A NOVEL ARCHITECTURE TO MODEL NON-LINEAR SYSTEMS

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ABSTRACT*

This paper shows a new architecture specially thought to model non-linear systems (NLSs). At first, it was applied only to memoryless systems but then it developed to solve a more general problem, NLSs with memory. The result is a new filter, based on the Fourier transform, that the authors have named "K-filter". Important features of the K-filter are its nonlinear behaviour and second, that it profits from a temporal diversity of the input signal in order to provide itself with memory. At the end of the paper, the K-filter is used to solve an identification problem of a communication system which behaves nonlinearly due to the response of the amplifiers and which also has memory introduced basically by the channel response.The simulation results will provide an evaluation of the K-filter.

1. INTRODUCTION

In the last few years non-linear signal processing has been emerged due basically to the saturation produced in the linear processing field during the 80's. At the beginning, the efforts were centred on the Volterra/Wiener approach [1,2]. Afterwards, the new philosophy of high order statistics found a wide and interesting field into the non-linear processing [3]. Both are important topics in relation to non-linear signal processing developed after other classic methods, as phase-plane analysis or techniques based on differential equations, were introduced. Nevertheless, all of them share a joint feature, that is the complexity both in the formulation of the problem and also in the solution. The authors propose a new and less complex architecture to model NLSs.This filter is also a nonlinear system and it is suitable to future development as adaptive filtering.

The organisation of the paper follows the temporal evolution of this filter, named "K-filter". To this effect, the paper starts presenting the filter used to model memoryless NLSs. Afterwards, it faces to the problem of how to generalise the architecture achieved previously to the new situation of NLSs with memory. This subject is discussed in the third section where the K-filter is showed. Together with it, some guide-lines of possible future work are pointed out. All this theoretical part is supported by some results obtained from computer simulation where the K-filter is used to solve an identification problem of a communication system. This system behaves nonlinearly due to the response of the amplifiers and it also has memory introduced basically by the channel response. In order to evaluate the performance, the K-filter is compared to an equaliser and also to a Volterra filter.

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2. THE K-FILTER MODELLING MEMORYLESS NLSs

Suppose a memoryless NLS characterised by its input/output relation, g(.).

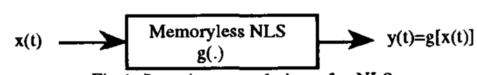

Fig.1.-Input/output relation of a NLS.

Keeping x(t) in the range [-Xmax,Xmax], the output of this system can be written as a Fourier series developed in the x domain. Therefore, an approximation of the output could be a shorted version of it (Eq.1).

$$\hat{y}(t)=\sum_{n=0}^{N} a_n \exp(-jn\omega_0 x(t)) \quad ;\omega_0 \ge \frac{\pi}{Xmax} \qquad \text{(Eq.1)}$$

The above expression corresponds to figure 2. This figure makes clear some features of the transformation which are quite important from our point of view.

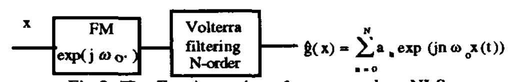

Fig.2.-The Fourier series of a memoryless NLS.

The scheme depicted on (Fig.2) shows the Fourier series as an exponential transformation applied to the input signal followed by a block named "Volterra filtering", which performs basically a memoryless Volterra series (Fig.3).

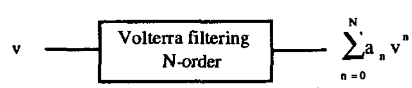

Fig.3.-The Volterra filtering block.

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As it is well-known, the Volterra series and the Taylor one share the same drawbacks because of the first one is a generalisation of the second.. An important problem is the convergence when the independent variable goes further from the point around which the series has been developed. Paying attention to figure 2, the exponential function included in the scheme can be understood as a previous transformation which bounds the input signal in magnitude. Obviously, this transformation will help to avoid the problem of convergence mentioned before.

Nevertheless, figure 2 does not represent the K-filter, yet. It is well-known that the coefficients of the Fourier series, denoted by a_n, are obtained from sampling the Fourier transform of g(.) at multiples of the main frequency ωo. The idea was then to check if it would perform better a filter which implements a Fourier series whose coefficients were obtained from sampling g(.) in the frequency domain nonuniformly [4]. To this effect, another two branches were added to the original scheme. One of them is also a Fourier series but built using a slightly different main frequency, $\omega o'$. The second one adds a continuous component which has been removed from the other two branches. The result is the architecture depicted in (Fig.4).

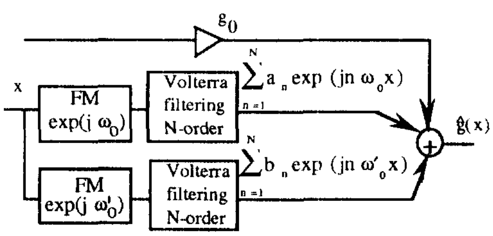


Fig.4.-The K-filter modelling a memoryless NLS.

At the beginning, the main reason to adopt the scheme of figure 4 instead of the previous one (Fig.2) was the purpose of adjusting the filter to the Kolmogorov's theorem. Although the aim of this paper is not go through this topic, some bibliography has been included [5.6].

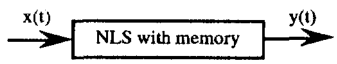
As it will be seen in the next section, this scheme (Fig.4) has been used to design the K-filter which also applies to NLSs with memory. Future work will be focussed on improving the behaviour of the filter by means of this extra branch. For instance, an open question would be the role of $\omega o'$; that is, how to select this parameter in order to achieve a better performance. From the results we have obtained, it seems that the scheme of figure 4 performs better than the one of figure 2.

An important aspect to remark from the K-filter (Fig.4) is that the quality of the approximation can be controlled by both the number of terms of the series, N, and the principal harmonics, ωo and $\omega o'$. By increasing the order of the Volterra series or decreasing the frequencies ωo and $\omega o'$, it is possible to improve the performance of the filter, since the Fourier transform of g(.) is usually a low pass spectrum.

3. THE K-FILTER APPLIED TO NLSs WITH MEMORY

Once it has been found a structure able to model memoryless NLSs, the next step is to face to the same problem but dealing with NLSs with memory. The filter represented in figure 4 is not longer fit for modelling the new systems because it does not include memory. Therefore, the main challenge is how to generalise the filter achieved in the previous point (Fig.4) to the new situation and to support the result by formal expressions.

To this effect, it is necessary to characterise the system under modelling. Taking into account that the input signal will be discrete, the output of a NLS with finite memory could be expressed as a function which depends not only on the value of the input at those moment but also on some previous values of it.



$$y(t) = g[x(t), x(t-\tau), \ldots x(t-(Q-1)\tau)]$$

Fig.5.-The output of a NLS with memory.

In consequence, the function needed to characterise a NLS with memory depends on a Q-dimensional vector built from a temporal diversity applied to the input signal, $[x(t), x(t-t), \ldots x(t-(Q-1)t)]$. The Q parameter, which varies from one system to another, measures how long is the memory of the NLS.

As it had been done in the case of memoryless NLSs, the function which characterises the system, g(.), can be expressed as a Q-dimensional Fourier series and in consequence, a shorted version of it is suitable to approximate the output of the NLS.

$$\hat{y}(t) = a_0 + \sum_{n_1}^{N1}\sum_{n_2}^{N2}\cdots \sum_{n_Q}^{NQ} a_{n_1 n_2 \ldots n_Q} e^{-j(n_1\omega_1 x(t) + n_2\omega_2 x(t-\tau) + \ldots + n_Q\omega_Q x(t-(Q-1)\tau))} \qquad \text{(Eq.2)}$$

The coefficients of this series, $a_{n1n2\ldots nQ}$, depend on Q variables since they are obtained from sampling the Q-dimensional Fourier transform of the g(.) function.

$$a_{n_1 n_2 \ldots n_Q} = F[g(.)]\Big|_{(n_1\omega_1, n_2\omega_2, \ldots n_Q\omega_Q)}$$

$$n_i = 1 \ldots Ni \qquad i = 1 \ldots Q \qquad \text{(Eq.3)}$$

From (Eq.3) it is easy to conclude that every axis of the Fourier transform is sampled at multiples of the corresponding main frequency, ωq.

Nevertheless, the filter proposed does not correspond exactly to (Eq.2). In order to build the K-filter from the shorted Q-dimensional Fourier series (Eq.2), two aspects must be taken into account. The first one is that only the terms of the Fourier series which belong to an axis of the Q-dimensional Fourier transform are taken. That is, the points which obey condition 4.

$$(0,\ldots,0,n_i\omega_q,0,\ldots,0)\quad;n_i=1\ldots N_i\quad q=1\ldots Q \qquad \text{(Eq.4)}$$

On the other hand, note that all the points which correspond to the same q belong to an axis and this axis is sampled at multiples of ωq. As it had been done with memoryless NLSs, each axis will be sampled nonuniformly. In consequence, each component of the temporal diversity vector will be used to compute two different Fourier series in order to achieve the non-uniform sampling of the corresponding axis.

Both conditions lead us to the scheme depicted on figure 6.

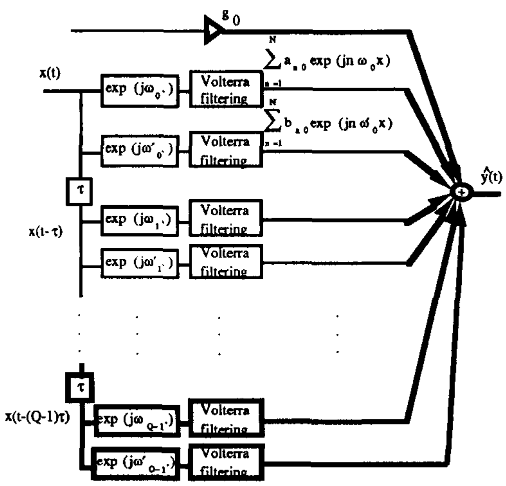


Fig.6.-The K-filter to model NLSs with memory.

Being the output of the filter equal to

$$\hat{y}(t)=a_0+\sum_{q=0}^{Q-1}\Big[\sum_{n=1}^{N} a_{nq}\exp(jn\omega_q x(t-q\tau))+ \sum_{n=1}^{N} b_{nq}\exp(jn\omega'_q x(t-q\tau))\Big] \qquad \text{(Eq.5)}$$

As it can be seen, the new K-filter (Fig.6, Eq.5) is obtained from applying the filter of figure 2 to each one of the components of the time diversity vector [x(t),x(t-t), ...x(t-Qt)]. It will keep the same properties of the K-filter of memoryless NLSs that we have discussed before but, a part form this, the new K-filter profits from a temporal diversity vector in order to introduce an important feature of the system under modelling: "memory".

Thinking in future work, the coefficients of the K-filter, that is, the coefficients of the Volterra filtering blocks, are suitable for learning by means of gradient techniques. This property is also shared by a classic Volterra series [2] but the K-filter shows an advantage in front of the other. The signal space generated by the K-filter at the output of the exponential transformations is bounded in magnitude. This is not accomplished by the Volterra filtering since the power of each component of the signal space will vary enormously from one to another. This feature will not help too much in an adaptive version of this filter when setting the step parameter, μ, of the LMS algorithm.

Another important aspect to remark is that the main reason to have chosen the K-filter not as the implementation of the shorted Q-dimensional Fourier series but a simple one, is the complexity and computer load that the full expression would suppose. The authors are conscious that by adding terms to the K-filter (Eq.5) in order to be nearer to the complete expression (Eq.2), the results achieved would improve. This is one possible way to focus our future efforts, including an evaluation the trade-off between improvement and additional computer load.

4. SIMULATION RESULTS

In this section, the authors propose to solve the identification of a communication link which behaves nonlinearly due to the response of the amplifiers located at both the receiver and transmitter and which also has memory introduced by the channel response.

In this way, the input to the system has been selected as a sampled band-pass normal distributed noise by means of the transfer function $H(z)=(z^{-4}+2.7607z^{-3}+3.8106z^{-2}+2.6535\,z^{-1}+0.9238)^{-1}$, being the response of the amplifiers equal to $y_{amp}=\mathrm{sign}(x)\,\exp(x^2/0.05)$. Finally, the channel is modelled by a filter whose transfer function is $Hc(z)=(z^{-3}+0.0928z^{-2}-0.3158z^{-1}+0.2)/(z^{-1}-0.5)$.

In order to identify the system a K-filter has been chosen. A part from it, two other filters have been also used to model the system. They will help us to evaluate and compare the performance of the K-filter. The first one is an equaliser of N coefficients (N-1 tap-delays) and the second one is a filter which implements a Volterra series with memory as the following one.

$$y(t)=\sum_{p=0}^{P} h_p[\underline{x}]\quad;\underline{x}=[x(t),x(t-\tau),\ldots x(t-(N-1)\tau]$$

$$h_p[\underline{x}]=\sum_{n_1=0}^{N-1}\cdots\sum_{n_p=0}^{N-1} h_{n_1\ldots n_p}\,x(t-n_1\tau)\ldots x(t-n_p\tau) \qquad \text{(Eq.6)}$$

The parameters which characterise this Volterra filter are N and P. The first one, N, measures the memory and P is the order of the nonlinear filter. Important to remark is that $h_p(x)$ takes one time the repeated terms

The solution we will present consists of designing the coefficients of each filter off-line by a minimum square error criterion which lead us to the Wiener solution.

Figure 7 shows the result of one simulation of 100 samples. It consists of 3 different plots, each one of them corresponds to the output of one of filters(dashed line) together with the output of the real system communication (continuous line). In this case the equaliser has only 3 coefficients, the Volterra filter 20 (all possible combinations with P=N=3) and the K-filter 25 (N=2, Q=3). A part from this, the SNR is of 10dB.

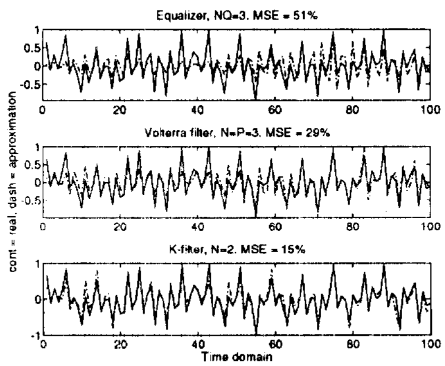


Fig.7. Results of the simulations. The first one corresponds to an equaliser, the second one to a Volterra filter with memory and the third one to the K-filter.

Note that the figures include the mean square error achieved by each filter. From these values it can be concluded that the K-filter performs much more better than the other two filters. The following table shows more results in order to discussed certain aspects which can be misunderstood.

Ns	EQUA. N=3	EQUA. N=25	Volt. N=P=3	K-filter, N=2
100	51%	45%	25%	15%
1000	—	52%	38%	25%
5000	—	52%	41%	29%
6000	—	51%	40%	29%

Table.1 Mean square error of the three filters depending on the number of samples of the input signal.

From figure 7 it can be thought that the K-filter has a much more better results than the equaliser because of the number of coefficients, since the K-filter has 25 and the other one only 3. This is the reason to have enclosed in table 1 the mean square error achieved by an equaliser of 25 coefficients.

As it can be seen, the error got by the equaliser is already higher than the one of the K-filter, 45% in front of 15%.

Another aspect we would like to remark is that if the number of samples increases, the K-filter is already better than the other ones, but in relation to the previous results (100 samples) the mean square error has increased. This behaviour was expected to be present because the relation between coefficients and conditions (now is 1000 and before was 100) has decreased. Nevertheless, the values of the corresponding mean square errors become stable as Ns increase (simulation of 5000 and 6000 values).

REMARKS

A new architecture, named K-filter, has been presented and it has been proved to be useful when modelling nonlinear systems, both with memory or memoryless. From the simulation results showed previously, we can conclude that it performs quite better than the other filters to which it has been compared. Future work will be focussed on, first of all, studying the behaviour of the K-filter but in different situations. The next one will be to solve the same identification problem but using adaptive techniques to update the coefficients of the K-filter. As it has been said before, gradient methods are suitable for this purpose. Furthermore, some changes, pointed out before in the paper, will be deeply studied since the authors expect that they will improve the results of the K-filter.

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