

High-Resolution Multipath Time Delay Estimation for Broad-Band Random Signals

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Abstract—The resolution limits to multipath time delay estimation for broad-band random signals are examined. First, appropriate Cramer–Rao lower bounds are derived to establish baseline performance for unbiased estimation. The bounds are then compared to computer simulation results. For the two-path case, a maximum likelihood estimator is implemented, while for the three-path case, the modified forward–backward linear prediction algorithm, developed for high-resolution frequency estimation, is used. It is shown that both of these techniques can achieve performance close to the appropriate CRLB. In particular, we demonstrate that reliable multipath estimation can be achieved even when the multipath time delays are much closer than could be resolved by conventional autocorrelation processing. A criterion, based on a binary hypothesis test for deciding whether one or two paths are present, is developed and shown to be useful in predicting when the Cramer–Rao bound is no longer reachable. The effect of having *a priori* knowledge of the value of the attenuation of the delayed path is studied via theory and simulation; it is shown that even relatively poor *a priori* knowledge of the attenuation can significantly help the estimate of time delay for small delays.

I. INTRODUCTION

MULTIPATH propagation, in which one or more attenuated and delayed versions of the same radiated signal are received at a single sensor (or a beam-formed array of sensors), is a frequent occurrence in ocean acoustics [1, p. 95] and in other fields where a signal and its “echo” are present. In ocean acoustics, the time of arrival difference between the various multipaths can provide source localization information if an acoustic propagation model is available; conversely, the time differences can be used to estimate the propagation structure if the source and receiver locations are known [2]. Thus, multipath time delay estimation is a problem of practical concern.

We are interested here only in random signals (not pulsed-like signals of known or unknown waveshape). As one application, we note that the broad-band component of radiated ship noise is well modeled as a random process [3, p. 328]. If such multipath arrivals are received on a single sensor (or a beam-formed array of sensors), one straightforward way to estimate the time delays is to autocorrelate the output signal. The resulting correlogram will have peaks corresponding to the various delay differences; if these peaks are resolvable, their locations can be

used as estimates of the time delays. The fundamental performance limits for estimating a single, resolvable time delay were examined in [4] where the performance of the maximum likelihood estimator (MLE) and the autocorrelator were compared to the appropriate Cramer–Rao lower bound (CRLB) and large error probability results.

The multipath peaks on the autocorrelogram are resolvable as long as the time delay differences are much greater than the width of the signal correlation function (or an inverse signal bandwidth). When the delays are closer than this, the peaks merge, and the delays are said to be non-resolvable. This is analogous to the estimation of the frequencies of two sinusoids in noise where, if the frequency separation is less than an inverse record length, the frequencies are not resolvable by Fourier methods. As is well known, however, this is not a fundamental limitation on resolution capability since the fundamental limit must depend on signal-to-noise ratio (SNR) as well as record length [5], [6]. The optimum processor for sinusoids in white noise performs a least squares search over all possible frequencies, amplitudes, and phases [7]. If the number of possible frequencies is large, this nonlinear least squares search becomes prohibitively time consuming and other, so-called high-resolution, methods which minimize the computational effort have been sought [7]. One such method, called modified forward–backward linear prediction (MFBLP) (Tufts and Kumaresan [8]), appears especially promising since it has been shown to work well down to nearly the threshold SNR (below which all methods fail). The multipath resolution problem is analogous to the frequency resolution problem with the bandwidth playing the part of the record length and the delay differences playing the part of the frequency differences. As in the frequency estimation problem, the optimum processor performs a nonlinear least squares search over all possible time delays. With a large number of possible time delays, this search can become prohibitively time consuming; thus, alternative high-resolution techniques are desirable. We will show below that MFBLP can be used for this problem.

The resolution of deterministic pulses has received considerable attention (see [9]–[11] and their references), but the resolution of random multipath signals has not. Hou and Wu [12] appear to be the only authors to have previously suggested techniques for high-resolution multipath time delay estimation with random signals. They suggest a method by which autoregressive techniques can

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be used to obtain high-resolution time delay estimates. They observe that since the received spectrum of a multipath signal (without noise) is just the signal spectrum times a sum of cosines (with periods equal to the time delays and their differences), then with known signal spectrum, the sequence of spectrum estimates in frequency can be regarded as a time series, and known high-resolution time series algorithms can be used to obtain high-resolution estimates of the multipath time delays. We use this idea in employing the MFBLP technique mentioned above, except we operate directly on power spectral estimates, unlike Hou and Wu who suggest operating on the real part of an estimated transfer function. We also provide here detailed analyses of optimum performance which Hou and Wu do not. Friedlander [13] has briefly examined multipath resolution analytically. We note that there are several points of similarity between our work and some of the pulse work, especially that in [10].

In the next section, we describe the signal models to be used. We then derive appropriate CRLB's. Next we implement an MLE for the two-path case, and compare its performance to the appropriate CRLB. Then we introduce the MFBLP technique and use it to estimate delays for the three-path case. These estimates are also compared to appropriate theoretical predictions. Throughout the paper, we assume that both signal and noise have known, flat, low-pass spectra. This is the simplest case, especially for MFBLP. We also assume throughout that the number of delay paths is known. Simultaneous detection and estimation of the number of paths and the delay has been investigated by several authors (see [14] and references), and many of those techniques may be applicable to this problem.

II. MODEL DESCRIPTION: THE RESOLUTION PROBLEM

As typical examples, we will consider both a two-path and a three-path model. For the two-path model, the received signal $r(t)$ is given by

$$r(t) = s(t) + as(t - D_0) + n(t), \quad 0 < t < T \quad (1)$$

where $s(t)$ and $n(t)$ are uncorrelated Gaussian processes, a is a frequency-independent attenuation coefficient, D_0 is the multipath time delay, and T is the observation record length. For all specific results, we will assume that both $s(t)$ and $n(t)$ have flat spectra in the band $|f| < B/2$, with spectral levels S_0 and N_0 , respectively. We will assume that the delayed path is attenuated, and thus that $a \leq 1$, although this is not necessary. The autocorrelation function of $r(t)$, $R_r(D)$, is

$$R_r(D) = (1 + a^2)R_s(D) + aR_s(D - D_0) + aR_s(D + D_0) + R_n(D) \quad (2)$$

where $R_s(D)$ and $R_n(D)$ are the autocorrelation functions of $s(t)$ and $n(t)$; with the assumed spectra, both $R_s(D)$ and $R_n(D)$ go through their first zeros at $D = \pm B^{-1}$. $R_r(D)$ has a peak at $D = 0$ and peaks at close to $D =$

$\pm D_0$. As D_0 decreases, the distinct peaks gradually become less recognizable until at $D_0 \cong 5/(4B)$, only one peak is distinguishable in the region of D_0 . Thus, $D_0B < 1$ is normally considered as a limit to resolution; this is the two-path resolution problem.

The spectrum of $r(t)$, $S_r(f)$, is

$$S_r(f) = [(1 + a^2) + 2a \cos(2\pi f D_0)]S(f) + N(f), \quad |f| < B/2 \quad (3a)$$

where $S(f)$ and $N(f)$ are the signal and noise spectra. With $S(f)$ and $N(f)$ flat, $S_r(f)$ is composed of a mean value plus a cosine term having a "period" (in frequency) of D_0^{-1} . The total bandwidth is B Hz. If $S_r(f)$ is viewed as a time series of "record length" B , then we will have difficulty estimating the "frequency" D_0 if the period is long compared to the record length, i.e., if $D_0^{-1} > B$ or $D_0B < 1$. This provides motivation for the later use of the MFBLP technique on the spectral estimates produced from the time series.

For the three-path model, the received signal is

$$r(t) = s(t) + a_1s(t - D_1) + a_2s(t - D_2) + n(t) \quad (4)$$

where $D_1 < D_2$ are the two delays (later we let the amplitudes of the two paths be equal for convenience; this is reasonable for many oceanic conditions). The autocorrelation function now has peaks at $D = 0, \pm D_1, \pm D_2$, and $\pm |D_2 - D_1|$. If D_1 or D_2 or both are close to zero, we have a resolution problem similar to that discussed above. To obtain a different problem, we will only consider the case where D_1 and D_2 are well removed from zero, but D_1 and D_2 are close to one another.

The spectrum of $r(t)$ for the three-paths case is

$$S_r(f) = [(1 + a_1^2 + a_2^2) + 2a_1 \cos(2\pi f D_1) + 2a_2 \cos(2\pi f D_2) + 2a_1a_2 \cos(2\pi f [D_2 - D_1])] \cdot S(f) + N(f), \quad |f| < B/2. \quad (3b)$$

Again, with flat spectra, $S_r(f)$ is made up of a mean value plus three cosines, with "periods" of D_1^{-1} , D_2^{-1} , and $(D_2 - D_1)^{-1}$. There will be a resolution problem if $(D_2 - D_1)B < 1$.

III. CRAMER-RAO LOWER BOUND

A. Two-Path Problem

We wish here to find the CRLB on the estimate of D_0 . Realistically, there are three other parameters that we do not know: the amplitude a , the signal spectral density S_0 , and the noise spectral density N_0 . To incorporate this lack of knowledge, we will assume that a , S_0 , and N_0 are fixed, but unknown, and we will estimate them along with D_0 . The evaluation of the CRLB is formally accomplished by computing the Fisher information matrix [16, p. 79, eq. (6)]. We first assume that D_0 only is unknown; then that

D_0 and a or S_0 are unknown; and then that D_0 , a , or S_0 are unknown. In this way, we determine the effect of lack of knowledge of the various parameters. The details of the CRLB calculations are given in [15].

In Fig. 1, we show normalized $(\text{CRLB})^{1/2}$ versus normalized BD_0 for various values of the parameter $R = 2aS_0/[(1+a^2)S_0 + N_0]$ for delay only unknown. (The various symbols are simulation results and will be discussed later.) For large values of BD_0 , the CRLB levels off to a value found in [4] for a single highly resolvable path:

$$B(BT \text{CRLB})^{1/2} = \left\{ \frac{6}{\pi^2} \left[\frac{1}{(1-R^2)} - 1 \right] \right\}^{1/2}. \quad (5)$$

As BD_0 decreases for R fixed, the CRLB oscillates about the large BD_0 value until approximately $BD_0 = 0.5$ at which point it rises rapidly.

For D_0 only unknown and small $WD_0 = 2\pi BD_0$, it is shown in [15] that

$$(2\pi B)^2 BT \text{CRLB} = 5 \left(\frac{1+R}{R} \right)^2 \frac{32}{(WD_0)^2}. \quad (6)$$

Thus, for small WD_0 , the CRLB behaves as $(WD_0)^{-2}$. Equation (6) is the same as a result in [13].

We have also found normalized $\text{CRLB}^{1/2}$ versus normalized separation for D_0 and a unknown. This result is similar to the D_0 only unknown result for large WD_0 . For small WD_0 , it is shown in [15] that (for both D_0 and a unknown and D_0 and S_0 unknown)

$$(2\pi B)^2 BT \text{CRLB} = \frac{45}{4} \left(\frac{1+R}{R} \right)^2 \frac{32}{(WD_0)^2}; \quad (7)$$

thus, lack of knowledge of either a or S_0 increases the small BD_0 value of the CRLB by a factor of 9/4. We also found that the CRLB for D_0 and S_0 unknown was very similar to that for D_0 and a unknown for all values of BD_0 . (It is shown in [15] that these two cases are identical for $a = 1$; we have shown above that they are identical for large and small WD_0 for all a ; thus, there is not much room for them to differ.)

In Fig. 2, we show normalized $\text{CRLB}^{1/2}$ versus BD_0 and D_0 , a , and S_0 unknown (N only known). For large BD_0 , the CRLB is similar to the previous two cases, but for BD_0 less than about 1, the bound increases more rapidly than before. Thus, lack of knowledge of both a and S_0 makes it more difficult to get accurate unbiased estimates of D_0 . It is shown in [15] that for small WD_0 , with D_0 , a , and S_0 all unknown, and for small R ,

$$(2\pi B)^2 BT \text{CRLB} = 315^2 \frac{1}{R^2} \frac{128}{(WD_0)^6}. \quad (8)$$

Thus, when D_0 , a , and S_0 are all unknown, the CRLB shifts from a $(WD_0)^{-2}$ dependence to a $(WD_0)^{-6}$ dependence for small WD_0 .

We note that all the cases discussed above do not depend on S_0 (except through R). The cases [D_0 unknown]

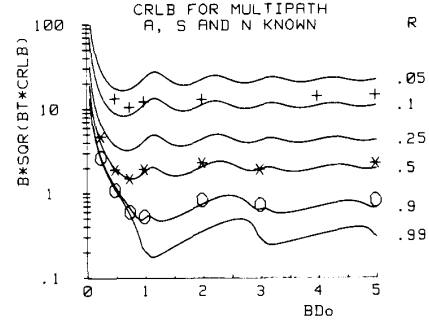


Fig. 1. CRLB for a , S_0 , and N_0 known.

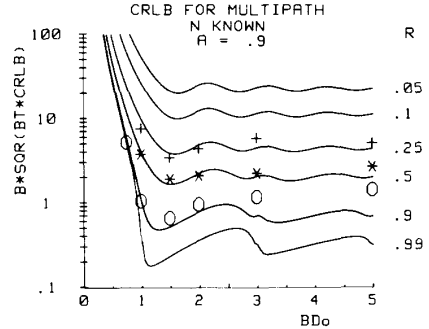


Fig. 2. CRLB for N_0 only known.

and [D_0 , a , S_0 unknown] do not depend on a (except through R) and the cases [D_0 and a or D_0 and S_0 unknown] depend weakly on a . Finally, we note that we have found the CRLB for all parameters [D_0 , a , S_0 , N_0 unknown]; this is very similar to the [D_0 , a , S_0 unknown] case so we do not discuss it further.

B. Three-Path Problem

For three paths, there are six unknown parameters: D_1 , D_2 , a_1 , a_2 , S_0 , and N_0 . We proceed as in the two-path case by analyzing the CRLB, first for D_1 and D_2 only unknown, then adding S_0 , a_1 , a_2 , and N_0 . The results are similar to the two-path case. For D_1 and D_2 only unknown, the CRLB remains close to the highly resolvable result until the normalized separation $B(D_1 - D_2)$ is about 0.25, at which point the bound rises rapidly. With either D_1 , D_2 , S_0 , or D_1 , D_2 , a_1 , a_2 as unknowns, the bound is close to the D_1 , D_2 only unknown bound. For D_1 , D_2 , S_0 , a_1 , a_2 unknown, the bound changes dramatically, and now, as for the two-path case, the CRLB increases rapidly for separations less than $B(D_1 - D_2) < 1$. Finally, adding N_0 as an unknown does not change the CRLB markedly.

In Fig. 3, we show the normalized three-path CRLB on the estimate of D_1 for $B(D_1 + D_2)/2 = 5.5$ versus normalized separation $B(D_2 - D_1)$. Fig. 3 shows a high SNR result where $a_1 = a_2 = a = 1$ and $R = (2aS_0)/[(1+2a^2)S_0 + N_0] = 0.6$. Note that, with $a < 1$, R cannot exceed 2/3 for the three-path case. The peak at $B(D_2 -$

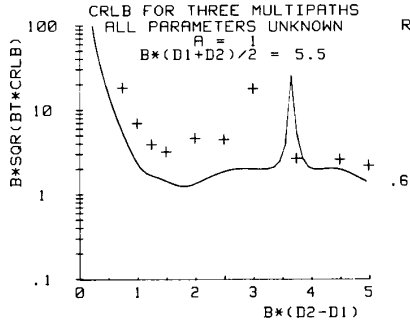


Fig. 3. CRLB for MFBLP.

$D_1) \cong 3.5$ occurs where, for these parameters, $D_2 - D_1 = D_1$.

IV. LIMITS TO ACHIEVING CRLB

The CRLB is a lower bound on the variance of an unbiased estimator; it is not, however, guaranteed to be reachable. The conditions under which the CRLB is no longer reachable are, thus, of fundamental interest. A complete analysis of this problem appears to be quite difficult; we approach it here first by following a suggestion from Root [17]. We suppose two paths actually exist. We are interested in comparing possible explanations of the observed data on the hypothesis that only one path is present with good explanations of the data on the hypothesis that two paths are present. We, thus, consider a binary hypothesis test to decide between H_0 and H_1 :

$$H_0: r(t) = s(t) + as(t - D_0) + n(t)$$

$$H_1: r(t) = bs(t) + n(t) = z(t) \quad (9)$$

where under H_1 , we simply represent the received data as a random process $z(t)$, with arbitrary power. Under H_0 , we assume the process has the true parameters D_0 and a since we are interested in good two-path explanations. To get a measure of the limits of resolution, we determine the minimum probability of error P_e in deciding between H_0 and H_1 when the parameters of H_1 (the total power in this case) are chosen to maximize the probability of error. It is shown in [15] that, for small R , P_e is given by

$$P_e = \Phi \left\{ \frac{1}{2(2)^{1/2}} R(BT)^{1/2} \left[\frac{1}{2} \left(1 + \frac{\sin WD_0}{WD_0} \right) - \left(\frac{\sin \frac{1}{2} WD_0}{\frac{1}{2} WD_0} \right)^2 \right]^{1/2} \right\} \quad (10)$$

where $\Phi(x)$ is the integral from x to infinity of the normalized Gaussian density function. Note that for small R , BT , or WD_0 , the argument goes to 0; thus, in these limits, P_e goes to $1/2$, as is reasonable, since a good processor need not make errors more than half the time at low SNR. We cannot claim any lower bounding properties for (10), but it should be a reasonable criterion for the limit of res-

olution. Later we will compare this theoretical result to simulation results. Similar results for the three-path case are derived [15].

It is shown in [15] that for small WD_0 , the argument of the $\Phi(\cdot)$ function in (10) becomes

$$\text{Arg} = \frac{1}{24(10)^{1/2}} (BT)^{1/2} R(WD_0)^2. \quad (11)$$

Then using (6), we find for small R

$$\text{Arg} = \frac{1}{6} \frac{D_0}{\text{CRLB}^{1/2}}. \quad (12)$$

Equation (12) shows that for small WD_0 , the important parameter for determining P_e is $D_0/\text{CRLB}^{1/2}$. This parameter is the ratio of the mean (true value D_0) normalized by the standard deviation of the best estimate of D_0 . When this parameter is large, P_e is small, and conversely. This conclusion makes sense in terms of the CRLB. If $D_0/\text{CRLB}^{1/2}$ is small, the probability density function of the errors cannot possibly be Gaussian (as is required for the CRLB to be met [16, p. 71]); hence, minimum variance unbiased estimates of the parameter are not possible. Thus, in this sense, the ability to resolve two paths from one path is linked to the ability to provide unbiased minimum variance estimates.

Using (10) and (12), we can conclude that with $D_0/\text{CRLB}^{1/2} = 7.7, 9.8, 14.0$, and 18.6 , $P_e = 0.1, 0.05, 0.01$, and 0.001 , respectively. These values of $D_0/\text{CRLB}^{1/2}$ seem large when viewed from the following point of view. If $D_0/\text{CRLB}^{1/2}$ were equal to 3.1 , then if the errors were Gaussianly distributed, estimates of D_0 less than zero would occur with a probability of 0.001 . Thus, it seems very likely that with these conditions, nearly unbiased estimates of D_0 with minimum variance (restricting estimates to be positive) could result. In fact, even if $D_0/\text{CRLB}^{1/2}$ were 2.3 , estimates of D_0 less than zero would still only occur with a probability of 0.01 if the errors were truly Gaussian. Thus, it seems that a value of $D_0/\text{CRLB}^{1/2}$ on the order of $2-3$ would be required for minimum variance unbiased estimates. This is a factor of roughly five times smaller than the values of $D_0/\text{CRLB}^{1/2}$ predicted from the error probability analysis for the same error probability. It will be shown below via simulation that the first point of view, based on probability of error, leads to more reliable conclusions.

We would also like to establish the limits to the ability to achieve the CRLB for the other cases of interest: D_0 and S_0 unknown, D_0 and a unknown, and D_0, a, S_0 unknown. It is difficult to carry out a probability of error analysis for these cases so we will simply examine the normalized CRLB. Thus, from (7), we find

$$\frac{D_0}{\text{CRLB}^{1/2}} = \frac{BT^{1/2}}{3} \frac{(WD_0)^2}{45^{1/2}}. \quad (13)$$

The right-hand side of (13) is a factor of $2/3$ smaller than the right-hand side of (14); thus, the CRLB (or D_0) must be correspondingly smaller (larger) to provide the same

probability of achieving unbiased estimates as in the D_0 only unknown case. Finally, when D_0 , a , and S_0 are all unknown, we find from (8) that the normalized CRLB is

$$\frac{D_0}{\text{CRLB}^{1/2}} = \frac{BT}{315} \frac{R(WD_0)^4}{2^{7/2}}. \quad (14)$$

For small WD_0 , the right-hand side becomes quite small, and correspondingly larger values of D_0 or smaller values of CRLB are needed to achieve unbiased, minimum variance estimation.

V. EFFECT OF A PRIORI KNOWLEDGE OF a

We have seen in the previous section that going from the D_0 and a unknown or the D_0 and S_0 unknown cases to the D_0 , a , and S_0 unknown case causes a radical change in the CRLB for small WD_0 . We wish to investigate this transition. We are motivated by the following observation. In ocean acoustics, we probably will not have good *a priori* knowledge of the received signal spectrum level; however, a reasonable estimate for the value of the reflection coefficient a can often be made [3, p. 128]. Thus, we wish to examine the way that the [D_0 , a , and S_0 unknown] case changes to the [D_0 and S_0 unknown] case. We do this by including the effects of prior knowledge about a in our derivation of the CRLB.

There are well-established procedures for introducing such *a priori* information into the Cramer-Rao theory [16, p. 84]. Thus, the Fisher information matrix J can be written as $J = J_D + J_p$ where J_D represents information obtained from the data and J_p represents prior information. J_D requires averaging over both the random parameters and the prior statistics, while J_p is only averaged over the prior statistics. As a result of this averaging, we require an approximation to evaluate J_D simply. We first assume that a is Gaussianly distributed with mean $E[a]$ and variance σ^2 ; this simplifies the form of J_p . Next, we assume that σ^2 is small enough so that we can take the probability density function of a as essentially a delta function centered on $E[a]$. With this, J_D reduces to our earlier result for J_D with known a .

With these assumptions, it is shown in [15] that

$$\text{CRLB}(D_0) = \text{CRLB}(D_0/a) \frac{1 + [\sigma^2/\text{CRLB}(a/D_0)]}{1 + [\sigma^2/\text{CRLB}(a)]}. \quad (15)$$

Note that if the prior knowledge of a is good compared to knowledge obtained from the data [i.e., if $\sigma^2 \ll \text{CRLB}(a/D_0)$ and $\text{CRLB}(a)$], then $\text{CRLB}(D_0) = \text{CRLB}(D_0/a)$ and the bound goes to the known a result. Conversely, if prior knowledge is poor compared to knowledge obtained from the data, the prior knowledge is ignored.

In [15], we obtain approximate results for $\text{CRLB}(D_0)$ with prior knowledge for small WD_0 and small SNR. It is shown there that under these conditions,

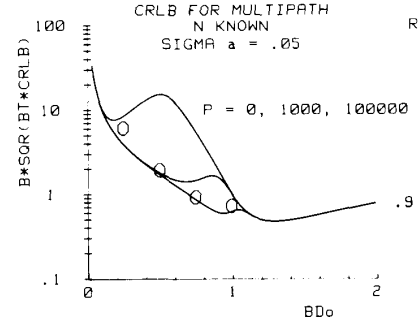


Fig. 4. CRLB with *a priori* knowledge.

$$\begin{aligned} \text{CRLB}(D_0) &= \frac{1}{BT} \frac{1}{R^2} \frac{360}{W^2(WD_0)^2} \\ &\cdot \frac{1 + \sigma^2 BT \left(\frac{S_0}{N_0}\right)^2 (1 - a^2)^2 (WD_0)^4 F(a, WD_0)}{1 + \sigma^2 BT \left(\frac{S_0}{N_0}\right)^2 (1 - a^2)^2 (WD_0)^8 G(a, WD_0)} \end{aligned} \quad (16)$$

where

$$\begin{aligned} F(a, WD_0) &= 360 \left[(1 + a)^4 - \frac{a(1 + a)^2 (WD_0)^2}{6} \right. \\ &\quad \left. + \frac{a(1 + 8a + a^2)(WD_0)^4}{4 \cdot 5!} \right] \end{aligned}$$

and

$$\begin{aligned} G(a, WD_0) &= \frac{7}{2} 10! \left[(1 + a)^4 \right. \\ &\quad \left. + \frac{(1 + a)^2 (1 - 12a + a^2)}{56} \right. \\ &\quad \left. \cdot (WD_0)^2 + 0(WD_0)^4 \right]. \end{aligned}$$

Equation (16) shows the way the CRLB behaves as a function of prior knowledge. Several parameters enter, but basically, for small σ , (16) reduces to (7) (the known a result), while for large σ , (16) reduces to (8) (the unknown a result). The transition between these two cases depends on σ^2 , BT , S_0/N_0 , a , and WD_0 . Large BT and large S_0/N_0 make prior knowledge less important (because they improve the knowledge obtainable about a from the data). Small WD_0 makes prior knowledge more important since this hurts the ability to obtain knowledge about a from the data. Note that, as a goes to plus or minus one, prior knowledge about a becomes extremely important. This is because it is impossible to obtain unbiased estimates of both a and S_0 , given that both are unknown, when a goes to plus one or minus one (see [15]).

Fig. 4 shows the CRLB with *a priori* knowledge of a .

The parameter $P = \sigma^2 R^2 BT / a^2$ is inversely proportional to the importance of *a priori* knowledge. Thus, with $P = 0$, the CRLB corresponds to that for the $[D_0 \text{ and } S_0 \text{ unknown}]$ case, while for large P , the CRLB goes to the $[D_0, S_0, \text{ and } a \text{ unknown}]$ case. The $P = 0$ curve is the lowest one in Fig. 4. Note the rapid transition for small BD_0 to the $[D_0 \text{ and } S_0 \text{ unknown}]$ case, even for relatively large P .

VI. DESCRIPTION OF ESTIMATION PROCEDURES

A. Two-Path Case—MLE

As described in [4], the MLE maximizes the function

$$\begin{aligned} \text{MLE} = & - \sum_{k=1}^K \left\{ \frac{|R_k|^2}{([1 + \hat{a}^2 + 2\hat{a} \cos w_k \hat{D}_0] \hat{S}_0 + N_0)} |F_k|^2 \right. \\ & \left. + \ln [(1 + \hat{a}^2 + 2a \cos w_k \hat{D}_0) \hat{S}_0 + N_0] |F_k|^2 \right\} \end{aligned} \quad (17)$$

by choice of \hat{D}_0 (and \hat{a} and \hat{S}_0 if unknown). R_k is the k th Fourier coefficient of the data $r(t)$, $K = 1/2BT$, and $|F_k|^2$ is the magnitude of the filter transfer function used to generate the data. In the simulation results, to be described later, we maximized MLE by first getting estimates of a and S_0 (if these were unknown) and then searching an *a priori* known region of delay to find the value of \hat{D}_0 which maximized MLE.

Estimates a and S_0 were obtained by setting the derivative of MLE with respect to the variables equal to zero and solving for the minimizing values of these parameters for each value of delay in the *a priori* region. For low SNR, explicit solutions can be obtained. Thus, if S_0 and D_0 only are unknown, a low SNR estimate for S_0 is (see [15])

$$\hat{S}_0 = \sum_{k=0}^K [|R_k|^2 - N_0] H_k \left/ \left(\sum_{k=0}^K H_k^2 \right) \right. \quad (18)$$

where $H_k = [1 + a^2 + 2a \cos w_k D]$.

If a and D_0 are unknown, a low SNR estimate for a is (see [15])

$$\hat{a} = \sum_{k=0}^K [|R_k|^2 - N_0] H'_k \left/ \left(\sum_{k=0}^K H_k H'_k \right) \right. \quad (19)$$

where H'_k is the derivative of H_k with respect to a .

Finally, if both a and S_0 , as well as D_0 , are unknown, we can simultaneously solve for a and S_0 . Solving for a and S_0 , however, introduces an unnecessary nonlinearity into the problem. It is just as satisfactory, and simpler, to solve for $(1 + a^2)S_0$ and $2aS_0$ as independent variables. For low SNR, this requires solution of the set of equations (see [15])

$$\begin{aligned} \sum_{k=0}^K [|R_k|^2 - N_0] &= \frac{1}{2} BT [(1 + \hat{a}^2) \hat{S}_0] \\ &+ \sum_{k=0}^K \cos w_k \hat{D}_0 [2a \hat{S}_0] \end{aligned}$$

$$\begin{aligned} & \sum_{k=0}^K [|R_k|^2 - N_0] \cos w_k D_0 \\ &= [(1 + \hat{a}^2) \hat{S}_0] \sum_{k=0}^K \cos w_k \hat{D}_0 \\ &+ \sum_{k=0}^K \cos^2 w_k \hat{D}_0 [2a \hat{S}_0]. \end{aligned} \quad (20)$$

Equations (21a)–(21c) can, thus, be used to estimate a , S_0 or $(1 + a^2)S_0$, and $2aS_0$ for each value of D_0 . The resulting estimates can be substituted into (20) and a value of MLE can be calculated for each D_0 . The value of D_0 which maximizes MLE in the *a priori* region is taken as a coarse estimate of delay. A finer estimate is obtained by parabolic interpolation using successively smaller intervals until convergence to specified accuracy.

B. Three-Path Case—MFBPL

For the three-path case, we could in principle compute the MLE and search in two dimensions for the maximizing values of D_1 and D_2 . Ultimately, however, we are interested in many paths; thus, a high-order search would become practically unworkable. At this point, we are, therefore, interested in finding a high-resolution technique which circumvents this need for a high-order nonlinear maximization. For this purpose, we will use the technique developed by Tufts and Kumaresan [8] called modified forward-backward linear prediction (MFBPL).

We use the technique directly as described in [8], with the exception that instead of processing the time series, we process the spectral estimates produced from the time series as discussed in Section II. Several special issues arise, however. The main issue is whether the raw periodogram or averaged periodograms should be analyzed. Multipath spectral estimates, viewed as a time series, differ from the normal problem of sinusoids in white, Gaussian noise in two important ways. First, since the variance of a spectral estimate is proportional to the true value of the spectrum, the cosines multiplying the spectrum [see (3b)] cause the variance of the spectral estimates to depend on the frequency index. This corresponds to nonstationary noise for a time series. The effect is reduced at low SNR since the (assumed flat) noise is the main contributor to the spectrum. A second difference is that the noise in the raw periodogram is chi-squared, not Gaussian. Spectral averaging will tend to make the estimates Gaussian. Since least squares algorithms are degraded by large outliers (common in non-Gaussian noise), it seems best to do some averaging.

We first give the following background for our MFBPL analysis. Assume there are N sampled points in the time series $r(t)$. We divide the time series into N/NP non-overlapping sequences of NP points each. We then perform 50 percent overlapped spectral averaging to generate the averaged spectral estimates which we ultimately analyze via MFBPL. We note that a multipath delay $D_0 = J\Delta T$ multiplies the spectrum by $\cos(2\pi f J\Delta T) = \cos$

$(2\pi K\Delta f J\Delta T) = \cos(2\pi KJ/NP)$. The "frequency" of the delay D_0 is, thus, equal to J/NP and we must require $J/NP < 1/2$. For the spectrum used in the simulations to be described below, the upper cutoff frequency ($B/2$) corresponded to one-eighth of Nyquist. Thus, using this knowledge, there are a total of $2NP/8 = NP/4$ frequency samples to process. It was found that performance of the MFBLP algorithm was much better if both positive and negative frequency samples were processed.

Our MFBLP analysis thus proceeded as follows. The spectrum of the received signal $r(t)$ [see (4)] was estimated using 50 percent overlapped (unwindowed) FFT processing by averaging the periodograms of the $(2N/NP - 1)NP$ point records. The $NP/4 - 1$ frequency points corresponding to the band $\pm B/2$ were then treated as the time series, as in [8]. A linear prediction filter length L was chosen (a value of L of roughly 0.5–0.66 times $NP/4$ was found to give the best results [4]). A value of $M = 3$ corresponding to the three "frequencies" of D_1 , D_2 , and $D_2 - D_1$ was specified. After subtracting the mean value of the frequency data from each frequency point, the data were read into a data matrix (the A matrix of [8]). Then, following [8], the eigenvalues and eigenvectors of the correlation matrix $R = A^*A$ were found. The minimum norm prediction filter was then found from the $2M$ principal eigenvalues and eigenvectors of R as described by [8, eq. (41)]. The complex zeros of the prediction error filter were then found via a polynomial rooting routine. The angle of the zeros, which lies between $\pm\pi$, was then converted to a time delay estimate by multiplying by NP/π . The magnitude of the root, which indicates how close the root is to the unit circle, was then used to pick the time delay estimates, which correspond to the M largest values.

VII. SIMULATION RESULTS

Computer simulation results were obtained for both the two- and three-path cases. The received signals [see (1) and (4)] were generated as described in [4].

A. Two-Path Case

The MLE described above was used to estimate time delay for the two-path case. The main purpose of the simulation was to determine when the CRLB can no longer be reached. Several cases were simulated; these were [D_0 only unknown], [D_0 and a , unknown], [D_0 , a , and S_0 , unknown], and [D_0 and S_0 , unknown, some prior knowledge of a].

In Fig. 1, the simulation results for [D_0 only unknown] are shown. The 0's are the rms error for $R = 0.9$, the stars are for $R = 0.5$, and the crosses are for $R = 0.1$. The vertical extent of the symbols is roughly a 95 percent confidence interval. All data were taken using $a = 0.9$. The data for $BD_0 = 0.25$ and 0.5 were taken for $BT = 4096$; the data for $BD_0 = 0.75$ and 1 were taken for $BT = 2048$; all the other data used $BT = 512$. These results show that the CRLB can be reached by the MLE even for

quite small values of BD_0 and R as long as a sufficiently large BT product is employed.

This result is not surprising. A more important outcome is to determine when the CRLB is no longer reachable. From examination of a large number of simulations made for different values of R , BT , and BD_0 , it appears that unbiased estimates of small values of BD_0 (i.e., $BD_0 < 1$) require that $D_0/\text{CRLB}^{1/2}$ be on the order of 10 or larger. This shows that the conclusions reached in Section IV above using the probability of error approach are more reliable than the conclusions based on a simple treatment using the assumed Gaussian distribution of the estimates. Simulation results were also run for the [D_0 and a , unknown] case. Again, the CRLB could always be met with a sufficient BT product, and a value of $D_0/\text{CRLB}^{1/2}$ of about 10 was required for unbiased estimation.

In Fig. 2, the simulation results for the [D_0 , a , and S_0 , unknown] case are shown. The 0's are the rms error for $R = 0.9$, the stars are for $R = 0.5$, and the crosses are for $R = 0.25$. All data again were taken for $a = 0.9$. The data for $BD_0 = 0.75$ and 1 used a BT of 4096, while the data for all of the other points were run using a BT of 1024. Reliable estimates for $BD_0 = 0.5$ and 0.25 could not be obtained using a BT of 4096 or less (which is the largest conveniently achievable BT product for the MLE on the computer used). This result is consistent with the earlier observation that $D_0/\text{CRLB}^{1/2}$ must be 10 or larger to achieve reliable estimates.

In Fig. 4, we show the simulation results for the rms error assuming *a priori* knowledge of a . Each data point shown in the figure came from 100 runs, each of which used a random value of a , which had a mean of 0.9 and a standard deviation of 0.05, to generate the data. The MLE algorithm used a constant value of $a = 0.9$ and estimated the values of S_0 and D_0 . The 0's are the rms error for $R = 0.9$ and were obtained using $BT = 4096$. These results are thus directly comparable to the small BD_0 , $R = 0.9$ results of Fig. 2. The value of P for these data is about 10 so the experimental data agree quite well with the theory; nonetheless, it seems remarkable that even with a σ as large as 0.05, the *a priori* knowledge allows far better estimation at these low BD_0 values than was obtainable with no knowledge of a .

The crosses in Fig. 3 show the simulation results for the rms error for the MFBLP technique. These are compared to the CRLB for all parameters unknown. The data for $B(D_2 - D_1) = 0.75$ and 1 are for a BT of 7429; those for $B(D_2 - D_1) = 1.25$ and 1.5 are for a BT of 4096; the remaining data are all for a BT of 2048. The data in the figure are for $R = 0.6$. The rms error agrees reasonably well with the CRLB; unlike the MLE runs, however, the estimates generally had a small bias. Other simulation runs for a lower signal-to-noise ratio $R = 0.14$ showed similar performance. As expected, a larger BT product was needed to get reliable results as the SNR was decreased.

Extensive results were also obtained using the MFBLP technique for the two-path, highly resolvable case to determine the SNR threshold for this technique. Results were

compared to the large error performance bounds described in [4]. The probability of large error versus SNR for the MFBLP technique closely followed the lower bound described in [4]; the MFBLP had a larger error probability than either the lower bound or the autocorrelator, the exact difference depending on BT . For $BT = 512$, the threshold SNR for MFBLP was about 2.5 dB larger than for the autocorrelation, while for $BT = 4096$, the threshold SNR for MFBLP was only 1 dB larger than for the autocorrelator. This shows that, at least for this highly resolvable case, the MFBLP technique can come quite close to the minimum achievable threshold SNR.

VIII. FINAL COMMENTS

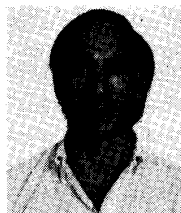
We have shown that reliable multipath time delay estimation can be achieved when the multipath time delays are closer than an inverse bandwidth. We have also shown that the MFBLP technique can be used to estimate multipath delays; it is especially promising when many delay paths are present. The major limitation of the simulation results given here is that they apply only to flat spectra. This is an especially crucial limitation for the MFBLP. Future work in this area should address ways to treat unknown, nonflat spectra. We have also assumed that the number of paths is known. If it is assumed that a single source is present, then for some propagation modes (for example, single-order bottom bounce), the number of paths is known; for other propagation modes, the number of paths will not be known, and techniques for addressing an unknown number of paths must be developed.

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REFERENCES

- [1] C. B. Officer, *Introduction to the Theory of Sound Transmission*. New York: McGraw-Hill, 1958.
- [2] R. C. Spindel, "Signal processing in ocean tomography," in *Adaptive Methods in Underwater Acoustics*, H. G. Urban, Ed. Dordrecht/Boston: Reidel, 1985, pp. 687-710.
- [3] R. J. Urick, *Principles of Underwater Sound*, 3rd ed. New York: McGraw-Hill, 1983.
- [4] J. P. Ianniello, "Large and small error performance limits for multipath time delay estimation," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 245-251, Apr. 1986.
- [5] W. Munk and K. Hasselmann, "Super resolution of tides," in *Studies in Oceanography*, K. Yoshida, Ed. Seattle, WA: Univ. Washington Press, 1955, pp. 339-344.
- [6] D. Rife and R. Boorstyn, "Multiple tone parameter estimation from discrete time observations," *Bell Syst. Tech. J.*, pp. 1389-1410, Nov. 1976.
- [7] S. M. Kay and S. L. Marple, "Spectrum analysis—A modern perspective," *Proc. IEEE*, vol. 69, pp. 1381-1419, Nov. 1981.
- [8] D. W. Tufts and R. Kumaresan, "Estimation of frequencies of multiple sinusoids: Making linear prediction perform like maximum likelihood," *Proc. IEEE*, vol. 70, pp. 975-989, Sept. 1982.
- [9] T. E. Landers and R. T. LaCoss, "Some geophysical applications of autoregressive spectral estimates," *IEEE Trans. Geosci. Electron.*, vol. GE-15, Jan. 1977.
- [10] R. J. P. DeFigueiredo, "Separation of superimposed signals by a cross-correlation method," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-31, pp. 1084-1189, Oct. 1983.
- [11] G. B. Marchisio and W. S. Hodgkiss, "Deconvolution applied to a near-bottom seismic profiler," *J. Acoust. Soc. Amer.*, vol. 72, pp. 1478-1491, Nov. 1982.
- [12] Z.-Q. Hou and Z.-D. Wu, "A new method for high resolution estimation of time delay," in *ICASSP '82 Conf. Proc.*, Paris, France, May 1982, pp. 420-423.
- [13] B. Friedlander, "On the Cramer-Rao bound for time delay and Doppler estimation," *IEEE Trans. Inform. Theory*, vol. IT-30, pp. 575-580, May 1984.
- [14] M. Wax, "Detection and estimation of superimposed signals," Ph.D. dissertation, Stanford Univ., Stanford, CA, Mar. 1985.
- [15] J. P. Ianniello, "High resolution multipath time delay estimation for broadband random signals," NUSC TM 871002, Mar. 30, 1987.
- [16] H. L. VanTrees, *Detection, Estimation, and Modulation Theory*. New York: Wiley, 1968.
- [17] W. L. Root, "Radar resolution of closely spaced targets," *IRE Trans. Mil. Electron.*, vol. MIL-6, pp. 197-204, Apr. 1962.



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