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Two-stage joint BEM-OTFS Channel Estimation Algorithm Based Sparse Bayesian Learning Algorithm

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***Abstract*—In this paper, a two-stage BEM-OTFS channel sparse Bayesian learning (SBL) estimation algorithm is proposed to address the problem of increasing error of the compressed sensing channel estimation algorithm at low resolution when OTFS is embedded in the pilot frequency mode and the pilot frequency overhead is too large. The algorithm uses a complex exponential basis expansion model (CE-BEM) at the transmitter side to design a design mode with lower pilot overhead, and a linear solution is used to obtain the roughly estimating channel matrix. After detecting the rough estimated channels, some of data symbols detected are used as pseudo-pilot for the second stage estimation. In the second stage, a sparse expression about the basis parameters is constructed using the discrete prolate spheroidal basis expansion model (DPS-BEM), which is solved by the SBL algorithm to calculate a more accurate channel after the sparse Bayesian learning (SBL) algorithm. The simulated numerical analysis shows that the present algorithm has ideal performance for OTFS symbols with lower Resolution, which is better than the SBL algorithm which is based on the DD domain pilot input-output relationship in general. Meanwhile, the present algorithm also has some advantages in spectral efficiency.**

Keywords—sparse Bayesian learning, basis expansion model, embedded pilot design

I. INTRODUCTION

The proposed planning of 6G network in [1] contains requirements for intelligent three-dimensional communication established in the full frequency bands everywhere in air, sky, earth, and sea, which includes intelligent three-dimensional communication proposed for high-speed mobility management, and higher requirements for signal reliability in different types of communication scenarios. Orthogonal Frequency Division Multiplexing(OFDM) technology is widely used in current 5G communication, and many researchers have proposed different OFDM modulation and demodulation schemes for different channel environments. However, OFDM often needs to consume a certain number of protection intervals and cyclic prefixes in the face of time-frequency double-selected channels, and its demodulation process also needs to face problems such as excessive complexity and estimation of the corresponding guide frequency design affected by the symbol alignment.

In 2017, R HADANI proposed orthogonal time frequency space (OTFS) modulation technique [2] as the main modulation alternative to OFDM in the future. It arranges the signals in a specific Delay-Doppler (DD) domain, where each symbol is represented by a pair of orthogonal basis functions in the time-frequency domain, and the time delay under multipath channels as well as the Doppler frequency shift in the DD domain will lead to a shift in the delay domain and the Doppler domain, respectively, which greatly improves the spectrum utilization and estimation accuracy of OTFS in the demodulation process. Therefore, research on OTFS is emerging, and many researchers are exploring OTFS channel characteristics to meet the requirements of 6G communication scenarios.

Many researchers have used the sparse property of double-selected channels under OTFS modulation for this purpose and applied compressive sensing to their channel estimation. Among them, Sparse Bayesian Learning (SBL), as one of the methods of sparse channel information recovery algorithms, is widely used in OTFS channel estimation because it does not require setting regularization parameters. For example, [3] proposed a method for estimation by sparse Bayesian under embedded guide frequency. In [4], a Bayesian estimation algorithm using multiple measurement vectors is designed by subspace channel estimation method, and then the team in [5] improves the

Bayesian estimation algorithm using row and group sparsity properties and applies it to MIMO channels. Then [6] uses block-sparse Bayesian estimation in MIMO scenarios and combined it with block reorganization to obtain better channel estimation accuracy by updating the size of non-sparse blocks at iteration time. [7] explores the Doppler-angle domain in terms of a local β-process to make it transformed into a sparse signal solution problem and follows this with sparse Bayesian estimation for large-scale MIMO uplinks.

Although the SBL algorithm has achieved some results in OTFS channel estimation, it is limited by the coding and decoding process. Usually the actual solution process obtains an approximation and does not solve the true fractional Doppler offset, and its effect will be greatly reduced when the OTFS frame grid size is small. In addition the actual double-selected fading channel causes signal frequency and phase variations due to the propagation distance difference problem, called Doppler expansion, few studies have proposed solutions for it.

Basis expansion model (BEM) parameterizes the time-varying channel as a weighted combination of basis functions to reduce the number of unknown channel coefficients, thus improving spectral efficiency, and is widely used in OFDM high-speed scenarios. However, the application of BEM in OFDM faces weaknesses such as restricted model usage conditions and difficulties in modeling in millimeter wave environment. To this end, this paper investigates the SBL algorithm based on the BEM-OTFS modeling environment and designs an algorithm to save the guide frequency overhead in this environment.

II. OTFS SYSTEM MODEL

The Modem link of OTFS model is shown in Fig 1. Assuming that binary symbols are passed through mQAM or mPSK(m denotes the number of mapping orders and $m \geq 2$) and mapped as MN data symbols arranged in DD domain. OTFS symbols are arranged in a grid size of $N \times M$, the spacing of delay axis and doppler axis of each grid are$\Delta\tau = 1/(M\Delta f)$ and $\Delta\upsilon = 1/(NT)$ respectively, where N and M denotes Grid size of the Doppler and time-delay domains in the DD domain respectively.

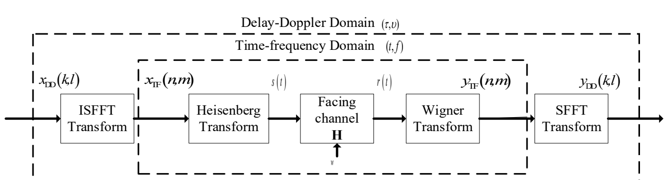

Fig. 1. OTFS channel model

The ranked signal is following inverse symplectic finite Fourier transform (ISFFT) and Heisenberg transformed to obtain the time domain transmit signal $s(t)$. Then it is converted to a receiver-side signal $r(t)$ after suffering multipath double-selected fading channel $\mathbf{H}$ and additive Gaussian white noise $\mathbf{w}$. After receiver's wigner transform and symplectic finite Fourier transform (SFFT), DD domain signal $y_{DD}(k,l)$ can be received.

The vectorized expression of the input-output relationship can be written as: $\mathbf{r} = \mathbf{H}_t\mathbf{s} + \mathbf{w}$,where $\mathbf{H}_t \in \mathbb{C}^{MN\times MN}$ denotes the time domain channel matrix, which its specific structure is shown in Equation (1),where $h[t,l']$ denotes The channel gain at the t-th moment under the l'-th normalized delay, and $l' = 0,1\ldots,L$, L denotes the maximum normalized time delay. Usually, the maximum normalized delay L is known at the receiver side for a given channel environment.

$$\mathbf{H}_t = \begin{bmatrix} h[0,0] & 0 & 0 & \cdots & h[0,2] & h[0,1] \\ h[1,1] & h[1,0] & 0 & \cdots & \vdots & h[1,2] \\ \vdots & h[2,1] & \ddots & \ddots & h[L-2,L] & \vdots \\ h[L,L] & h[3,2] & \vdots & \ddots & 0 & h[L-1,L] \\ 0 & \vdots & \vdots & \vdots & \ddots & 0 \\ \vdots & 0 & \vdots & \vdots & h[MN,1] & h[MN,0] \end{bmatrix} \tag{1}$$

The delay and doppler offsets of the i-th path under the DD domain grid are $l_i = \tau_i \times M\Delta f$ and $k_i = \upsilon_i \times NT$respectively. Meanwhile, the maximum delay and the maximum Doppler shift of the channel in the DD domain can be written as $l_\tau = \tau_{max} \times M\Delta f$ and $k_\upsilon = \upsilon_{max} \times NT$ respectively. Usually the actual number of paths $L \leq P \ll N \times M$, and the sampled values can be compressed, so there is sparsity in OTFS.

III. TWO-STAGE CHANNEL ESTIMATION ALGORITHM BASED ON BASE EXPANSION MODEL

A. Pilot Design And Linear Estimation Based On CE-BEM

the expression of $\mathbf{H}_t$ after BEM modeling can be written as: $\mathbf{H}_t = \sum_{q=0}^{Q} \mathrm{diag}\{\mathbf{b}_q\}\mathbf{C}_q + \mathbf{E}_{mod}$,where $\mathbf{b}_q$ denotes the q -th basic function vector, which its values are related to the type of model used. The q-th basis function vector of CE-BEM, for example, is $\mathbf{b}_q = \left[1, e^{j\omega_q}, \ldots, e^{j\omega_q(MN-1)}\right]^T$;$\mathbf{E}_{mod}$ denotes the modeling error of the base expansion model, which is related to the model selection. The size of ω_q determines the type of BEM. $\boldsymbol{c}_q = \left[c_q[0], c_q[1], \ldots, c_q[L]\right]$ denotes the q-th unknown BEM parameter vector.$c_q[l'], l' = 0,1,2,..,L$ denotes the BEM confident of l'-th delay taps. Q denotes the order of BEM, which determines the modeling size. The size of Q varies with the resolution R .Typically it has the lower limit $Q \geq RN[f_{\max}/\Delta f]$, where $f_{\max}/\Delta f$ denotes the biggest doppler spread with f_{max} under the subcarrier spacing Δf .The conversion expression between $\mathbf{c}_q$ and its matrix structure of $\mathbf{C}_q$as shown in (2),where $\mathbf{c}_q = \left[c_q[0], c_q[1], \ldots, c_q[L]\right]^T$, $c_q[l'], l' = 0,1,2,..,L$ denotes the cofficient of l'-th delay taps,which is the value to be solved in the channel estimation process. Thus, the number of unknown parameters to be solved using BEM channel estimation is reduced fromMN to $(Q+1)(L+1)$.

$$\begin{aligned} \mathbf{C}_q &= \boldsymbol{F}_{MN}^H \mathrm{diag}\{\boldsymbol{F}_{MN\times L}\boldsymbol{c}_q\}\boldsymbol{F}_{MN} \\ &= \mathrm{circ}\,\underbrace{\{c_q[0], c_q[1], \ldots, c_q[L], 0, \ldots, 0\}}_{(L+1)(Q+1)} \end{aligned} \tag{2}$$

Considering the problem that the usage of BEM modeling methods should ensure a lower pilot overhead compared to [8], there is a two stage estimation algorism is designed. The detection is performed on the basis of the previous CE-BEM and the results are used as pseudo-pilots to achieve spectral occupancy reduction. The second estimation of which uses DPS-BEM as a modeling method, whose basis function has a stronger energy concentration , difficult to cause significant frequency leakage and adaptable in high doppler environments compared to the former,[9]. And the structure schematic of the whole algorithm is shown in Fig 2.

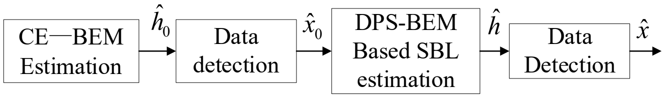

Fig. 2. Two-Stage BEM algorithm Principle diagram

Let the CE-BEM order of the first stage is $Q_s = N[f_{max} / \Delta f]$.therefore ,the Tx symbols in DD domain and estimation area as shown in Fig 3, and let the area of second stage estimation is $\mathbf{y}_{p2}$.With the help of CE-BEM framework from [10],and considering $N_p M_p$ pilots' input-output relationship in the area of size $\mathbb{C}^{(N_p+Q_s)\times(M_p+L)}$ of $\mathbf{H}$, the l'-th basis coefficient of CE-BEM can be solved by (3), and then $\tilde{\mathbf{C}}_\mathrm{q}^0$ and $\widetilde{\mathbf{H}}_0$ can be solved .

$$\tilde{c}_\mathrm{q}^0[l'] = \sum_{i_k=0}^{M_p} \frac{y[k,l]}{x_p} e^{-j2\pi\frac{q'(l+i_k)}{MN}} \tag{3}$$
$$k = k_p - N_p/2 + q', l = l_p + l'$$

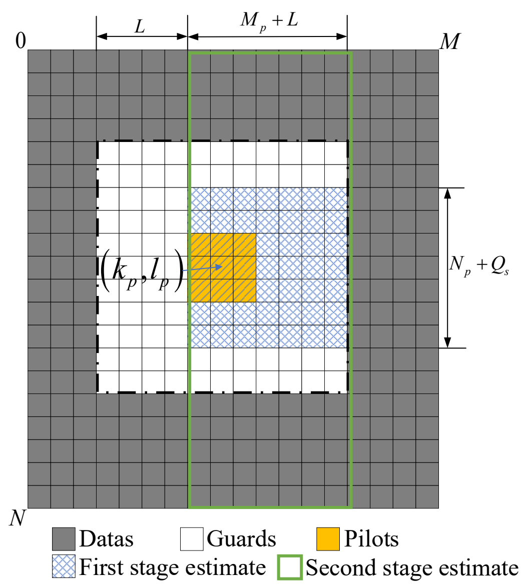

Fig. 3. Tx symbols in DD domain and estimation area

$\mathbf{y}_{p2}$ is constructed as shown in Equation(),where $\hat{\mathbf{x}}_1^0$ and $\hat{\mathbf{x}}_2^0$ denotes data symbols in matrix $\hat{\mathbf{x}}^0$ recovered by the first detection in its $k \in (1{:}\,k_p - Q_s, l_p{:}\,l_p + l_\tau)$ and $l \in (k_p + Q_s{:}\,N, l_p{:}\,l_p + l_\tau)$ respectively.

$$\mathbf{y}_{p2} = \left[\hat{\mathbf{x}}_1^{0\,T}, \mathbf{y}_{p1}{}^T, \hat{\mathbf{x}}_2^{0\,T}\right]^T \tag{4}$$

B. Sparse Bayesian learning based on DPS-BEM

Before utilizing Sparse Bayesian learning algorithm, the DPS-BEM is first introduced. Let the order under the DPS-BEM estimation be $Q_L = 4N[f_{max} / \Delta f]$.The acquisition of the basis functions of the DPS-BEM requires first establishing the kernel functions$\mathfrak{C} \in \mathbb{C}^{MN\times MN}$,the $[n,m]$ element calculating formula of $\mathfrak{C}$ is:

$$\mathfrak{C}[n,m] = \frac{\sin[2\pi(n-m)f_d T_s]}{[\pi(n-m)]} \tag{5}$$

Where $n, m = 0,1\ldots, N\times M$.Then $\mathbf{B}_t$, the eigenvector group of $\mathfrak{C}$ can be solved, and the basis function matrix $\mathbf{B}$ by extracting first $Q_L + 1$ vector of $\mathbf{B}_t$. That is $\mathbf{B} = \mathbf{B}_t(:, 1{:}\,Q_L + 1)$.

To facilitate the next analysis, the channel link matrix needs to be redescribed by the BEM modeling idea, which transforms the original underdetermined problem of channel estimation into an overdetermined problem of sparse signal recovery. Therefore, in order to match symbols of $\mathbf{y}_{p2}$, Let the pilot offset is on the range in delay domain is $M_\mathrm{DPS} = l_{\tau_\mathrm{est}} + 1$, the basis function matrix corresponding to compressed sensing, that is $\mathbf{B}_{pilot} = \left[\mathbf{b}_0^{pilot}, \mathbf{b}_1^{pilot}, \ldots, \mathbf{b}_Q^{pilot}\right]$, where $\mathbf{b}_q^{pilot}$ can be obtained by truncate the first $Q_L + 1$ column elements of $\mathbf{b}_{\boldsymbol{q}}^{pilot}$, that is:

$$\begin{aligned} &\mathbf{b}_{\boldsymbol{q}}^{pilot}(M_\mathrm{DPS}(i-1) + 1{:}\,M_\mathrm{DPS}i + 1) \\ &\quad = \mathbf{b}_q^{pilot}\left(l_p + Mi{:}\,l_p + l_{\tau_\mathrm{est}} + Mi\right) \end{aligned} \tag{6}$$

Firstly, considering the channels under each time delay tap L as it respective independent linear time-varying channels according to the BEM expression (6), that is $\mathbf{h}(:,l) = (h[0,l], h[1,l], \ldots, h[MN-1,l])^T \in \mathbb{C}^{MN\times 1}$ .The resulting BEM modeling of the matrix yields:

$$\mathbf{h}(:,l) = \sum_{q=0}^{Q} \mathbf{b}_q(:,q)\mathbf{c}_q(q,l) \tag{7}$$

Following the structure of $\mathbf{h}$, from $\mathbf{y}_{p2}$,let each Column vectors of $\mathbf{y}_{p2}$ are $\hat{\mathbf{x}}_p^1(:,m)$,where $m = 1,2,\ldots, M_\mathrm{DPS}$,and then each $\hat{\mathbf{x}}_p^1(:,m)$ can be constructed as a circular matrix $\boldsymbol{\Delta}_i = circ\left[\hat{\mathbf{x}}_p^1(:,m)^T\right]$,which is shown as (8).

$$\mathbf{W}_l = \begin{pmatrix} \boldsymbol{\Delta}_1 & & & \\ & \boldsymbol{\Delta}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{\Delta}_M \end{pmatrix} \in \mathbb{C}^{M_\mathrm{DPS}N\times M_\mathrm{DPS}N} \tag{8}$$

Utilizing the matrix $\mathbf{W} = \left[\mathbf{W}_0, \mathbf{W}_1, \ldots, \mathbf{W}_l, \ldots, \mathbf{W}_{l_{\tau_\mathrm{est}}}\right] \in \mathbb{C}^{MN\times MN(l_{\tau_\mathrm{est}}+1)}$, The observation matrix expression can then be derived as:

$$\mathbf{A} = \mathbf{W}\left(\mathbf{I}_{M_\mathrm{DPS}} \otimes \boldsymbol{\Psi}\right) \tag{9}$$

Where $\boldsymbol{\Psi} = 1/\sqrt{N}\left(\mathbf{F}_N \otimes \mathbf{I}_{M_\mathrm{DPS}}\right)\mathbf{B}_{pilot}, \mathbf{F}_N$ is the N-order DFT matrix. Therefore, the sparse DPS-BEM input-output relation for OTFS receiver can be written as：

$$\mathbf{y}_p = \mathbf{A}\mathbf{c}_\mathrm{DPS} + \mathbf{w}_p \tag{10}$$

Among them, $\mathbf{c}_\mathrm{DPS} = \left[\mathbf{c}_0{}^T, \mathbf{c}_1{}^T, \ldots, \mathbf{c}_{Q_L}{}^T\right]^T$ denotes BEM coefficient vectors to be solved in the sparse expression. In the response of the biggest delay we assumed in pilot design is lower than actual biggest delay L ,therefore BEM most of taps coefficients of $\mathbf{c}_\mathrm{DPS}$ can be regarded as 0 except only L taps exist larger parameters. Thus, the BEM modeling solution problem is converted into a sparse signal solution problem, and the basis parameter vector **c** will be solved next from the sparse solution perspective.

Sparse Bayesian learning is based on statistical assumptions on sparse signals, where the signals are considered to be probability distributions in a finite dimensional space. These assumptions usually involve prior distributions of signals and sparsity constraints. According to [11] , [12] and [13],The principle of sparse Bayesian estimation is as follows.

Assuming the actual channel noise $\mathbf{w} \sim CN(0, \delta^2 \mathbf{I}^{MN})$, where C is the of $\mathbf{w}$, and $N(\cdot)$ denotes a Gaussian distribution with mean 0 and variance δ^2.Considering in other SBL algorism the equivalent channel matrix to be solved $\mathbf{h}$ obeying Gaussian distribution with mean $\mathbf{A}$ and variance $\delta^2 \mathbf{I}^{MN}$,the conditional probability distribution of $\mathbf{c}_{\mathrm{DPS}}$ can be decomposed as:

$$\begin{aligned} \mathrm{p}(\mathbf{c}_{\mathrm{DPS}} \mid \boldsymbol{\Gamma}) &= \prod_{l=1}^{(L+1)(Q_L+1)} p(c_l \mid \gamma_l) \\ &= \prod_{l=1}^{(L+1)(Q_L+1)} CN(c_l; 0, \gamma_l^{-1}) \end{aligned} \tag{11}$$

Where $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_i\}_{i=0}^{L-1} \in \mathbb{R}^{L \times L}$ is a hyperparameters vector, of which element γ_i ,the i-th symbol corresponding to c_i, is the i-th value of $\mathbf{c}$.Meanwhile,L is the magnitude of observation value, which is used as observation vector through the iterating process. Assuming that $\boldsymbol{\Gamma}$ obeys gamma distribution and is constrained by the shape parameter a and the inverse scale parameter b, their values need to be initialized before estimation, and usually the initialized values are related to the signal-to-noise ratio and the signal power.

It's worth noting that above we resolved is basic basis coefficient vector **c** after BEM modeling instead of equivalent channel vector **h** .Each c_i of $\mathbf{c}_{\mathrm{DPS}}$ are jointly controlled by Laplace prior with hyperparameter λ[14], that is $p(c_i|\lambda) = \mathrm{Laplace}(0, 1/\sqrt{\lambda})$,where the size of λ is set as 10^{-4} .So combining the Bayesian hierarchical modeling of each stage, the joint probability density function about the whole variable can be obtained (PDF）：

$$p(\mathbf{c}) = p(\mathbf{c} \mid \mathbf{y}_p, \boldsymbol{\Gamma}, \gamma_0) = N(\mathbf{c} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{12}$$

Where $(\mathbf{c} \mid \boldsymbol{\mu})$ and $\boldsymbol{\Sigma}$ denotes mean vector and variance matrix of corresponding mean vector and variance matrix, and its expression can be obtained as:$\boldsymbol{\mu} = \boldsymbol{\Sigma}\mathbf{A}^{\mathrm{H}}\mathbf{Y} \in \mathbb{C}^{L \times 1}$ and $\boldsymbol{\Sigma} = (\mathbf{A}^{\mathrm{H}}\mathbf{R}_{\mathrm{v}}^{-1}\mathbf{A} + \boldsymbol{\Gamma}^{-1})^{-1} \in \mathbb{C}^{L \times L}$.Calculating $\boldsymbol{\mu}$ needs to have $\boldsymbol{\Gamma}$ and γ_0 firstly. For this reason, using expected to maximize (EM) algorithm solution, the formula is expressed as:

$$(\boldsymbol{\Gamma}^{\mathrm{new}}, \gamma_0^{\mathrm{new}}) = \underset{\boldsymbol{\Gamma}, \gamma_0}{\operatorname{argmax}} E\{\ln p(\mathbf{Y}, \mathbf{c}, \boldsymbol{\Gamma}, \gamma_0)\} \tag{13}$$

According to [15], $\boldsymbol{\Gamma}^{\mathrm{new}}$ and γ_0^{new} can be solved as shown in the equation (14).

$$\begin{cases} \gamma_{\mathrm{i}}^{\mathrm{new}} = \dfrac{\sqrt{1 + 4\lambda(\boldsymbol{\Sigma}[i,i] + \boldsymbol{\mu}^2[i])} - 1}{2\lambda} \\ \boldsymbol{\Gamma}^{\mathrm{new}} = \boldsymbol{\Gamma}^{\mathrm{new}} \cup \gamma_{\mathrm{i}}^{\mathrm{new}} \end{cases} \tag{14}$$

However, after the iterative process finished, $\tilde{\mathbf{c}}_{\mathrm{DPS}}$ is not sparse. Therefore, we need to extract the parameters from $\boldsymbol{\Gamma}$ by threshold determination of i-th element $\boldsymbol{\Gamma}[i]$, as shown in equation (16).

$$\tilde{c}_i = \begin{cases} \boldsymbol{\mu}(j), \boldsymbol{\Gamma}[i] \geq \varepsilon_c \\ 0 \quad , \boldsymbol{\Gamma}[i] < \varepsilon_c \end{cases} \tag{15}$$

Where the size of $\varepsilon_c$is planned according to the size of $\tilde{\mathbf{c}}_{\mathrm{DPS}}$, to ensure that each $\tilde{c}[j]$ obtained matches the actual channel environment and is sufficiently sparse. Then $\widetilde{\mathbf{H}}$ can be recovered by $\tilde{\mathbf{c}}$ and $\mathbf{B}_{\mathrm{DPS-BEM}}$, and $\widetilde{\mathbf{X}}$ can be recovered by $\mathbf{Y}$. The above sparse SBL-based BEM-OTFS channel estimation algorithm is as follows:

Algorithm 1 SBL-Based Channel Estimation Algorithm

Input:$\mathbf{Y}, \boldsymbol{\phi}, P, L, Q$.
Output: Channel basis parameter matrix $\tilde{\mathbf{c}}_{\mathrm{DPS}}$
Initialize :$\gamma_0, \boldsymbol{\Gamma}, \boldsymbol{\Gamma}^{\mathrm{new}}, \mathrm{a}, \mathrm{b}, \mathrm{j} = 0$
While not satisfied:1. $\frac{\|\boldsymbol{\Gamma}^{\mathrm{new}} - \boldsymbol{\Gamma}\|_2^2}{\|\boldsymbol{\Gamma}\|_2^2} < 1\mathrm{e}-5$ and 2. $j \geq 500$ then
    $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^{\mathrm{new}}$
$\boldsymbol{\mu} = \boldsymbol{\Sigma}\mathbf{A}^{\mathrm{H}}\mathbf{Y}_{\mathrm{p}}$
$\boldsymbol{\Sigma} = (\mathbf{A}^{\mathrm{H}}\mathbf{R}_{\mathrm{v}}^{-1}\mathbf{A} + \boldsymbol{\Gamma}^{-1})^{-1}$
$\gamma_0^{\mathrm{new}} = \frac{2a-2+P}{2b+\mathbb{E}\{\|\mathbf{Y}-\boldsymbol{\phi}\boldsymbol{\mu}\|_2^2\}}$
For $i = 1: (L+1)(Q_L+1)$
    $\gamma_i^{\mathrm{new}} = \frac{\sqrt{1+4\lambda(\boldsymbol{\Sigma}[i,i]+\boldsymbol{\mu}^2[i])}-1}{2\lambda}$
    $\boldsymbol{\Gamma}^{\mathrm{new}} = \boldsymbol{\Gamma}^{\mathrm{new}} \cup \gamma_{\mathrm{i}}^{\mathrm{new}}$
End for
$j = j + 1$
End while
Get the vector of basis parameters to be solved $\tilde{\mathbf{c}}_{\mathrm{DPS}}$ from $\boldsymbol{\mu}$ by equation (16)

IV. SIMULATION RESULTS

In this section, we use 3GPP TS 36.141 extended vehicular A(EVA) model 0 to simulate. The communicating system parameters of the simulation process as shown in table Ⅰ.

TABLE I. SYSTEM SIMULATION PARAMETERS

Parameter	Index
Carrier frequency	4GHz
Subcarrier spacing	15KHz
N	128
M	64
Vehicular speed	700km/h
Pulse type	rectangular pulse
Mapping	4QAM

In the two-stage estimation algorism. the Massage passing(MP) algorism[16] can be used to recover symbols. And in order to Compare the practical performance of channel estimation in communication systems, both the SBL algorithm and the orthogonal matching tracking (OMP) algorithm with a similar pilot design mode as in [3] are used for simulation. And in order to verify SBL performance, let OMP algorithm applied in the two-stage method. The simulation results are shown in Figure 4 and Figure 5.

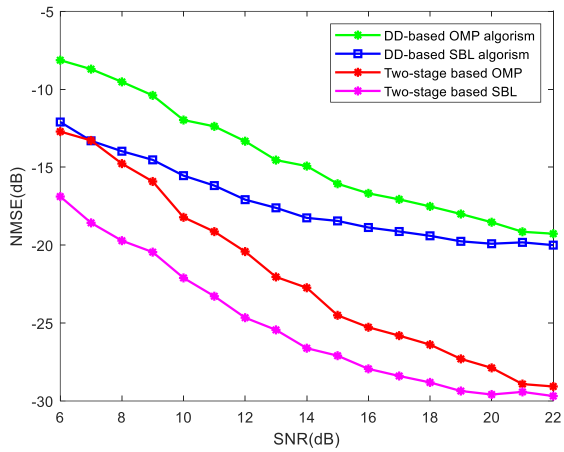

Fig. 4. NMSE variation curve with SNR for four algorithms

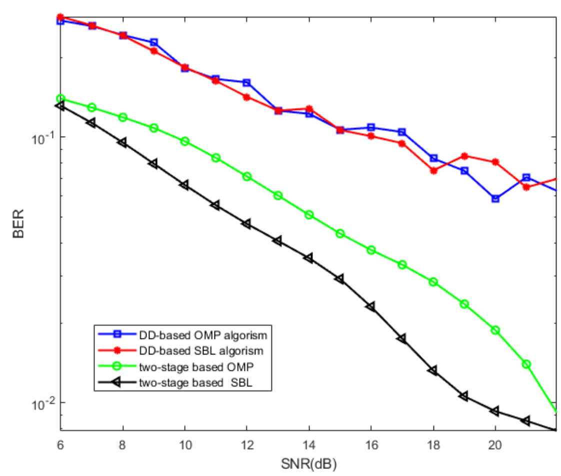

Fig. 5. NMSE variation curve with SNR for four algorithms

From the two curves, although the performance has been weakened a lot, the error of two-stage algorism is lower than that constructed sparsity in DD domain. It can be seen that The Two stage-based OMP algorithm and SBL algorithm have been significantly improved in performance. And SBL algorithm in BEM has a better performance in the above. The proposed two-stage SBL algorism has better performance than those three algorisms.

V. CONCLUSION

In this paper, we propose a two-stage BEM-OTFS SBL channel estimation algorithm to solve the problem of decreasing estimation accuracy of compressed sensing based OTFS channel estimation algorithm at low resolution, and the algorithm uses CE-BEM-based Coarse estimation combined with symbol detection to establish pseudo-pilots, and has the same estimation accuracy while reducing the algorithm leading frequency overhead. The Bayesian learning algorithm is applied to the proposed BEM-OTFS channel estimation by establishing sparsity using the method of setting the maximum delay about guard intervals much larger than the actual maximum delay. Simulation results show that this algorithm has a more promising estimation performance than the SBL and OMP algorithms on the DD domain at a lower resolution.

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