MEASURING TRUE SPECTRAL DENSITY FROM ML FILTERS (NMLM AND q-NMLM SPECTRAL ESTIMATES)

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ABSTRACT

Starting from the classical procedure reported by Capon for power level estimation from ML filters, the authors present how this method can be modifyed in order to obtain a power spectral density estimate. The basic idea is to compute the effective bandwidth of the ML filter, and normalize the power level estimate, at the output of a quadratic detector following the filter, with it. The effective bandwidth has been obtained by an equal area constraint criteria.

Furthermore, the above mentioned estimate, we called NMLM, converges in the distributional sense to the true spectral power density. This suggest the use of new estimates, denoted in the text as q-MLM, which improves the mentioned convergence both in 1-D as well as 2-D problems of SPA.

1. INTRODUCTION

The bank filter theory is widely used in simultaneous spectral |2| analysis even after parametric methods appears with their impressive results |3|. Within the category of filter bank spectral analyzers the most popular is the well-known Welch's procedure |4|. No matter the apparent superiority of parametric methods the robustness and the low complexity of the filter bank methods give to them certain preponderance in practical problems in many fields of application.

Using a bank of filters, each output will provide, after a quadratic detector, an estimate of the power from the input signal which is in the frequency range within the effective bandwidth of the analysis filter. These concepts appear briefly summarized in Fig 1.

It should be noted that, either the power level $P_{\omega 0}$ or the power spectral density P_{ω_0}/B_ω, the measure will be so adequate depending on up to what degree the filter bandwidth is reduced. The drawback,

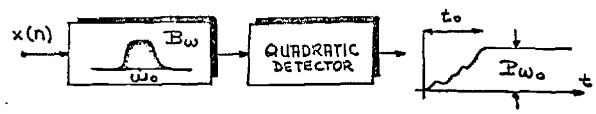

Fig. 1. Filter bank analysis.

of using small bandwidths in the analysis, is the increase in the transient response t_0 (approximately $1/B_\omega$) that will require at least t_0 sec. of the input signal x(n) to allow the correct measure of the power level at the output of the quadratic detector. Thus, given t_0 accordingly with the available data sample x(n) (n=0, N-1), from the uncertainly principe a maximum value for B_ω is allowed, resulting in the leakage from the nearest frequency bands to the center frequency ω_0.

Designing the analysis filter to detect the power level of a pure complex sinusoid in white noise, will requiere: Maximum output at the frequency where the sinusoid is, and minimum noise bandwidth. The well-known solution is a matched filter that, under these assumptions, will result in a DFT of the input signal with length M. The DFT can be viewed as a bank of N matched filters with Maximum outputs at n=N/2 (transient time) and "steered" or matched to equally spaced frequencies $\omega_k=2\pi k/N$; (k=0,N-1).

The optimum character of the periodogram is mixed when the noise is not white or there are more than one spectral line in the input signal. It is clear that in this case each filter should remove more o less the other spectral lines, viewed as interferences by the analysis filter, and

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whitening the colored noise. The constraints are unity response at the frequency where the filter is steered ω_0 and a finite impulse response if a transient time t_0 should be keeped within margins.

Formulating the concept described in the previous paragraph, will result in the well-known procedure MLM reported by Capon |1| that will be briefly described here after. Given the vector $\underline{A}$ with M coefficients as its impulse response,

$$\underline{A} = (a(0),\ a(1),\ \ldots\ ,\ a(M))^t \tag{1}$$

we would like that its frequency response at ω_0 will be unity

$$A(\omega_0) = \underline{S}^t . \underline{A} = 1 \tag{2}$$

being

$$\underline{S}^t = (1,\ \exp j\omega_0, \ldots, \exp jM\omega_0) \tag{3}$$

the so-called steering vector, and t denotes transpose and complex conjugate.

In other sense and to better reflect the leakage, it can be seen in |4| that the input power density spectrum $S(\omega)$ can be formulated as it is shown in (4)

$$S(\omega) = \tilde{S}(\omega) + S_0(\omega) \tag{4}$$

where $S_0(\omega)$ is the s.p.d. included in the effective bandwidth of the analysis filter, which power $P_{\omega 0}$ we are dealing for, and $\tilde{S}(\omega)$ is the interference s.p.d. which produces the leakage in measuring $P_{\omega 0}$ due to the use of a non-ideal bandpass filter. It is worthwhile to note that as concerns with the measure of $P_{\omega 0}$, using (4) is equivalent to (5).

$$S(\omega) \cong \tilde{S}(\omega) + P_{\omega 0}\ S(\omega - \omega_0) \tag{5}$$

From (5) it can be concluded that minimizing (a) results in the same solution than minimizing (b), because they differ in a constant i.e. the power level $P_{\omega 0}$.

$$\begin{array}{ll} \int \tilde{S}(\omega)\,|A(\omega)|^2\, d\omega & (a)\ , \\ \int S(\omega)\,|A(\omega)|^2\, d\omega & (b) \end{array} \tag{6}$$

In conclussion, we can derive the following two equation to resume the ML method of bank filtering power level estimate

$$\int S(\omega)\,|A(\omega)|^2\, d\omega \Big|_{min.} = \underline{A}^t\ \underline{\underline{R}}\ \underline{A}\Big|_{min} \tag{7}$$

$$\underline{S}^t\ \underline{A} = 1$$

The well-known solution is ( )

$$\underline{A} = \underline{\underline{R}}^{-1} \cdot \underline{S}\ /\ \underline{S}^t\ \underline{\underline{R}}^{-1}\ S \tag{8}$$

and the output of the quadratic detector following the ML filter is:

$$\tilde{P}_{\omega 0} = \int S(\omega)\,|A(\omega)|^2 d\omega = 1/(\underline{S}^t\ \underline{\underline{R}}^{-1} \underline{S}) \text{ watts.} \tag{9}$$

The above estimate will differ from the actual $P_{\omega 0}$ depending on up to what degree the effects of $\tilde{S}(\omega)$ head been reduced

$$P_{\omega 0} = \tilde{P}_{\omega 0} - \int \tilde{S}(\omega)\,|A(\omega)|^2\ d\omega \tag{10}$$

2. THE NORMALIZED ML METHOD (NMLM)

At this point and starting from the power spectral density can be derived by dividing or normalizing the mentioned power level by the bandwidth which has been used in the measure. We select to estimate the filter bandwidth by using the equal area constraint that is depicted in Fig. 2.

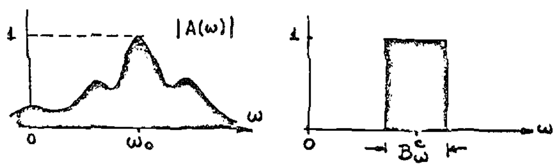

Fig. 2. Equal area constraint in computing the equivalent filter bandwidth.

In other words (11) will provide B^e_ω.

$$B^e_\omega = \frac{1}{2\pi} . \int |A(\omega)|^2\ d\omega = \underline{A}^t\ \underline{A} \tag{11}$$

Seems to be clear that this estimate will improve the rought approximate currently used for B^e_ω (i.e. B^e_ω equal to the inverse of the filter length M).

Using the bandwidth shown in (11) result in the so-called normalized maximum likehood spectral estimate.

$$S_{NMLN}(\omega) = \frac{P_{\omega 0}}{B^e_\omega} = \frac{P_{\omega 0}}{\underline{A}^t . \underline{A}} = \frac{\underline{S}^t \underline{\underline{R}}^{-1} \underline{S}}{\underline{S}^t \underline{\underline{R}}^{-2} \underline{S}} \tag{12}$$

The reader can see in Fig. 3 the comparison of the Capon's procedure, ME and the proposed here for the same order. From this figure it can be concluded that:

a.- The spectral estimate have resolution similar to ME and the low side-lobe level of MLM.

b.- The estimate proposed for B^e_ω improves the resulting quality of the Capon's estimate which uses 1/Q. Other estimates for this bandwidth, more accurate than the proposed here, could further improve the final quality obtained.

In the same sense, it can be seen in the Figure that the associated bandwidth to the ME estimate is narrower than in the nMLM estimate. In order to provide a theoretical support to this effect, we can reformulate the ME method as *minimize* $\underline{A}_{ME}^{t}\ \underline{\underline{R}}\ \underline{A}_{ME}$ constrained to $\underline{A}_{ME}^{t}.\underline{1}=1$ being $\underline{1}$ equal to $(1,0,\ldots,0)^{t}$.

The solution for the ME filter, which minimums results in the corresponding spectral peaks, is (12.a), and its bandwidth is (12.b).

$$A_{ME}=\frac{\underline{1}^{t}\underline{\underline{R}}^{-1}\underline{1}}{\underline{1}^{t}\underline{\underline{R}}^{-1}\underline{1}}\ (12a);\quad B^{ME}=\frac{(\underline{1}^{t}\underline{\underline{R}}^{-1})^{2}}{(\underline{1}^{t}\underline{\underline{R}}^{-2}\underline{1})}\quad (12b)$$

If a sinusoid in noise is the signal under consideration, the bandwidth of the

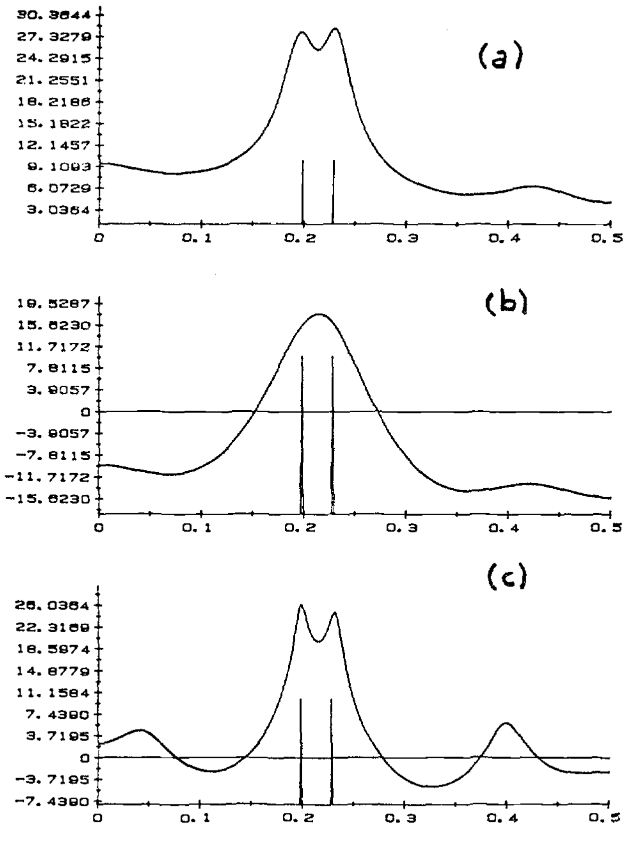

Fig. 3. (a) NMLM method, (b) Capon's method, (c) ME. Data sample of 256 points, with two sinusoids in white noise at frequencies located at $f_1=0.2$, $f_2=0.23$ and $SNR_1=SNR_2=10$dB. Order 10 in the three procedures.

zeros in the analysis filter involved in MEM, is smaller than the analysis filter in MLM. This is the way to check over which method provides better resolution; anyway, the high complexity of 2-D MEM appraises the proposed procedure.

3. THE QMLM ESTIMATE

Using singular value decomposition for matrix $\underline{\underline{R}}^{-1}$, it can be shown that the spectral estimate, proposed in the previous section, will converge to the true spectral power density in the distributional sense as the order M increases to infinity.

In other sense, it is well-known that an approximate procedure to reduce matrix $\underline{\underline{R}}^{-1}$ to their principal components is to form $\underline{\underline{R}}^{-q}$ with q greater than one. In such a way, and being λ_i the largest N (less than M) eigenvalues of $\underline{\underline{R}}^{-1}$, matrix $\underline{\underline{R}}^{-q}$ can be rewritten in the approximate form given by (13).

$$\underline{\underline{R}}^{-q} \simeq \sum_{i=0}^{N-1} \lambda_i\ \underline{u}_i.\underline{u}_i^{t} \qquad (13)$$

So that, keeping in mind that maintaining the convergence, previously mentioned, is an important property in any candidate for an spectral estimator, we suggest (14) as a generalized version of the Capon's method and the normalized one.

$$S_{qMLM}(\omega) = \frac{\underline{S}^{t}\underline{\underline{R}}^{-q+1}\underline{S}}{\underline{S}^{t}\underline{\underline{R}}^{-q}\ \underline{S}} \qquad (14)$$

Note that q=1 will arise to the classical MLM procedure and q=2 to the so-called normalized maximum likehood method. Before to show some simulation results, it is worthwhile to mention that q, canot be considered as a factor which controls the 'cosmetic' resolution of the estimate, it is just a parameter which increases the convergence rate of the estimate in the distributional sense to the true espectral power density. In other words, using q=2 instead q=1 the resulting estimate could exhibit spectral peaks not shown with an unity exponent as the reader can see in Figure 3.

Using hardware implementation of this procedures for spectral analyzers, the designer should limit the exponent q according to the finite register length in order to avoid distortion in the measured peaks of the power density spectrum under processing. In fact, and from a few experiments carried out by the authors powers greater than q=4 for orders 8 and 16 introduces serious errors in the peak's magnitude. Usually the resulting estimate provides the correct location of peaks in these circumstances.

In order to show the performance of increasing q, Fig. 4 represents the resulting two-dimensional spectral estimate for a 5x5 basic correlation support (3x3 mask filter) which consists in two sinusoids at (0.2,0.2) and (0.25,0.25) in 0dB SNR each.

It should be pointed out that many of the reported properties for (14) are mixed if the numerator is removed from it.

4. CONCLUSIONS

It has been shown that a spectral power density can be obtained from ML filters by using the effective bandwidth of the analysis filter. This results in a nice trade-off between the resolution of MEM and the low sidelobe level of MLM.

Furthermore, on the basis of the statistical properties associated with the proposed estimate, we suggest the use of high order quadratic forms in order to improve the convergence of the estimate to the true power spectral density as the order of the analysis filters increases.

2-D plots of the estimate exhibits the great performance of the method that is hard to obtain with currently reported methods.

5. REFERENCES

|1| J. Capon. "High-Resolution Frequency-Wavenumber Spectrum Analysis". Proc. IEEE, vol. 57, pp. 1408-1418, Aug. 1969.

|2| A. Papoulis. "The Fourier Integral and its applications", Chapter 8, Mc.Graw-Hill, 1962.

|3| J.P. Burg. "A New Analysis Technique for Time Series Data". Proc. NATO Advance Study Inst. on Signal Proc., Enschede The Netherlands, 1968.

|4| P.D. Welch. "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging over Short Modified Periodograms. IEEE Trans. AU-15, pp.70-73, June 1967.

|5| M.A. Lagunas and A. Gasull. "An Improved Maximum Likehood Method for Power Spectral Density Estimation". To appear in the February Issue of the ASSP Trans 1984.

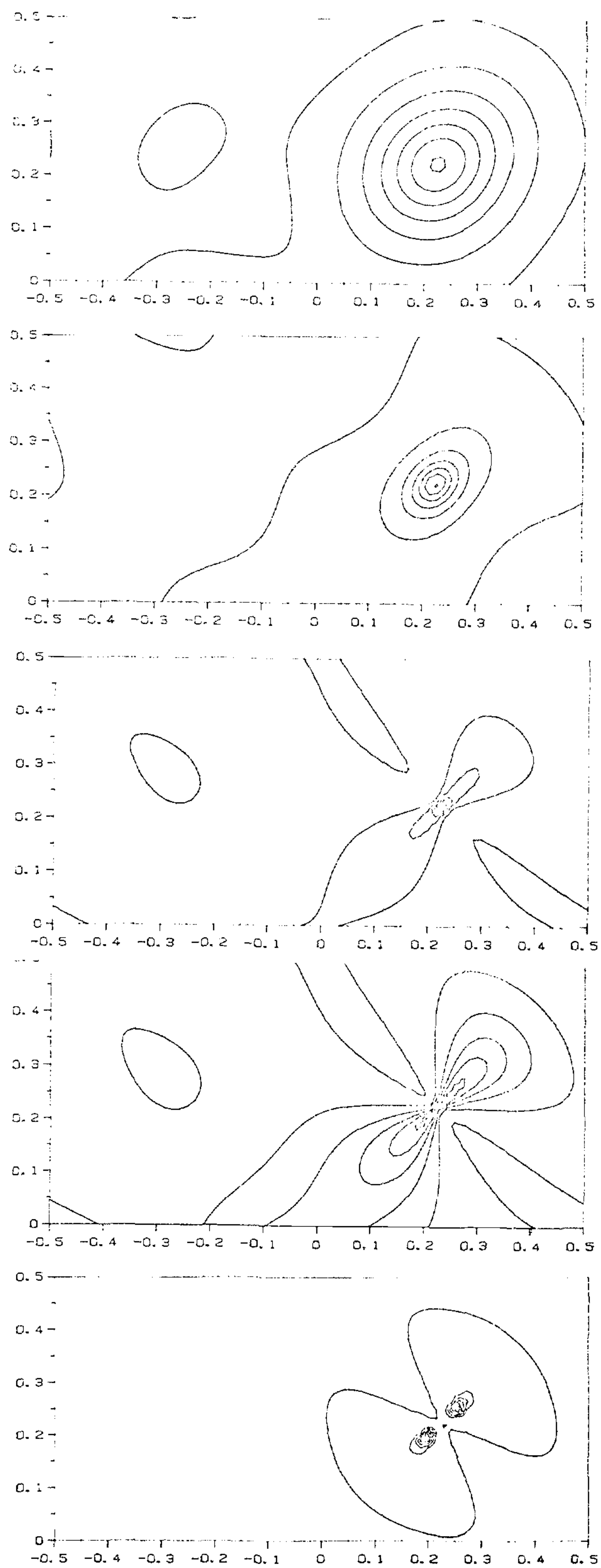

Fig. 4. $S_{qMLM}(\omega_1,\omega_2)$. Contour plotd from -5.7 to 3.3dB. (Increment 1.5dB).