NON-LINEAR SPECTRAL ESTIMATION

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ABSTRACT

This work describes the use of constraints in variational procedures for spectral analysis.

It is reported how the designer in a variational approach for spectral estimation has to select a set of constraints. At the same time it is shown that the selected set of constraints becomes more relevant, as concerns with the resulting quality of the estimate, than the objective function minimized or maximized in the procedure.

Finally, some examples are presented which are the result of considering correlation and envelope constraints and minimizing the correlation extrapolation energy.

INTRODUCTION

The use of variational methods in spectral analysis is, doubtless the most powerful tool to give formalism to already reported procedures of great interest.

All the interesting methods of spectral analysis can be regarded as variational statements in the time or frequency domain /1/, /2/. On the other hand, the duality or possibility of introducing design conditions in the objective or constraints of the variational problem, makes it an exhaustless source of new methods. It is sure that viability, robustness and other features can seriously decrease the interest of new procedures. At the same time, it is important to point out that the variational procedure gives the estimador structure but it does not guarantee its existence, in other words, the Lagrange multipliers solution which defines it can not exist.

This work has been supported by CAYCIT, grant number 21096/84

As an example, let us say that the designer freedom with regard to the constraints is not such and it must be taken into account their compatibility in order to guarantee the existence of any solution besides the objective function itself.

It is clear that the above mentioned problem is enough important itself so as to find out an answer before studying all the possibilyties which arise from the variational procedure. Let us see how to undertake the problem from its beginning. The constraints must be understood in the sense that some part of the previous knowledge we have about the actual power spectrum must be reflected by the obtained estimator.

Constraints can be either linear or non-linear; even more, among the first case it can be found those which are equalities or inequalities. Thinking over constraints which are in the equality form, they will have the form (1),

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}(S_x(w))e^{jmw}\,dw = \emptyset(m) \qquad (1)$$

on the other hand, those in the inequality form will be like it is shown in (2).

$$\beta(S_x(w)) > 0 \qquad (2)$$

It must be pointed out that a forced constraint must be $\beta(.)=$. due to the positivity character of the resulting power spectral estimate.

When selecting the constraints of type (1) it must be managed, as it has already been underlined, that the following three characteristics are provided as close are possible:

- Not being contradictories.
- Robustness to data record length.
- The less redundant as possible.

First feature is usually fulfilled when all the constraints arise from the same $P_x(w)$ function. Yherefore, both sets of constraints could be written in (3)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\Phi(S_x(w))e^{jmw}\,dw = \phi_m$$
$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\Psi(S_x(w))e^{jnw}\,dw = \varphi_n \qquad (3.a)$$

$$\phi_m = \frac{1}{2\pi}\int_{-\pi}^{\pi}\Phi(P_x(w))e^{jmw}\,dw$$
$$\varphi_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}\Psi(P_x(w))e^{jnw}\,dw \qquad (3.b)$$

Assuming that $P_x(w)$ is an available function from the data x(n) (n=0,N-1), we can just choose among two basic options: either that function is the periodogram or we have some previous estimator of higher quality. It seems to be obvious that in most of the cases our candidate will be the periodogram, or any other alternative coming directly from the data Fourier transform, that will provide the $P_x(w)$ function used above.

Concerning with the next desired feature in the constraints, that is, the robustness over the data record length, there are two basic possibilities. One way of getting more statistical estability over ϕ_m or φ_n constraints is to use smoothed versions of $P_x(w)$, as the so called WOSA, STUSE, B.T., etc. In other words, some versions obtained by means of averages or just smoothing of the squared magnitude of the DFT of the data, could be a useful procedure in order to obtain more robustness of ϕ_m and φ_n regardless the data record length N. The second possibility is that of using $\Phi(.)$ or $\Psi(.)$ functions in (3.b) such that stable labels are made about the information reflected by $P_x(w)$, which is obtained from the data record.

It is in this second line where it can be enlarged the main contribution of this current work.

The last feature concerning to use constraints with the smallest redundancy between them, is related with the fact that the global number of constraints P+Q, in a variational procedure, and bebause of reasons based on the second feature, is quite lower to the data record length N. It means that there exists a great data reduction in passing from N data samples to P+Q parameters. If besides this reduction, the Q constraints are redundant from the others P, the numerical complexity of the procedure is being increased with no information improvement over the data record information.

To finish this section, just a few lines devoted to the objective function.

In a variational procedure, the objective to be maximized or minimized will be like (4).

$$\left.\frac{1}{2\pi}\int_{-\pi}^{\pi} G\left[S_x(w)\right]dw\right|_{\text{MAX or MIN}} \qquad (4)$$

A lot of works have been written regarding the special interest of G(.). This great deal, in what is just a cosmetic factor, or a way out over what is really important which are the selected set of constraints, is disordered under our point of view.

To reinforce the above idea, this work will be devoted to the study of constraints. Just in the last section it will be shown the need, and how artificial it is, of finding an objective function to complete all together with the constraints, the variational procedure.

CORRELATION CONSTRAINTS AND THE ANALYTICAL SIGNAL SPECTRA

If there exists some clear constraints in variational methods, those are the correlation ones.
In short, to reflect the property of verifying (5)

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} S_x(w)e^{jmw}dw = r(m);\ |m|<Q \qquad (5.a)$$

or

$$S_x(w) = \sum_{-\infty}^{\infty} r_e(m)e^{-jmw} \qquad (5.b)$$

where $r_e(m)$ equals to r(m) when $|m|<Q$.

Thus, an almost obligated election for $\Phi(.)$ would be $\Phi(.)=.$; however, let us see an alternative available to what is regarded as an standard in spectral estimation and that was already used by Cadzow some years ago but in another context.

The power spectrum, as a real signal, has an analytical signal $A_{s(w)}$ of quick definition from (5.b) and that is given according to (6).

$$A_s(w)=r_e(0)+\sum_{m=1}^{\infty} 2.r_e(m)e^{-jmw} \tag{6}$$

So, an alternative to establishing constraints would be over $A_s(w)$ accordingly to (7).

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} A_s(w)e^{jmw}dw = \begin{cases} r(0) \ ; \ m=0 \\ 2r(m); \ m=1,Q \\ 0 \ ; \ m=-Q,-1 \end{cases} \tag{7}$$

Clearly, this lattest statement leads us to the use of what we could name the analytical spectrum instead of the spectral density.

It is interesting to point out that in the case of an ARMA (P,Q) model, the analytical spectrum will have the same pole location as the associated model does. Let us say, if N(exp(jw))/M(exp(jw)) is the spectral density arising from using white noise input to an invariant linear system of rational transfer function A(z)/B(z), then it can be written (8).

$$\frac{N(z)}{M(z)} = \frac{A(z)}{B(z)} \cdot \frac{A(1/z)}{B(1/z)} \tag{8}$$

Likewise, decomposing N(z)/D(z) into single fractions and grouping them into those having poles inside and outside the unit circle, expression (9) evolves.

$$\frac{N(z)}{M(z)} = \frac{A(z)\,A(1/z)}{B(z)\,B(1/z)} = \frac{C(z)}{B(z)} + \frac{C(1/z)}{B(1/z)} \tag{9}$$

From (9), it follows that $A_s(z)=2C(z)/B(z)$ and, through (6), it is obtained the expression for S(w) as S(w)=2.Real C(exp(jw))/B(exp(jw)) . So, in this way, it is seen that the ARMA model poles arise directly in $A_s(z)$ denominator as well. At last, and before closing this section it is important to remind that the usage of analytic signals, decreases aliasing effects due to non-linearities not foreseabled from a theoretical statement.

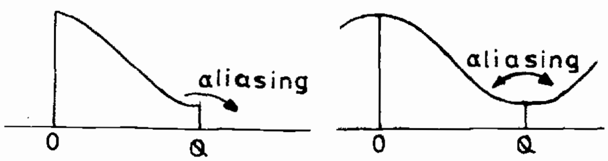

Fig. 1. Aliasing effects are decreased since half length is mantained free at energetic content.

ADDITIONAL CONSTRAINTS: ENVELOPE

It is clear that summarizing N data values of the signal under test x(n) (n=0,N-1) into just Q values, does not guarantee, in general, that all information of interest has been reflected into the variational procedure. In other words, from the N available correlation lags, just Q have been retained assuming that the remaining N-Q lags have low statistical stability, and this fact, does not compensate their information contribution to the procedure.
In a previous paper /3/,/4/ it was managed the use of cepstrum constraints, in the feeling that those constraints retain global properties from the overall N correlation values.

Likewise it was shown that so as correlation constraints in the maximum entropy method contributes to pole location, cepstrum constraints does with respect the zero location. In this last way, it was shown that the derivative of a cepstra coefficient with respect to the corresponding correlation lag, was mainly dependent on the zeroes of the associated spectrum.

However, the usage of cepstrum as additional constraints, introduces two important problems. Te first of both is the non-linear character associated to the problem of finding the Lagrange multipliers. The second one becomes from the involved knowledge assumed over the given N data points of the data periodogram. Definitively, taking the logarithm in the frequency domain, due to its non-linear character, extends the duration length in the other domain.

Few functions $\Phi(.)$ or $\Psi(.)$ like those outlined in (3.a) will fulfill the last stated problem. Nevertheless, it exists one which besides of being of higher order than the correlation, does not reflects this problem. This function is the spectrum squared envelope which we will denote as $|E_s^2(w)|$. This envelope is evaluated accordingly to (10),

$$|E_s(w)|^2 = H_s^2(w)+S^2(w) \tag{10}$$

where $H_s(w)$ is the Hilbert Transform of the spectral density S(w).

On the other hand, $|E_s(w)|$ is the magnitude of the so-called analytical spectrum.

$$|E_s(w)|^2 = A_s(w).A_s^*(w) \tag{11}$$

then

$$E_s(z)E_s(1/z) = \frac{C(z)\ C(1/z)}{B(z)\ B(1/z)} \qquad (12)$$

From (12) it can be seen that this function shares again the pole location of the ARMA associated model, it does not involve any non-linear operation and is a second order function since its inverse Fourier Transform is the convolution of the autocorrelation causal image with itself.

This second order charact er at the squared envelope gives to its constraints a greater robustness in front of measurement noise in data acquisition and a smaller sensitivity of the constraints with respect the data record length.

This underlined effect is depicted in figure 2, where it can be observed how periodogram morphology changes while its squared envelope $|E_p(w)|^2$ remains unaltered when data record length is decreased.

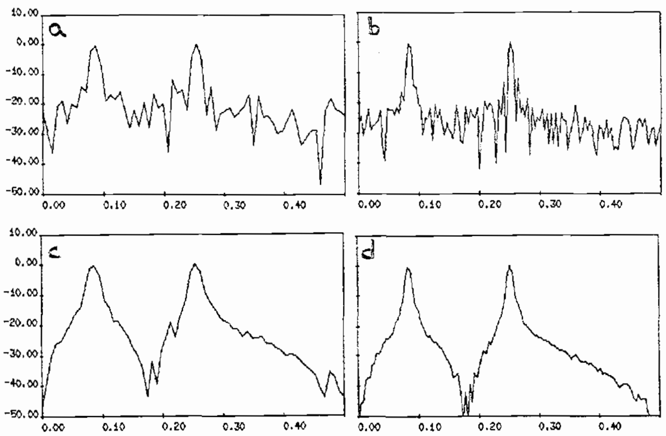

Fig. 2. Two sinusoids in white noise of normalized frequencies $f_1=0.08$ and $f_2=0.25$ respectively and SNR of 10 db; a) and b): 64 and 128 data points periodograms respectively; c) and d) the corresponding envelopes.

Take in mind those above comments, because of being a second order function, the relative sensitivity to the data record length does not depend over the spectrum energy.

This situation is not the case for the peridogram, which sensitivity in low energy zones tends to infinity.

From the above ideas, it seems to be clear that the other candidate to conform the set of constraints, all together with the correlations ones, are the constraints coming from the spectral envelope.

RESULTS AND CONCLUSIONS

In our oppinion, the main conclussion is that between the great number of possibilities that arise from variational procedures, the most interesting are those coming from correlation and envelope constraints.

In /7/ is considered one of that possibilities which gets as objective the minimization of the $\varphi(0)$ value. Besides of this, it is also studied the problem of existence and possitivity of the resulting spectra (as suggested in /5/ and /6/).

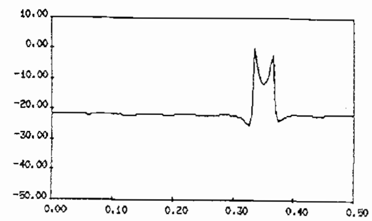

Fig. 3. Two sinusoids of normalized frequencies $f_1=0.33$ and $f_2=0.36$ and SNR = 10 db, analyzed taking as constraints 5 correlation lags and 10 envelope ones.

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