CEPSTRUM CONSTRAINTS IN ME SPECTRAL ESTIMATION

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ABSTRACT

Current trends in spectral estimation techniques follow basicaly two guidelines; by introducing modifications in the objetive function to be minimized or, keeping the objetive, by modifying the set of constraints in the minimization procedure. This work can be encompassed in the second sense, showing the potential of cepstrum constraints in spectral estimation methods. The special features associated with this constraints allow to use raw approximations in order to linearize or simplify the computations involved in the procedure maintaning within margins of adequate quality the resulting estimate.

THE ME PROCEDURE

Being $S_x(\omega)$ the spectral estimate of a random process $\{x\}$ and given a data sample $x(n)$ $(n=0,M-1)$, an ME estimate have to minimize the objetive function (1),

$$\frac{1}{2\pi}\cdot\int_{-\pi}^{\pi}\log S_x(\omega)\,d\omega \qquad (1)$$

in accordance with a set of constraints.

The set of constraints use to be the autocorrelation estimate (2), which is usually obtained from the IDFT of the periodogram $P(\omega)$ corresponding with the data sample |1|,

$$\frac{1}{2\pi}\cdot\int_{-\pi}^{\pi}S_x(\omega)\exp(jn\omega)d\omega=r_x(n);\ |n|\leq Q \qquad (2)$$

where $|.|$ denotes absolute value and,

$$r_x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}P(\omega)\exp(jn\omega)d\omega;\ |n|\leq Q \qquad (3)$$

By using Lagrange multipliers $\lambda_n(n=-Q,Q)$ for the constraints (2), after forming the new function to be minimized and from variational concepts, the well known all-pole estimate is obtained by setting derivative with respect $S_x(\omega)$ equal to zero.

This work deals with the effect of adding cepstrum constraints to (2) in the ME procedure. Of course, increasing the number of constraints represents a trade-off between complexity and the amount of information, provided by the data set which is represented in the constraints of the optimization procedure.

CEPSTRUM CONSTRAINTS AND ME ESTIMATE

Let us suppose that 2M points of the cepstrum $\hat{c}_x(m)$ $(m=1,M$ and $-1,-M)$ are available from the IDFT of $\log|P(\omega)|$ and they are included as new constraints (4) added to (2) in order to find the corresponding ME estimate.

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\{\log S_x(\omega)\}.\exp(jm\omega)d\omega=\hat{c}_x(m);\ \underset{m\neq 0}{|m|<P} \qquad (4)$$

where

$$\hat{c}_x(m)=\frac{1}{2\pi}\int_{-\pi}^{\pi}[\log P(\omega)]\exp(jm\omega)d\omega;\ \underset{m\neq 0}{|m|<M} \qquad (5)$$

Using the same procedure as in |2|, multipliers λ_n and μ_m are set for correlation and cepstrum constraints respectively. Thus the new function to be minimized is (6), where the terms not depending on $S_x(\omega)$ have been removed.

$$\int_{-\pi}^{\pi}\Big[\{\log S_x\}.\Big(1+\sum_{|m|=1}^{M}\mu_m\exp(jm\omega)\Big)+ S_x.\Big(\sum_{|n|=0}^{Q}\lambda_n.\exp(jn\omega)\Big)\Big]d\omega \qquad (6)$$

By setting the derivative with respect to $S_x(\omega)$ in the braket term of (6) equal to zero; then, the desired estimate with zeros and poles is obtained.

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$$S_x(\omega)=\frac{1+\sum_{|m|=1}^{M}\mu_m.\exp(jm\omega)}{\sum_{|n|=0}^{Q}\lambda_n.\exp(jn\omega)} \qquad (7)$$

Trying to find the Lagrange multipliers μ_m and λ_n will face a non-linear problem. In the next section an approximate approach in order to obtain the coefficients of (7) will be provided.

ALTERNATIVE REPRESENTATION OF CEPSTRUM CONSTRAINTS

The basic approximation to be considered in this section is to assume that both $S_x(\omega)$ and $P(\omega)$ have an effective time limited cepstrum and M is selected long enough to cover this effective time duration of $\hat{c}_x(n)$. After this assumption and bearing in mind that $S_x(\omega)$, the desired estimate, have to verify the cepstrum constraints, expression (8) can be used for $S_x(\omega)$ where $c_x^e(n)$ denotes the extrapolated values by the ME estimate.

$$S_x(\omega)=\exp\{c_x^e(0)+\sum_{m>0}^{M}2.\hat{c}_x(m)\cos n\omega + \sum_{m>M}^{\infty}2c_x^e(m)\cos m\omega\} \qquad (8)$$

Now, defining $\tilde{c}_x(m)$ as the difference of the extrapolated cepstrum and the complete cepstrum derived from $P(\omega)$, we have:

$$S_x(\omega)=P(\omega).\exp\{\tilde{c}_x(0)+\sum_{m>M}^{\infty}2.\tilde{c}_x(m)\cos m\omega\} \qquad (9)$$

In this formula we will made the following considerations:

a.- the term $\tilde{c}_x(0)$ is small enough to consider $\exp\{\tilde{c}_x(0)\}$ close to one.

b.- The summatory of (9) can be reduced to the first lag $\tilde{c}_x(M+1)$, and due to its small value the term

$$\exp\{2\tilde{c}_x(M+1)\cos(M+1)\omega\}$$

can be approximated by

$$\{1+2\tilde{c}_x(M+1)\cos(M+1)\omega\}.$$

After these considerations the cepstrum constraints can be denoted by:

$$S_x(\omega)\simeq P(\omega)\{1+2\tilde{c}_x(M+1)\cos(M+1)\omega\} \qquad (10)$$

SOLUTION FOR PARAMETERS λ_n AND μ_m

Note that (10) implies that, in some sense, the cepstrum constraints can be considered as like-correlation constraints obtained from the IDFT of the righ hand side of the above expression. To clarify this point, note that the correlation constraint implies that coefficients λ_n and μ_m have to verify (11), where m=0,M and $|.|$ indicates absolute value.

$$\mu_m=\lambda_0 r_x(m)+\sum_{n=1}^{Q}\lambda_n.\{r_x(m+n)+r_x(|m-n|)\} \qquad (11)$$

Equation (11) needs for 2Q+1 auto-correlation lags and there are more unknows than equations. With respect to the number of equations it is clear that the new equations, that the procedure needs, have to stem from the cepstrum constraints. In other words, Q values from the IDFT of the equation (10) will allow to obtain λ_n and μ_m together with the previous equations. So that the new set of equations which complete (11) are:

$$\mu_m=\lambda_0\tilde{r}_x(m)+\sum_{n=1}^{Q}\lambda_n\{\tilde{r}_x(m+n)+\tilde{r}_x(|m-n|)\} \qquad (12)$$

where

$$\tilde{r}_x(m)=\mathrm{IDFT}\{P(\omega)\left[1+\tilde{c}_x(M+1)\cos(M+1)\omega\right]\} =r_x(m)+\tilde{c}_x(M+1).\{\bar{r}_x(m+n)+\bar{r}_x(|m-n|)\} \qquad (13)$$

and

$$\bar{r}_x(q)=r_x(M+1+q)+r_x(M+1-q) \qquad (14)$$

To make it clear, by substracting (11) from (12) the resulting set of equations shows how parameters λ_n can be obtained like in an all-pole fashion from the pseudocorrelation $\bar{r}_x(q)$

$$0=\lambda_0\bar{r}_x(m)+\sum_{n=1}^{Q}\lambda_n\{\bar{r}_x(m+n)+\bar{r}_x(|m-n|)\} \qquad (15)$$

It should be noted that because the number of equations (12) is Q, solving (15) is equivalent to solve for polynomial B(z), where $B(z).B(1/z)=\lambda(z)$, using Levinson algorithm and the pseudo-correlation $\bar{r}_x(q)$. After coefficients λ_n are found, coefficients μ_m are obtained using (11), which completes the procedure.

The required approximations needed in the previous sections may be are not correct or without adequate theoretical support, but it seems to be the only way to avoid the use of iterative techniques that the non-linear character of cepstrum constraints

produces. In general, these iterative techniques have not well-known convergence properties getting worse in this case where positivness to the associated estimate have to be controled in two functions (i.e. the extrapolated auto-correlation and the extrapolated cepstrum).

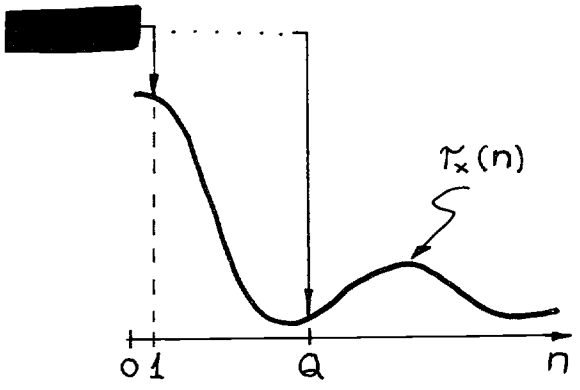

Fig. 1a. Design equations in the all-pole ME procedure.

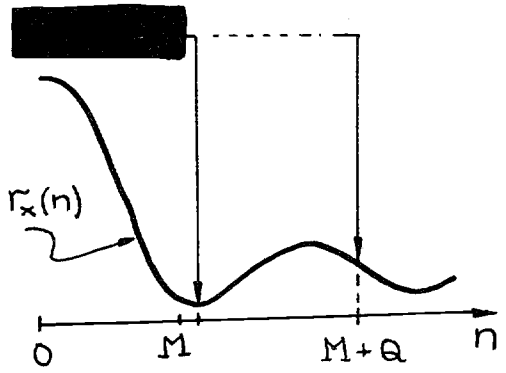

Fig. 1b. Design equations in the extended Yules-Walker method.

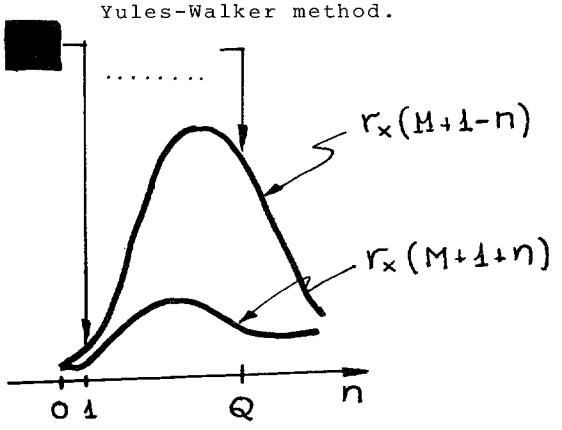

Fig. 1c. Design equations in the proposed technique.

FURTHER DETAILS OF THE PROCEDURE

In this section some details of the previously described procedure will be discussed.

First at all it is worthwile to compare the procedure with classical all-pole or Yules-Walker and the so called extended Yules-Walker procedure. In Figure 1 the reader can see the three procedures in an exact autocorrelation prediction fashion, where the arrows indicate the lags to be predicted and the black boxes the data support to be used in the prediction. Thus the design equations for coeficients in all-pole ME and points to be predicted exactly.

Note that the proposed technique is similar to the extended Y-W method but due to the special character of pseudo-correlation $\bar{r}_x(q)$ the design equation can be named as exact central autocorrelation prediction.

With respect to the use of cepstrum constraints, the intention of the author was to show that, under his personal point of view, the fundamental limitations of parametric procedures stem from the amount of information provided by the data sample, which is represented in the procedure. Clearly it seems to be that from N data samples by representing this information in M constraints with M<<N is where the procedure fails no matter what is the objetive function selected. It is in this last sense that the inclusion of cepstrum constraints is adequate because it is well known that a few values of cepstrum can summarize global properties of the data sample.

From the proposed approach in the previous sections it can be concluded that the main property of cepstrum constraints has been partially destroyed when after the approximations the global behavior of the procedure is reduced to P+Q+1 autocorrelation lags. This is a consequence of the approximation made over expression (9); but, if more terms $\tilde{c}_x(m)$ are under consideration, they will increase considerably the number of correlation lags involved in the procedure. In fact, this preceeding comment supports the possibility, after considering $\tilde{c}_x(M+1), \tilde{c}_x(M+2), \ldots, \tilde{c}_x(M+R)$ different from zero, of and averaged pseudocorrelation version in computing parameters λ_n.

$$\tilde{r}_x(m)=\frac{1}{2\pi}\cdot\int_{-\pi}^{\pi} P(\omega)\{1+2\sum_{r=1}^{R}\varepsilon_r\cos(M+r)\omega\}e^{jm\omega}d\omega \qquad (16)$$

where ε_r are the cepstrum error $\tilde{c}_x(M+r)$

and some a priori distribution for them are available. For example, assuming a high level of noise in small values of the autocorrelation estimate an adequate selection for ε_r will be close to zero and the oposite for high absolute autocorrelation values; in other words an adequate empirical rule to select ε_r is (17)

$$\varepsilon_r = \frac{r_x(M+r)}{r_x(0)} \qquad (17)$$

Finally, is interesting to remark that an MA estimate of maximum entropy is derived from the cepstrum constraints in the same way that the B-T estimate is derived from the autocorrelation ones. Note that even in this case the perspectives of the procedure looks correct because in the MA case the entropy as objetive implies the truncation of the given cepstrum which is less important, in general, than made over the autocorrelation function. For an AR design with only autocorrelation constraints the classical ME procedure remains.

AN EXAMPLE

In order to test the procedure with the given approximations, the familiar AR(5) was used. In the first case an ARMA (5,5) was done from the proposed approach in order to test the sensibility to wrong order choice. Figure 2 shown the result obtained

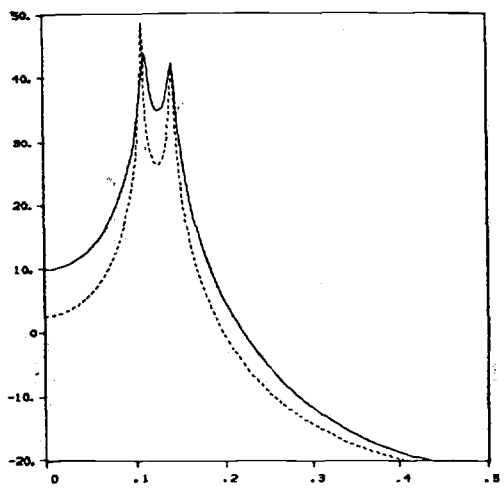

Fig. 2. Resulting estimate for an ARMA (5,5) choice (---) and actual AR(5) (—)

In the second case a white noise was added to the AR(5) process with a global signal to noise ratio of 5dB. The resulting estimate can be viewed in Fig.3

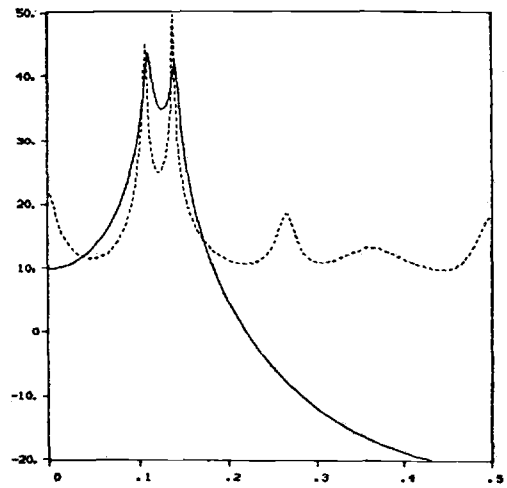

Fig. 3. The same as fig. 2 using an ARMA (15,15) and adding white noise to the original AR(5) process (SNR=5dB).

In this figure 3 the deshed line indicates the resulting estimate and the continuous line the actual power spectral density before the white noise was added. Also there are an important difference between Fig. 2 and 3 the length of the data sample was 100 and 200 respectively.

CONCLUSSIONS

The work presents the use of cepstrum constraints in parametric spectral estimation, concrectly, in the ME method the cepstrum introduces zeros in the associated model for the resulting estimate. In addition, in order to obtain an easy to compute algorithm, a set of approximations are provided in the text.

The more remarkable feature of the work seems to be the relevance of constraints that in a few parameters concentrate global properties of the data sample or its associated autocorrelation. In most of the cases, this kind of functions, like cepstrum, will made the objetive function irrelevant.

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