A COMPARATIVE EVALUATION OF SEVERAL BEARINGS-ONLY TRACKING FILTERS

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ABSTRACT

Eleven different filter configurations were studied in order to determine their nominal relative performance and stability when passively tracking an air target with ESM direction finding equipment. The characteristics of the various filters are: The composition of the state elements (Cartesian, polar with polar rates, polar with course and speed, or polar with Cartesian rates); the number of state elements (four or three); the type of estimation used (all but two being Kalman filters); and the type of target model assumed (all but one being constant velocity Cartesian). The filters were tested in a simulation against non-maneuvering targets assuming zero-mean uncorrelated measurement errors of known typical variance. Four filters were found to perform well and one other is possibly acceptable. A final choice would need to consider performance against a maneuvering target and performance with measurements from the actual sensor under consideration.

INTRODUCTION

Most algorithms designed to track targets using bearings-only measurements were developed by the sonar community for tracking surface and sub-surface targets. The highly nonlinear nature of this problem, however, indicates that different algorithms might be more suitable to the ESM passive tracking of an air target where the target dynamics and bearing measurement errors and rates differ significantly. This study was designed to help answer the question of optimal filter configuration for this application.

There are several questions and considerations that one should take into account to design such a filter. It becomes immediately obvious that the selection of a filter type for passive tracking is, in many ways, a repetition, under different circumstances, of an old problem which occurs in radar tracking filter development. That is, where should one put the nonlinearity in order to minimize its adverse effects? This problem occurs in tracking situations since most targets are best, in the sense of most linearly, described in Cartesian coordinates whereas observations are made in polar coordinates. The analogy ends here, however, since range measurements are not available and therefore the position of the target is not completely observable. The effect of this is to dramatically lengthen the probability density contour of target position in the range direction. Coupled with the generally greater (relative to radar) bearing measurement error standard deviation, the problem of the nonlinearity is, therfore, more adverse since, for the ESM problem, it is more difficult to compute in the neighborhood where the necessary linearizations are valid. Perhaps more importantly, the stability of the nonlinear filter in such a situation becomes questionable. The stability problem, common to many types of nonlinear algorithms, results from the particular type of feedback structure built into every filter; wherein, once the filter state has an "incorrect" value, due to observation or other type of error, this value is fed back into the filter structure, which is necessarily dependent on the state estimate, so that the next observation is processed incorrectly. Once this problem occurs, it happens on occasion that the algorithm never recovers but proceeds further from the correct value even when given good observations, thus effecting the nonlinear filter instability. As will be shown later in this report, it is apparently not too difficult to construct unstable algorithms designed to process such ESM measurements.

Another consideration for the Kalman optimal estimators in this study is the error covariance matrix. This paper assumes that the reader has a general familiarity with the concepts of a Kalman filter. It is necessary that the calculated error covariance of a Kalman filter be realistic in the sense that it is consistent with the true estimation errors. If this is not the case, then one should probably not bother to use such a filter since the generally excellent performance of this optimal estimator is derived from the error covariance and the largest portion of its computation is usually due to calculating this covariance. Failure to achieve a reasonable consistency between the true error and the calculated covariance can result in many different types of problems such as divergence, instability, or merely generally poor performance. In order to ensure that the Kalman filters used here do not suffer

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from such problems, the ratio of the actual errors to calculated filter error covariance was monitored by the simulation.

The objective, simply stated, is the development of a stable and efficient algorithm with close to the best possible performance. Fourteen different passive tracking configurations were initially considered in this study. Three of these algorithms were discarded on the basis of either stability problems or during preliminary testing leaving eleven for which results are reported. They are organized on the basis of the composition of their state vectors into four types: Cartesian, polar, course and speed, and hybrid (or mixed Cartesian/polar) state. The simulation is described and results for each filter presented. This paper is a condensation of a much larger report and, due to space limitations, necessarily can not go into the detail that might be desired by some readers. The interested reader is therefore referred to Clark (15) for such details as computer program listings, etc.

INITIALIZATION

Initialization of all filters is based on exactly the same basic information (when possible) so that all filters remain comparable. This infomration originates in the coordinate frame of bearing, course, speed, and range. The initial bearing estimate is simply the first measurement, i.e.

$$\hat{\theta}(1) = \theta_m(1) \tag{1}$$

which has an error standard deviation equal to the measurement standard deviation σ, i.e.

$$\sigma_\theta(1) = \sigma \tag{2}$$

The target is initially assumed to be radially inbound so that the initial course estimate is

$$\hat{c}(1) = \theta_m(1) \pm \pi \tag{3}$$

The standard deviation of initial course errors is calculated assuming a uniform distribution from 0 to 2π radians around the estimate giving

$$\sigma_c(1) = \pi/\sqrt{3} \tag{4}$$

The estimated speed is calculated from an assumed Cartesian zero-mean bivariate symmetric normal distribution converted to polar coordinates yielding an equivalent Rayleigh distribution with mean

$$\hat{s}(1) = \sigma_{vc}\sqrt{\pi/2} \tag{5}$$

and standard deviation

$$\sigma_s(1) = \sigma_{vc}\sqrt{2 - \pi/2} \tag{6}$$

where σ_{vc} is the assumed velocity standard deviation in Cartesian coordinates. The value chosen for σ_{vc} was 0.33 kilometers/second which is approximately Mach 1. The initial range was assumed to be

$$\hat{r}(1) = 175 \text{ kilometers} \tag{7}$$

whereas the true initial range happens to be 141 kilometers for all trajectories. The standard deviation of the initial range error is assumed to be

$$\sigma_r(1) = 81.65 \text{ kilometers} \tag{8}$$

as in references 1 through 4. Having established the standard intial conditions in this reference frame, it is relatively simple to write them in any other coordinate system. This exercise will not be included in this paper due to space considerations.

CARTESIAN FILTERS

In this section, five variants of passive tracker based upon a Cartesian state vector are described. The state vector for each of these filters is given as

$$X = [x \quad \dot{x} \quad y \quad \dot{y}]^T \tag{9}$$

Since from the basic assumption, target motion is linear in a Cartesian frame, the state extrapolation equation is simply

$$\hat{X}' = \Phi\hat{X} \tag{10}$$

and the extrapolated error covariance is

$$P' = \Phi P \Phi^T + Q \tag{11}$$

where the prime denotes the new time and the hat denotes the estimate. The process noise matrix Q is used to account for unmodelled effects and is zero unless otherwise noted. The transition matrix for this case is simply

$$\Phi = \begin{bmatrix} 1 & \Delta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

where Δ is the extrapolation interval or time between measurments. The differences in each of the Cartesian filter variants lie only in the manner the measurement updates are effected. We will now consider each of these in detail.

Extended Kalman Filter

The EKF is the textbook approach to dealing with nonlinearities in the update stage. The author built no simulation for this particular filter and no results are presented here. The

EKF is discussed here mainly for completeness and also to expose the reader to several references which treat the EKF in the passive tracker situation. To start with, the EKF has been applied--apparently with success--to the passive sonar problem. For example, Hunter and McDonald (7) used the EKF with no reference to any serious stability problems encountered, but mention large sensitivities to data interval, correlated measurements and maneuvers. In a very extensive Ph.D. thesis, Mitschang (8) used the EKF to process doppler shifted frequency as well as sonar bearings with no mention of problems. The author therefore concludes that, for the conditions and parameters associated with passive sonar tracking, the EKF can often be utilized successfully. The question must then be asked: "Will the same hold true for the case of ESM tracking of an aircraft?"

Recent simulation experience indicates that there are stability and performance problems associated with the EKF when used in the ESM passive tracking situation. Quoting from (1): "The processing of bearing-only data by the non-linear filter was unsatisfactory with [the assumed measurement error level] believed due to linearization problems with this error value." Even the iterated EKF demonstrated slow convergence if it converged at all. "The filter proved to be close to instability since divergence was found under some conditions." In more recent years, even the sonar people have attacked the conventional EKF for use with their passive tracking problem. Aidale (9) demonstrated via simulation that the EKF is "potentially unstable" and devised a "linear solution"--a modified EKF--which he feels provides a "viable automatic technique for bearings-only target motion analysis." This approach was not examined by the author but might be worthy of consideration in the future. Tenny et al (10) discuss a large number of ad hoc modifications required to prevent divergence of the EKF when dealing with bearing observations of poor quality. Alspach's paper (11) points out the difference in performance between a simple EKF and an optimal bayesian filter in pictorial form and explains why EKF's do not perform satisfactorily. His bayesian algorithm is not (and was not intended to be) a computationally tenable approach to the problem. Finally, Chou (12) explains the fundamental drawbacks of the EKF for bearings-only work and quantitatively analyzes the serious problem of "range collapse", the common form of instability for this problem and one the author has encountered in this work. Chou formulated two new approaches that avoid the source of the problem. The first approach he calls the Alternating-Coordinate-System Filter which is identical to the Hybrid Polar/Cartesian Kalman Filter developed independently in (2). This filter will be discussed and then further developed later in this report. Chou's second approach is a stripped-down version of the Gaussian sum method devised by Alspach and, as mentioned, will not be considered further in this report. Therefore, the classical EKF having been dispensed with, we will now move along to other Cartesian forms.

Pseudo-Linear Filter

The Pseudo Linear Filter, developed in (4) by Blaydes and Holmes, manufactures a linear measurement by using the measured bearing and an estimated range to create the pseudo-linear Cartesian observation. That is

$$Z = \begin{bmatrix} \hat{r} \sin \Theta_m \\ \hat{r} \cos \Theta_m \end{bmatrix} \tag{13}$$

and

$$H = [1 \quad 0 \quad 1 \quad 0] \tag{14}$$

where $\quad Z = HX + V \quad (15)$

The measurement error vector V corresponding to Equation (15) can be found by differentiating Z and substituting finite errors for differentials. By squaring and taking expected values, the pseudo-measurement error covariance matrix R can easily be found. The usual linear Kalman filter update equations can now be used. The state update equation is

$$\hat{X}'' = \hat{X}' + K(Z - H\hat{x}') \tag{16}$$

where $\quad K = P'H^T(HP'H^T + R)^{-1} \quad (17)$

The error covariance is updated as

$$P'' = (I - KH)P' \tag{18}$$

The filter model at this point represents the Pseudo Linear Filter as presented in (4). It only remains to specify the value of estimated range to used in the pseudo-measurements and the associated range error variance. The problem is, of course, that there is no independent range information available. Blades and Holmes recommend using the current value of $\hat{r}$ and σ_r which the author therefore used. The Kalman filter assumes, however, that the error associated with each new "measurement" is independent of the previous errors and, of course, this is not the case. The filter believes that range estimates are improving when, in fact, there is no basis for this and the range variance becomes unrealistic.

Correlated Pseudo-Linear Filter

An algorithm that accounts for serial correlation of the measurement error was developed in Reference 5 from a smoother/filter originally published by Sage and Melsa (6). It uses the basic measurement Equation (15) as before but also assumes that the measurement noise is the output of a linear discrete system driven by zero-mean white noise. The development of this filter is rather lengthy and will not be reproduced here but the interested reader is referred to Clark (5). In order to use this model, we assume that the range measurement error obeys a linear, first-order model, i.e.

$$V_r = \rho V_{ro} + \xi \tag{19}$$

where ρ is the range error correlation coefficient for a one time interval Δ displacement. The variance of ξ, the uncorrelated zero-mean discrete Gaussian random variable, is

$$\sigma_\xi^2 = \sigma_r^2(1 - \rho^2) \tag{20}$$

Notice that for the special case of $\rho = 1$, which corresponds to the situation when the range error is completely correlated (a simple bias), σ_ξ vanishes and the only random portion of the measurement is that which originates for the observed bearing.

The results of the Correlated Pseudo-Linear Filter as a function of the assumed range error correlation coefficient ρ are unfortunately not what one might expect. The performance degraded consistently as ρ moves away from zero toward one. In fact, no results were obtained for the $\rho = 1$ case due to the loss of the positive definite property for the error covariance matrix. Apparently, there is nothing--except an observed improvement in error to covariance matching--to be gained by trying to account for the serial correlation of the range error in the Pseudo-Linear Filter. So, we therefore discard the Correlated Pseudo Linear Filter.

An alternate approach to the range error correlation problem considered the effect of measurement error correlation with the state vector error. Unfortunately, this method suffers from the range collapse problem similar to the Extended Kalman Filter. Therefore, it also was discarded from consideration.

Pseudo-Linear Alpha Beta Filter

The Pseudo-Linear Alpha Beta Filter is very similar to the Pseudo-Linear Filter in that it uses estimated range to form Cartesian pseudo-measurements. The main difference is that it is not a Kalman filter, as are the others in this study, and does not require error covariance to compute the gains. For this reason, it is one of the fastest of all the filters tested. Also, by not using error covariance, the Psuedo-Linear Alpha Beta Filter also avoids a potential source of instability due to nonlinear coupling of the state and gains through the covariance matrix. Although the filter is still nonlinear in the state due to the use of estimated range in the measurements, it did not in fact display any stability problems in the simulated runs of this study. The alpha-beta filter is also presently the most widely used filter type for tracking and smoothing applications, and for this reason alone, no tracking filter study is really complete without, at least, considering it. On the negative side of the ledger, without the use of the error covariance, there is no easy method of coupling the two spatial dimensions and this factor, because of the strong cross correlation present in the bearing-only problem in Cartesian coordinates, limits the accuracy obtainable from this method relative to the coupled Kalman filter.

The alpha-beta filter is usually considered to be a recursive formulation of the global least squares solution--"fitting" a straight line to the data--obtained from the normal equations. The gains used can be obtained from numerous sources such as Quigley and Holmes (13) and they are for the k-th measurement

$$\alpha = \frac{2(2k-1)}{k(k+1)} \tag{21a}$$

$$\beta = \frac{6}{k(k+1)} \tag{21b}$$

Transformed Alpha Beta Filter

The Transformed Alpha Beta Filter takes a different approach to Cartesian filtering than the previously described four filters. Conceptually, this filter starts with four stages: (1) Cartesian extrapolation; (2) transform to polar; (3) update bearing and bearing rate assuming alpha-beta gains; and (4) transform to Cartesian. The last three stages can then be combined into one nonlinear Cartesian update step.

By eliminating the intermediate polar states, the entire nonlinear Cartesian update can then be written compactly in vector-matrix form as

$$\hat{X}'' = G\hat{X}' \tag{22}$$

where

$$G = \begin{bmatrix} C_v & 0 & S_v & 0 \\ -\gamma S_v & C_v & \gamma C_v & S_v \\ -S_v & 0 & C_v & 0 \\ -\gamma C_v & -S_v & -\gamma S_v & C_v \end{bmatrix} \tag{23}$$

$$v = \theta_m - \arctan(\hat{x}'/\hat{y}') \tag{24a}$$

$$C_v = \cos \alpha v \tag{24b}$$

$$S_v = \sin \alpha v \tag{24c}$$

$$\gamma = \beta v/\Delta \tag{24d}$$

Even though written in this matrix form, which appears linear, the nonlinearity is embedded in the G matrix through the residual v term. In fact, this particular form of Cartesian filter presents the nonlinearity in a much clearer fashion than any of the others.

POLAR FILTERS

Polar filters locate the tracking nonlinearity in the extrapolation step of the filter process. As bearing is necessarily an element of the state vector of a polar filter, the update step is always linear. Three polar Kalman filters are presented in this section. The first one, the Transformed Polar Filter, is identical in concept and perfomrance to the Hybrid Polar-Cartesian Kalman Filter of Blaydes (2) but, by combining several steps of this filter into one, is several times faster. The second filter, which has been called the Compact Polar Filter, is a three-state contracted version of the transformed filter. Finally, the Random Acceleration Filter takes a different approach to the target kinetics as viewed in the polar frame. Let us now consider each of these three filters in turn.

Transformed Polar Filter

As mentioned in the introduction, the Hybrid Polar-Cartesian Kalman Filter generally performs quite well but suffers in implementation due to its considerable computational load. It appears to be several times slower than any of the other algorithms studied. Therefore, if a method could be found to speed up this filter, it could be very useful. In this section, a modification to the Hybrid Polar-Cartesian Kalman Filter is made which serves to accomplish this very aim. The resulting algorithm appears to operate at a rate comparable to the other filters. The idea for this approach originated from discussions between John Holmes of ASWE and the author.

The orginial Hybrid Polar-Cartesian Kalman Filter was thought of as being basically Cartesian with a transformation to polar after the extrapolation. Upon executing the linear polar update, the state and covariance were transformed back to Cartesian form. In order to understand the modification to obtain the Transformed Polar Filter, it is better to think of the Hybrid Polar-Cartesian Kalman Filter as basically polar. The polar elements are therefore transformed to Cartesian for the linear extrapolation and then back to polar for update. The modification, which is called the Transformed Polar Filter, then merely combines these three steps into one nonlinear state extrapolation and a combined effective transition matrix.

The polar and Cartesian state vectors are defined as

$$X = [\theta \quad \dot{\theta} \quad r \quad \dot{r}]^T \tag{25}$$

and

$$X_c = [x \quad \dot{x} \quad y \quad \dot{y}]^T \tag{26}$$

If the transformation from polar to Cartesian is given by a vector nonlinear equation $X_c = X_c(X)$, then the polar covariance is then transformed to Cartesian via

$$P_c = DPD^T \tag{27}$$

where the Jacobian of $X_c(X)$ is $D(X) = \partial X_c/\partial X$.

The Cartesian state and covariance are then extrapolated as before

$$X_c' = \Phi_c X_c \tag{28}$$

$$P_c' = \Phi_c P_c \Phi_c^T \tag{29}$$

where Φ_c is simply the same Cartesian transition matrix as defined in Equation (12). The subscript c is used again to distinguish Cartesian quantities from unsubscripted polar elements. Finally, the extrapolated Cartesian state is transformed back to polar by $X' = X'(X_c')$ and the extrapolation polar covariance is found to be

$$P' = C'P_c'C'^T \tag{30}$$

where $C(X) = \partial X/\partial X_c$

Notice that Equation (36) uses $C' = C(X')$ and not C.

It now becomes clear that the state equations can be combined to form a single, nonlinear polar state extrapolation function which turns out to be

$$X' = X'(X) = \begin{bmatrix} \theta' \\ \dot{\theta}' \\ r' \\ \dot{r}' \end{bmatrix} = \begin{bmatrix} \theta + \alpha \\ \dot{\theta}/\gamma^2 \\ \gamma r \\ b\dot{r} + a\tau \end{bmatrix} \tag{31}$$

where
$$\rho = \dot{r}/r \tag{32a}$$

$$\beta = 1 + \Delta\rho \tag{32b}$$

$$\tau = \Delta\dot{\theta} \tag{32c}$$

$$\gamma = \sqrt{\tau^2 + \beta^2} \tag{32d}$$

$$\alpha = \arctan(\tau/\beta) \tag{32e}$$

$$b = \cos\alpha = \beta/\gamma \tag{32f}$$

$$a = \sin\alpha = \tau/\gamma \tag{32g}$$

It is now easy to see that the covariance can be extrapolated in one step

$$P' = \Phi P \Phi^T \tag{33}$$

where the equivalent transition matrix Φ is the product

$$\Phi = C'\Phi_c D \tag{34}$$

After a not insignificant amount of algebra, it can be shown that

$$\Phi = \begin{bmatrix} 1 & \Delta c & -\Delta\rho d & -\Delta d \\ 0 & c^2-a^2/\gamma^2 & 2\rho cd & 2cd \\ 0 & a\Delta r & b+a\tau & \Delta b \\ 0 & ar(1+c) & a\dot{\theta}(1+\Delta\rho/\gamma^2) & b-a\tau \end{bmatrix} \tag{35}$$

where $c = b/\gamma$ (36a)

$d = a/(\gamma r)$ (36b)

Actually, as it turns out, forming the matrix triple product in Equation (34) is not necessary. It can be shown--as it was determined after the fact--that the resultant matrix Φ is exactly identical to the Jacobian of the nonlinear extrapolation function in Equation (31). That is,

$$\Phi = \partial X'(X)/\partial X \tag{37}$$

In retrospect, this is not too surprising since one is dealing with linearization. It is not the type of result, however, that one could simply write down without comfirmation. In any case, this linearization technique is the one used in the remaining filters of this type, i.e., those with nonlinear state extrapolation equations.

Compact Polar Filter

Upon examination of the polar state extrapolation, Equation (31), it can be determined that: (a) $\dot{r}$ and r can be combined into one state variable $\rho = \dot{r}/r$; and (b) the remaining variables θ and $\dot{\theta}$ are functions only of ρ and not r or $\dot{r}$. This is another way of saying that r and $\dot{r}$ are perfectly correlated in the extrapolation stage. For passive tracking with no measurements of r or $\dot{r}$ available, nothing in the update step serves to separate these variables. Therefore, we can say that there are really only three independent variables present in the bearings-only tracking problem. By eliminating a fourth, dependent state element from the estimation algorithm, one should be able to improve the performance of the filter. Also a 44 percent reduction in the number of covariance elements results in an obvious computational advantage to be enjoyed by a three state filter over the ones with four state variables. Let us therefore investigate this filter.

The state extrapolation equation is readily obtained from Equation (31) and can be written

$$X' = X'(X) = \begin{bmatrix} \theta' \\ \dot{\theta}' \\ \rho' \end{bmatrix} = \begin{bmatrix} \theta + \alpha \\ \dot{\theta}/\gamma^2 \\ (\beta\rho + \tau\dot{\theta})/\gamma^2 \end{bmatrix} \tag{38}$$

where the definitions of the various extra parameters are identical to those just given. The transition matrix can again be obtained by the differentiation process of Equation (31) and will not be repeated here.

Since this is a three-state filter, it is not possible to completely specify position and velocity. This is due to the fact that the complete state is not observable in the bearings-only tracking problem. In order to estimate the complete state for comparison purposes, it is necessary to make an assumption about the state. For all the three-state filters in this study, it was decided that the speed, which is a constant and for which one might make a reasonable estimate, would be the most desirable parameter to assume and the initial estimated value was used. It is then relatively easy to determine the range estimate.

Random Acceleration Filter

The Random Acceleration model represents a different approach to the problem of describing the kinetics of the target. All the other filters in this report assume a constant velocity Cartesian target. While this model is not, of course, strictly accurate as targets can and do maneuver, it generally is applicable the greater part of the time or at least piecewise between maneuvers. The problem for polar filters is that this linear Cartesian motion is nonlinear in the polar frame as can easily be observed in the previous two models. Polar accelerations appear which are sometimes referred to as "pseudo" maneuvers in the literature. The Random Acceleration Filter, rather than propagating these pseudo maneuvers in an exact nonlinear fashion, instead represents the observed angular acceleration as a sample trajectory from a population of random trajectories with first order temporal correlation. [For a detailed discussion of this model, the reader is referred to Quigley and Holmes (13) or Clark (5).] In other words, the filter assumes the angular acceleration is a serially correlated random variable with known statistics. Specifically, the auto correlation for the acceleration is

$$\begin{aligned}\Omega(\Delta) &= E[\ddot{\theta}(t)\ddot{\theta}(t+\Delta)] \\ &= \sigma_{\ddot{\theta}}^2 \, \mathrm{EXP}(-\Delta/\tau_{\ddot{\theta}})\end{aligned}$$

We therfore require only two parameters ($\sigma_{\ddot{\theta}}$ and $\tau_{\ddot{\theta}}$) to characterize the target maneuverability. The parameter $\sigma_{\ddot{\theta}}$ is the standard deviation of target angular acceleration and $\tau_{\ddot{\theta}}$ is the angular acceleration characteristic time (approximately 0.8 times the mean time between zeroes or the inverse of angular acceleration frequency). One does not necessarily require that any particular trajectory will be well matched by this model but that one would expect that trajectory to be "contained" statistically in the random acceleration model and thus be a "reasonable" sample from the assumed population.

The state vector for this model is

$$X = [\theta \quad \dot{\theta} \quad \ddot{\theta}]^T \tag{40}$$

with transition matrix

$$\Phi = \begin{bmatrix} 1 & \Delta & \eta \\ 0 & 1 & \lambda \\ 0 & 0 & \mu \end{bmatrix} \tag{41}$$

where $z = \Delta/\tau_{\ddot{\Theta}}$ (42a)

$$\eta = \exp(-z) \quad (42b)$$

$$\lambda = \tau_{\ddot{\Theta}}(1-\eta) \quad (42c)$$

$$\mu = \tau_{\ddot{\Theta}}^2(\eta+z-1) \quad (42d)$$

Another distinction of the Random Acceleration Filter is that, unlike the other filters in this report, it has non-zero process noise which is given as a function of the two parameters. The equations for the symmetric process noise matrix can be found in Clark (5).

COURSE AND SPEED POLAR FILTERS

This section deals with another type of polar filter in which the original polar velocity components, angular and range rates, are replaced by course and speed. Three different filter configurations are developed in turn based on this basic form of state vector. Let us now consider the first filter.

Course and Speed Filter

The state vector for this filter is given as

$$X = [\Theta \quad c \quad s \quad r]^T \quad (43)$$

The target state can be extrapolated a time interval Δ via an exact nonlinear function. The extrapolated state can be written

$$X' = \begin{bmatrix} \Theta' \\ c' \\ s' \\ r' \end{bmatrix} = \begin{bmatrix} \Theta + \arccos u \\ c \\ s \\ fr \end{bmatrix} \quad (44)$$

where $u = (1+\varepsilon C_\delta)/f$ (45a)

$$f = \sqrt{1+\varepsilon^2+2\varepsilon C\delta} \quad (45b)$$

$$\delta = c-\Theta \quad (45c)$$

$$C_\delta = \cos\delta \quad (45d)$$

$$\varepsilon = \Delta s/r \quad (45e)$$

It can be seen that Equation (44) is of the general nonlinear form and the linearized matrix Jacobian can be used to approximate the transition matrix used for covariance extrapolation.

Speed Over Range Filter

As in the case of the Compact Polar Filter, a simpler variation of the previous Course and Speed Filter can be obtained by noting that speed and range do not appear independently and, except for a priori initial conditions, no independent information is garnered from the bearings-only tracking process on either variable. In fact, only the ratio of these variables is inferred and speed and range are highly correlated. The combination of these variables to eliminate this correlation therefore appears to be a logical modification.

The state vector for this filter was chosen to be

$$X = [\Theta \quad \varepsilon \quad c]^T \quad (46)$$

where the dimensionless quantity ε was used previously in the Course and Speed Filter. The state extrapolation equation can easily be obtained from Equation (44) and is found to be

$$X' = \begin{bmatrix} \Theta' \\ \varepsilon' \\ c' \end{bmatrix} = \begin{bmatrix} \Theta + \arccos u \\ \varepsilon/f \\ c \end{bmatrix} \quad (47)$$

where u and f are defined as before.

Point of Closest Approach Filter

By calculating the point of closest approach (PCA), one can construct a version of the Speed Over Range Filter which is even simpler in that it has two constant parameters and only bearing itself as a function of time. First, we find the range at PCA by differentiating range as a function of time and setting the derivative to zero. Solving for time yields the time at which range is a minimum and substituting this time back into the original range equation gives us the range at PCA.

$$r_c = r(t) \cdot |\sin\delta| \quad (48)$$

Equation (48) can be used to eliminate r(t) in favor of the constant r_c in the bearing equation. Let us define a new state variable

$$\tau = r_c/(\Delta s) \geq 0 \quad (49)$$

The reader might recognize the ratio s/r_c as the maximum bearing rate which occurs at PCA.

The state vector for this filter is then defined as

$$X = [\Theta \quad \tau \quad c]^T \quad (50)$$

which can be extrapolated via

$$X' = \begin{bmatrix} \Theta' \\ \tau' \\ c' \end{bmatrix} = \begin{bmatrix} \Theta + \arctan w \\ \tau \\ c \end{bmatrix} \quad (51)$$

where $w = \sin\delta/(h + \cos\delta)$ (52b)

$$h = \tau/\sin\delta = r/(\Delta s) = 1/\varepsilon \qquad (52c)$$

Notice that for the radial target case, both τ and $\sin\delta$ vanish but h can still be calculated using the later part of Equation (52c). Notice that for this situation h assumes a large value and the transition matrix Φ approaches the identity matrix which is appropriate for the radial target case.

HYBRID STATE FILTERS

The filters in this section are distinct from all the others presented in that the state vectors, in an effort to minimize the filter nonlinearities, are chosen as a mixture of polar and Cartesian elements. Specifically, bearing is chosen as one element so that the update step will be linear. Ideally, one would like the remaining elements to be Cartesian so that the extrapolation stage is, to the extent possible, also linear.

Hybrid State Filter

The state vector for this filter is chosen to be

$$X = [\Theta \quad r \quad \dot{x} \quad \dot{y}]^T \qquad (53)$$

which can again be extrapolated by an exact nonlinear function.

$$X' = \begin{bmatrix} \Theta' \\ r' \\ \dot{x}' \\ \dot{y}' \end{bmatrix} = \begin{bmatrix} \Theta + \arctan(w) \\ fr \\ \dot{x} \\ \dot{y} \end{bmatrix} \qquad (54)$$

where

$$w = (\eta_x\cos\Theta - \eta_y\sin\Theta)/(1 + e) \qquad (55a)$$

$$\eta_x = \Delta\dot{x}/r \qquad (55b)$$

$$\eta_y = \Delta\dot{y}/r \qquad (55c)$$

$$e = \eta_x\sin\Theta + \eta_y\cos\Theta \qquad (55d)$$

$$f = \sqrt{1 + 2e + \varepsilon^2} \qquad (55e)$$

$$\varepsilon = \Delta s/r = \sqrt{\eta_x^2 + \eta_y^2} \qquad (55f)$$

The reader might note that the definition of f and ε are identical to their original defintions. They look somewhat different, however, because of the different functional dependence on the new variables.

Compact Hybrid Filter

It is again obvious that range and the Cartesian velocities do not appear independently in the Hybrid State Filter. These variables can therefore be combined to form a compact (three state) filter as before. The obvious new variables are η_x and η_y.

$$X = [\Theta \quad \eta_x \quad \eta_y]^T \qquad (56)$$

The state extrapolation equation is then

$$X' = \begin{bmatrix} \Theta' \\ \eta_x' \\ \eta_y' \end{bmatrix} = \begin{bmatrix} \Theta + \arctan(w) \\ \eta_x/f \\ \eta_y/f \end{bmatrix} \qquad (57)$$

The transition matrix is now smaller but more complex than for the Hybrid State Filter.

DESCRIPTION OF THE SIMULATION

The simulation is written in the BASIC computer language for use on the Hewlett-Packard 9830 computer. The simulation program was developed as a tool to evaluate the relative performance of the various passive filter configurations against three representative straight line targets observed by a bearings-only tracker with given measurement error statistics. For the interested reader, complete details of the simulation with listings and flow diagrams can be found in Clark (15).

The trajectories were obtained by integrating from an initial position (x = -100 km, y = 100 km) with given constant velocity Cartesian components. One trajectory is directly closing (C = 135°), one is initially crossing (C = 45°) and one is in between (C = 90°). The starting point for each is identical and the velocity of each is 0.60 km/s (approximately Mach number 1.61). There are 50 data points generated which are separated by 4 second time intervals. The true bearings are calculated from the true Cartesian coordinates and simulated errors are added to generate the measurements. Gaussian random numbers are created from uniform random numbers using the Box-Muller transformation. A particular different seed is selected for the uniform random number generator for each of the three trajectories so that the measurements errors for each trajectory are different but that each filter configuration sees the same measurements for each of the trajectories. This procedure ensures that the results for the various filters are directly comparable without resorting to long Monte-Carlo type simulations.

RESULTS

The previous sections have described 11 different bearings-only tracking filter configurations, the computer simulation used to stimulate them and the detailed performance results which were monitored by the program. This section evaluates the filters on a relative basis in terms of their estimation accuracy, stability and covariance behavior and finally their relative computational load. One filter appears to have an edge as the best on the basis of these tests and three or four others appear to be good enough to also merit further consideration. Recommendations for additional work on these four filters are then suggested.

Since this study is basically a relative comparison of several filter forms, the absolute performance of each particular filter is not highly relevant as a single entity but can best be interpreted when directly compared with other filters operating with the same data set against each particular target. With this thought in mind, one finds in Table 1 an overall summary of error performance in an absolute sense (except for a normalizing factor which is constant for each variable). In fact, all the bearing and course errors are normalized by the bearing measurement standard deviation and the range and speed errors by their respective initial error standard deviations. It should be noted that the use of the normalized errors does not imply that the error results will necessarily scale for other error levels because the filters in this report are nonlinear and, in all probability, so are the results. This form of dimensionless presentation is nonetheless useful in that results should be approximately scaleable over some reasonable neighborhood of the nominal values. In Table 1 across the top, one finds the three target numbers--with a reminder below each one describing what kind of trajectory that target represents--and the last column represents the average of the three targets. Each of these columns is in turn divided into three columns representing the best root-mean-square error values of all the filters for that particular variable as well as the average of all the filters for RMS and bias errors. A few trends between the targets are certainly worth noting at this time. First of all, for the radial target (number 2), the bearing and course errors are much better than for the crossing targets while the range and speed are much worse. It is easy to see why this is the case since bearing and course are easily discernable for the radial case while range and speed are totally unobservable. In fact, Target 3, which starts out exactly crossing, has the worse course errors and the best range and speed errors. One also notices that the bias portion of the errors tend to be the significant factor in that they are relatively small when the filters are performing best and tend to dominate the worse error values. Looking at course errors, one also suspects that the radial inbound initial condition estimate for the Kalman filters must influence the final outcome. Similarly, the range and speed initial conditions would have influenced their results. Obviously, for the three state filters, the speed bias error represents the total error in all cases.

In order to present the results for each filter in a manner which does not require reference to the performance of other filters, it was decided to utilize the best RMS values to define a figure of merit as follows.

$$\text{FM} = \text{Figure of Merit} \equiv \text{Best RMS Value/Filter RMS Value} \tag{58}$$

This figure of merit necessarily lies between zero and one. The closer the value is to unity, the better is the filter and, if the figure of merit equals one, then that filter has equaled the best performance of any of the filters for that variable (say course) and that target. The bias errors will also be represented as a fraction of the corresponding root-mean-square values to make them easy to interpret on their own. Finally, another measure is defined for the Kalman type filters which is the covariance normalized state estimation errors. These values, called Covariance Factors, for each state variable and each trajectory, are the mean value of the square of the state error divided by the value of the respective Kalman filter error standard deviation. Therefore, if the Kalman filter is performing properly, the value of the Covariance Factors should be near unity.

The computational effort required to effect one cycle (an extrapolation and measurement update) was also estimated. This was done by examining each vector-matrix equation required to implement each configuration and counting or otherwise estimating the number of add/subtracts, multiply/divides and special "functions" such as trigometric functions (or their inverse) or square roots. Each of these computation types were totalled and then combined by more-or-less arbitrarily assigning a relative value, called a computational unit, to each type of calculation. Specifically, the add/subtract were assigned the value one, multiply/divides the number 10 and functions the value of 100 computational units. Obviously such assignments actually depend on the particular computer used and on the algorithms used to evaluate the functions. These values were thought to be somewhat representative, however, and were therefore assumed to serve the purpose of obtaining a single number for comparison purposes. Two methods were used to obtain the actual values for each phase. If the equation under consideration was nonlinear--such as the nonlinear state extrapolations--or if a multiplicative matrix was sparse, the algebraic form of the equation was used and the number of operations actually were counted. If, on the other hand, normal matrix algebra with more-or-less full matrices could be employed, then formulae from Mendel's paper (14) on Kalman filters was used. If the values for a particular matrix were constant (such as the Φ matrix for Cartesian filters) or were otherwise available, then it was assumed no calculations were necessary for that phase. As can be seen, the Random Acceleration and two alpha beta filters are the fastest of all the configurations while the Transformed Polar Filter is the worst taking about 10 times more computational units than the Random Acceleration Filter.

Table 2 is the overall summary evaluation of all the filter configurations studied here and includes a summary of all the performance and computational measures considered. The first seven columns are the average root-mean-square Figure of Merit values averaged over both parameters and targets. Again, more detailed results can be found in (15). The next-to-last column is a weighted combination of these seven average values where the weighting factors are somewhat arbitrary values reflecting the author's opinion of the relative importance of each of the values when considering the potential use to which the bearings-only tracker might be put. Briefly, the following is the reasoning used in selecting the weighting factors: (a) between the coordinates (bearing, course, range, and speed), bearing is obviously the most important because it can be independently estimated and can be particularly useful when associating these tracks with those from other sensors; (b) Between the targets, it is felt that, for tactical reasons, the closing target (Target 2) is most important and the weighting factors were reduced as the targets become oriented more toward crossing; (c) the coordinate weightings are more important than those for the target geometry. The final set of weighting factors for the seven columns are therefore as follows: (0.300, 0.225, 0.150, 0.075, 0.075, 0.125, 0.050). Note that these seven values add up to one. Applying these multiplicative weighting factors to the seven columns, one gets the weighted performance factor in the next to last column. The eighth column is the average fractional RMS bias where the average is taken over all the individual values in the results. Notice the high negative correlation between these values and the weighted performance values. It is obvious that the poor performance of some of the filters is driven by the proportionally high bias content in the root-mean-square errors. It also appears that, on average, even the best filters operating in these idealized (simulated) environments are going to contain about 50 percent bias content in their estimation errors! That is surprisingly high for supposedly unbiased estimators but it only serves to emphasize the difficulty of designing filters for this highly nonlinear situation. The next column of average Covariance Factors also displays a high degree of negative correlation with the weighted performance factors. This also could be expected since one cannot expect the Kalman filters to perform well in an absolute sense when the actual errors are large relative to their own calculated error covariance. The next column of computational requirements for the filters only serve to point out that good performance in this situation can only be obtained at the expense of heavy computational burden.

Pseudo Linear Kalman Filter

The estimation performance values for this filter are very good. In fact, the FM values for two of the cases are 1.00 and the average values, particularly for bearing, are excellent. Unfortunately, the Covariance Factors are not particularly good although they are not so bad as to indicate a divergence problem. The Pseudo Linear Kalman Filter does not perform nearly as well as it thinks it does and, given the correlated nature of the pseudo-measurement, this behavior is not unexpected. One possible method of eliminating the erroneous state error covariance is to artificially increase the assumed range measurement error variance used by the filter in the R matrix. This technique keeps P from falling so rapidly--thus making it more realistic--and partially accounts for the serial correlation of the range error. The results of this experiment are that if we double the assumed range error standard deviation, the desired effect is achieved, i.e., the Covariance Factors fall from an average of 1.41 to the almost perfect value of 1.01 while the performance in bearing and course degrades only slightly. In fact, the range and speed values actually improve. Tripling the range error does not further improve the Covariance Factor but starts to degrade performance more seriously. Presumably, one would want to optimize performance by varying the range error standard deviation although there seemed little point in doing that here. The main conclusion is that, by carefully choosing the assumed range error standard deviation, the Pseudo Linear Kalman Filter can provide stable bearings-only tracking with good performance and realistic error covariance.

Pseudo Linear Alpha Beta Filter

The performance of the Pseudo Linear Alpha Beta Filter is poor, the worst in fact of all the filters tested here, although the bearing estimates for Target 1 happened to be the best of all the filters. It is understandable that the course and bearing errors for Target 2 are so bad, relative to the Kalman filters, because all the Kalman filters are initialized assuming the correct radial inbound course. The *a priori* weighting of this information in the Kalman filter propagates the correct course estimate longer than the least squares filter which uses no *a priori* information. This same reasoning probably also explains the good bearing performance of the alpha beta filter for Target 1 when the Kalman filter propagates the wrong course estimate. The overall speed errors for this filter are very poor and no explanation for this behavior has been found. Summarizing, the Pseudo Linear Alpha Beta Filter, while apparently stable, generally yielded quite poor performance.

Transformed Alpha Beta Filter

The Transformed Alpha Beta Filter might be described as a higher order or "more nonlinear"--in the sense of not having been linearized--filter than the Pseudo Linear Alpha Beta Filter because, by utilizing the small angle approximation for the residual angle, one recovers exactly the Pseudo Linear Alpha Beta Filter. This interpretation of the effects of applying an approximation might lead one to feel justified in expecting better performance from the Transformed Alpha Beta Filter. On the other hand, the assumptions in the update step might, on reflection, indicate poorer performance. First of all, range and range rate are not explicitly updated but are implicitly modified via other changes in the system state vector. It requires a Kalman-type filter to calculate the necessary cross correlations with bearing to obtain the gains and actually update range and range rate with the bearing residual. Also, the alpha beta gains that are employed for the bearing channel were derived under circumstances that do not exist for this application. Namely, these are that bearing is a linear function of time and that bearing is not correlated with any other variable (like range). In fact, a comprehensive least squares approach to the bearings-only problem can apparently only be attempted using a global (nonrecursive), iterative nonlinear estimation method. In any case, the author has found from the experiences of this study that performance expectations based on arguments of this type as often as not are incorrect.

Let us therefore look at the results for the Transformed Alpha Beta Filter. On average, relative to the Pseudo Linear Alpha Beta Filter, the Transformed Alpha Beta Filter performed slightly better in bearing, course, and range but slightly worse with speed. Since the first variables are more important, the Transformed Alpha Beta Filter has to be considered a bit better than the Pseudo Linear Alpha Beta Filter. On the other hand, contrasting these results with the Pseudo Linear Filter, which is a Kalman filter, one finds the alpha beta filters coming off poorly indeed.

Transformed Polar Filter

Comparing the results for the Transformed Polar Filter with the best filer so far, the Pseudo Linear Filter, we find the performance of the Transformed Polar Filter equals that of the Pseudo Linear Filter for bearing and course and exceeds it for range and speed. Also, the Covariance Factor is much better--without resort to artificial means--with a value of 0.67 as compared to 1.41. Therefore, the Transformed Polar Filter provides the best overall performance of any filter discussed so far and it will be used as a standard for additional comparisons.

Compact Polar Filter

The results for the Compact Polar Filter are disappointing to say the least. While one would have expected an improvement in performance over the Transformed Polar Filter when a extraneous--or dependent--variable is eliminated, instead the performance is essentially the same for bearing and course but considerably worse for range and speed. The speed errors for a three-state filter are--using the basic constant speed assumed in this study--of course uncontrollable and it is likely that the speed error generated in turn the range error. The error in range over speed should have been monitored as well in the simulation and it is unfortunate that this was not done. It is interesting that this filter produced all the best results for the closing case (Target 2). The results only serve to emphasize that, for a highly nonlinear filtering application such as bearings-only tracking, it is not always possible to anticipate the performance of a particular form of implementation. The average Covariance Factor is unity which is exactly what one would like to see. Based on these results then, the Compact Polar Filter should be considered a viable candidate tracking filter that should be considered further.

Random Acceleration Filter

The original description of the Random Acceleration Filter did not specify the values of the two parameters, $\sigma_{\ddot{\theta}}$ and $\tau_{\ddot{\theta}}$. Normally the selection process would involve an examination and/or analysis of the target scenario to choose a set of parameters or an upper and lower bound for each parameter that might be used in some type of adaptive filter as in Clark (5). Rather than go through this procedure, it was decided that a more expedient approach for the purposes of this study--that is a quick answer to determine if the filter offers any promise--would be to perform a rough optimization on the two parameters over the targets in the simulation. As an initial guess, it was decided to vary $\sigma_{\ddot{\theta}}$ between 10^{-6} and 10^{-2} radians/second^2 and $\tau_{\ddot{\theta}}$ between 20 and 100 seconds. The bearing and other estimation errors were then compared to find roughly the best combination of parameters. Surprisingly, it was found that the results were not particularly sensitive to $\tau_{\ddot{\theta}}$ but that a value of approximately 50 seconds apparently gave the best overall results. On the other hand, the results, particularly for course, were much more sensitive to the value of $\sigma_{\ddot{\theta}}$ and a strong and defnite error minimum was found in the neighborhood of 10^{-4} radians/second^2. Therfore, these were the parameter values used for the results reported in Table 2 and used for purposes of comparison with the other filters. The overall tracking performance of this filter is relatively poor as compared to the Transformed Polar Filter. The Covariance Factors are very good with an overall average of 0.60. Therefore, the Transformed Polar Filter is the best of all the polar filters with the Compact Polar Filter a close second.

Course and Speed Filter

The only linearizing approximation that appears in the Course and Speed Filter is in the extrapolation of the error covariance for bearing and range. Everything else is exact whether linear or nonlinear. The basic idea of this filter was to minimize the amount of approximation required of course. As error covariance is only approximate in any case and since an unknown amount of process noise must eventually be added to account for target maneuver, it would seem that this approach of putting the approximating linearization in the covariance extrapolation to be relatively insensitive. The use of course and speed should also be advantageous since they are constants for the non-maneuvering targets considered here and are ideal target-oriented parameters for which to specify maneuver statistics when a maneuver does occur. All these arguments supporting the apparent advantages of this configuration tend again to lead one to expect good performance. Unfortunately, as the results indicate, the a priori arguments are once again misleading, or at least partially so. The absence of results for Target 2 are due to an instability that will be discussed shortly. The irony of the situation is that the results for Targets 1 and 3 are excellent--among the best of all the filters. In fact, for Target 1, the results for bearing, range and speed, and the average were the best of any filter. For Target 3, the course was the best of all filters and all the averages were quite good. Also, the average Covariance Factor was excellent with a value of 0.70. Unfortunately, the instability of Target 2 spoils the other results as it obviously renders the filter unusable.

Target 2 is the directly closing target and the instability is of the type known as "range collapse" which is known to plague other bearings-only trackers such as the Cartesian Extended Kalman Filter. The problem, of course, is that a radial target displays no information on either range or speed when observed with bearings-only so that the filter can only hope to "coast" in range motion over the course of the trajectory. The Course and Speed filter, in fact, properly did just that over the first half of the trajectory. Eventually, however, the bearing error effects in the state vector crept into the nonlinear state transition matrix and accelerated the range decrements and increased the closing speed at an ever increasing rate until eventually range vanished and even went negative. With no range observations to contain range estimates, this is a serious potential problem for any bearings-only tracker. Therefore, the Course and Speed Filter, while offering much promise, is unacceptable in its present form due to range instability.

Speed Over Range Filter

In the Speed Over Range Filter, although the range instability problem has disappeared, the overall performance is very poor. In fact, the performance is so poor and inconsistent with the calculated covariance that this filter could really be described as unstable, but the instability is not strong enough to terminate the computer program. The Covariance Factors are so exceptionally large that it is surprising the absolute errors are as good as they are. This filter was, by far, the worst from the covariance viewpoint and is obviously unacceptable on this basis alone. The author could find no factor in either the design of the filter or in the computer program that would explain the eratic behavior of this filter. Therefore, we must discard from consideration the Speed Over Range Filter.

Point of Closest Approach Filter

The results for the Point of Closest Approach Filter are again disappointing in that overall performance, in particular bearing, is quite poor. Again, arguments that might lead one to expect a priori good performance--such as the simplicity of the configuration, the constant parameters, the linearity--have again led the author down the wrong path. The Covariance Factor, with an average value of 1.71, is not at all good. In fact, erroneous error covariance may be responsible for the overall poor performance. This filter model (or a close variant) with its constant parameters, is probably the best candidate of all the models for implementation with a global nonlinear least squares estimation technique. In any case, the current Kalman filter configuration of the Point of Closest Approach Filter is unacceptable on the grounds of poor performance.

Hybrid State Filter

The results for the Hybrid State Filter are very fine indeed--particularly for bearing, range, and speed. Course errors are acceptable but not nearly as good as the Transformed Polar Filter. The Covariance Factor, with an average value of 0.78, is very good. Apparently, the idea combining polar and Cartesian elements in one state vector has produced the desired results.

Compact Hybrid Filter

Again, the idea of combining dependent states has not yielded the desired improvement for the Compact Hybrid Filter. Although, course shows a slight improvement, the overall performance degrades relative to the original Hybrid State Filter. Again, one reason for the lower performance might be due to a covariance problem since the average Covariance Factor here is 1.81.

CONCLUSIONS

Of the 11 filter configuration studies, therefore, we conclude that four (or maybe five) yield a performance good enough to merit further consideration. Four others should probably be discarded on the basis of poor performance relative to what can be obtained. One filter--the Course and Speed Filter--must be discarded due to stability problems when tracking the important closing target. The last filter, the Speed Over Range Filter, should be discarded on the basis of a totally unrealistic Covariance Factor. On the basis of only the results reported here, the best filter is the Transformed Polar Filter. This filter is similar to the Alternating-Coordinate-System Filter recommended by Chou (12) and is a more developed (and quicker) version of Blaydes Hybrid Polar/Cartesian Kalman Filter (2).

Since this study does not provide an ultimate choice as to which of the five filter configurations to recommended, three suggestions for further work appear pertinent at this time. (1) Maneuvering Targets. A representative selection of maneuvering targets should be added to the targets considered here to determine which, if any, of the recommended filters offer any advantages in this situation. It is important to keep the non-maneuvering targets in the scenario as well, of course, in order to measure the probable degradation of performance due to false alarms in any adaptation logic that may be implemented to deal with the maneuvers. The author would be surprised if typical performance against maneuvering targets is not considerably worse than against the constant velocity targets considered here. If this is the case and if it is anticipated that the target scenario is dominated by such maneuvering target, it could call into question some of the conclusions presented here. It may be then that one of the simpler, but poorer, performing filters might be adequate for that situation. (2) Parametric Studies. Some of the parameters held fixed in this work should be each varied over some realistic specified range of values to determine if the performance of the various filters remains basically the same and particularly if the same filters that perform best here remain the best under other conditions. Two parameters that come immediately to mind here are measurement error level and measurement update rate over which the passive bearings-only receiver has no control. (3) Sensitivity studies. Studies should be made to determine the expected degradation of performance when the assumed measurement error level in the Kalman filters does not match the real error level input to the filter. This a particularly troublesome problem to any maneuver detection and adaptation logic that may be employed. Also, ability to degrade gracefully in the presence of spatially and temporally varying biases and serially correlation should be examined. Ultimately, of course, while such general studies as this provide the general base of information for decision-making, the final implemented algorithm must reflect consideration of the realities of the actual sensor data, the actual target motions and the allocated computer time and storage.

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Table 1. Normalized Best RMS and Average RMS and Bias Errors for All Errors

	TARGET 1 Crossing Approaching PCA			TARGET 2 Radial Closing			TARGET 3 Crossing Leaving PCA			Average		
	Best RMS	Average RMS	Average Bias	Best RMS	Average RMS	Average Bias	Best RMS	Average RMS	Average Bias	Best RMS	Average RMS	Average Bias
Bearing	0.42	0.64	-0.29	0.25	0.24	-0.12	0.31	0.44	-0.15	0.33	0.44	-0.19
Course	5.57	7.76	-0.94	2.39	5.88	-0.29	10.17	14.14	12.01	6.04	9.26	3.59
Range	0.28	0.49	0.22	0.66	0.84	0.80	0.20	0.44	-0.18	0.38	0.57	0.28
Speed	0.43	0.73	-0.23	0.86	1.44	-0.95	0.32	0.63	-0.38	0.54	0.93	-0.52

Table 2. Summary Evaluation

Type	Filter Name	Average Root-Mean-Square Figure of Merit: Bearing	Course	Range	Speed	Target 1	Target 2	Target 3	Average Fractional RMS Bias	Average Covariance Factor	Computational Requirements	Weighted Performance Factor	Overall Rating
Cartesian	Pseudo Linear	0.94**	0.86*	0.83*	0.83	0.84	0.92	0.83	0.53	1.41[m]	2509	0.88*	Good
	Pseudo Linear Alpha Beta	0.79	0.58	0.69	0.37	0.68	0.57	0.57	0.51*	1.10[3]	648	0.65	Poor
	Transformed Alpha Beta	0.83	0.61	0.74	0.35	0.64	0.51	0.75	0.50*	—[3]	617	0.67	Poor
Polar	Transformed Polar	0.94**	0.86*	0.86*	0.85	0.76	0.97*	0.90*	0.53	0.67	3318	0.89**	Best
	Compact Polar	0.94**	0.87**	0.70	0.62[2]	0.78	1.00**	0.57	0.67	1.00	1763	0.84*	Good
	Random Acceleration	0.75	0.56	0.77	0.62[2]	0.58	0.68	0.77	0.62	0.60	333	0.68	Poor
Course and Speed	Course and Speed	0.93*[1]	0.80[1]	0.87**[1]	0.79[1]	0.90**	—[u]	0.79	0.58	0.70	3078	0.86[1]	u
	Speed Over Range	0.51	0.42	0.30	0.62[2]	0.38	0.66	0.35	0.81	10.77[u]	1724	0.47	u
	Point of Closest Approach	0.53	0.67	0.64	0.62[2]	0.47	0.83	0.57	0.76	1.71[m]	1106	0.62	Poor
Hybrid	Hybrid State	0.93*	0.79	0.86*	0.96**	0.89*	0.87	0.90**	0.47**	0.78	2063	0.88*	Good
	Compact Hybrid	0.87	0.81	0.66	0.62[2]	0.80	0.89	0.54	0.66	1.81[m]	1932	0.79	OK
Best of All Filters		0.94	0.87	0.87	0.96	0.90	1.00	0.90	0.47			0.89	
Average of All Filters		0.81	0.71	0.72	0.66	0.70	0.79	0.69	0.60			0.74	

** Best of All Filters
* Good - Within 0.05 of Best
m Marginal
u Unacceptable

1. Average of Targets 1 and 3 only
2. Three State Filter - Speed not estimated
3. Covariance not required