EFFECTIVENESS OF SUCCESSIVE INTERFERENCE CANCELLATION AND ASSOCIATION POLICIES FOR HETEROGENEOUS WIRELESS NETWORKS

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ABSTRACT

The densification of the network infrastructure is a possible solution to meet the explosive growth of mobile data demand. In the resulting interference-limited networks, interference management techniques are of interest to increase the spectral efficiency. Successive interference cancellation (SIC) provides modest gains when users are connected to the access point (AP) which provides the maximum average received signal power. In this paper, we focus on alternative association policies where SIC gives rise to a substantial performance gain. Specifically, we present a probabilistic framework to evaluate the performance of heterogeneous networks with SIC capabilities considering the minimum load association policy and range expansion. Numerical results show the effectiveness of SIC for these association policies.

Index Terms— successive interference cancellation, cellular network, stochastic geometry, association policy

1. INTRODUCTION

By reducing the distance between access points (APs) and user nodes, the deployment of heterogeneous networks is an effective way to enhance the spectral efficiency. Nevertheless, a massive deployment of APs leads to a significant increase of the network energy consumption [1], while the network performance is severely affected by interference. To further improve the network capacity, different interference management techniques have been analysed in literature. Successive interference cancellation (SIC) is a technique that decodes signals according to descending signal power and subtracts the decoded signals from the incoming signal resulting in an increase of the signal-to-interference ratio (SIR) [2, 3]. As only the first cancellation has a significant effect on the performance, the computational requirements related to SIC are limited and hence, SIC qualifies for DL transmissions [4]. It has further been shown that SIC gives modest results for SIR values above 0 dB, while there are distinct benefits for bad signal conditions. Hence, it is valuable to evaluate association policies for future heterogeneous networks where users are not connected to the AP that provides the best signal quality.

To describe the network performance, we propose in this work an analytical framework that accommodates for the heterogeneity that characterizes future wireless networks. To this end, we include different association policies for heterogeneous wireless networks, for which SIC yields distinct performance gains. Specifically, we include the minimum load association policy and range expansion. The proposed framework accounts for all essential network parameters and provides insight in the achievable gains of SIC in multi-tier heterogeneous networks. The minimum load policy can be used to evaluate the feasibility of load balancing [5], while range expansion with SIC capabilities can allow for efficient traffic offloading [6].

2. SYSTEM MODEL

We consider a multi-tier heterogeneous network composed of K tiers. For every tier $k \in \mathcal{K} = \{1, ..., K\}$, the access points (APs) are distributed according to a homogeneous Poisson point process (PPP) Φ_k in the Euclidean plane with density λ_{k} such that $\Phi_k \sim \mathrm{PPP}(\lambda_{\mathrm{k}})$. While it is natural to use the Poisson model as the underlying spatial stochastic process for irregularly deployed APs such as picocells and femtocells, modeling the location of regularly deployed macrocell base stations (MBSs) by means of a PPP has been empirically validated and yields conservative bounds on the network performance [7]. More recently, also theoretical evidence has been given for modeling the deterministic locations of MBSs by means of a PPP, provided there is sufficiently strong log-normal shadowing [8]. All APs apply an open access (OA) policy, such that users can be served by each AP of each tier. The mobile users are spatially distributed as $\Psi \sim \mathrm{PPP}(\mu)$ over $\mathbb{R}^2$. Each AP of tier k transmits with power P_k over the total bandwidth W. The total available spectrum W is divided in subchannels by aggregating a fixed number of consecutive subcarriers of bandwidth B, such that the total number of available subchannels equals $\lfloor W/B \rfloor$.[1] We denote the subchannel index as j, where $j \in \mathcal{J} = \{1, 2, \ldots, \lfloor W/B \rfloor\}$. In order to maximize frequency reuse and throughput, each AP has access to the entire available spectrum. We represent the i-th AP of tier k as $x_{k,i}$. Hence, denoting the available channels of $x_{k,i}$ as $\mathcal{J}^{(x_{k,i})}$, we have $\mathcal{J}^{(x_{k,i})} = \mathcal{J},\ \forall i, k$. A user receives a signal from $x_{k,i}$ with signal power $P_k h_u g_\alpha(u - x_{k,i})$, where h_u represents the power fading coefficient for the link between the user u and $x_{k,i}$, and $g_\alpha(x) = \|x\|^{-\alpha}$ is the power path loss function with path loss exponent α. For notational convenience, u and x will be used to denote network nodes as well as their location. The association of a user to $x_{k,i}$ is typically based on the following association metric

$$x_{k,i} = \arg\max_{k,i} A_k \|u - x_{k,i}\|^{-\alpha}, \tag{1}$$

where A_k represents the association rule. For $A_k = 1$, the user is associated to the nearest base station. For $A_k = P_k$, the association is based on the maximum average received signal power, where the

[1]Without loss of generality, we assume $B = 1$.

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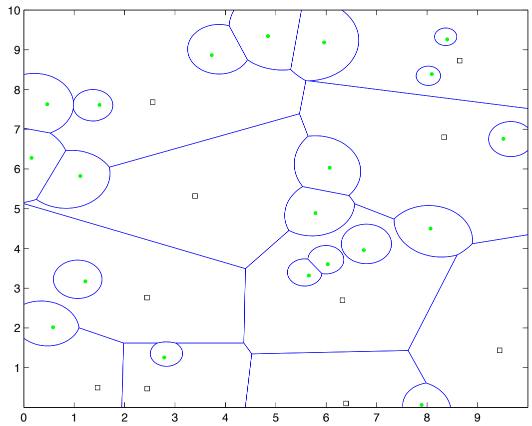

Fig. 1. Multiplicatively weighted Voronoi tessellation for a two-tier network.

averaging is done with respect to the fading parameter h.[2] Using this association rule, the set of APs forms a multiplicatively weighted Voronoi tessellation on the two dimensional plane, where each cell $C_{k,i}$ consists of those points which have a higher average received signal power from $x_{k,i}$ than from any other AP, as depicted in Fig. 1. Formally, we define the cells as

$$C_{k,i} = \{y \in \mathbb{R}^2 \mid \|y - x_{k,i}\| \leq (A_k/A_l)^{1/\alpha}\|y - x\|, \\ \forall x \in \Phi_l \backslash \{x_{k,i}\},\, l \in \mathcal{K}\}. \quad (2)$$

According to the association rule in (1), users will connect to different tiers and the density of users connected to tier k is given by μ_k. Considering a K-tier network in downlink (DL), each tier $k \in \mathcal{K}$ is characterized by the set $\{P_k, \lambda_k, \mu_k\}$ consisting of the DL transmission power, AP density, and associated user density. The sets of transmission powers and densities are denoted as $\mathbf{P} = \{P_1, \ldots, P_K\}$, $\boldsymbol{\lambda} = \{\lambda_1, \ldots, \lambda_K\}$, and $\boldsymbol{\mu} = \{\mu_1, \ldots, \mu_K\}$, respectively. Within a Voronoi cell, mobile users are independently and uniformly distributed over the cell area. Fairness between users is accomplished by proportional allocation of the time and frequency resources. We consider an orthogonal multiple access scheme, which ensures that at any given time and frequency, only a single user per cell is active. As interference dominates noise in modern cellular networks, we consider the network to be interference-limited. For the link between base station $x_{k,i}$ and user u, we define the signal-to-interference ratio (SIR) on channel j in DL as

$$\text{SIR}_j(x_{k,i} \to u) = \frac{P_k h_u g_\alpha(x_{k,i} - u)}{\sum_{k\in\mathcal{K}} \sum_{v\in\Phi_{k,j}\backslash\{x_{k,i}\}} P_k h_v g_\alpha(v - u)}. \quad (3)$$

Let $\Phi_{k,j}$ denote the APs active on channel j in tier k. A transmission is successful if the SIR of the intended link exceeds a prescribed threshold η_t, which reflects the required quality-of-service (QoS) in terms of transmission rate. Hence, the success probability can be written as $\mathbb{P}_\text{s}(\eta_\text{t}) = \Pr\{\text{SIR}_j(x_{k,i} \to u) \geq \eta_\text{t}\}$.

[2]The association rule can further be adjusted to accommodate for cell range expansion by defining $A_k = b_k P_k$, where b_k represents an association bias for tier k.

3. ASSOCIATION POLICIES AND SIC GAINS

In this section, we analyze multi-tier networks with different association policies. Recent work shows that a generic heterogeneous multi-tier network can be represented by a single-tier network where all system parameters such as the transmission power, fading parameter, and path loss exponent are set to constants, while the determinative parameter is an isotropic (possibly non-homogeneous) AP density [9]. For a constant path loss exponent, the isotropic density of the equivalent network reduces to a homogeneous value, as such generalizing previous results where the dispersion of the aggregate interference depends on a single moment of the transmission power and the fading distribution [10]. For the performance evaluation of SIC in Section 3.1, we will relax the system model to the stochastically equivalent single-tier network with density given by $\lambda_\text{eq} = \sum_{k\in\mathcal{K}} \lambda_k P_k^{2/\alpha}$, which follows from Campbell's theorem, and where the transmission power is equal to one. In Section 3.1, the aim is to deepen the understanding of heterogeneous networks accounting for the concepts of coverage area and load. In Section 3.2, we condition on the association to a specific tier. In this case, we will not resort to the single-tier stochastic equivalent of the network as this would result in a loss of physical insight related to the differences between the tiers.

3.1. Minimum load association policy

Considering fairness between users, in case of data sensitive applications it can be preferential to connect to the AP with the lowest load, rather than to the AP that offers the highest SIR. The same observation holds for networks that apply a load balancing policy and where users are actively transferred to lightly loaded APs different from the AP of their own Voronoi cell [5]. In the following, we consider the association policy where a user connects to the AP with the lowest load for a given connectivity range R_con with respect to the user. In this scenario, the performance metric of interest is the rate per user, which reflects the quality of service (QoS) and depends on the AP load, defined as the number of users M connected to the AP. This scenario leads to interesting trade-offs between APs where the loss of SIR can be compensated by the gain of available resource blocks per user. We consider a single tier network and we model explicitly the load of the APs by considering the marked PPP $\tilde{\Phi} = \{(X_i, L_i) \,|\, X_i \in \Phi(\lambda),\, L_i \sim F_L(l)\}$, with L_i the load of X_i and $F_L(l)$ the load distribution. We consider a typical user at the origin of the Euclidean plane and we compare the performance of the max-SIR association policy with the minimum load association policy for DL transmissions in terms of rate per user. Let $\mathcal{R}$ denote the rate per user and we define the coverage probability as the CCDF of the rate $\mathbb{P}_\text{c}(\rho) = \mathbb{P}[\mathcal{R} > \rho]$ [6], which is given by

$$\mathbb{P}_\text{c}(\rho) = \Pr\left[\frac{1}{M}\log(1 + \text{SIR}) > \rho\right] \\ = \Pr[\text{SIR} > 2^{M\rho} - 1] = \mathbb{E}_M[\mathbb{P}_\text{s}(2^{M\rho} - 1)] \quad . \quad (4)$$

To calculate the coverage probability, we need to characterize the distribution of M. The load of a cell depends on the distribution of the Voronoi cell area $\mathcal{A}$, represented by $f_\mathcal{A}(x)$, for which an approximation has been proposed in [11]. Using this approximation, the probability mass function of M is given by

$$f_M(m) = \int_0^\infty \Pr[M = m \,|\, \mathcal{A} = x] f_{\mathcal{A}}(x)\mathrm{d}x = \frac{3.5^{3.5}}{m!}\frac{\Gamma(m+3.5)}{\Gamma(3.5)}\left(\frac{\mu_j}{\lambda}\right)^m\left(3.5+\frac{\mu_j}{\lambda}\right)^{-(m+3.5)}. \quad (5)$$

In case of the max-SIR policy, the coverage probability conditioned on the number of associated users is given by

$$\begin{aligned}\mathbb{P}_\mathrm{c}^{(\text{MAX SIR})}(\rho|M) &= \Pr[\mathrm{SIR} > 2^{M\rho}-1 \,|\, M] \\ &\overset{(a)}{=} \int_0^\infty 2\lambda\pi r \exp(-\pi\lambda r^2(1+\varsigma^{2/\alpha}C(1/\varsigma^{2/\alpha},\alpha)))\mathrm{d}r \\ &= \frac{1}{1+\varsigma^{2/\alpha}C(1/\varsigma^{2/\alpha},\alpha)}, \end{aligned} \quad (6)$$

with $\varsigma = 2^{M\rho}-1$ and $C(b,\alpha) = \int_b^\infty \frac{1}{1+w^{\alpha/2}}\mathrm{d}w$, and where the coverage probability in (a) is calculated similar to [7]. Deconditioning over M, the rate coverage can be written as

$$\mathbb{P}_\mathrm{c}^{(\text{MAX SIR})}(\rho) = \sum_{m\geq 0} f_M(m)\mathbb{P}_\mathrm{c}^{(\text{MAX SIR})}(\rho|m+1), \quad (7)$$

where the load of the cell under consideration includes the admitted user.

Lemma 1. *For a typical user that connects to the AP with the lowest load within the range R_con, the coverage probability is given by*

$$\mathbb{P}_\mathrm{c}^{(\mathrm{MINL})}(\rho) = \sum_{m\geq 0} f_{M_{(1)}}(m)\mathbb{P}_\mathrm{c}^{(\mathrm{MINL})}(\rho|m+1), \quad (8)$$

where $f_{M_{(i)}}(m)$ represents the probability mass function (PMF) of the i-th order statistic of the load.

Proof. For the minimum load association policy, the typical user is appointed to the AP with the lowest load which is uniformly distributed over $b(0,R_\mathrm{con})$ with distance distribution $f_R(r) = 2r/R_\mathrm{con}^2$. We assume that there are $N = \lfloor \lambda\pi R_\mathrm{con}^2 \rfloor$ APs within the connectivity range. The coverage probability for the minimum load scheme conditioned on the load is given by

$$\begin{aligned}\mathbb{P}_\mathrm{c}^{(\mathrm{MINL})}(\rho|M) &= 1/R_\mathrm{con}^2 \int_0^{R_\mathrm{con}^2} \exp(-\pi\lambda\varsigma^{2/\alpha}C(0,\alpha)r^2)2r\mathrm{d}r \\ &= \frac{1-\exp(-\pi\lambda\varsigma^{2/\alpha}C(0,\alpha)R_\mathrm{con}^2)}{\pi\lambda\varsigma^{2/\alpha}C(0,\alpha)R_\mathrm{con}^2}. \end{aligned} \quad (9)$$

The i-th order statistic of the load is given by [12]

$$f_{M_{(i)}}(m) = \frac{1}{\mathcal{B}(i, M-i+1)}\int_{F_M(m-1)}^{F_M(m)} w^{i-1}(1-w)^{m-i}\mathrm{d}w, \quad (10)$$

where $\mathcal{B}(a,b)$ represents the beta function. Deconditioning (9) with respect to the first order statistic of the load, the proof is concluded. □

3.2. Range expansion

While the higher tiers in a multi-tier network are intended to offload data traffic from the macrocell network, this target is impeded considerably due to the relatively small coverage area of the higher tiers, which are usually denoted as small cells. To encourage users to connect to the small cells, range expansion has been proposed which applies an association policy based on a biased received signal power [13]. Although range expansion mitigates the UL cross-tier interference, users in DL experience bad signal conditions in the range expanded areas since they are not connected to the base station that provides the highest average SIR. It is therefore meaningful to study the benefit of SIC in DL for those users located in the range expanded areas (REAs). To calculate the success probability of the users belonging to the REA, we need to define the distance distribution of these users with respect to the serving AP. The following lemma is an extension of [14] for a K-tier network.

Lemma 2. *Let $B = \{b_k\}$ be the set of biases corresponding to each tier. The distance distribution of users located in the REA to the serving AP is given by*

$$\begin{aligned}f_{X_k^{(\mathrm{RE})}}(x) = \frac{2\pi\lambda_k}{p_{\mathrm{a},k}^{(\mathrm{RE})}}x\Bigg[&\exp\left(-\pi\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\frac{b_i}{b_k}\right)^{2/\alpha}x^2\right) \\ &- \exp\left(-\pi\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\right)^{2/\alpha}x^2\right)\Bigg] \end{aligned} \quad (11)$$

where the association probability to the REA of tier k is $p_{\mathrm{a},k}^{(\mathrm{RE})} = 1-\sum_{i\neq k} p_{\mathrm{a},i}(b) - p_{\mathrm{a},k}(B \,|\, b_k = 1)$ and the association probability to the k-th tier is given by

$$p_{\mathrm{a},k}(B) = \frac{\lambda_k}{\sum_{i\in\mathcal{K}}\lambda_i\left(\frac{P_i}{P_k}\frac{b_i}{b_k}\right)^{2/\alpha}}. \quad (12)$$

For the proof, we refer to [15] and [14]. Note that a user, which belongs without biasing to tier $i \neq k$, is located in the REA $C_k^{(\mathrm{RE})}$ of tier k if the relationship $P_k x_k^{-\alpha} < P_i x_i^{-\alpha} < b_k P_k x_k^{-\alpha}$ holds. In order to calculate the benefit of canceling the strongest interferer for the users located in the REA, we provide the following lemma.

Lemma 3. *After canceling the strongest AP, the success probability of the users located in $C_k^{(\mathrm{RE})}$ is given by*

$$\begin{aligned}\mathbb{P}_{\mathrm{s,IC}}(\eta_\mathrm{t}, 1 \,|\, x_k \in C_k^{(\mathrm{RE})}) &= \frac{1}{p_{\mathrm{a},k}^{(\mathrm{RE})}} \\ \times \Bigg[&\frac{1}{\sum_{i\in\mathcal{K}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha}\left(\eta_\mathrm{t}^{2/\alpha}C((1/\eta_\mathrm{t})^{2/\alpha},\alpha)+\left(\frac{b_i}{b_k}\right)^{2/\alpha}\right)} \\ &- \frac{1}{\sum_{i\in\mathcal{K}}\left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha}\left(\eta_\mathrm{t}^{2/\alpha}C((1/\eta_\mathrm{t})^{2/\alpha},\alpha)+1\right)}\Bigg]. \end{aligned} \quad (13)$$

Proof. The success probability of a mobile node belonging to $C_k^{(\mathrm{RE})}$ and connected to the k-th tier conditioned on the distance can be written as

$$\mathbb{P}_\mathrm{s}(\eta_\mathrm{t} \,|\, x_k \in C_k^{(\mathrm{RE})}, x_k) = \prod_{i\in\mathcal{K}}\mathcal{L}_{I_{\Phi_i}}\left(\frac{\eta_\mathrm{t}x_k^\alpha}{P_k}\right),$$

where

$$\mathcal{L}_{I_{\Phi_i}}\left(\frac{\eta_\mathrm{t}x_k^\alpha}{P_k}\right) = \exp\left(-\pi\lambda_i\eta_\mathrm{t}^{2/\alpha}\left(\frac{P_i}{P_k}\right)^{2/\alpha}C((b_i/\eta_\mathrm{t}b_k)^{2/\alpha},\alpha)x_k^2\right).$$

The integration interval of the integral in $C(b,\alpha)$ is determined noting that the location of the user in the REA $P_i x_i^{-\alpha} < b_k P_k x_k^{-\alpha}$ yields the interferer exclusion region $x_i > (P_i/b_k P_k)^{1/\alpha} x_k$. Applying the change of variables $(P_k/\eta_{\mathrm{t}} P_i)^{2/\alpha}(x_i/x_k)^2 \rightarrow u$ and deconditioning on x_k, we can write

$$
\begin{aligned}
&\mathbb{P}_{\mathrm{s}}(\eta_{\mathrm{t}} \,|\, x_k \in C_k^{(\mathrm{RE})}) \\
&= \int_0^\infty \exp\Big(-\pi \eta_{\mathrm{t}}^{2/\alpha} \sum_{i\in\mathcal{K}} \lambda_i (P_i/P_k)^{2/\alpha} C((b_i/\eta_{\mathrm{t}} b_k)^{2/\alpha}, \alpha) x_k^2\Big) \\
&\quad \times f_{X_k^{(\mathrm{RE})}}(x_k) \mathrm{d}x_k \\
&= \frac{1}{p_{\mathrm{a},k}^{(\mathrm{RE})}} \\
&\times \left(\frac{1}{\sum_{i\in\mathcal{K}} \left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha} \left(\eta_{\mathrm{t}}^{2/\alpha} C((b_i/\eta_{\mathrm{t}} b_k)^{2/\alpha}, \alpha) + \left(\frac{b_i}{b_k}\right)^{2/\alpha}\right)} \right. \\
&\left. - \frac{1}{\sum_{i\in\mathcal{K}} \left(\frac{\lambda_i}{\lambda_k}\right)\left(\frac{P_i}{P_k}\right)^{2/\alpha} \left(\eta_{\mathrm{t}}^{2/\alpha} C((b_i/\eta_{\mathrm{t}} b_k)^{2/\alpha}, \alpha) + 1\right)} \right).
\end{aligned} \tag{14}
$$

Applying SIC to a user located in the REA, the highest unbiased received signal power of tier i is canceled. As a result the interference cancellation radius relative to the i-th tier increases from $(P_i/b_k P_k)^{1/\alpha} x_k$ to $(P_i/P_k)^{1/\alpha} x_k$, and hence, the success probability after canceling the strongest AP can be written as (13). □

From Lemma 3, the bias factors of the different tiers can be determined to guarantee a given performance for the mobile users belonging to the REA.

4. NUMERICAL RESULTS

In this section, we present some numerical results that illustrate that SIC, although not very effective in networks applying the association policy based on the maximum average received signal power, can have distinct advantages in scenarios with other association policies. Figure 2 depicts the coverage probability and compares the max-SIR association policy with the minimum load association policy. From the numerical results, we see that the coverage probability decreases significantly for the minimum load policy when no SIC is applied. This means that the loss in SIR cannot be compensated by the lower load of the AP. However, when SIC is applied ($n = 1$) based on the scheme presented in [4], the minimum load association outperforms max-SIR association in terms of rate per user. From this figure, we conclude that when SIC is applied, users can be offloaded to nearby APs without loss of capacity, which paves the way for more advanced load balancing techniques.

Figure 3 depicts the success probability of a typical user in the REA in a two-tier network with densities $\lambda_1 = 10^{-5}\mathrm{m}^{-2}$ and $\lambda_2 = 10^{-4}\mathrm{m}^{-2}$ for different values of the range expansion factor b. The figure illustrates how the success probability decreases as the REA gets larger with increasing values of b. Moreover, from the numerical results we observe that the increase of success probability due to SIC is substantial. This scenario is a realistic example where SIC can provide high performance gain.

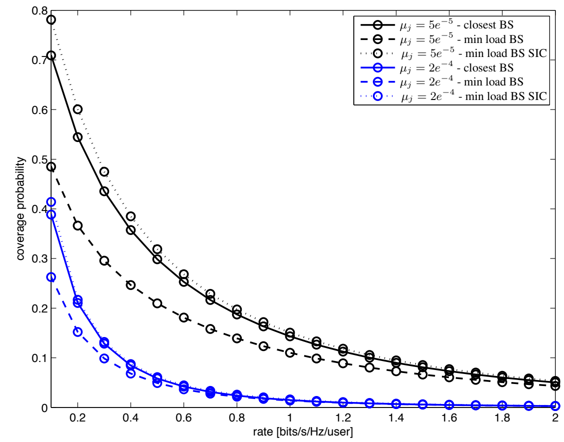

Fig. 2. The coverage probability is depicted for the max-SIR association policy (solid lines), the minimum load association policy (dashed lines), and the minimum load policy with SIC (dotted lines) for $\lambda = 10^{-5}$ and $\alpha = 4$.

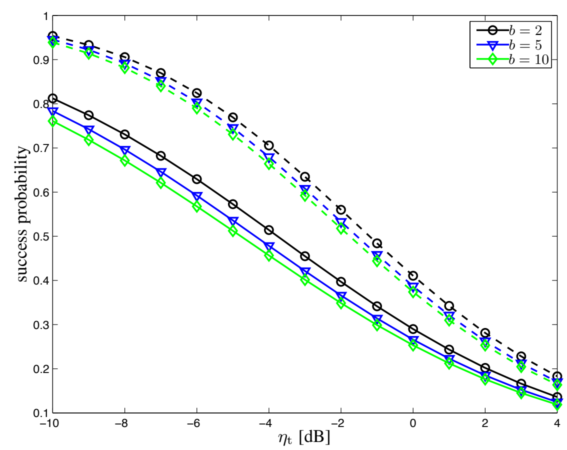

Fig. 3. Success probability for users belonging to the REA without SIC (solid lines) and canceling the strongest AP (dashed lines) for power ratio $P_1/P_2 = 10$.

5. CONCLUSIONS

In this work, we proposed a probabilistic framework for the performance analysis of heterogeneous wireless networks with SIC capabilities. The framework considers different association policies and performance metrics and addresses the heterogeneity of multi-tier networks by considering the differences in load and coverage between access points. We presented two deployment scenarios for future multi-tier networks where users are not connected to the AP that provides the best signal quality, i.e. minimum load association and range expansion. Numerical results show that SIC yields distinct performance gains for these scenarios. This work deepens the understanding of SIC by defining the achievable gains for different association policies in multi-tier heterogeneous networks.

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