

DETECTION OF PARTITIONED SIGNALS BY DISCRETE CROSS-SPECTRUM ANALYSIS

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ABSTRACT

Partitioning of the observation space using quantiles of the underlying noise distribution is a useful technique to achieve distribution-free performance. A set of quantiles and related functions essentially determine a nonlinearity which depends on the underlying noise and precedes the standard detector. In this paper, the received data in each channel of the array is first partitioned by a set of quantiles and then transformed by a discrete Fourier transform into discrete frequency estimates which are then multiplied by complex weights. The cross-spectrum for each frequency is then obtained by the direct segment method. The results show, based on a modified version of the asymptotic relative efficiency measure, that performance gains can be achieved in non-Gaussian noise compared with detectors designed assuming Gaussian noise.

I. INTRODUCTION

The purpose of this paper is to formulate the theory for detecting partitioned signals by cross-spectrum analysis. Partitioned detectors have been studied in some detail for detecting signals in non-Gaussian noise (1). The partitioned detector is distribution-free by virtue of estimating a set of quantiles from the unknown noise distribution. They have been shown to be robust and simple to implement (2). Most of the work on partitioned detectors has assumed independent and identically distributed (i.i.d.) noise samples. We assume that the quantiles and related functions from each channel of an array are obtained from i.i.d. samples, so that the nonlinearities are memoryless.

The data is first passed through the nonlinearities and then each channel of the array is discrete Fourier transformed and multiplied by complex weights. The cross-spectrum for each frequency component is obtained by the direct segment method of spectrum analysis (3). Within each segment, the data is assumed dependent and appropriate correlation functions

are defined, however, subsequent processing of segments are assumed to be independent. This assumption is necessary to assure consistency in the performance measure. We also assume that the noise in each channel is independent from all other channels for all time delays. An example is given assuming independent but non-Gaussian noise samples. The results indicate the possibility of performance improvement in non-Gaussian noise by partitioned arrays compared to arrays designed assuming Gaussian noise.

Other approaches, which employ nonlinearities to improve performance in non-Gaussian noise, were confined to the time domain (4, 5 and 6). In fluctuating ambient noise fields, the optimum processor was shown to be nonlinear (?). It was concluded, in reference (8), that deep sea ambient noise was in some cases non-Gaussian and non-stationary for time periods as short as a few minutes. In addition, biological noise was found to be non-Gaussian (8). A recent experiment (as discussed in reference (9)) with a high gain acoustic array reported that the noise was non-stationary at the output of extremely narrow beams for periods greater than 15 seconds. Shipping noise has also been reported to be non-Gaussian (10). Measurements of underwater ambient noise beneath sea ice, in a frequency band from 12.5 Hz. to 6.4 KHz., were found to be impulsive and highly non-Gaussian (11). A summary of the results of measurements of under-ice noise, up to 1968, can be found in reference (12). In the non-acoustic area, the major sources of interference to signals in the extremely low frequency ELF band are atmospheric noise, power line radiation, and jamming (13). Apparently, the dominant source of interference in the ELF band is attributed to lightning which causes the noise characteristics to be non-Gaussian. The Sanguine communication system attempts to communicate with submerged submarines in the ELF band (14), and therefore, could be highly susceptible to non-Gaussian noise activity.

For the above reasons, it is desirable to consider detection systems which can be optimized (adaptable) to the noise conditions.

II. RECEIVER STRUCTURE

We assume that the receiver sampled data vector is composed of additive signal and noise which are statistically independent, both spatially and temporally, and can be written as

$$H_1: X = S + N$$

$$H_0: X = N$$

$$\text{where } X = \{X_{j,i+(q-1)N_0}\}_{i=0,\dots,N_0-1};$$

$j=1,\dots,M$; $q=1,\dots,n_T$ is the received sampled data from an array of M sensors. The components $S_{j,i+(q-1)N_0}$, $N_{j,i+(q-1)N_0}$ represent signal and noise at the j th sensor and $(i+(q-1)N_0)$ th time sample respectively. The signal is assumed to be a stationary plane wave that arrives at an angle ϵ with respect to the axis of the array and is defined as $S_{j,i+(q-1)N_0} =$

$S((i+(q-1)N_0)\Delta - \epsilon_j)$ where Δ is the interval between consecutive temporal samples in seconds and ϵ_j represents the time delay in seconds of the signal at the j th sensor. The noise will be assumed isotropic, but not necessarily Gaussian. In all cases, the signal will be assumed much weaker than the noise intensity which is not an unreasonable assumption in underwater acoustic signal processing problems.

The receiver is defined for each frequency component as a sum of n_T independent and identically distributed (i.i.d.) components and can be expressed as,

$$R(n_T, p) = \sum_q^{n_T} R(q, p) \quad (1)$$

where

$$R(q, p) = \left| \sum_j^M A_j(p) B_j^m(p, q) \right|^2 \quad (2)$$

and $A_j(p)$ are complex weights to be determined. Also,

$$B_j^m(p, q) = \frac{1}{\sqrt{N_0}} \sum_i^{N_0-1} \left(\sum_k^m b_{jk} n_{jk i q} \right) e^{-j i p}, \quad (3)$$

where $j = \sqrt{-1}$, $p = 2\pi f \Delta$ (f is the frequency), $n_{jk i q} = u(X_{j,i+(q-1)N_0} - a_{jk-1}) -$

$u(X_{j,i+(q-1)N_0} - a_{jk})$, ($u()$ is the unit step function), and a_{jk} represents the k th

quantile of the j th sensor, and b_{jk} are called scores which may be fixed or allowed to adapt to the underlying distribution (see reference (15)). However, the scores and quantiles are always held constant during any decision interval. The term in parenthesis in equation (3) represents the nonlinearity which precedes the standard Gaussian detector (16). We have also neglected a data window and omitted a sampling rate scaling in equation (3) for convenience. In the following development, we will only be concerned with the cross-channel performance measure. In general, all the terms can be used in the performance prediction, however, if M is large, the cross-channel terms probably dominate the results and this technique greatly simplifies the analysis. Also, to simplify the notation, the dependence on p (normalized frequency) will sometimes not be explicitly shown.

III. MEAN AND VARIANCE

The performance measure will only depend upon the mean and variance (under H_0) of equation (2).

The mean is given by

$$E(R(q, p)/H_1) = \sum_{j_1}^M \sum_{j_2}^M A_{j_1} A_{j_2}^* b_{j_1}^m b_{j_2}^m \tilde{S}(p) e^{-j p D} \quad (4)$$

where

$$b_{j_1}^m = \sum_k^m (b_{jk})^2 G(a_{jk}),$$

$$\tilde{S}(p) = \sum_{v=-z+D}^{z+D} (1 - |v-D|/N_0) R^S(v) e^{-j p v},$$

and we set $\Delta=1$ for convenience, $z = \pi N_0 - 1$, $D = (\epsilon_{j_1} - \epsilon_{j_2})$, $G(a_{jk})$ is the probability of

the data sample at the j th sensor being in the interval partitioned by the quantiles $(a_{jk}; a_{jk-1})$. Also, $R^S(v)$

represents the auto-correlation function of the signal. It should be emphasized that the result in equation (4) only applies for weak signals. As $N_0 \rightarrow \infty$

$\tilde{S}(p) \rightarrow S(p)$, which is the auto-spectral density of the signal. Both $R^S(v)$ and $S(p)$ are real functions. Under $H_0(j_1 \neq j_2)$

the mean is exactly zero.

The variance, under H_0 , for $j_1 \neq j_2$ assuming isotropic noise and using a similar procedure given in reference (3), is as follows,

$$\text{VAR}(R(q,p)/H_0) = \sum_{j_1=1}^M \sum_{j_2=1}^M \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} f_{j_1 j_1}^m(x)$$

$$f_{j_2 j_2}^m(x') \left(|A_{j_1}|^2 |A_{j_2}|^2 |w_{N_0}(x-p)|^2 \right.$$

$$\left. |w_{N_0}(x'-p)|^2 + A_{j_1} A_{j_1}^* A_{j_2}^* A_{j_2} w_{N_0}^*(x-p) \right.$$

$$w_{N_0}(x+p) w_{N_0}^*(x'+p) w_{N_0}(x'-p) \Big) dx dx' \quad (5)$$

where

$$w_{N_0}(x-p) = \frac{1}{\sqrt{N_0}} \sum_{i=1}^{N_0-1} e^{-j \cdot i(x-p)}$$

and we have defined the Fourier transform pair (assuming they exist) of the noise as:

$$R_{jj}^m(v) = \left(\frac{1}{2\pi}\right) \int_{-\pi}^{\pi} f_{jj}^m(x) e^{j \cdot v x} dx$$

$$= \sum_{k_1=1}^m \sum_{k_2=1}^m b_{jk_1} b_{jk_2} E(n_{jk_1 i_1} n_{jk_2 i_2}) / b_j^m,$$

is the normalized auto-correlation function and the partitioned auto-spectral density is given by the expression

$$f_{jj}^m(p) = \sum_{v=-\infty}^{\infty} R_{jj}^m(v) e^{-j \cdot v x}$$

where $f_{jj}^m(p)$ and $R_{jj}^m(v)$ are real functions and as $N_0 \rightarrow \infty$

$$\text{VAR}(R(q,p)/H_0) \rightarrow \sum_{j_1=1}^M \sum_{j_2=1}^M |A_{j_1}|^2 |A_{j_2}|^2 f_{j_1 j_1}^m(p) \cdot f_{j_2 j_2}^m(p) b_{j_1}^m b_{j_2}^m, \quad (6)$$

For $p=0$, equation (6) is multiplied by a factor of 2. We shall ignore the factor of 2 when expressing the variance since the performance measure will ultimately be a ratio and therefore, the factor of 2 will cancel out.

IV. RESULTS

We will define a signal-to-noise ratio parameter, assuming the conditions already discussed hold, for sufficiently

large N_0 as:

$$d = \frac{(E(R(q,p)/H_1) - E(R(q,p)/H_0))^2}{\text{VAR}(R(q,p)/H_0)} \quad (7)$$

The normal procedure to obtain the asymptotic relative efficiency ARE would be to expand the mean with respect to $S(p)$ and then form a ratio of d 's for two different receivers, and then let $S(p) \rightarrow 0$. However, since we have already assumed the signal to be weak, in our derivation equation (7) can be used directly with an equivalent result.

Now, let $A_j = \frac{\sqrt{S(p)}}{f_{jj}^m(p)} e^{j \cdot p \cdot j}$, which is

similar to the optimum filter, except that the result uses the partitioned auto-spectral density. If $m \rightarrow \infty$ and the noise is Gaussian, then the result reduces to the classical optimum filter.

Equation (7) reduces to

$$d_m = (S(p))^2 \sum_{j_1=1}^M \sum_{j_2=1}^M b_{j_1}^m b_{j_2}^m / (f_{j_1 j_1}^m(p) f_{j_2 j_2}^m(p)) \quad (8)$$

$$= M(M-1) (S(p)/f_{jj}^m(p))^2 (b_j^m)^2$$

If we let d_m represent an array designed assuming Gaussian noise, the ARE can be defined as,

$$\text{ARE}_{m,G} = (N(p)/f_{jj}^m(p))^2 (\sigma_N^2)^2 (b_j^m)^2, \quad (9)$$

where $\sigma_N^2 < \infty$ is the underlying noise variance and $N(p)$ represents the corresponding normalized auto-spectral density.

To simplify matters further, let the sampling rate be such that the noise samples are independent, then the ARE reduces to,

$$\text{ARE}_{m,G} = (\sigma_N^2)^2 (b_j^m)^2 \quad (10)$$

In the figure we have plotted $5 \log(\text{ARE})$ versus c , based on equation (10), using a generalized Gaussian density (15), where c is a positive constant controlling the decay rate of the tails. For example, if $c=2$ the density is Gaussian and for $c=1$ the density is Laplacian. When $m=2$, the partitioned detector is equivalent to a sign detector or a detector employing hard clipper as nonlinear elements. For smaller values of $c \leq 1$, the partitioned detector for $m=2$ and $m=10$ perform substantially better than the standard linear correlator. At $c=.5$, the partitioned detector for $m=10$ is about 3 dB better than the partitioned detector

for $m=2$. However, at $c=10$, the partitioned detector for $m=10$ is about 7 dB better than the partitioned detector for $m=2$.

V. CONCLUSIONS

We have derived a detector structure which is similar to the classical Gaussian detector, except that the data is first partitioned by a set of quantiles based on the underlying noise statistics. If the number of quantiles used approaches infinity and if the noise is Gaussian, the partitioned detector approaches the classical detector. In non-Gaussian noise, the performance of the partitioned array detector as expressed in equation (10), can be better, based on the ARE performance measure, than an array designed assuming Gaussian noise.

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