this was illustrated in [7] in order to give a toy example of a cellular automaton on this grid.

We can do the same here and Figure 7 gives the 36<sup>th</sup> step of execution of this automaton starting from an initial configuration in which the seven 2-triangles of place 2 of a heptagon are in the same state, in red in the figure: we call this the core-2 configuration. As we can see, the automaton has a non-small number of states: 18 of them. In [7], we had 5 states only. In fact, it is possible to have 4 states in the case of the heptagrid if we do not need to differentiate the two white sons of a white node. We need much more states here as we wish to diffuse the structure of the tree with its two types of rules. For programming reasons, it was easier to program the automaton by implementing the following strategy: when the automaton enters a heptagon, it goes as soon as possible to the 2-triangles with place 2. There, by a counting process, it determines the directions of the sons from the direction of the father which is the direction from which the automaton entered the cell.

The way the automaton is working can be seen as an animation on the slides which are deposited on [8].



Figure 7 The propagation of the tree structure of the heptagrid. Left-hand side: with the indication of the grid. Right-hand side: the grid has been removed.

It seems to us that the result has a striking similarity with pictures about the growth of colonies of bacteria in highly stressed conditions.



**Figure 8** Left-hand side: propagation of the tree structure of the heptagrid in the heptatrigrid. Right-hand side, propagation of a bacteria colony, picture by courtesy of Professor Ben-Jacob.

We focus on this topic in the next sub-sections trying to reduce the number of states and to define more acceptable rules from a biologic point of view.

## 3.2 The other simulations

In this sub-section, we successively examine three attempts to simulate the propagation of colonies of bacterias. We shall consider the number of states we use as well as the information that the cells are assumed to know about themselves. We shall try to give the states and these assumptions a kind of biological flavour. We shall have to keep in mind the specificity of the cellular automaton programming. In particular, a cell cannot directly act upon another one. Such an action has to be 2-stepped: the cell c which wants to act on a neighbour n has to signal the intention of this action by taking a particular state. Seeing this state on c, and possibly seeing an additional information displayed by its other neighbours, n can interpret the intention and take the state wished, so to say, by its neighbour c. This is why we shall say that the cell does this and that in our sequel, although, strictly speaking, the cell can perform such actions in an indirect way, as was just sketched before.

#### 3.2.1 With two states

We start with a very rudimentary situation. We have two states: white for the medium, black for the colony. The cells want to propagate, but competition is not encouraged. In these conditions the rules can be stated in a simple way as follows:

- (a) A black cell remains black.
- (b) A white cell becomes black if and only if it has exactly one black neighbour at this time.

From condition (a), once a white cell c becomes black at time  $t_0$ , it is black for all times t with  $t \ge t_0$ .



Figure 9 Simulation of a diffusion process with 2 states.

Figure 9 illustrates the 36<sup>th</sup> time of this situation starting from the core-2 configuration. We can see that the colony invades almost all the space, leaving holes unoccupied. The condition on the change of the white cell to a black one has, as a consequence, that a white cell which has two black neighbours exactly remains white. This the reason of the pairs of adjacent white cells which are regularly produced in the evolution of the automaton.

#### 3.2.2 With four states: version 1

In this sub-subsection, we give us more states: four of them, W, R, Y and V calling them white, red, yellow and vermilion respectively. As previously, white represents the medium. Red is almost the initial configuration which is, here again, the core-2 configuration. Then yellow or vermilion can represent the colony: either one of them or both.

This time too, we formulate the rules in the same style as previously, by properties on the number of states which are around the cell.

- (a) A red, yellow or vermilion cell remains in its colour.
- (b) A white cell becomes red, yellow or vermilion if and only if, at this time, it has exactly one neighbour which is red, yellow or vermilion respectively.

Here too, when a white cell becomes non-blank, it keeps the new colour for ever.



Figure 10 Simulation of a diffusion process with 4 states, based on a local knowledge of the colony.

Figure 10 illustrates the  $36^{\text{th}}$  step of computation starting from a configuration where the non white cells occupy a heptagon exactly, with the pattern we have, in Figure 10 for the central heptagon: the 2-triangles with places 2 or 3 are red, the 2-triangles with place 0 are vermilion, those with place 1 are yellow. We call this the heptagonal core configuration. We can notice that in this case also, the cells which remain white are the same as those of the previous automaton. We also notice that, thanks to the heptagonal core of the initial configuration, the red state no more occurs in the computation. After the initial time, the computation outside the heptagonal core involves three states only: the white, the yellow and the vermilion one.



Figure 11 Comparing the diffusion process with 4 states based on local knowledge of the colony with a colony of Figure 1, picture by courtesy of Professor Ben-Jacob.

If we look carefully at the computation outside the heptagonal core, we can see that yellow and vermilion areas are very similar. Each one covers a larger and larger area. Consequently, we have to reasonably consider that both of them represent the colony, see Figure 11 for a comparison with one of the colonies illustrated in Figure 1. We can view them as two sub-species of the colony which do not mix.

#### 3.2.3 With four states: version 2

In this sub-subsection, we again have four states. However, we have an additional assumption. We assume that the colony has some knowledge of the geometry of the space. This can be viewed as an acquired experience of the space by the colony. The knowledge we assume is that a cell knows its place and whether it is in slice 1 or not. We can see that this is a very limited information: 1 bit for the slice and two ones for the space, which is rather few. We take the same colours as previously, with white as the state for the space. Here too, the initial configuration is the heptagonal core.

This time too, the formulation of the rules is in the same style as previously but it becomes more intricate, as it involves the place and the slice of a 2-triangle.

(a) A red, yellow or vermilion cell remains in its colour.

- (b) If a white cell has two white neighbours and if its slice is 1, then it takes the colour of its third neighbour.
- (c) If a white cell has two white neighbours and if its slice is not 1 but its place is 2 then if its third neighbour is red, yellow or vermilion, it becomes yellow, vermilion or red respectively.
- (d) If a white cell has two white neighbours and if its third neighbour is red then, if its place is 3, 0 or 1, it becomes red, vermilion or yellow respectively.
- (e) If a white cell has the states white, yellow and vermilion among its three neighbours, if its slice is 1 and if its place is 3, then it becomes red.

With these rules, the cellular automaton behaves in a somewhat different manner. As can be seen from Figure 12, although the initial configuration is the same as previously, the four states are now involved during the whole computation. Moreover, the colony does not occupy the whole space: the branches which regularly are spread out are far away from each other, which avoid any kind of competition. Also, we can see that this time we have a cooperation between the states. The knowledge whether a cell is in a slice 1 or not allows



**Figure 12** Simulation of a diffusion process with 4 states, based on a local knowledge of the colony plus a slight insight on the global structure.

the colony to take advantage of the topology in order to invade the center of a heptagon, according to the scenario contained in the condition c of the rule. Next, the condition d and e allow to occupy the slices 4 and 5 of the heptagon and those ones only, without knowing the number of the slice. This obtained by the combination of the conditions c, d and e. Once this is checked for one heptagon around the central cell, this is repeated for all the heptagons which are the 4- and 5-neighbours of a heptagon. This way, we obtain seven binary trees which grow from the heptagonal core. In Figure 13, we compare the growth of Figure 12 with a picture of another bacteria colony. Note that the left-hand side picture of Figure 13 is obtained from Figure 12 by removing the drawing of the heptatrigrid. The computer program which draws the figure writes down a PostScript file from the information obtained by performing the simulation up to the 36<sup>th</sup> step, starting from the heptagonal core. In this writing, the program simply removes the drawing commands used to draw Figure 12, simply keeping the filling commands which allow to paint close areas defined for drawing the same figure.



Figure 13 Left-hand side: simulation of the growth of a bacteria colony with the heptatrigrid. Right-hand side, propagation of another bacteria colony, picture by courtesy of Professor Ben-Jacob.

# 4 Conclusion

It should be remarked that in all the previous simulations, the computation may be as long as wished within the time and memory limits of a computer. Due to the exponential growth of the number of 2-triangles as we go away from the central heptagon, these limits are rapidly reached and improvements in technology may perhaps allow us by one round of 2-triangles further each time the capacity is multiplied by 3. However, for simulations of actual colonies of bacteria, this is not a problem as their growth is not only finite but also small in the hyperbolic scale.

It seems to us that this hyperbolic simulation gives an interesting approximation of the phenomenon observed in real experiments. The above discussion about the space of computation indicates that it could be interesting to investigate generation 3 of triangles and so, to look at what we obtain for 3-triangles. Most probably, we could get a finer simulation but certainly at the price of a bigger number of states. The interpretation of these states from a biological point of view is of course a question as well as how much of the knowledge of the space could be allowed for 3-triangles where a third parameter within the place is necessary.

These are directions for further work on this topic.

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