Medical Imaging Research Laboratory
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SIMULTANEOUS CODED PLANE WAVE IMAGING IN ULTRASOUND: PROBLEM FORMULATION AND CONSTRAINTS

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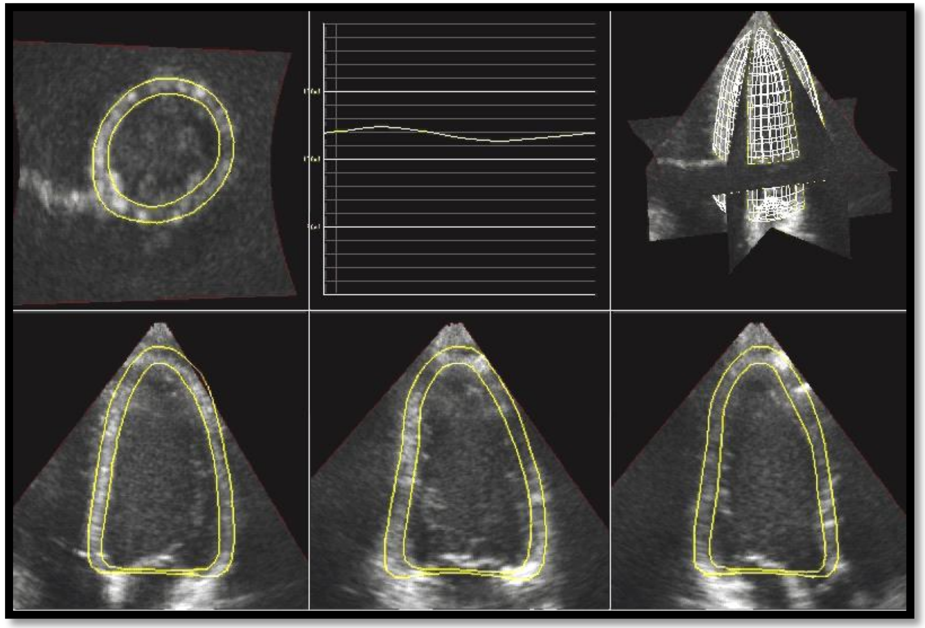
Context - Motivation

Ultrasound is often used to image fast transient events

Elastography
(@: www.institut-langevin.espci.fr)

Flow imaging
(Courtesy of Alfred Yu,
University Waterloo, Canada)



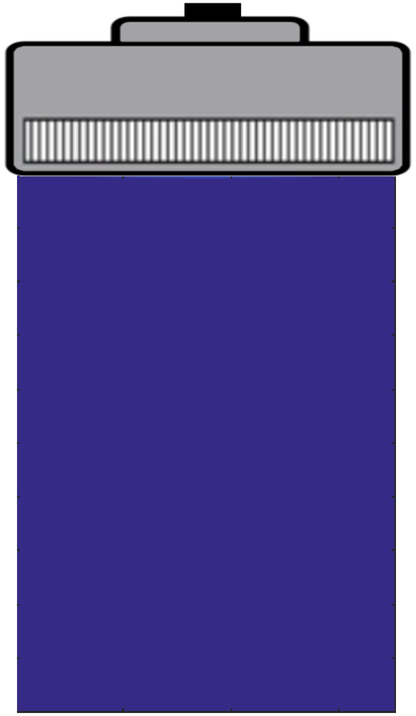
Strain imaging
(Courtesy of Lasse Lovstakken
NTNU Trrondheim)

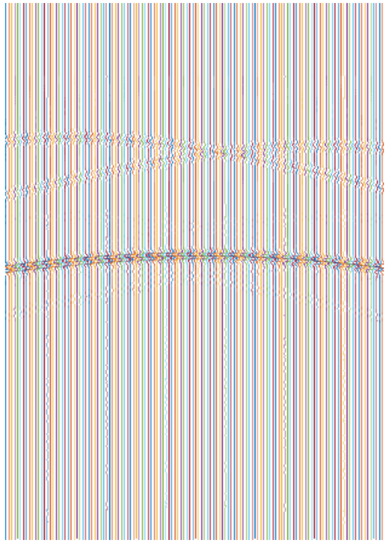
High frame rate ultrasound could improve such techniques in 2D[1] [2]

Higher acquisition rate of 2D frames => Higher frame rate of 3D frames

[1] Hansen, H. H. G., Saris, A. E. C. M., Vaka, N. R., Nillesen, M. M., & de Korte, C. L. (2014). Ultrafast vascular strain compounding using plane wave transmission. *Journal of biomechanics*, *47*(4), 815-823.
[2] Vappou, J., Luo, J., & Konofagou, E. E. (2010). Pulse wave imaging for noninvasive and quantitative measurement of arterial stiffness in vivo. *American journal of hypertension*, *23*(4), 393-398

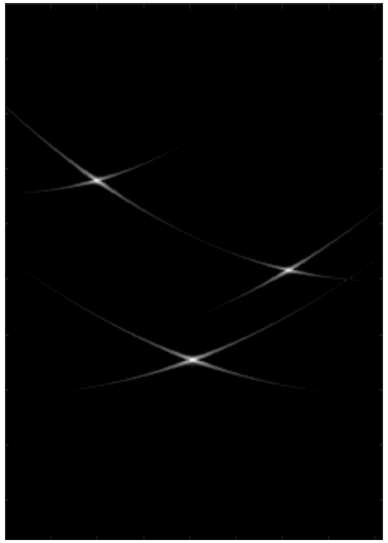
Plane wave imaging[1]





Received signals

Beamforming
Log compression



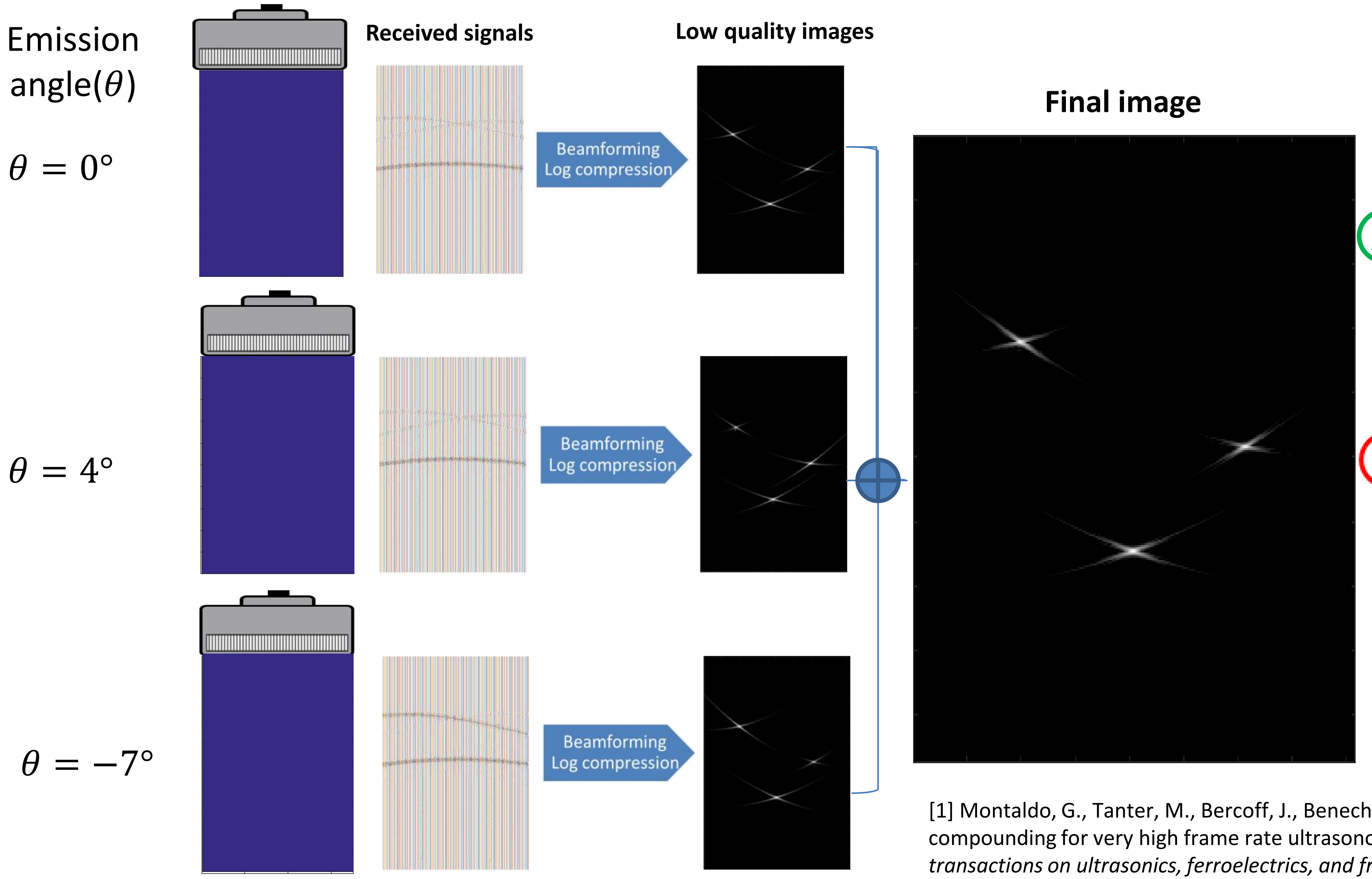
Final Image

Ultrafast acquisition rate $(\sim {}^{2*depth}/_{c})$

Presence of artefacts

[1] Sandrin, L., Catheline, S., Tanter, M., Hennequin, X., & Fink, M. (1999). Time-resolved pulsed elastography with ultrafast ultrasonic imaging. *Ultrasonic imaging*, *21*(4), 259-272.

Plane wave coherent compounding imaging[1]

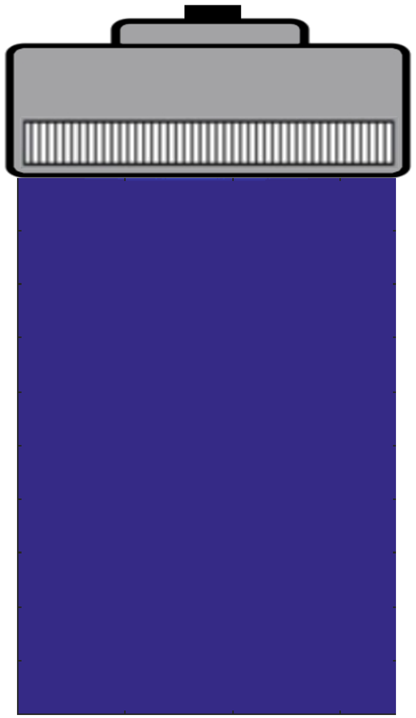


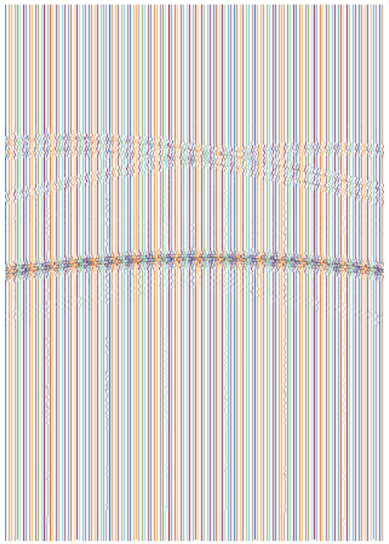
Better image quality

Decrease of the acquisition rate

[1] Montaldo, G., Tanter, M., Bercoff, J., Benech, N., & Fink, M. (2009). Coherent plane-wave compounding for very high frame rate ultrasonography and transient elastography. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, *56*(3), 489-506.

Our proposal: Simultaneous emission of the plane waves

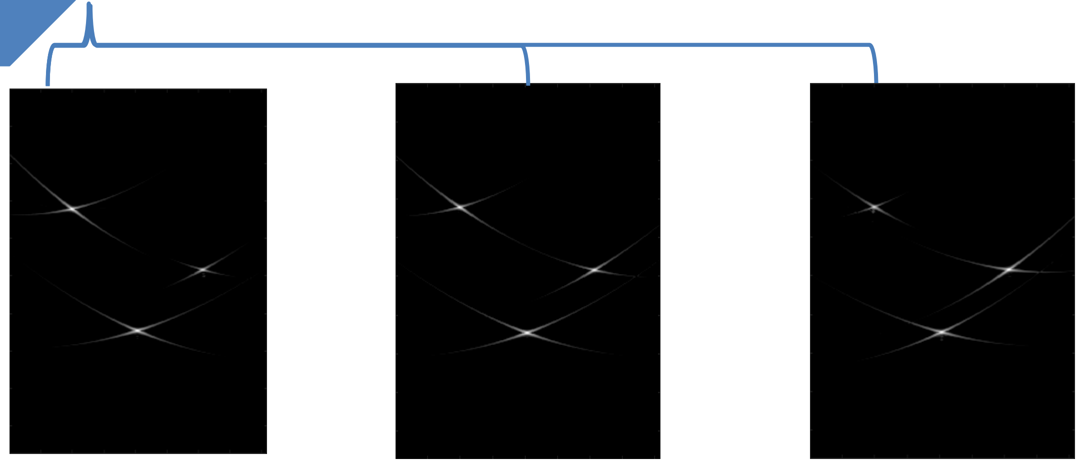




~~Beamforming~~
~~Log compression~~

Direct beamforming
=> artefacts

Inverse problem approach



Q: What is the mathematical model of the system?

Q: What inverse problem approach to use in order to solve the model?

Q: What excitation signals to use?

At the end of this presentation you will know:

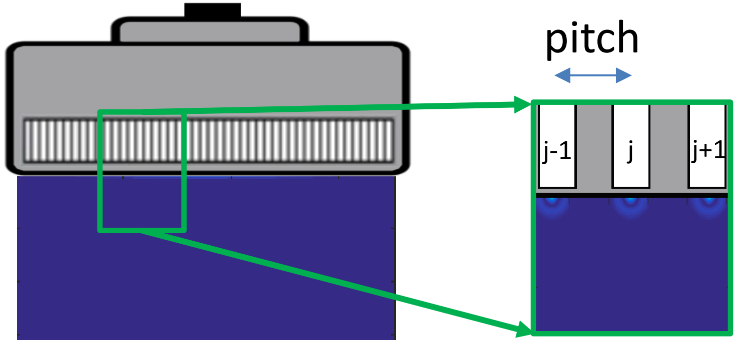
The mathematical model of our system

The estimator that we use to solve our system

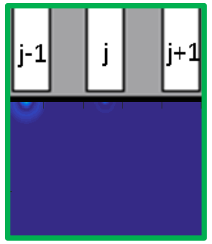
The conditions under which our system becomes well-posed

The excitation signals that allow the system to be well-conditioned – Codes

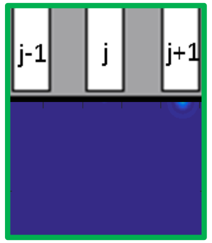
Emission/reception of a plane wave carrying an arbitrary signal a(t)



$t_j = 0, \forall j \in [1, ..., N_e], \theta = 0°$



$t_j < t_{j+1}, \forall j \in [1, ..., N_e], \theta = 4°$



$t_j > t_{j+1}, \forall j \in [1, ..., N_e], \theta = -7°$

jth Emitted signal:

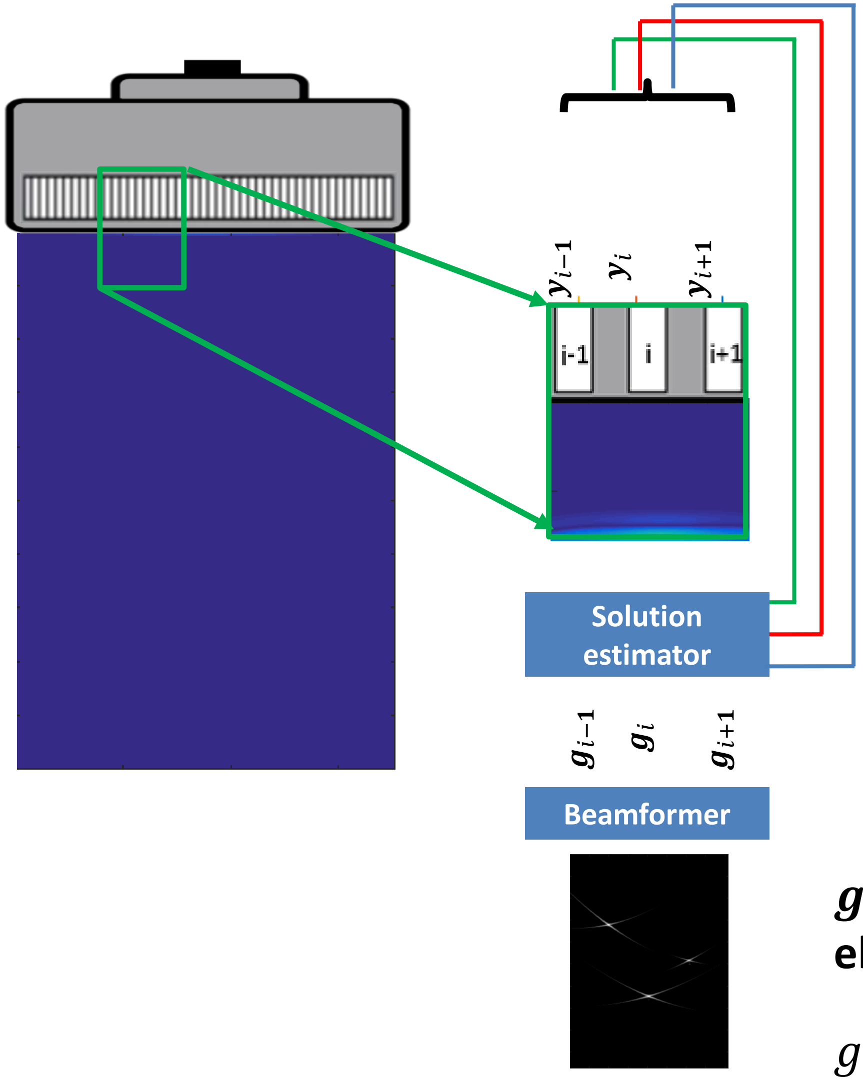
$$a_j(t) = a(t) * \delta(t - t_j)$$

$a(t)$ – signal carried by the plane wave

$$t_j = (j-1) \times pitch \times tan(\theta)$$

θ – tilt of the plane's wave wavefront

Emission/reception of a plane wave carrying an arbitrary signal a(t)



ith Received signal:

$$y_i(t) = \sum_{j=1}^{N_e} a_j(t) * h_{je}(t) * g_{ji}(t) * h_{ir}(t) + v_i(t)$$

$h_{je}(t), h_{ir}(t)$ – acousto-electrical impulse responses of elements at emission, reception

$g_{ji}(t)$ – impulse response of the medium when element j emits and i receives

$$y_i(t) = a(t) * g_i(t) + v_i(t)$$

$$g_i(t) = \sum_{j=1}^{N_e} \delta(t - t_j) * h_{je}(t) * g_{ji}(t) * h_{ir}(t)$$

$g_i(t)$ is the pulsed plane wave response of the medium seen by the ith element of the probe

$g_i(t)$ – raw signals to be beamformed into the image of the medium

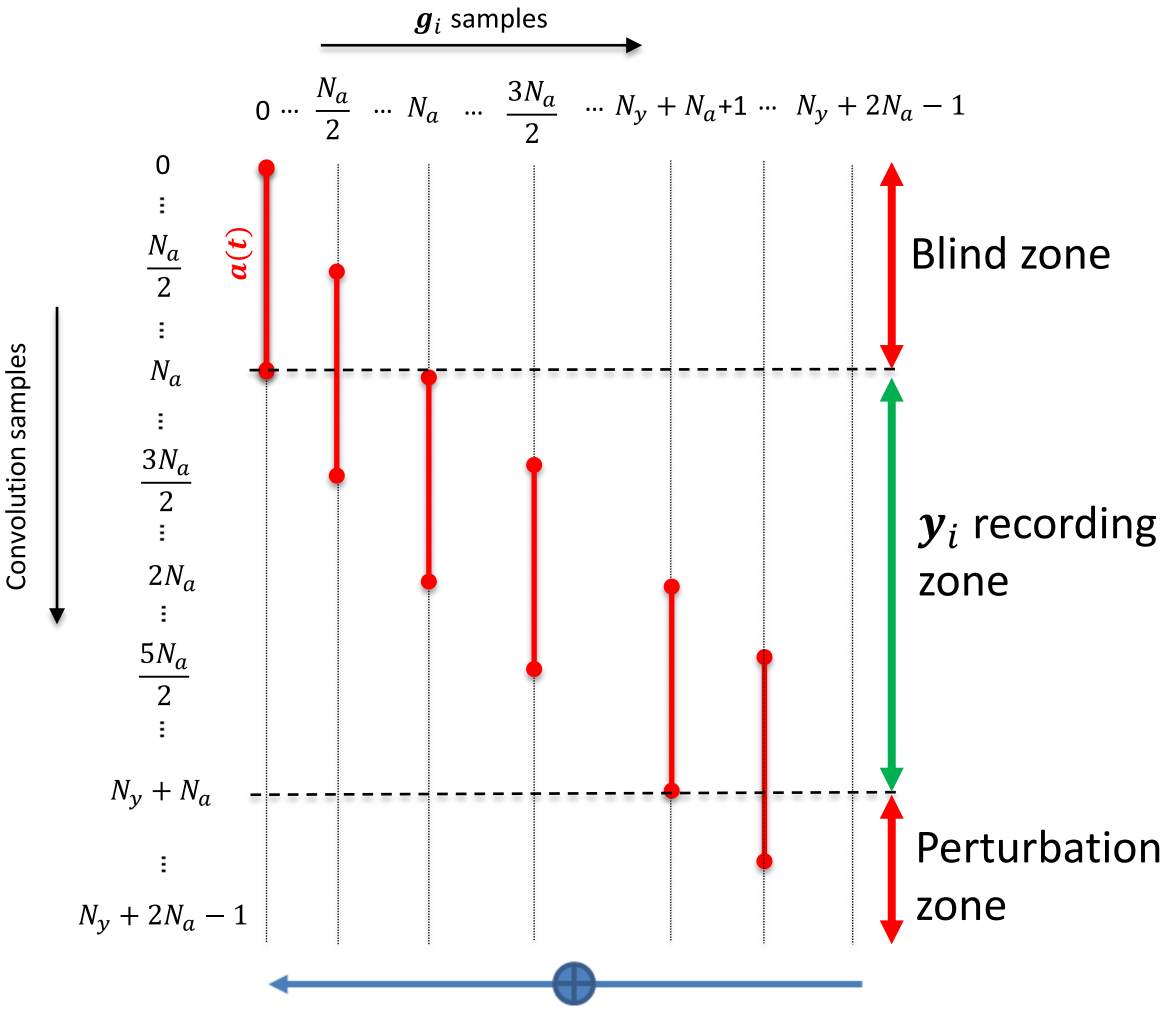
One plane wave model discretization

$$y_i(t) = a(t) * g_i(t) + v_i(t) \xrightarrow{\times \uparrow\uparrow\uparrow\uparrow} \mathbf{y}_i = \mathbf{A} \times \mathbf{g}_i + \mathbf{v}_i$$

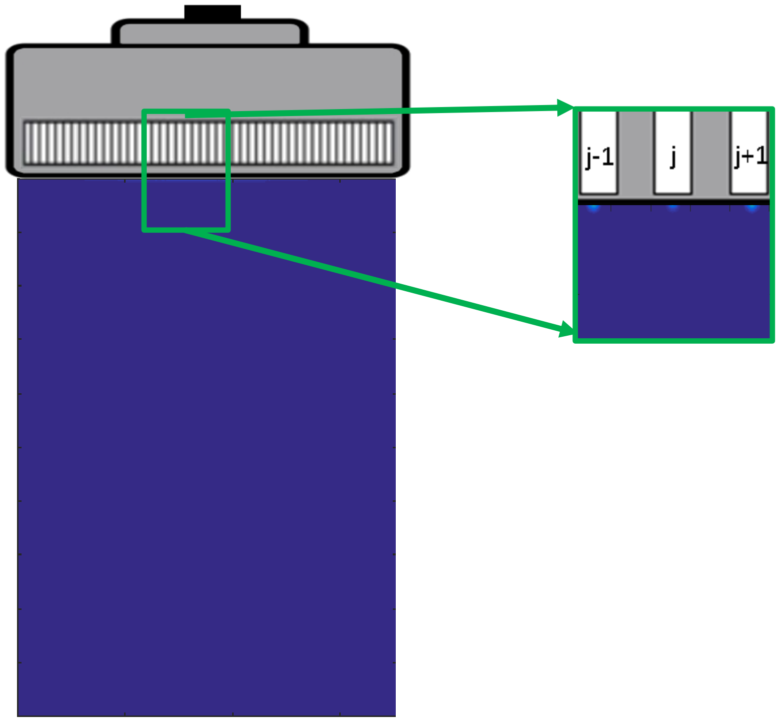
$$\mathbf{y}_i = \begin{bmatrix} y_i[0] & y_i[1] & y_i[2] & y_i[3] & \cdots & y_i[N_y] \end{bmatrix}^T$$
$$\mathbf{g}_i = \begin{bmatrix} g_i[0] & g_i[1] & g_i[2] & g_i[3] & \cdots & g_i[N_g] \end{bmatrix}^T$$
$$\mathbf{v}_i = \begin{bmatrix} v_i[0] & v_i[1] & v_i[2] & v_i[3] & \cdots & v_i[N_y] \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} a_{N_a-1} & a_{N_a-2} & \cdots & a_0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & a_{N_a-1} & \cdots & a_1 & a_0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & a_{N_a-1} & a_{N_a-2} & \cdots & a_0 \end{bmatrix}$$

$\mathbf{A}$ is a N_y by $N_y + N_a - 1$ matrix



Emission/reception of N_{pwi} plane waves carrying signals $a^k(t)$



$$a_j(t) = \sum_{k=1}^{N_{pwi}} a^k(t) * \delta(t - t_j^k)$$

k – plane wave index

$$y_i(t) = \sum_{k=1}^{N_{pwi}} a^k(t) * g_i^k(t) + v_i(t)$$

$$\mathbf{y}_i = \sum_{k=1}^{N_{pwi}} \mathbf{A}^k \times \mathbf{g}_i^k + \mathbf{v}_i$$

Replacing: $\begin{cases} \mathbf{A}_c = \begin{bmatrix} \mathbf{A}^1 & \mathbf{A}^2 & \mathbf{A}^3 \cdots & \mathbf{A}^{N_{pwi}} \end{bmatrix} \\ \overline{\mathbf{g}}_i = \begin{bmatrix} \mathbf{g}_i^1 & \mathbf{g}_i^2 & \mathbf{g}_i^3 \cdots & \mathbf{g}_i^{N_{pwi}} \end{bmatrix} \end{cases}$

$$\mathbf{y}_i = \mathbf{A}_c \overline{\mathbf{g}}_i + \mathbf{v}_i$$

Estimate $\overline{g_i}(t)$ to reconstruct the ultrasound images

For this paper we used Linear Square Estimator: $\hat{\overline{\mathbf{g}}}_i = (\mathbf{A}_c^T \mathbf{A}_c)^{-1} \mathbf{A}_c^T \mathbf{y}_i$

Constraints for a well-posed system

$$\mathbf{A}_c = \left[\mathbf{A}^1 \quad \mathbf{A}^2 \quad \mathbf{A}^3 \cdots \quad \mathbf{A}^{N_{pwi}}\right]$$

A_c size : N_y rows, $N_{pwi}(N_y + N_a - 1)$ columns

Constraint 1, on the medium:

$$g_i[z] = 0, \forall z \in [0 \ldots N_a] \cup [N_y + N_a - 1 \ldots N_y + 2N_a - 1]$$

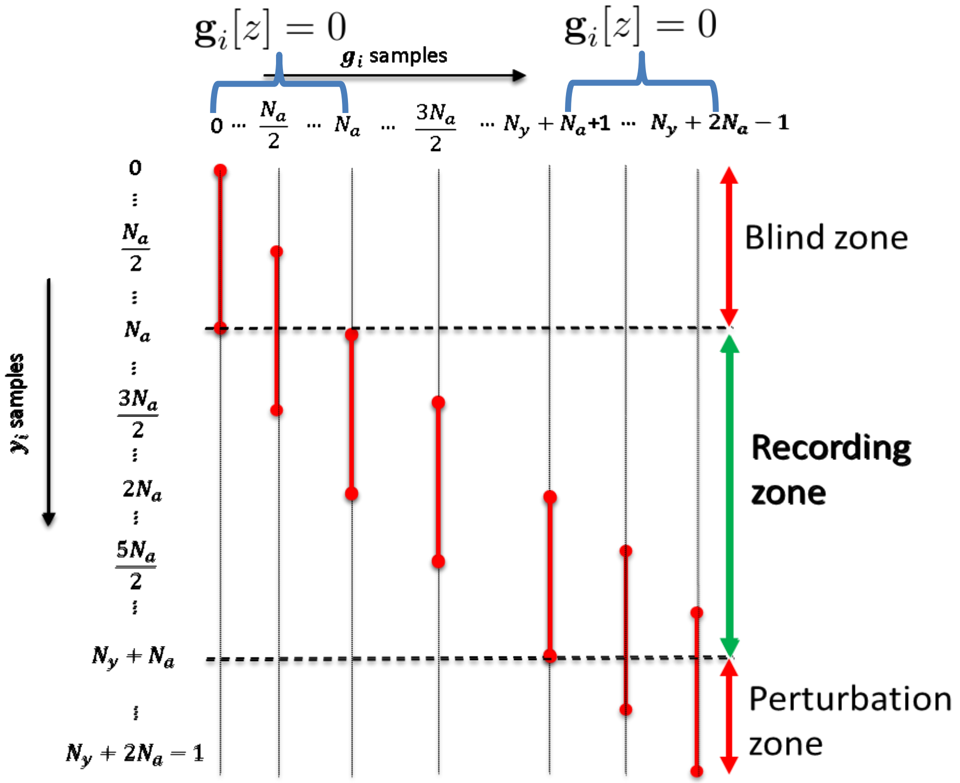
New A_c size: N_y rows, $N_{pwi} \times N_g$ columns

with: $N_y = N_a + N_g - 1$

Constraint 2, on the length of $a^k\ (t)$:

$$N_a = (N_{pwi} - 1)N_g + 1$$

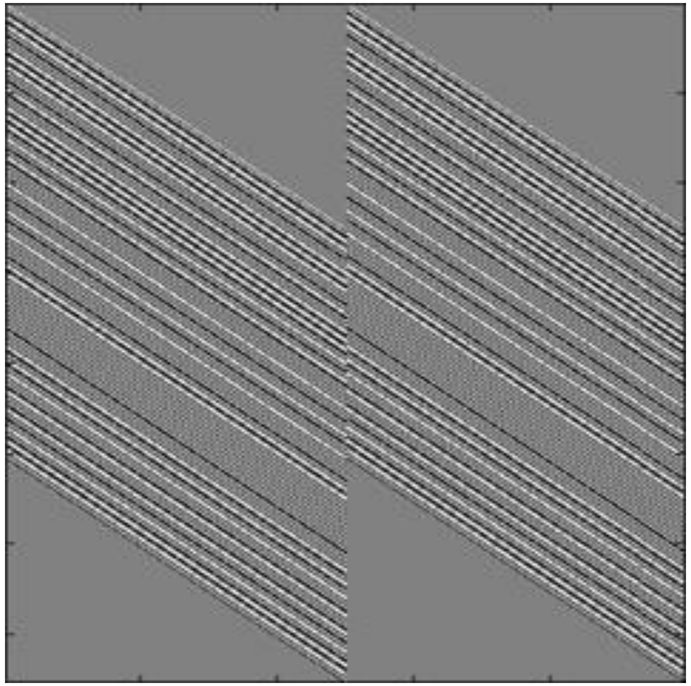
The emitted signals a^k must be $N_{pwi} - 1$ times longer than the impulse response of the medium



$$\mathrm{A}^k = \begin{bmatrix} a^k[0] & 0 & 0 & \cdots & 0 \\ a^k[1] & a^k[0] & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a^k[N_a-1] & \ddots & \ddots & \ddots & a^k[0] \\ 0 & a^k[N_a-1] & \ddots & \ddots & a^k[1] \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a^k[N_a-1] \end{bmatrix}$$

Constraints on the correlation of the emitted signals

$\mathbf{A}_c = \left[\mathbf{A}^1 \quad \mathbf{A}^2\right]$, N_y rows by $2N_g$ column matrix



Low mutual coherence of A_c => A_c well conditionned

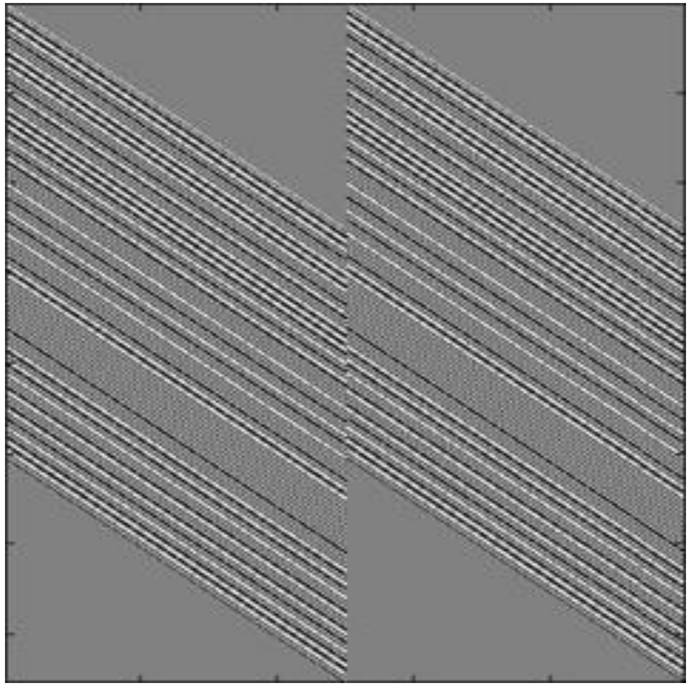
Excitation signals: $a^k(t) = s^k(t)sin(2\pi f_0 t)$

$s^k(t)$ **is a pseudo-orthogonal code**

Frequency shift to f_0 using BPSK modulation

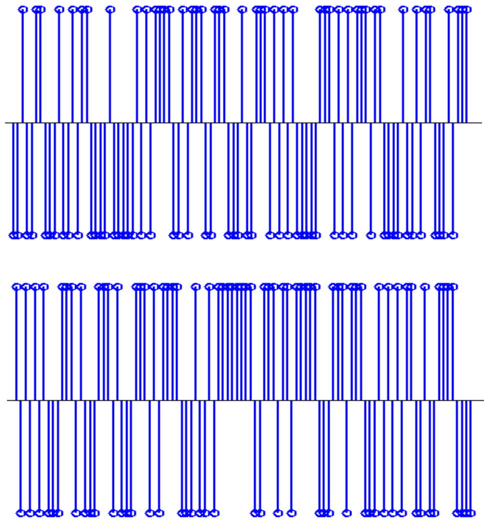
Constraints on the correlation of the emitted signals

$\mathbf{A}_c = \begin{bmatrix} \mathbf{A}^1 & \mathbf{A}^2 \end{bmatrix}$, N_y rows by $2N_g$ column matrix



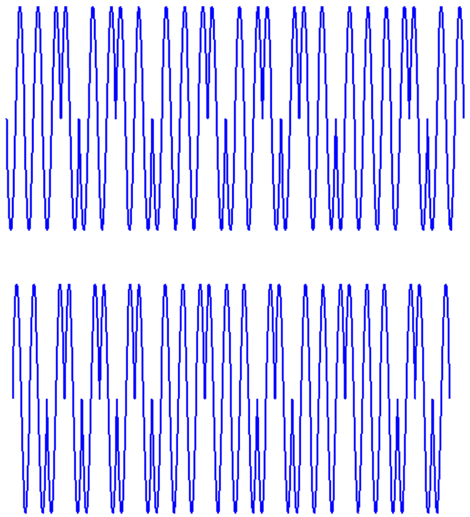
$s^1(t)$

$s^2(t)$

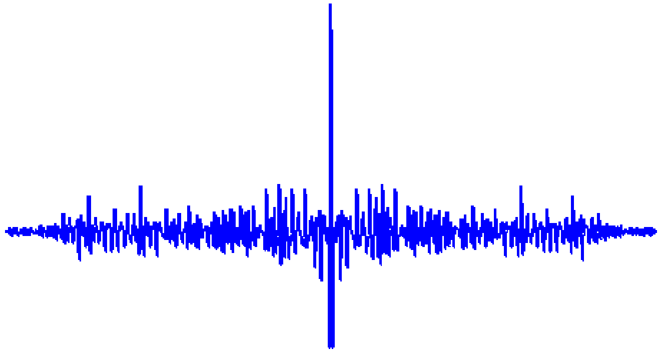


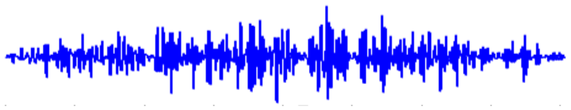
$a^1(t)$

$a^2(t)$



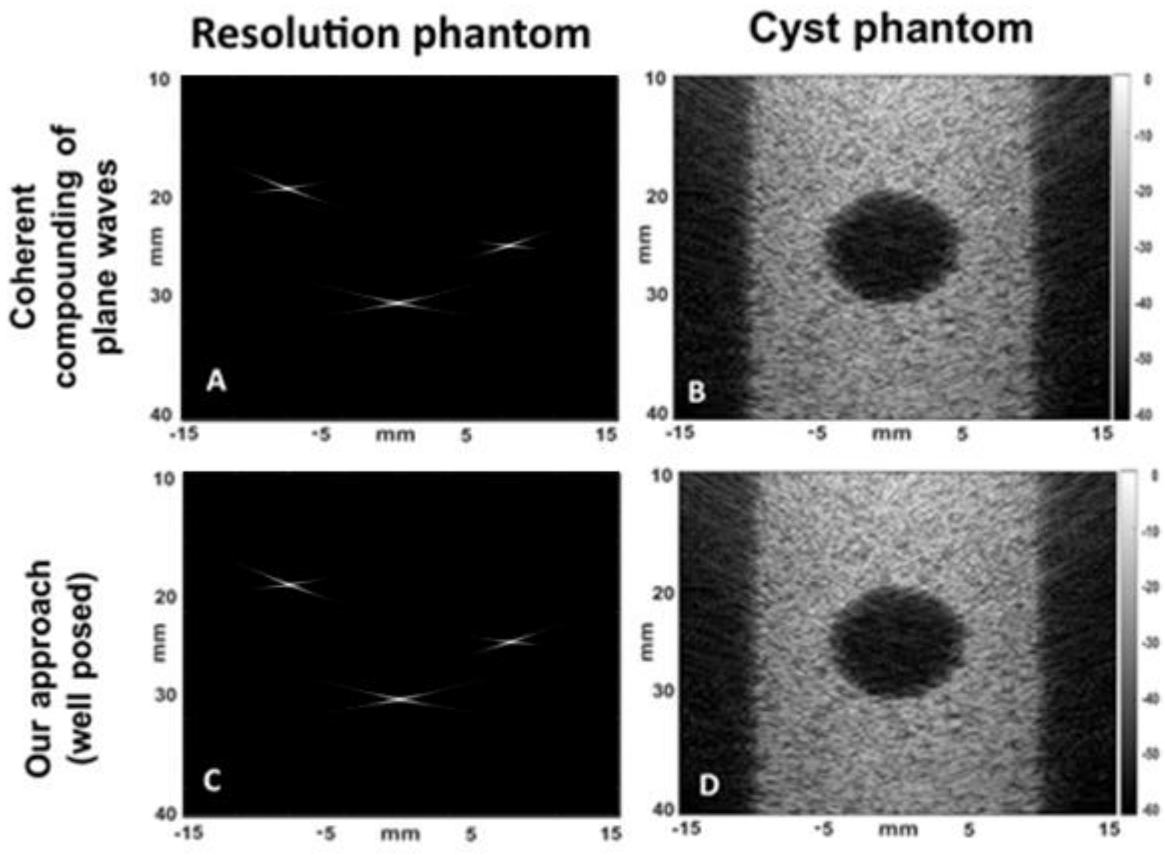
$a^1(t)$ auto-corr.





$a^1(t), a^2(t)$ cross-corr.

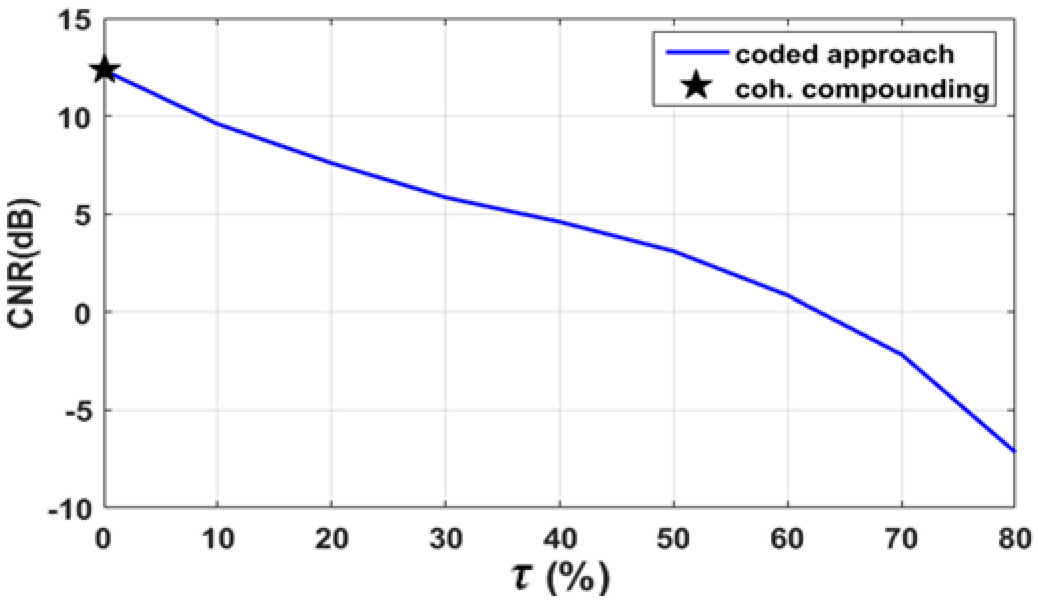
FieldII[1] [2] simulations results



$N_{@5pwi}$ – recording time, classical approach

N_y – recording time, coded approach

$$\tau = \frac{N_{@5pwi} - N_y}{N_{@5pwi}} * 100\%$$ – time gain obtained with coded approach



Reducing the observation time adds a Perturbation Zone, that makes the system ill-posed thus the CNR drops

[1] J. A. Jensen, MBEC, 1996
[2] J. A. Jensen et al., TUFFC, 1992

Conclusion

A mathematical model of the simultaneous coded plane wave imaging

The physical constraints on the medium and emitted signals for a well posed system

Validation of the imaging process model without time gain

Increasing the frame rate leads to image quality decrease

Q: What's next?

Implement regularization to solve the inverse problem

Thank you!