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14. Abstract/Notes

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METHODS FOR IMAGE INTERPOLATION THROUGH FIR FILTER DESIGN TECHNIQUES

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ABSTRACT

Interpolation methods in image processing are necessary in various applications. In this work the problem of image interpolation is approached from the viewpoint of digital signal processing. This paper presents a two-dimensional extension of earlier work in one dimension. A class of image interpolators is thus obtained and may be compared with the more common ones, such as nearest-neighbor, bilinear and cubic convolution.

I. INTRODUCTION

The use of interpolation techniques in digital image processing is extensive, including resampling, geometric correction and scale magnification problems. The great amount of data involved in image processing makes it mandatory to develop efficient algorithms - - resulting from a compromise between interpolation quality and computational cost. Therefore, a good understanding of the interpolation process is needed to design algorithms which meet the desired requirements.

In what follows, the problem of Image Interpolation is approached from a Digital Signal Processing (DSP) point of view. In one dimension, the DSP approach to interpolation has been well-studied [1-4] and this work represents a two-dimensional extension of the former ones; a class of image interpolators is analysed: these functions are obtained by FIR filter design methods. Their behavior is then compared with the more common image interpolators such as bilinear nearest-neighbor, and cubic convolution [5]. The aim of the comparison was to establish practical performance limits and to subsidize the choise of adequate interpolation functions.

II. FIR IMAGE INTERPOLATION: DESIGN AND COMPARISON

Design methods

The Digital Signal Processing approach shows that the process of interpolation can be well-formulated as a low-pass linear filtering problem [1]. The linear phase restriction makes FIR realizations the usual choice, instead of IIR realizations. In the two-dimensional case, a simplifying solution is to make the design problem separable: The two-dimensional filter $H(w_1, w_2)$ may be expressed as a product

$$H(w_1, w_2) = H(w_1).H(w_2) \qquad (1)$$

The two-dimensional interpolator is thus obtained in two steps: the one-dimensional filter is designed, and the two-dimensional interpolation is made by convolution of the filter response in the horizontal and vertical directions. Three design methods were used:

1) Window design;
2) Optimum equiripple linear phase designs ;
3) Special FIR interpolators with minimum RMS error in the time domain.

All obtained filters use 4x4 regions (16 pixels) for interpolating one pixel in the resulting image. In the window design, a finite weighting sequence $w[k]$ is used to modify the ideal Nyquist interpolator $\tilde{h}[k]$ - the sinc function-so as to obtain the finite filter

$$h[k] = \tilde{h}[k].w[k] \qquad (2)$$

which can be efficiently implemented.

Among the used windows were Hamming, Cosine, Kaiser and Papoulis. A parabolic window - due to Shlien [6] - was also used. It is of the form:

$$w[k] = 1 - \left(\frac{2\,k}{N-1}\right)^2 \quad , \; k = -\frac{N-1}{2}, \ldots, \frac{N-1}{2}, \qquad (3)$$

for an N-sized filter.

A more sophisticated design technique is the equiripple design. In our case, we used a well-known computer program [7] to obtain filters with that property.

Interpolation filters have also been proposed which minimize the mean-square-error between the interpolated samples and the output of the ideal interpolator [2]; the filter parameters depend on the input autocorrelation. It was later shown [3] that this design procedure could also minimize the maximum normalized error in the time domain if the input signal is supposed band-limited and spectrally flat.

COMPARISON MEASURES

In order to compare the performance of the FIR image interpolators, it is necessary to use measures which take into account the specific structure of the image interpolation problem. To this end, Pratt [8] has proposed two measures:

(a) resolution loss, expressed as:

$$\varepsilon_R = \frac{E_I - E_R}{E_I} \qquad (4)$$

where:

$$E_R = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} W_I(\omega_x,\omega_y)\left|I(\omega_x,\omega_y)\right|^2 d\omega_x\, d\omega_y \qquad (5)$$

represents the actual interpolated image energy, and

$$E_I = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} W_I(\omega_x,\omega_y)\, d\omega_x\, d\omega_y, \qquad (6)$$

the energy of the ideally interpolated image.

In the above, $W_I(\omega_x,\omega_y)$ stands for the power spectral density of the image to be recovered in the process, and $I(\omega_x,\omega_y)$, the frequency response of the interpolator.

(b) interpolation error, defined as:

$$\varepsilon_A = \frac{E_A}{E_T}, \qquad (7)$$

where:

$$E_T = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_I(\omega_x,\omega_y)\left|I(\omega_x,\omega_y)\right|^2 d\omega_x\, d\omega_y \qquad (8)$$

indicates the total energy of the interpolated image, and

$$E_A = E_T - E_R \qquad (9)$$

represents the portion of the energy interpolated outside Nyquist limits.

The results of using those comparison measures to the above-mentioned image interpolators are outlined on Table 1.

The power spectral density was obtained as the Discrete Fourier Transform of the separable markovian autocorrelation function, which has the form:

$$R_I(j,k) = R_I(j).R_I(k) = (.953)^j.(.953)^k \qquad (10)$$

TABLE 1

INTERPOLATOR ERRORS

	RESOLUTION LOSS ε_R(%)	INTERPOLATION ERROR ε_I(%)
NEAREST-NEIGHBOR	23.4	24.3
BILINEAR	38.6	3.4
CUB. CONVOL.	18.6	2.6
SHLIEN	15.1	1.9
HAMMING	22.0	3.1
KAISER	24.9	3.6
PAPOULIS	29.3	5.8
SINC	18.3	2.4
COSINE	15.6	2.0
EQUIRIPPLE(f_p=0,05		
f_r=0,07; δ_1/δ_2 = 1	41.9	12.3
EQUIRIPPLE(f_p=0,055;		
f_r=0,09; δ_1/δ_2 = 50	20.9	0.6
MINIMAX ERROR	17.2	1.4

From Table 1, it can be seen that the simple Shlien (parabolic) and Cosine windows had the best behavior even if compared with filters with optimality properties.

To verify the above results, a scale magnification experiment was made: a test image - the "Kodak girl" - was reduced 8 times from its original 512x512 size to a 64x64 image. At this stage, the reduced image was interpolated to its original size by means of the various interpolators discussed. The parameters calculated were the error (mean and variance) and the absolute value of the error (mean and variance as percentage of the original image): The experiment was conducted on an Image-100 system that runs under control of a PDP 11/45 computer.

The parameters are shown in Table 2.

TABLE 2

INTERPOLATOR PERFORMANCES FOR SCALE MAGNIFICATION

INTERPOLATOR	ERROR		ABS.ERROR(%)	
	MEAN	VAR	MEAN	VAR
NEAREST-NEIGHBOR	0.20	181	10.6	7.0
BILINEAR	0.80	154	11.2	5.4
CUBIC CONVOLUTION	-0.40	144	9.8	5.3
SHLIEN	-0.40	145	9.9	5.3
HAMMING	0.10	164	9.8	5.4
KAISER	-0.50	165	9.9	5.4
PAPOULIS	-5.60	167	12.7	6.8
SINC	1.0	145	9,9	5.3
COSINE	0.7	145	9.9	5.3
EQUIRIPPLE(f_p=0,05;	-19.8	719	37.1	22.9
f_r=0.07; δ_1/δ_2 = 1)				
EQUIRIPPLE(f_p=0,055:	15.4	216	22.9	9.3
f_r=0.09; δ_1/δ_2=50)				
MINIMAX ERROR	-1.5	145	9.9	5.4

These results corroborate the findings of Table 1 in that the more accurately designed interpolators do not have a better performance than the simple ones designed by the windowing method. In visual terms, the images generated by Shlien, sinc, cosine, and cubic convolution interpolators are superior to the others. The visual results can perhaps be interpreted in terms of the complexity of the human vision, that certainly does not rely on square error type of fidelity criterion. In fact, a very important part on an image visual information is contained in its contours.

III. HEURISTIC TECHNIQUES FOR INTERPOLATOR DESIGN

The previous section suggests that image texture properties can be used as additional information in the interpolation process. The idea in what follows is to obtain a measure of the local edge density, data which indicate the most suitable interpolator for that region. Areas which do not have a great edge density may be interpolated by a simpler process than those having many countours. In this fashion, a great economy in the interpolation process may be obtained.

The edge density measure used was the maximum variation operator [9]. For a 2x2 image region - as shown in Figure 1 - the maximum variation operator is defined as:

$$MV = \max \left\{ (|a-b|+|c-d|), (|a-c| + |b-d|) \right\}. \tag{11}$$

```
   :  :
... a b ...
... c d ...
   :  :
```

Figure 1 - 2x2 IMAGE REGION

An heuristic procedure for image interpolation was developed that, for each 2x2 image region, calculates the maximum variation (MV). If the edge density MV is less than a certain threshold, interpolation is made by means of the bilinear function.

Otherwise, a higher-order interpolator such as Shlien or cubic convolution is used. At the end of each output line, the percentage of bilinear interpolation up to that point is calculated. If that percentage does not fit within a predeterminad range the threshold is modified accordingly.

Using the above heuristic procedure, the scale magnification experiment was repeated, using the "Kodak girl" reduced to a 64x64 size to interpolate back to its 512x512 size. By fixing the desired bilinear interpolation percentage at (70 ± 3)%, the obtained images had the same visual quality as the earlier ones, but the computation time was cut by 40%.

The above algorithm was applied to the problem of resampling of LANDSAT imagery, that is sampled on a 57m x 79m rectangular grid. By interpolation that resolution was changed to a more convenient one, for example, 50m x 50m. The good performance of the heuristic algorithm was confirmed in this experiment, since time savings up to 50% were observed with no apparent visual difference.

IV. CONCLUSIONS

This work represents an attempt to study the problem of Image Interpolation from a Digital Signal Processing point of view. In this context, the problem is equivalent to the design of low-pass FIR digital filters. Three design methods were used: windowing, equiripple and minimax error for interpolation. Based on comparisons made - including analysis in the space and frequency domains - it can be stated that:

(a) The use of more accurate design methods than that of windowing did not result in a better quality of the interpolated images. This result shows the importance of the edge information - which consists in high frequencies - in the image information content. It should be expected that filters designed through criteria that take into account the characteristics of the human visual system could display a better performance.

(b) The introduction of additional information - local edge density - made it possible to reduce computational cost, while maintaining the same visual quality as higher-order interpolation.

APPENDIX

Two photographs are included to illustrate the above work. In Figure 1, the original image is shown, and in Figure 2, the image interpolated by the sinc window is displayed.

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