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THIRD ORDER WIDELY NON LINEAR VOLTERRA MVDR BEAMFORMING

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ABSTRACT

It is now well known that time invariant (TI) linear beamformers, such as the Capon's beamformer, are only optimal for stationary Gaussian observations whose complex envelope is necessarily second order (SO) circular. However in many applications such as in radiocommunications, most of the signals are nonGaussian and their complex envelope presents very often some SO and/or higher order (HO) non circularity properties. For this reason we propose in this paper a third order widely non linear Volterra minimum variance distortionnless response (MVDR) beamformer taking into account both the potential non Gaussian character and the potential HO non circularity (up to sixth order) of interferences. Some properties, performance and adaptive implementation of this new beamformer are presented in the paper. Illustrations shows the great interest, with respect to the existing beamformers, of this new beamformer for non Gaussian and HO non circular interferences, omnipresent in practice.

Index Terms— Beamformer, non circular, non Gaussian, higher order, interferences, MVDR, Volterra, widely non linear, widely linear, third order, fourth order, sixth order.

1. INTRODUCTION

Conventional time invariant (TI) linear beamformers, such as the Capon's beamformer [1], are only optimal for stationary Gaussian observations whose complex envelope is necessarily second order (SO) circular. However in many applications such as in radiocommunications, most of the signals are nonGaussian and their complex envelope presents very often some SO and/or higher order (HO) non circularity properties [2]. For example a M-PSK (Phase Shift Keying with M states) signal is a nonGaussian signal whose complex envelope is noncircular at an order q such that $q \geq M$. For this reason, a third order Volterra generalized sidelobe canceller (GSC) [3] structure has been proposed in [4] for a few years to improve the performance of the Capon's beamformer in nonGaussian context. However this Volterra beamformer does not exploit the potential noncircularity of the interferences. To overcome this drawback, a widely linear [5] MVDR beamformer has been proposed recently in [6] to improve the performance of the Capon's beamformer in SO noncircular context. However this beamformer does not exploit the potential nonGaussian character of the interferences. In order to keep the advantages of these two beamformers, we propose in this paper a so-called third order widely nonlinear Volterra MVDR beamformer which takes into account both the potential nonGaussian character and the potential HO noncircularity (up to sixth order) of interferences. Some properties, performance and adaptive implementation of this new beamformer are presented in the paper. Illustrations show the great interest of this new beamformer with respect to the existing ones for nonGaussian and HO noncircular interferences, omnipresent in practice. Note that widely nonlinear Volterra complex filters have already been used in [7] and [8] but for mean square estimation and detection respectively.

The paper is organized as follows. Section 2 introduces hypotheses and formulates the problem. Section 3 introduces the new third order widely nonlinear Volterra beamformer. Section 4 presents some properties and computes the output signal to interference plus noise ratio (SINR) at the output of this new beamformer. Finally section 5 compares the performance of the proposed beamformer to those of the existing ones for both BPSK and QPSK interferences.

2. HYPOTHESES AND PROBLEM FORMULATION

2.1. Hypotheses

Let us consider an array of N narrowband sensors. Each sensor is assumed to receive a signal of interest (SOI) corrupted by a total noise potentially composed of interference and background noise. Under these assumptions, the vector $\mathbf{x}_t$ of complex amplitudes of the signals at the output of these sensors can be written as follows

$$\mathbf{x}_t = s_t\mathbf{s} + \mathbf{n}_t, \tag{1}$$

where s_t and $\mathbf{s}$ correspond to the complex envelope, assumed zero-mean, and the steering vector, assumed perfectly known and such that its first component is equal to one, of the SOI respectively. The vector $\mathbf{n}_t$ is the total noise vector, assumed to be zero-mean, potentially nonGaussian and noncircular and independent of the SOI s_t.

2.2. Problem formulation

The problem addressed in this paper is to estimate the unknown signal s_t from the observation $\mathbf{x}_t$. Naturally, the best estimate y_t of s_t according to the minimum mean square error (MMSE) is the conditional expectation $y_t = \mathrm{E}(s_t/\mathbf{x}_t)$. Note that for respectively circular or noncircular mutually Gaussian distributions of $(s_t, \mathbf{x}_t)$, this conditional expectation becomes linear or widely linear [5]. But for nonGaussian distribution of $(s_t, \mathbf{x}_t)$, the derivation of this conditional expectation needs this distribution which is unknown in practice.

For this reason, we try in this paper, to find an approximation of this conditional expectation through the analysis of a particular class of nonlinear beamformer corresponding to what we call in this paper complex TI widely nonlinear Volterra beamformers, introduced for the first time in [7]. The general input/output relation of a full M-th order complex TI widely nonlinear Volterra beamformer is defined by

$$y_t = \sum_{m=1}^{M} \sum_{q=0}^{m} \mathbf{w}_{m,q}^H (\mathbf{x}_t^{\otimes q} \otimes \mathbf{x}_t^{*\otimes(m-q)}) \tag{2}$$

where $(\mathbf{w}_{m,q})_{0\leq q\leq m, 1\leq m\leq M}$ is a $N^m \times 1$ TI complex filter and $\otimes$ denotes the Kronecker product. (2) defines a widely linear filter [5] for $M = 1$ and a full complex linear-quadratic filter [8] for $M = 2$.

Note that the best Mth-order complex approximation corresponds to the orthogonal projection of s_t on the subspace generated by (2). Consequently the term $\mathbf{w}_{m,q}^H(\mathbf{x}_t^{\otimes q} \otimes \mathbf{x}_t^{*\otimes(m-q)})$ brings information only if $\mathbf{w}_{m,q}^H(\mathbf{x}_t^{\otimes q} \otimes \mathbf{x}_t^{*\otimes(m-q)})$ is correlated with s_t, which requires that the $(m+1)$th-order moment $\mathrm{E}(s_t^q s_t^{*(m+1-q)})$ is not zero. In practice, the distribution of s_t is often symmetric and consequently the odd order moments of s_t are zero. To take advantage of this Mth-order complex approximation for arbitrary distribution, we only consider complex TI Volterra filters such that M is odd, containing only polynomial terms of odd order.

3. TI VOLTERRA MVDR BEAMFORMING

To simplify the notations, we only consider in the following that $M = 3$ for which

$$\begin{aligned} y_t &= \underbrace{\mathbf{w}_{1,1}^H \mathbf{x}_t}_{0} + \underbrace{\mathbf{w}_{1,0}^H \mathbf{x}_t^*}_{1} \\ &+ \underbrace{\mathbf{w}_{3,0}^H(\mathbf{x}_t^* \otimes \mathbf{x}_t^* \otimes \mathbf{x}_t^*)}_{2} + \underbrace{\mathbf{w}_{3,1}^H(\mathbf{x}_t \otimes \mathbf{x}_t^* \otimes \mathbf{x}_t^*)}_{3} \\ &+ \underbrace{\mathbf{w}_{3,2}^H(\mathbf{x}_t \otimes \mathbf{x}_t \otimes \mathbf{x}_t^*)}_{4} + \underbrace{\mathbf{w}_{3,3}^H(\mathbf{x}_t \otimes \mathbf{x}_t \otimes \mathbf{x}_t)}_{5} \qquad (3) \\ &\stackrel{\mathrm{def}}{=} \widetilde{\mathbf{w}}^H \widetilde{\mathbf{x}}_{13,t}, \qquad (4) \end{aligned}$$

where $\widetilde{\mathbf{w}} \stackrel{\mathrm{def}}{=} [\mathbf{w}_{1,1}^T, \mathbf{w}_{1,0}^T, \mathbf{w}_{3,0}^T, \mathbf{w}_{3,1}^T, \mathbf{w}_{3,2}^T, \mathbf{w}_{3,3}^T]^T$ and $\widetilde{\mathbf{x}}_{13,t} \stackrel{\mathrm{def}}{=} [\mathbf{x}_t^T, \mathbf{x}_t^H, (\mathbf{x}_t^* \otimes \mathbf{x}_t^* \otimes \mathbf{x}_t^*)^T, (\mathbf{x}_t \otimes \mathbf{x}_t^* \otimes \mathbf{x}_t^*)^T, (\mathbf{x}_t \otimes \mathbf{x}_t \otimes \mathbf{x}_t^*)^T, (\mathbf{x}_t \otimes \mathbf{x}_t \otimes \mathbf{x}_t)^T]^T$ is the augmented observation [9].

Analyzing the different terms of (3), it is possible to show that all the terms convey information for a rectilinear interference such as an ASK or a BPSK interference whereas only terms 0 and 4 convey information for a circular interference. For QPSK interferences, which is SO circular but fourth-order noncircular, terms 0, 2 and 4 convey information. The third order Volterra beamformer associated only with the terms 0 and 4 is a so called linear-cubic beamformer, used in [4]. When at least one of the terms between 1, 2, 3, 5 is taken into account in addition to terms 0 and 4, the third order beamformer is so called widely nonlinear. In this case as long as the terms 1, 2, 3, 5 are not all taken into account, the beamformer is a partial third order widely nonlinear Volterra beamformer. It becomes a full third order widely nonlinear Volterra beamformer when both 1, 2, 3 and 5 terms are taken into account in addition to 0 and 4 terms.

To introduce, our Volterra MVDR beamformer, we focus now only on the particular partial structure $y_t = \mathbf{w}_{1,1}^H \mathbf{x}_t + \mathbf{w}_{3,2}^H(\mathbf{x}_t \otimes \mathbf{x}_t \otimes \mathbf{x}_t^*)$, because the other partial and full structures follow the same derivation. This beamformer is a *linearly constrained minimum variance beamformer* [10] which uses only $\mathbf{s}$ as a prior knowledge. It corresponds to the filter $\widetilde{\mathbf{w}}$ that minimizes its output power

$$\widetilde{\mathbf{w}}^H \mathbf{R}_{\tilde{x},13} \widetilde{\mathbf{w}} \tag{5}$$

with $\mathbf{R}_{\tilde{x},13} \stackrel{\mathrm{def}}{=} \mathrm{E}(\widetilde{\mathbf{x}}_{13,t} \widetilde{\mathbf{x}}_{13,t}^H)$ under multiple linear constraints. Apart from the constraint $\mathbf{w}_{1,1}^H \mathbf{s} = 1$ to preserve the SOI, s_t, we must elaborate a little bit to consider the third-order term

$$\mathbf{w}_{3,2}^H[(s_t\,\mathbf{s} + \mathbf{n}_t) \otimes (s_t\,\mathbf{s} + \mathbf{n}_t) \otimes (s_t\,\mathbf{s} + \mathbf{n}_t)^*] \tag{6}$$

because the total noise $\mathbf{n}_t$ is random. A solution consists in decomposing $\mathbf{n}_t$ on a deterministic orthogonal basis $(\mathbf{s}, \mathbf{u}_1, ...\mathbf{u}_{N-1}) \in \mathbb{C}^N$ because $\mathbf{s}$ is known: $\mathbf{n}_t = \alpha_{0,t}\mathbf{s} + \sum_{i=1}^{N-1} \alpha_{i,t}\mathbf{u}_i$. Hence, excluding the terms $\mathbf{u}_i \otimes \mathbf{u}_j \otimes \mathbf{u}_k^*$ of (6) which will be considered in the minimization of the output power (5), a solution to avoid correlated terms with s_t in y_t is to impose the $1 + 3(N-1) + 3(N-1)^2 = N^3 - (N-1)^3$ following constraints:

$$\begin{aligned} \mathbf{w}_{3,2}^H(\mathbf{s} \otimes \mathbf{s} \otimes \mathbf{s}^*) &= 0 \\ \mathbf{w}_{3,2}^H(\mathbf{u}_i \otimes \mathbf{s} \otimes \mathbf{s}^*) &= 0, \quad \mathbf{w}_{3,2}^H(\mathbf{s} \otimes \mathbf{u}_i \otimes \mathbf{s}^*) = 0 \\ \mathbf{w}_{3,2}^H(\mathbf{s} \otimes \mathbf{s} \otimes \mathbf{u}_i^*) &= 0, \quad \mathbf{w}_{3,2}^H(\mathbf{u}_i \otimes \mathbf{u}_j \otimes \mathbf{s}^*) = 0 \\ \mathbf{w}_{3,2}^H(\mathbf{s} \otimes \mathbf{u}_i \otimes \mathbf{u}_j^*) &= 0, \quad \mathbf{w}_{3,2}^H(\mathbf{u}_i \otimes \mathbf{s} \otimes \mathbf{u}_j^*) = 0 \end{aligned}$$

where $i, j \in \{1, .., N-1\}$. Putting all the constraints in a matrix form, we obtain the following constraint:

$$\mathbf{C}^H \widetilde{\mathbf{w}} = \mathbf{f} \tag{7}$$

with $\mathbf{C} = \begin{bmatrix} \mathbf{s} & \mathbf{O}_{N\times[N^3-(N-1)^3]} \\ \mathbf{0}_{N^3\times 1} & \mathbf{A} \end{bmatrix}$, where the columns of $\mathbf{A}$ contain the previous vectors $\mathbf{s} \otimes \mathbf{s} \otimes \mathbf{s}^*, ..., \mathbf{u}_i \otimes \mathbf{s} \otimes \mathbf{u}_j^*$ and $\mathbf{f} \stackrel{\mathrm{def}}{=} (1, \mathbf{0}_{N^3-(N-1)^3}^T)^T$.

We note that $\mathbf{R}_{\tilde{x},13}$ is singular due to the redundancies in the augmented observation $\widetilde{\mathbf{x}}_{13,t}$. Consequently, the filter $\widetilde{\mathbf{w}}$ that minimizes $\widetilde{\mathbf{w}}^H \mathbf{R}_{\tilde{x},13} \widetilde{\mathbf{w}}$ under constraint (7) cannot be directly derived. To solve this constrained minimization problem, the redundancies of $\widetilde{\mathbf{x}}_{13,t}$ have to be withdrawn to give the new augmented observation $\widetilde{\mathbf{x}}'_{13,t}$ with covariance $\mathbf{R}'_{\tilde{x},13}$ which is now not singular. And the new filter $\widetilde{\mathbf{w}}'$ associated with the new constraint matrix $\mathbf{C}'$ would be given by

$$\widetilde{\mathbf{w}}' = \mathbf{R}'^{-1}_{\tilde{x},13} \mathbf{C}' [\mathbf{C}'^H \mathbf{R}'^{-1}_{\tilde{x},13} \mathbf{C}']^{-1} \mathbf{f}.$$

Naturally all this presentation and those of the next section extends straightforwardly to arbitrary other partial or full structure. For example, for the full structure, $\mathbf{C}$ has dimension $(2N + 4N^3) \times (2 + 4(N^3 - (N-1)^3)$.

4. VOLTERRA GSC STRUCTURE

4.1. Equivalent Volterra GSC structure

It can be easily verified that our Volterra MVDR beamformer has this equivalent simpler to implement GSC structure [3]

$$\widetilde{\mathbf{w}}_0^H$$

proposed recently in [6] and to Capon beamformer [1], both implemented by SMI algorithms. Although less fast than the MVDR_1 and Capon beamformers, we see that the partially and fully structured Volterra MVDR beamformers practically converge from a number of 50 and 100 snapshots respectively. Furthermore, in this scenario the partially and fully structured Volterra MVDR beamformers largely outperform the MVDR_1 and Capon beamformers.

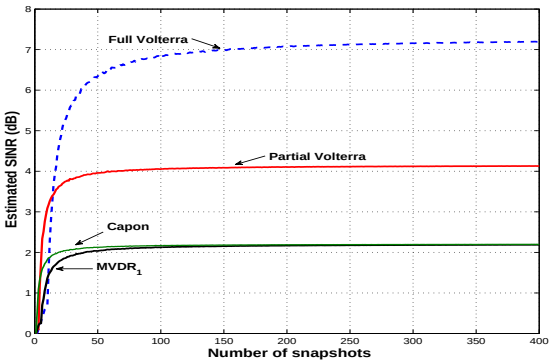


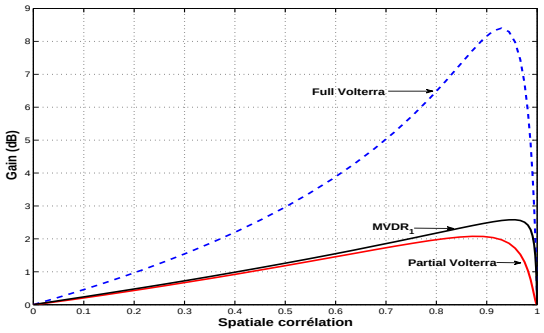
Fig.2 Estimated SINR at the output of different beamformers.

In the second experiment, we are interested by the gain in SINR with respect to the Capon algorithm in the steady state, as a function of the spatial correlation $|\alpha_{j,s}|$ defined by $\alpha_{j,s} = \frac{\mathbf{j}^H \mathbf{s}}{|\mathbf{j}||\mathbf{s}|} = |\alpha_{j,s}| e^{i\phi_{j,s}}$, The interference is a BPSK (a) or QPSK (b) modulation with SNR = 20dB and INR = 20dB. Naturally there is no gain for $|\alpha_{j,s}| = 0$ ($\mathbf{s}$ and $\mathbf{j}$ orthogonal, total rejection of the interference) and $|\alpha_{j,s}| = 1$ ($\mathbf{s}$ and $\mathbf{j}$ colinear, no rejection of interference).

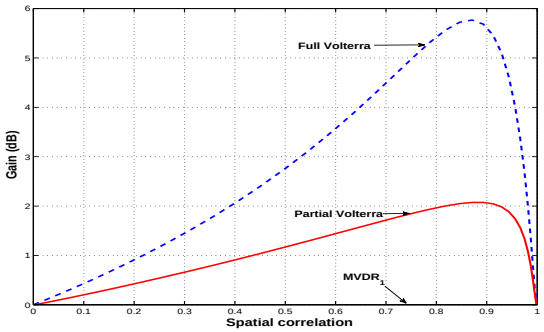
For a BPSK interference, term (1) of eq.(3) conveys more information than term (4) because the nonGaussianity of BPSK is relatively weak (kurtosis of -2) with respect to its SO noncircularity that is maximum (rectilinear). Consequently the MVDR_1 beamformer [6] that uses only term (1) outperforms the partial third order Volterra beamformer. In contrast, all the terms of eq.(3) convey information and thus the total third order Volterra beamformer largely outperforms the partially structured one. Furthermore, we note that the gain can be very large (over 8dB) in this scenario.

For a QPSK interference, the MVDR_1 beamformer [6] does not improve the Capon beamformer because term (1) of eq.(3) does not convey any information for SO circular interference. In contrast, for the partially [resp., totally] structured MVDR Volterra beamformer, term (4) [resp., terms (2) and (4)] of eq.(3), convey information for fourth-order noncircular interference. Consequently, these two MVDR Volterra beamformers outperform the Capon beamformer. Furthermore, we note that the gain can be large (about 2dB and 6dB) in this scenario.

In these two experiments, we see the interest to take into account, both nonGaussianity and noncircularity of the interference through our full third order widely nonlinear Volterra beamformer.



(a) BPSK interference



(b) QPSK interference

Fig.3 Gain of Volterra MVDR beamformers as a function of the spatial correlation.

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