A STRUCTURE FOR THE COMBINED REDUCTION OF BIAS AND VARIANCE IN ESTIMATING SOURCE LOCATION AND MOTION

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ABSTRACT

A structure is presented for passive estimation of range, bearing as well as velocity of a source from a linear array. It uses a quasi-optimal post-processor of the time delays, which are obtained from a generalized correlator with finite observation time. The post-processor ultimately maps the sequential time delay observations onto invariant source trajectory parameters over which smoothing is performed to reduce, jointly, the variance and the bias in the estimate of the source kinematics. The present approach remains viable for moving sources at long ranges, off-broadside source directions and high time delay variances. Analysis and simulation results are presented to justify its usefulness under the stated stringent conditions. Review of and comparison to existing approaches are made to highlight the viability of present approach in the estimation of source trajectory.

I. INTRODUCTION

Under study is the basic problem of passive localization and motion analysis of a source observed from a linear array. Passive localization is concerned with estimation of the range and direction to a source from differences in arrival times or time delays τ_1 and τ_2 measured respectively between sensors I-II and sensors II-III (Figure 1). When the measurements are perfect, the law of cosines may be applied to triangles SI II and SII III to solve for range R and direction B in terms of time delays τ_1 and τ_2:

$$R = \frac{D^2 - .5c^2 (\tau_1^2 + \tau_2^2)}{c(\tau_1 - \tau_2)} \qquad (1)$$

$$B = \sin^{-1}\left[\frac{c}{2D}(\tau_1 + \tau_2) + \frac{c^2}{4DR}(\tau_1^2 - \tau_2^2)\right]$$

The sound speed c is presumed 5000 ft/sec; the intersensors separation D is set at 100 ft. Source motion parameters may be estimated from sequential observations of τ_1 and τ_2 that are provided in the cross-correlation process by cross-correlating the sensor outputs. The observation time is chosen short enough to permit the assumption of local stationarity.

This paper estimates the location and motion of a source from noisy time delay observations. In section (II), bias and variance on the estimates due to noise are discussed, then an approach is described to minimize their effects. In section III, a structure for the estimation process is given. In section IV, applications of the structure are made to cases of interest, and results are shown.

II. NOISY TIME DELAYS AND ESTIMATION OF SOURCE MOTION

The time delay measurements are usually imperfect and this causes fluctuations in the range and direction values, with subsequent errors in the velocity estimates. When a Taylor expansion on the range is carried out and only the linear term is relevant, the mean of the source range and direction are considered unbiased and their variance is a linear function of the time delay variance. For an effective array length, minimization of the variance in source location leads to minimization of the time delay variances. To effect this minimization, Hannan and Thomson (1), Hahn and Stretter (2), Knapp and Carter (3), Hassab and Boucher (4,5), have used varied processors to estimate the time delays by including an optimum window or filter in the basic correlator. MacDonald and Schultheiss (6) developed a modified split beam tracker for time delay estimation. These techniques presume stationary source and array positions as well as signal and noise statistics.

This linear analysis of equation (1) is physically relevant at ranges close to the expansion point in the Taylor series and/or at small variances of the time delays. As the source range increases, as the source moves away from the array broadside, or as the time delay variance deteriorates with signal and noise conditions, the bias in the range is no longer negligible and the relation between the variances of the range to the time delays becomes quite non linear. Hassab and Boucher (7) have concluded that the problems of range bias and variance with the limited observation intervals in the cross-correlator, become intertwined and have to be minimized simultaneously through sequential smoothing of the time delays over sucessive observation intervals. Otherwise, the bias can be substantial in varied practical source locations relative to the receiving array. Recently, this bias has been calculated in various forms by Hassab and Boucher (7), Hilliard and Pinkos (8), and Ludeman (9). Guimond and Nardone, in an unpublished manuscript, illustrated the detrimental effect of the bias by keeping the nonlinear terms in the Taylor expansion; this yields for a zero mean Gaussian noise, a lower bound on both the range bias $\langle R_b \rangle$ and corresponding variance σ^2_R

$$\langle R_b \rangle = \frac{2\sigma_\tau^2 c^2 R^3}{D^4 \cos^4 B}, \quad \sigma^2_R = \frac{2\sigma_\tau^2 c^2 R^4}{D^4 \cos^4 B}\left(1+\frac{16\sigma_\tau^2 c^2 R^2}{D^4 \cos^4 B}\right) \quad (2)$$

where σ^2_τ is the time delay noise variance.

A minimization of the error in the preceding range estimates has been carried out by increasing array length D or by minimizing the time delay variances. Practical considerations such as array dynamics, available space, signal coherence eventually impose limitations on the permissible array size. With this in mind, there is an interest in pursuing the other alternative of extending the usefulness and effectiveness of an existing array by increasing the signal processing gain. This paper explores the latter alternative, includes source kinematics and array dynamics in the minimization process and demonstrates the accrued advantage in using this approach.

The simultaneous reduction of range bias and variance for a moving source constitute the subject of this paper. For variance reduction of a stationary source, Bangs and Schultheiss (10), Carter (11), and Hahn (12) have developed various maximum likelihood localization estimators with optimality conditioned on having a negligible bias and sufficiently long obervation time. Hahn uses a generalized correlator to estimate the various time delays, then applies a Gauss-Markov algorithm across the set of delays obtained from the several sensors to better the source location estimates. For improvement in delay measurements, Kirlin (13) assumes a model for the time delay variations and uses sequential estimation to allow for processing of new time delay measurements. In their source location estimator, Bangs and Schultheiss institute a search for a maximum by adjusting the various time delays under the condition that all delays point to a single hypothesized location. Carter adjusts the hypothesized source range and bearing then selects the corresponding delays until a maximum is obtained in the generalized correlator.

The preceding approaches are optimal when stationarity of the physical phenomena can be presumed over a long observation time. In practice, the signal and noise characteristics can slowly vary, and the time delays from a moving source may be considered quasi-stationary only over a finite observation interval. Those constraints limit the observation time of the above estimators, hence deteriorating their performance from the optimal condition. Knapp and Carter (14), Schultheiss and Weinstein (15) have dealt with the motion induced nonstationarity on τ through consideration of Doppler estimation problem. The ensued increase in the observation interval should remain short enough so that the time delays vary according to the chosen low order polynomial form as discussed in (15).

Now a different approach is presented to effectively reduce the variance along with the bias in the estimation of source motion over an extended number of the correlator observation intervals. Its implementation is carried out through a quasi-optimal post-processing of the time delays that are obtained from a generalized correlator with short observation interval in order to ensure stationarity of all elements in the problem. The post-processor is designed to ultimately map the time delay observations onto invariant and unbiased source motion parameters over which smoothing is performed to reduce both the variance and the bias in estimating source location. This mapping imparts stationarity to the problem, thus allowing an effective increase in the averaging time of the localization system beyond that allowed in a generalized correlator.

For illustration, let us discuss the classic problem dealing with estimation of the location $[R_x(t), R_y(t)]$ and motion $[V_x, V_y]$ of a constant velocity source. To ensure stationarity, each observation interval of the sensor outputs is of finite duration; then a generalized correlator is used to measure the time delays. Over successive observation intervals, a time delay function has the form $\tau(t) = \sum_{k=0}^{K} d_k t^k$

If estimation of the K-parameters in $\tau(t)$ is attempted, complications arise due to the presence of noise and the unknown order K of the polynomial. The order K is not known a-priori since it is a function of the relative motion and the number of observation intervals. Over a limited number of observation intervals, however, $\tau(t)$ is likely to vary in a linear fashion and parameter estimation may be carried out with a short memory filter (16). This filter has other benefits since it can aid in the estimation of time delays through the design of a gating mechanism (5,7). for the peak search in the correlator output. Additionally, the resulting decrease in time delay variance allows an extended region of operation away from a given array before the need arises to precede the triangulation scheme by spatial gating. Such gating can be helpful in the estimation process when independent information is available to define the most probable region of source location. For further smoothing beyond the few observation intervals in the short filter memory, the assumption of a certain type of source motion i.e. constant velocity can be very helpful. Then source motion is fixed by a pair of equations

$$\begin{aligned} R_x(t) &= R_x(0) + V_x t \\ R_y(t) &= R_y(0) + V_y t \end{aligned} \quad (3)$$

and there are only four unknowns to estimate over all the successive observation intervals. Then the noisy time delays are constrained within a processor to point to an estimate $\hat{R}_x(0), \hat{R}_y(0), \hat{V}_x, \hat{V}_y$ with a minimum mean square error. A description of a processor is given in section (III). The highly expanded memory system will yield an enhanced estimation of the unknown parameters as is demonstrated in section (IV).

III. ESTIMATOR STRUCTURE

Reduction of the bias and variance in source location and motion estimates can be accomplished by effective use of statistical estimation techniques over sequential observation intervals. The estimator is an expanding memory filter that uses successive time delay observations together with an appropriate physical model of source motion to obtain smooth source trajectory estimates.

Modeling of the physical dynamics of source motion can be accomplished using a variety of coordinate systems and models concerning source motion. Examples of often used coordinate systems include polar, modified polar and cartesian. Polar coordinates yield a state vector with elements consisting of source range, bearing and their respective derivatives, while the modified polar system develops a state vector using the reciprocal of range, bearing and the derivatives of range and bearing, all normalized by the range. Cartesian coordinates yield state vector elements consisting of the X-Y components of source range and velocity in developing a source motion model. A usual basic assumption concerning source motion is to consider it to be of constant velocity over the successive observation intervals. If this assumption is restrictive then more sophisticated source dynamic models can be used that actually characterize source maneuvers. Accordingly source motion is described as piecewise constant; that is consisting of non-maneuvering portions joined by maneuvering portions. The maneuvering portions can then be modeled as random velocity perturbations resulting in the use of adaptive filtering techniques (17) or can be modeled as an unknown but deterministic input resulting in the use of estimation/identification techniques (18). It might be pointed out that the constant velocity assumption is the basis for either technique and thus will be the approach taken in the paper. Finally, the cartesian coordinate system will be used in developing the constant velocity source motion. Assuming a unity sampling rate, the discrete time motion can be written as

$$X(k+1) = A(k)X(k) + U(k) + D(k)W(k) \qquad (4)$$

where the 4x1 state vector and 4x4 plant matrix are defined as

$$X(k) = \begin{bmatrix} V_x(k) \\ V_y(k) \\ R_x(k) \\ R_y(k) \end{bmatrix}, \quad A(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \qquad (5)$$

Here, $V_x(k)$, $V_y(k)$ and $R_x(k)$ and $R_y(k)$ are respectively the x and y components of source velocity and range relative to the array while U(k) is a 4x1 vector containing observer accelerations and the term D(k)W(k) is included to account for zero mean random perturbations in source and array motion.

Completion of the problem formulation requires that a relationship between the observables and desired state vector to be estimated be developed. For the linear array there exists three choices for observables. They consist of either using the measured time delays directly or of mapping the measured time delays to range and bearing via equation (1) or of finally converting the measured time delay to bearing and the reciprocal of range.

The particular choice of observables depend on the coordinate system being used as well as the effect of measurement noise on the observables. For example, the combination of bearing and the reciprocal of range is amenable to the modified polar coordinate model since these quantities are linearly related to the system states (however, the discrete time system model becomes nonlinear). Further, the choice of bearing and range in conjunction with the cartesian coordinate model can result in biased state estimates since in effect equation (2) concludes the range measurement is biased. This result will be demonstrated in a forthcoming section. Finally, the approach used in this paper is to relate the time delay observables τ_1, τ_2 to the cartesian coordinate system model. The result is

$$\begin{bmatrix} \tau_1(k) \\ \tau_2(k) \end{bmatrix} = \frac{1}{c} \begin{bmatrix} -R+[R^2+D^2-2RD \sin B]^{\frac{1}{2}} \\ R-[R^2+D^2+2RD \sin B]^{\frac{1}{2}} \end{bmatrix}_k + N(k) \qquad (6)$$

Here N(k) is a 2x1 vector whose components define zero mean gaussian noise on the individual time delay observations. The range R and bearing B defined in terms of the system states are

$$R(k) = [R_x^2(k) + R_y^2(k)]^{\frac{1}{2}}$$

$$B(k) = 90° - \tan^{-1}\left[\frac{R_x(k)}{R_y(k)}\right] + C_o(k)$$

where $C_o(k)$ is observer heading.

Examination of equations (4) and (6) reveal that we are faced with a nonlinear estimation problem, i.e., the plant model is linear in the states while the measurement relationship is nonlinear in the states. Application of statistical estimation techniques to nonlinear systems have resulted in the use of extended Kalman filters, nonlinear weighted least squares, maximum likelihood or stochastic approximation algorithms. The approach used in this paper is that of an extended Kalman filter which essentially performs a linearization of equation (6) and then uses linear Kalman filtering techniques (19).

IV. SIMULATION RESULTS

Three processing techniques are considered for simulation. The first scheme directly transforms the best time delays available from generalized cross-correlators to range and bearing; i.e., a direct localization solution. The second method utilizes the direct localization solution as the measurements for an extended Kalman filter (EKF) and obtains motion parameters in addition to smoothed localization estimates. The final processor, the one proposed here, uses time delays directly in the EKF to obtain the localization and motion solutions. The results are presented in figures 2-21 which show the accrued improvement when using the proposed processor.

The particular source-observer geometry selected for simulation has an initial contact at a range of 20 kiloyards and a true bearing of 180°. The source moves on a course of 90° at a speed of 20 knots while the observer maintains a constant course of 120° with a speed of 10 knots. All angles are referenced to north. The data is available at equal intervals and is in the form of time delays corrupted by additive, zero-mean, white-gaussian noise with a standard deviation of 5 μ seconds. The geometry is such that the source is initially 30° off broadside to the array and moves to a location at 60° by the conclusion of the run. The range rate is closing and modest, resulting in a 10% decrease in range. Observer maneuvers have been deliberately avoided to exclude localization by the bearings-only ranging mechanism (20). Although other geometries (including observer maneuvers) have been simulated, these particular results are selected because they are illustrative without being atypical. For the given source-observer encounter, Monte Carlo simulation is carried out varying only the noise sequence added to the time delays.

The errors in the direct localization results for the first technique are obtained by averaging the solutions from a 10^4 member ensemble. The ensemble mean and variance of the range and bearing errors are shown in Fig. 2,12,3,13 respectively. The geometric range solution exhibits a positive bias that is in good agreement with the theoretically predicted value in equation 2. The effect of the nonlinear transformation of time delay into range has been to map the zero mean time delay error into a biased (non-zero mean) range error. Additionally, since the range error in figure 2 is averaged it does not adequately convey the spread of the error in time experienced on a single run, which can be substantial and is reflected in the rms error plot. In contrast, the bearing error (figure 3) is small due to the inherently more accurate bearings obtainable from a long baseline array.

Mean errors in the Kalman filter localization and motion solutions obtained by processing first bearing and range measurements then time delays are displayed respectively in figures 4-7 and figures 8-11. Measures on the error estimates are obtained from a 10^2-member ensemble. The initialization of both filters is identical. Since the true range is unknown, initialization of the ranging states and optimal weighting of the range measurement via the variance of equation 2 is not possible. Hence, a nominal range value, equal to the range gate, was selected, in this case 30 kiloyards. It should be noted that processing of range and bearing is sensitive to both initialization and appropriate weighting of the range measurements which does lead to divergence of the estimates. The above measures only reflect the runs that converge. This is in contrast to processing the time delays directly where the variances are known. Inspection of figure 4 shows that the range error is biased in much the same way as it was for the direct localization solution. This is the case since the filter is processing biased range measurements according to equation 2. Again the bearing error is small, figure 5. The filter also provides estimates of course and speed, which are also biased, as can be seen in their respective error plots, figures 6 and 7. The variance in the state estimates is reduced by the filter and this results in smoother solution estimates than those obtained by a direct localization solution.

The results of the proposed post processor, using time delay data as measurements for the EKF, are shown in Fig. 8-11 and 18-21. The smoothing for the time delay data is seen to result in a minimized range error bias as well as a reduction in the variance, (Fig.8,18). Analogous results are observed for errors in bearing (Fig.9,19). course (Fig.10,20) and speed (Fig.11,21). This improvement results because the EKF essentially smoothes the time delays by estimating the source state that minimizes the residual difference between the measured and estimated time delay measurements. The filtering process results in an effective reduction of time delay variance and thereby reduces the range error bias and variance. Finally, although not shown, the EKF predictions of the error estimates are consistent with the actual errors from the simulation.

V. DISCUSSIONS AND CONCLUSIONS

A processing structure is derived to estimate the trajectory of a source based on indirect aspects (time delays) of the source motion. The proposed structure maps a non-stationary problem into a stationary one where appropriate smoothing in time is carried out. The addition of the proposed structure improves substantially on the techniques that process inappropriately mapped time delays or those that transform directly the best time delays available into source motion estimates. The latter approach can only be optimum when stationarity of all the elements in the problem including a static source and array can be presumed. For this reduced case the present approach converges also to the optimum estimates.

Though we have reported on the results of a cartesian system processing time delay observations τ_1 and τ_2, other representations have proven satisfactory. For instance, the modified polar system has been used with equivalent success as well as processing other time delays combinations such as $(\tau_1+\tau_2)$ and $(\tau_1-\tau_2)$ and 1/R and B. It is recognized that the modified polar formulation facilitates the initialization process of the filter. For now, the principal point is that smoothing be carried out over unbiased parameters.

The present study has dealt primarily with zero uncertainties in the time delay observations and the moving source or array. The form of the proposed structure however is amenable to further extensions and relaxation of these assumptions. Modeling non-zero mean uncertainties in array and source motion can be approached through inclusion of adaptive filtering and/or parameter identification techniques.

Finally, the total system gain in a localization and motion analysis system is derived from spatial or array gain (size, sensors placement, etc.) and temporal or processing gain (signal, data). System designers seek to optimize the use of the spatial and temporal dimensions. Practical considerations such as array stabilization, cost...eventually limit the achievable spatial gain. The usefulness of such systems can be enhanced through inclusion of appropriate temporal processing techniques. The present paper proposes such a technique.

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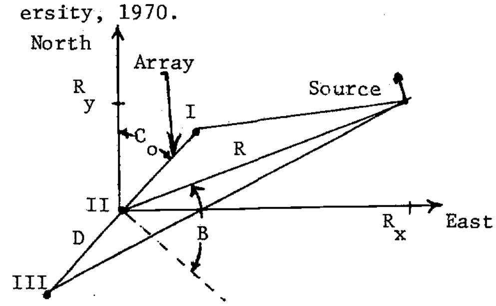

Fig.1 Source/Array Geometry (Array Size Magnified)

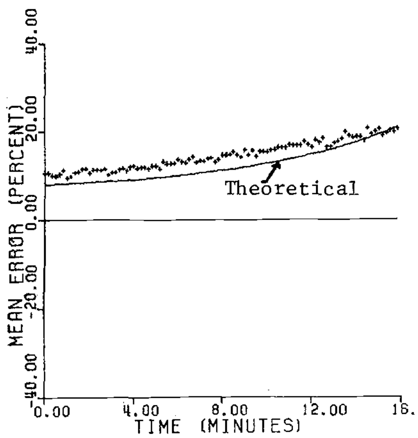

Fig. 2 Direct Localization - Range Estimate

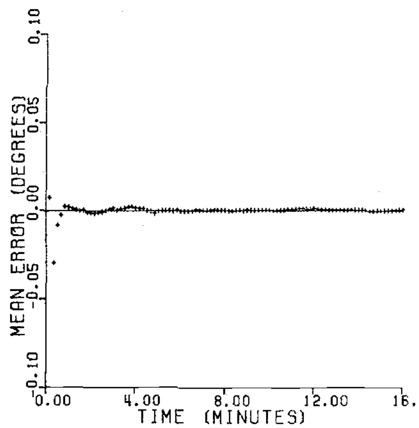

Fig. 3 Direct Localization - Bearing Estimate

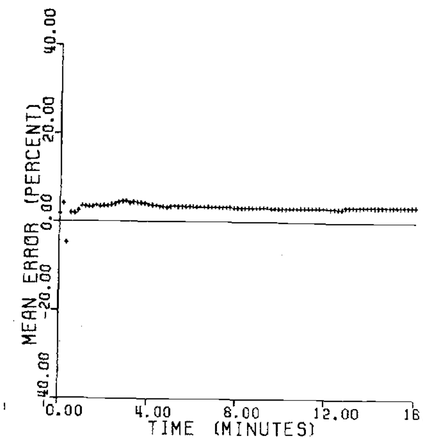

Fig. 4 Smoothing of Direct Localization - Range Estimate

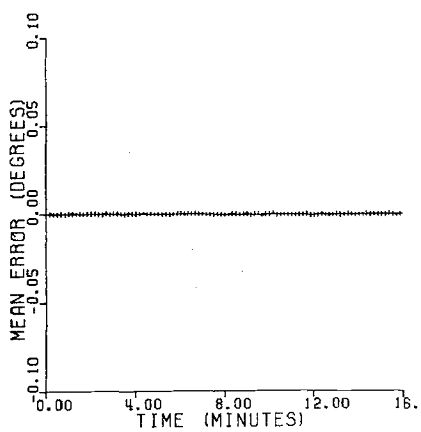

Fig. 5 Smoothing of Direct Localization - Bearing Estimate

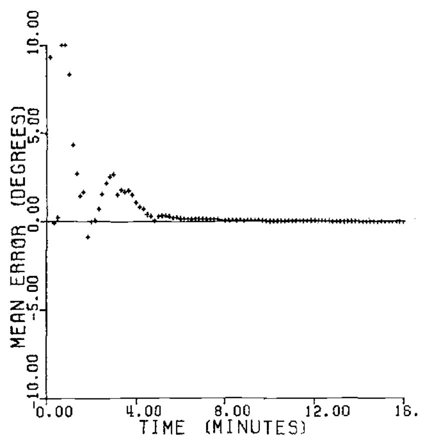

Fig. 6 Smoothing of Direct Localization - Course Estimate

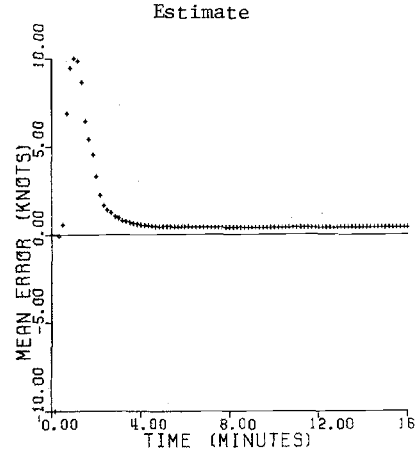

Fig. 7 Smoothing of Direct Localization - Speed Estimate

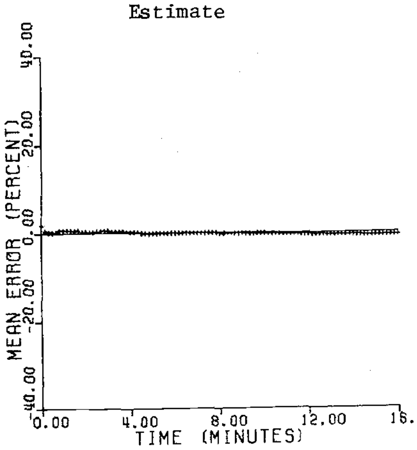

Fig. 8 Proposed Structure - Range Estimate

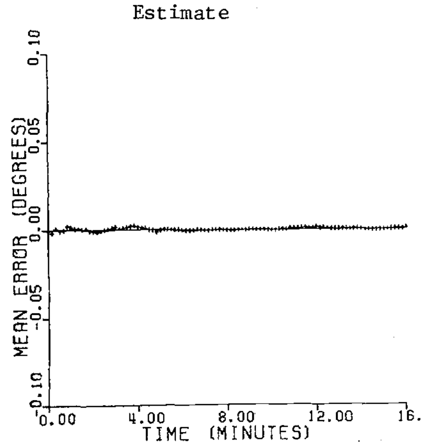

Fig. 9 Proposed Structure - Bearing Estimate

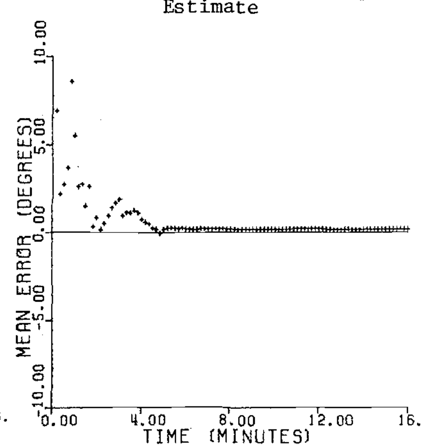

Fig. 10 Proposed Structure Course Estimate

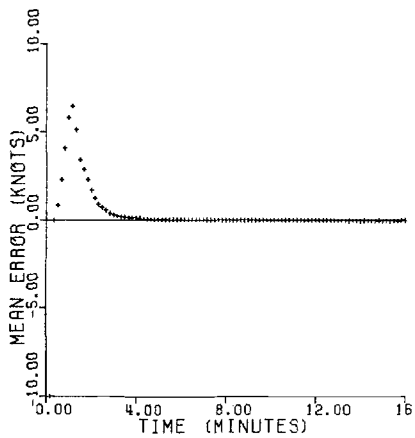

Fig. 11 Proposed Structure - Speed Estimate

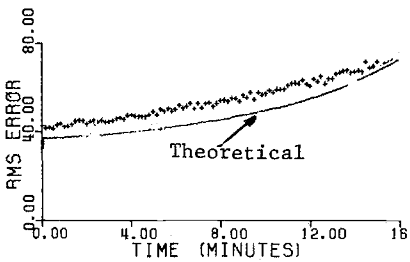

Fig. 12 Direct Localization - Range Estimate

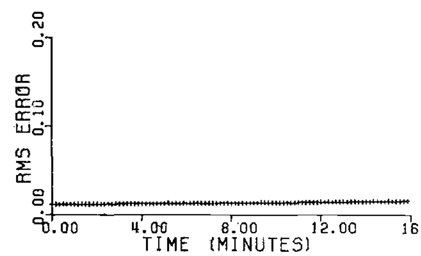

Fig. 13 Direct Localization - Bearing Estimate

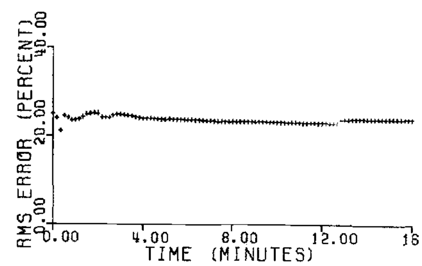

Fig. 14 Smoothing of Direct Localization - Range Estimate

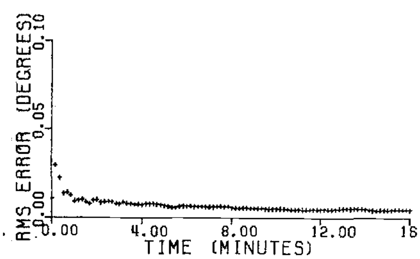

Fig. 15 Smoothing of Direct Localization - Bearing Estimate

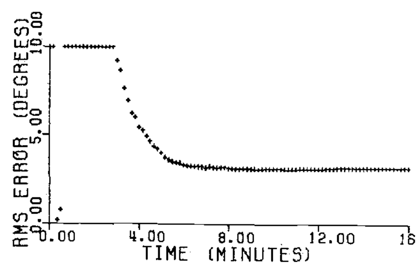

Fig. 16 Smoothing of Direct Localization - Course Estimate

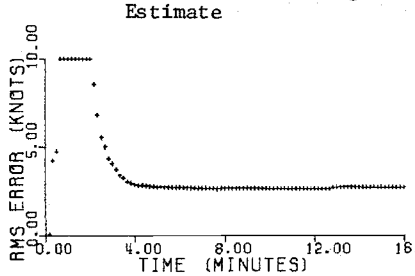

Fig. 17 Smoothing of Direct Localization - Speed Estimate

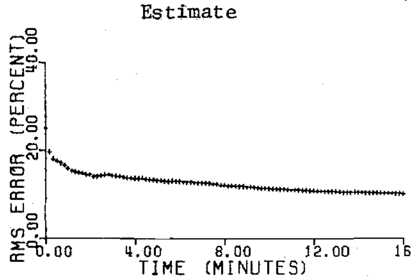

Fig. 18 Proposed Structure - Range Estimate

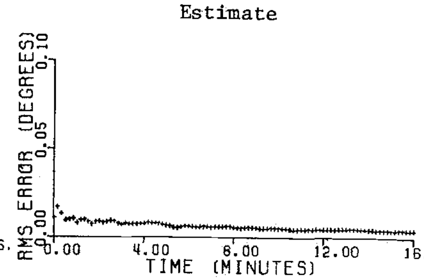

Fig. 19 Proposed Structure - Bearing Estimate

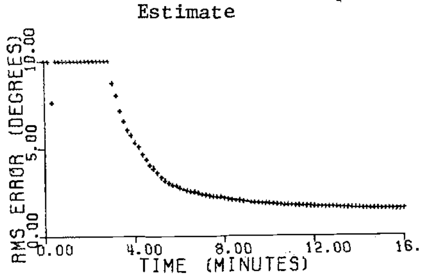

Fig. 20 Proposed Structure - Course Estimate

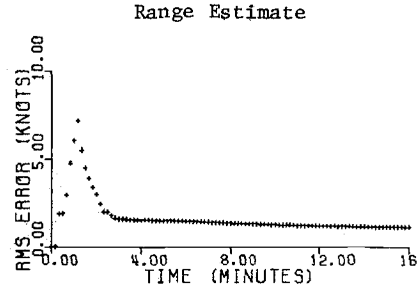

Fig. 21 Proposed Structure - Speed Estimate