

CYCLIC DELAY-DOPPLER SHIFT: A SIMPLE TRANSMIT DIVERSITY TECHNIQUE FOR DELAY-DOPPLER WAVEFORMS IN DOUBLY SELECTIVE CHANNELS

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ABSTRACT

Delay-Doppler waveform design has been considered as a promising solution to achieve reliable communication under high-mobility channels for the space-air-ground-integrated networks (SAGIN). In this paper, we introduce the cyclic delay-Doppler shift (CDDS) technique for delay-Doppler waveforms to extract transmit diversity in doubly selective channels. Two simple CDDS schemes, named time-domain CDDS (TD-CDDS) and modulation-domain CDDS (MD-CDDS), are proposed in the setting of multiple-input multiple-output (MIMO). We demonstrate the applications of CDDS on two representative delay-Doppler waveforms, namely orthogonal time frequency space (OTFS) and affine frequency division multiplexing (AFDM), by deriving their corresponding CDDS matrices. Furthermore, we prove theoretically and experimentally that CDDS can provide OTFS and AFDM with full transmit diversity gain on most occasions.

Index Terms— CDDS, doubly selective channels, transmit diversity, MIMO, OTFS, AFDM.

1. INTRODUCTION

The next generation wireless network (NGWN) is conceived to support ultra-reliable, high-efficiency, and low-latency communication in high-mobility scenarios of the space-air-ground integrated networks (SAGIN) [1]. Underlying these dynamic channels are the severe Doppler shifts caused by the high-mobility of the vehicles, such as satellites and planes, which impair greatly the orthogonality between the subcarriers in orthogonal frequency division multiplexing (OFDM) [2, 3]. One of the most prospective alternatives of OFDM is delay-Doppler (DD) genre waveform. In particular, two waveforms called orthogonal time frequency space (OTFS) and affine frequency division multiplexing (AFDM) have attracted substantial attention in the past few years [4–13]. The main idea of OTFS and AFDM is modulating the information symbols in the DD domain and the discrete affine Fourier transform (DAFT) domain, respectively, to acquire sparser and more stationary channel representations of the doubly selective channels (DSC), which bring

in strong resilience to delay-Doppler shifts and significant performance enhancement compared to OFDM.

The diversity order achieved by a specific waveform is a key indicator of its capacity to support reliable communications, which is essential for SAGIN applications. One regular approach to increase the diversity order of the systems is to explore the multiple-input multiple-output (MIMO) techniques. Compared to the received antenna diversity, which is quite straightforward to obtain as revealed by the researches on MIMO-OTFS and MIMO-AFDM systems in [14, 15], the acquirement of transmit diversity (TD) is of great challenge. The adaptation of space-time coding with Alamouti and cyclic delay diversity (CDD) structures to OTFS were investigated in [16] and [17], respectively. However, neither of them can provide satisfactory performance either because of the relatively slow-changing channels assumption or the inability to achieve full TD gain. Moreover, to the best of our knowledge, there is no investigation on TD extraction in the emerging AFDM literature.

In this paper, we introduce cyclic delay-Doppler shift (CDDS) for delay-Doppler waveforms to extract transmit diversity in doubly selective channels. The core idea of CDDS is performing delay-Doppler shifts in advance flexibly at different transmit antennas to augment the number of effective propagation paths of the wireless channels. We propose two types of CDDS, termed time-domain CDDS (TD-CDDS) and modulation-domain CDDS (MD-CDDS), which can be conducted by simply multiplying the transmit vector with a well-designed CDDS matrix. To provide a visualized insight into CDDS, we demonstrate its implementations in OTFS and AFDM systems.

2. BASIC CONCEPTS OF OTFS AND AFDM

In this section, the basic concepts of OTFS from [4, 5] and AFDM from [10, 15] are reviewed, which lays the foundations for the CDDS demonstration in Section 3.

2.1. OTFS system

Let $\mathbf{X}_{\text{OTFS}} \in \mathbb{A}^{N \times M}$ denotes a matrix of MN quadrature amplitude modulation (QAM) symbols in the DD domain, where $\mathbb{A}$ represents the modulation alphabet, N and M denote the number of samples in the DD plane correspondingly. Firstly,

This work was supported by Guangdong Natural Science Foundation under Grant 2019A1515011622. (* tangyq8@mail.sysu.edu.cn)

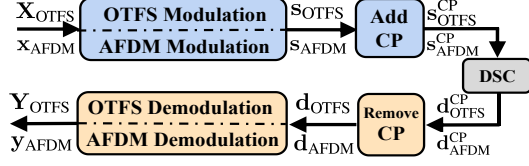


Fig. 1. Modulation/demodulation block diagrams of OTFS and AFDM systems.

the MN information symbols $X_{\text{OTFS}}[k, l]$ are mapped to the time domain symbols $s_{\text{OTFS}}[n]$ via OTFS modulation, where $k \in [0, N-1]$, $l \in [0, M-1]$, and $n \in [0, MN-1]$ denote the indices of Doppler, delay, and time domains, respectively.

After appending a cyclic prefix (CP) to $s_{\text{OTFS}}[n]$, $s_{\text{OTFS}}^{\text{CP}}[n]$ are transmitted into the DSC, which can be modeled in the DD domain as

$$h(\tau, \nu) = \sum_{i=1}^P h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i) \quad (1)$$

where P is the number of paths, h_i is the channel gain of the i -th path, integers $l_i \in [0, l_{\max}]$ and $k_i \in [-k_{\max}, k_{\max}]$ represent the indices of delay tap and Doppler tap corresponding to τ_i and ν_i , respectively ($\tau_i \triangleq \frac{l_i}{M\Delta f}$, $\nu_i \triangleq \frac{k_i}{NT}$), $l_{\max}$ and $k_{\max}$ denote the maximum delay and Doppler, respectively, T and Δf represent the time and frequency domain sample intervals of the time-frequency plane.

At the receiver, the received CP-free time domain symbols $d_{\text{OTFS}}[n]$ can be expressed in vector form as

$$\mathbf{d}_{\text{OTFS}} = \sum_{i=1}^P \mathbf{H}_i \mathbf{s}_{\text{OTFS}} + \mathbf{v} = \mathbf{H} \mathbf{s}_{\text{OTFS}} + \mathbf{v} \quad (2)$$

where $\mathbf{v} \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_{MN})$ is the time domain noise vector, $\mathbf{H} \in \mathbb{C}^{MN \times MN}$ denotes the effective time domain channel matrix, $\mathbf{H}_i = h_i \Delta_{MN}^{k_i} \Pi_{MN}^{l_i}$ represents the time domain subchannel matrix of the i -th path (each path can be viewed as one subchannel), Π_{MN} denotes the one-step $MN \times MN$ forward cyclic-shift matrix which models the delay shift, while the digital frequency shift matrix $\Delta_{MN}^{k_i} \triangleq \text{diag}(e^{j\frac{2\pi}{MN}k_i n}, n = 0, 1, \dots, MN-1)$ models the Doppler shift of the channels.

After performing the OTFS demodulation, $\mathbf{d}_{\text{OTFS}}$ are mapped to the DD domain as $\mathbf{Y}_{\text{OTFS}}$. Assuming bi-orthogonal transmit and receive pulses are used, the 2D circular convolution input-output relationship (IOR) of OTFS in the DD domain is given by (discarding the additive white Gaussian noise (AWGN) term) [5]

$$Y_{\text{OTFS}}[k, l] = \sum_{i=1}^P h_i e^{-j2\pi\nu_i\tau_i} X_{\text{OTFS}}[(k - k_i)_N, (l - l_i)_M] \quad (3)$$

with $(\cdot)_N$ being the N -modulus operator.

2.2. AFDM system

Let $\mathbf{x}_{\text{AFDM}} \in \mathbb{A}^{\hat{N} \times 1}$ denotes a vector of $\hat{N}$ QAM symbols that reside on the DAFT domain, where $\hat{N}$ denotes the number of subcarriers and is set to MN for the convenience of

making a fair comparison between OTFS and AFDM. Firstly, AFDM modulation is performed to map $\mathbf{x}_{\text{AFDM}}$ to the time domain $\mathbf{s}_{\text{AFDM}}$. Similar to the OTFS modulation, a CP should be added to $\mathbf{s}_{\text{AFDM}}$ before transmitting it into the same DSC modeled in (1). After the interaction with the DSC, which is identical to (2), the received CP-free time domain symbols $\mathbf{d}_{\text{AFDM}}$ can be obtained. Then the AFDM demodulation is implemented on $\mathbf{d}_{\text{AFDM}}$ to transform it back to the DAFT domain as $\mathbf{y}_{\text{AFDM}}$. Finally, the IOR of AFDM in the DAFT domain is given by (discarding the AWGN term) [10]

$$y_{\text{AFDM}}[m] = \sum_{i=1}^P h_i e^{j\frac{2\pi}{\hat{N}}(\hat{N}c_1 l_i^2 - m' l_i + \hat{N}c_2(m'^2 - m^2))} x_{\text{AFDM}}[m'] \quad (4)$$

where m denotes the index of the DAFT domain, two AFDM parameters $c_1 = \frac{2(k_{\max} + k_s) + 1}{2\hat{N}}$ (k_s is a non-negative integer which will be illustrated in Section 3.2) and c_2 is either an arbitrary irrational number or a rational number sufficiently smaller than $\frac{1}{2\hat{N}}$, index indicator $\text{ind}_i \triangleq (k_i + 2\hat{N}c_1 l_i)_{\hat{N}}$.

3. CYCLIC DELAY-DOPPLER SHIFT

In this section, we propose TD-CDDS and MD-CDDS for DD waveforms to acquire transmit diversity.

3.1. TD-CDDS

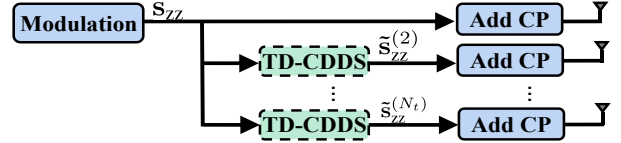


Fig. 2. Block diagrams of the transmitter in TD-CDDS-based systems.

As the block diagrams shown in Fig. 2, TD-CDDS is implemented after modulation at the transmitter with N_t transmit antennas (TA). Here we assume that $\mathbf{s}_{zz} \in \mathbb{C}^{\hat{N} \times 1}$ represents the CP-free transmitted time domain symbols, where “zz” represents any DD-genre waveform, e.g., OTFS and AFDM, $\hat{N}$ denotes the number of samples of $\mathbf{s}_{zz}$ (i.e., $\hat{N} = MN = \hat{N}$).

At the t -th TA ($t = 2, \dots, N_t$), we firstly multiple $\mathbf{s}_{zz}$ with $\Pi_{\hat{N}}^{\tilde{k}}$ and $\Delta_{\hat{N}}^{\tilde{l}}$ to perform $\tilde{l}$ -step cyclic delay shift and $\tilde{k}$ -step cyclic Doppler shift (referred as $[\tilde{k}, \tilde{l}]$ -step CDDS), respectively, which is given by

$$\tilde{\mathbf{s}}_{zz}^{(t)} = \Delta_{\hat{N}}^{\tilde{k}} \Pi_{\hat{N}}^{\tilde{l}} \mathbf{s}_{zz} = \mathbf{H}_{\text{TD-CDDS}}^{[\tilde{k}, \tilde{l}]} \mathbf{s}_{zz} \quad (5)$$

with $\mathbf{H}_{\text{TD-CDDS}}^{[\tilde{k}, \tilde{l}]} = \Delta_{\hat{N}}^{\tilde{k}} \Pi_{\hat{N}}^{\tilde{l}}$ being the $[\tilde{k}, \tilde{l}]$ -step TD-CDDS matrix. Then $\tilde{\mathbf{s}}_{zz}^{(t)}$ is transmitted into the channels at the t -th TA after adding a CP.

Finally, by replacing $\mathbf{s}_{\text{OTFS}}$ in (2) with $\tilde{\mathbf{s}}_{zz}^{(t)}$, we have the t -th TA component of the CP-free received time domain symbols $\mathbf{d}_{zz}$ at arbitrary receive antenna (RA), which is given by

$$\begin{aligned}
\mathbf{d}_{zz}^{(t)} &= \mathbf{H} \tilde{\mathbf{s}}_{zz}^{(t)} = \sum_{i=1}^P h_i^{(t)} \Delta_N^{k_i} \Pi_N^{l_i} \Delta_N^{\tilde{k}_i} \Pi_N^{\tilde{l}_i} \mathbf{s}_{zz} \\
&= \sum_{i=1}^P h_i^{(t)} e^{-j \frac{2\pi}{N} \tilde{k}_i l_i} \Delta_N^{k_i} \Delta_N^{\tilde{k}_i} \Pi_N^{l_i} \Pi_N^{\tilde{l}_i} \mathbf{s}_{zz} \quad (6) \\
&= \sum_{i=1}^P \tilde{h}_i^{(t)} \Delta_N^{\tilde{k}_i} \Pi_N^{\tilde{l}_i} \mathbf{s}_{zz}
\end{aligned}$$

where $\tilde{h}_i^{(t)} = h_i^{(t)} e^{-j \frac{2\pi}{N} \tilde{k}_i l_i}$ denotes the effective channel gain of the i -th path between the t -th TA and the RA, $\tilde{k}_i = k_i + \tilde{k}$, and $\tilde{l}_i = l_i + \tilde{l}$.

The equation (6) shows that the TD-CDDS operation on $\mathbf{s}_{zz}$ in (5) is equivalent to shifting the delay and Doppler spreads of all the original propagation paths simultaneously by $\tilde{l}$ and $\tilde{k}$, respectively. Therefore, TD-CDDS can be considered as the generalization of CDD in [17] from the frequency-selective channels to the doubly selective channels.

3.2. MD-CDDS

Different from TD-CDDS, MD-CDDS is performed before the modulation, as depicted by Fig. 3.

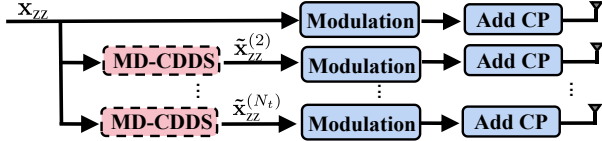


Fig. 3. Block diagrams of the transmitter in MD-CDDS-based systems.

1) MD-CDDS-OTFS: Let $\mathbf{x}_{\text{OTFS}} = \text{vec}(\mathbf{X}_{\text{OTFS}})$, where $\text{vec}(\cdot)$ and $\text{vec}^{-1}(\cdot)$ denote the column-wise vectorization and its reverse, respectively. Then the $\tilde{l}$ -step cyclic delay shift of $\mathbf{X}_{\text{OTFS}}$ can be obtained with $\text{vec}^{-1}(\Pi_{MN}^{N \times \tilde{l}} \mathbf{x}_{\text{OTFS}})$, and the $\tilde{k}$ -step cyclic Doppler shift counterpart can be acquired with $\text{vec}^{-1}((\mathbf{I}_M \otimes \Pi_N^{\tilde{k}}) \mathbf{x}_{\text{OTFS}})$, where $\otimes$ denotes the Hadamard product. Therefore, performing $[\tilde{k}, \tilde{l}]$ -step CDDS on $\mathbf{X}_{\text{OTFS}}$ is equivalent to multiplying $\mathbf{x}_{\text{OTFS}}$ with a MD-CDDS matrix $\mathbf{H}_{\text{MD-CDDS}}^{[\tilde{k}, \tilde{l}], \text{OTFS}}$, which is given by

$$\begin{aligned}
\tilde{\mathbf{X}}_{\text{OTFS}} &= \text{vec}^{-1}(\mathbf{H}_{\text{MD-CDDS}}^{[\tilde{k}, \tilde{l}], \text{OTFS}} \mathbf{x}_{\text{OTFS}}) \\
&= \text{vec}^{-1}(\Pi_{MN}^{N \times \tilde{l}} (\mathbf{I}_M \otimes \Pi_N^{\tilde{k}}) \mathbf{x}_{\text{OTFS}}). \quad (7)
\end{aligned}$$

Then the IOR of OTFS in (3) becomes

$$\begin{aligned}
Y_{\text{OTFS}}[k, l] &= \sum_{i=1}^P h_i e^{-j 2\pi \nu_i \tau_i} \tilde{X}_{\text{OTFS}}[(k - k_i)_N, (l - l_i)_M] \\
&= \sum_{i=1}^P h_i e^{j 2\pi (\nu_i \tilde{\tau} + \tilde{\nu} \tau_i + \tilde{\nu} \tilde{\tau})} e^{-j 2\pi (\nu_i + \tilde{\nu})(\tau_i + \tilde{\tau})} \\
&\quad \times X_{\text{OTFS}}[(k - (k_i + \tilde{k}))_N, (l - (l_i + \tilde{l}))_M] \\
&= \sum_{i=1}^P \tilde{h}_i e^{-j 2\pi \tilde{\nu}_i \tilde{\tau}_i} X_{\text{OTFS}}[(k - \tilde{k}_i)_N, (l - \tilde{l}_i)_M] \quad (8)
\end{aligned}$$

where $\tilde{h} = h_i e^{j 2\pi (\nu_i \tilde{\tau} + \tilde{\nu} \tau_i + \tilde{\nu} \tilde{\tau})}$ is the effective channel gain, $\tilde{l}_i$ and $\tilde{k}_i$ represent the indices of effective delay tap and Doppler tap corresponding to $\tilde{\tau}_i = \tau_i + \tilde{\tau}$ and $\tilde{\nu}_i = \nu_i + \tilde{\nu}$, respectively.

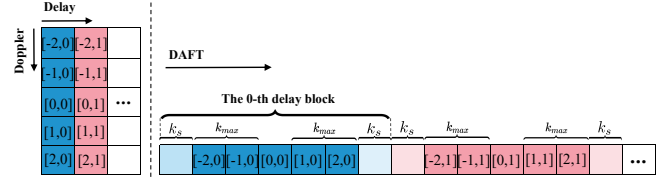


Fig. 4. The bijective relation between delay-Doppler (left) and DAFT (right) domain channel representations ($k_{\max} = 2$ and $l_{\max} = 1$).

2) MD-CDDS-AFDM: As pointed out in [15], the one-dimensional DAFT domain channel representation in AFDM can be viewed as the dimensionality reduction of the DD domain channel in terms of delay blocks splicing (each delay block is separated from its adjacency by two k_s -size spacing band), which is demonstrated in Fig. 4. Consequently, the $[\tilde{k}, \tilde{l}]$ -step MD-CDDS in AFDM can be done by performing Δm -step cyclic shift in the DAFT domain, where $\Delta m = \tilde{k} + [2(k_{\max} + k_s) + 1]\tilde{l}$. Denoting the maximum cyclic Doppler shift at all the TA as $\tilde{k}_{\max}$, then the parameter k_s in c_1 should be set as $\tilde{k}_{\max}$ to ensure that the indices of the new paths after cyclic Doppler shifts remain in the original delay blocks.

We next derive the MD-CDDS matrix of AFDM. To this end, we first multiply $\mathbf{x}_{\text{AFDM}}$ with $\Pi_N^{\Delta m}$ to perform Δm -step DAFT shift. Then the IOR in (4) converts to (9), where $\text{ind}_i \triangleq (\tilde{k}_i + 2\hat{N}c_1\tilde{l}_i)_{\hat{N}}$, $\mathcal{E}(m', \Delta m)$ is given by

$$\begin{aligned}
\mathcal{E}(m', \Delta m) &= \\
&\begin{cases} e^{j \frac{2\pi}{N} (m'(\tilde{l} + 2\hat{N}c_2(\hat{N} - \Delta m)) + \hat{N}^2 c_2(\hat{N} - \Delta m))}, & 0 \leq m' < \Delta m \\ e^{j \frac{2\pi}{N} (m'(\tilde{l} + 2\hat{N}c_2(\hat{N} - \Delta m))}, & \Delta m \leq m' \leq \hat{N} - 1 \end{cases} \quad (10)
\end{aligned}$$

and $\mathcal{C}(l_i, \Delta m) = e^{j \frac{2\pi}{N} (\Delta m(l_i + \hat{N}c_2\Delta m) - \hat{N}c_1\tilde{l}(2l_i + \tilde{l}))}$. It is important to notice that $\mathcal{E}(m', \Delta m)$ has no relevance to m, h_i, l_i , and k_i , which means we can eliminate it in advance at the transmitting end by multiplying $\mathbf{x}_{\text{AFDM}}$ with matrix $\mathbf{P}_N^{(\Delta m)} = \text{diag}(\mathcal{E}^*(m', \Delta m), m' = 0, 1, \dots, \hat{N} - 1)$, where $(\cdot)^*$ denotes the conjugate operator. After that, (10) can be further simplified as

$$y_{\text{AFDM}}[m] = \sum_{i=1}^P \hat{h}_i e^{j \frac{2\pi}{N} (\hat{N}c_1\tilde{l}_i^2 - m'\tilde{l}_i + \hat{N}c_2(m'^2 - m^2))} x_{\text{AFDM}}[m'] \quad (11)$$

where $\hat{h}_i = h_i \mathcal{C}(l_i, \Delta m)$ is the effective channel gain. Here we have the MD-CDDS matrix of AFDM as $\mathbf{H}_{\text{MD-CDDS}}^{[\tilde{k}, \tilde{l}], \text{AFDM}} = \Pi_N^{\Delta m} \mathbf{P}_N^{(\Delta m)}$. By comparing (8) with (3) and (11) with (4), we can notice that the $[\tilde{k}, \tilde{l}]$ -step MD-CDDS can also be considered as shifting the delay and Doppler spreads of all the original propagation paths simultaneously by $\tilde{l}$ and $\tilde{k}$, respectively.

$$\begin{aligned}
y_{\text{AFDM}}[m] &= \sum_{i=1}^P h_i e^{j \frac{2\pi}{N} (\hat{N} c_1 l_i^2 - (m' - \Delta m)_{\hat{N}} l_i + \hat{N} c_2 ((m' - \Delta m)_{\hat{N}}^2 - m^2))} x_{\text{AFDM}}[m'], \quad m' = (m + \text{ind}_i)_{\hat{N}}, \quad 0 \leq m \leq \hat{N} - 1. \\
&= \sum_{i=1}^P h_i \mathcal{E}(m', \Delta m) \mathcal{C}(l_i, \Delta m) e^{j \frac{2\pi}{N} (\hat{N} c_1 (l_i + \tilde{l})^2 - m' (l_i + \tilde{l}) + \hat{N} c_2 (m'^2 - m^2))} x_{\text{AFDM}}[m']
\end{aligned} \tag{9}$$

3.3. Performance analysis of TD-CDDS and MD-CDDS

Denote the DD profile set of the channel before performing CDDS as $\mathbb{P} = \{(k_1, l_1), \dots, (k_P, l_P)\}$, the effective DD profile set after $[\tilde{k}^{(t)}, \tilde{l}^{(t)}]$ -step CDDS at the t -th TA as $\mathbb{P}^{[\tilde{k}^{(t)}, \tilde{l}^{(t)}]}$, and $\tilde{\mathbb{P}} = \mathbb{P} \cup \mathbb{P}^{[\tilde{k}^{(2)}, \tilde{l}^{(2)}]} \cup \dots \cup \mathbb{P}^{[\tilde{k}^{(N_t)}, \tilde{l}^{(N_t)}]}$.

Theorem 1: Full transmit diversity gain can be achieved by OTFS and AFDM through CDDS in doubly selective channels if $|\tilde{\mathbb{P}}| = N_t P$ ($|\cdot|$ denotes the cardinality).

Proof: The diversity orders ρ of the $N_t \times N_r$ MIMO-OTFS (with phase-rotation precoding [14]) and MIMO-AFDM are proven to be $P N_r$ in [14] and [15], respectively. After performing TD-CDDS or MD-CDDS, the MIMO system degenerates into a single-input multiple-output (SIMO) system with $\tilde{P} = |\tilde{\mathbb{P}}|$ effective paths between the transmitter and the receiver, leading to the effective diversity order $\tilde{\rho} = \tilde{P} N_r$. Therefore, when condition $|\tilde{\mathbb{P}}| = N_t P$ is satisfied, $\tilde{P} = N_t P$ and $\tilde{\rho} = N_t P N_r$, i.e., full transmit diversity gain N_t can be acquired. This completes the proof of Theorem 1.

From the viewpoint of the ultimate effect, the difference between TD-CDDS and MD-CDDS lies in the extra complex exponentials of the new channel gains. Moreover, it is worth emphasizing that the MD-CDDS matrices of OTFS and AFDM are simply two sparse permutation matrices for phase compensation, which has no relevance to the DD profile of the real-time channels. It means that the TD-CDDS and MD-CDDS matrices can be calculated only once in advance with relatively low computation complexity at the transmitter despite the ever-changing channels, which is of extreme significance from the perspective of practical implementation, e.g., SAGIN applications. Furthermore, while TD-CDDS is suitable for all the DD waveforms, where all of them share the same TD-CDDS matrix, the MD-CDDS matrices should be designed elaborately according to the IOR of different waveforms. However, MD-CDDS can be combined with other precoding techniques to achieve joint precoding without introducing additional operation overhead to the transmitter.

4. SIMULATION RESULTS

In this section, we verify the effectiveness of the two proposed CDDS schemes through simulations. We adopt carrier frequency $f_c = 4$ GHz, $\Delta f_{\text{OTFS}} = 20$ kHz, $N_{\text{OTFS}} = 5$, $M_{\text{OTFS}} = 2$, $\Delta f_{\text{AFDM}} = 4$ kHz, $N_{\text{AFDM}} = 10$ to ensure the same resources are occupied by OTFS and AFDM ($M_{\text{OTFS}} N_{\text{OTFS}} = N_{\text{AFDM}}$, $M_{\text{OTFS}} \Delta f_{\text{OTFS}} = N_{\text{AFDM}} \Delta f_{\text{AFDM}}$). $P = 2$ paths with DD profile of $\{(-1, 0), (1, 0)\}$ (corresponding to a maximum Doppler shift of 4 kHz and a maximum UE speed of 1,080 kmph), BPSK and ML detector are used. $[1, 1]$ -step CDDS are applied at the second TA to

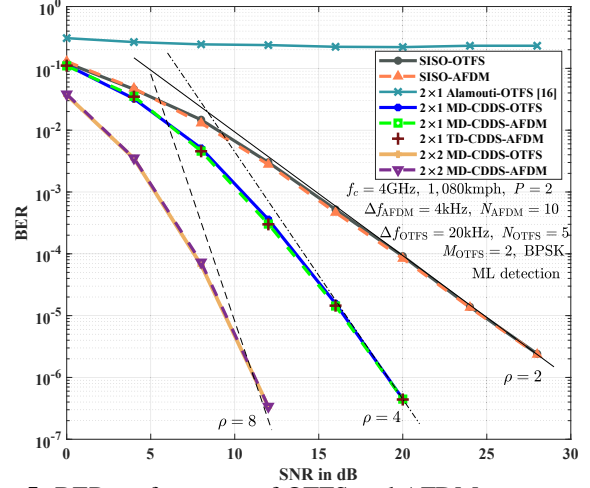


Fig. 5. BER performance of OTFS and AFDM systems with different MIMO configurations.

ensure full TD can be achieved. Fig. 5 shows the bit error ratio (BER) performance comparison between OTFS and AFDM systems with different MIMO configurations. We can observe that the diversity orders of the single-input single-output (SISO) systems, the 2×1 MD-CDDS systems, and the 2×2 MD-CDDS systems of OTFS and AFDM are 2, 4, and 8, respectively, which is associated with Theorem 1. Moreover, the 2×1 MD-CDDS-AFDM and the 2×1 TD-CDDS-AFDM systems establish the same performance. This is because the extra complex exponentials of the two new channel gains caused by TD-CDDS in (6) and MD-CDDS in (11) will not change the statistical characters of the initial channel gains. Furthermore, the 2×1 MD-CDDS-OTFS outperforms the 2×1 Alamouti-OTFS in [16] significantly, given that the DSC can no longer be approximate to unchanged within two successive frames under these high-mobility channels.

5. CONCLUSION AND FUTURE WORK

In this work, we present a novel transmit diversity technique, named cyclic delay-Doppler shift, for delay-Doppler-genre waveforms. The TD-CDDS and MD-CDDS matrices of OTFS and AFDM are derived, supporting the conclusion that full transmit diversity can be achieved with proper CDDS-step arrangements. It is worth emphasizing that CDDS is performed within only one transmit frame with relatively low complexity and no additional procedure at the receiver, which is particularly attractive for ultra-reliable and low-latency communications in high-mobility wireless systems, such as the space-ground-integrated communication systems. In the near future, we will analyze CDDS comprehensively in the condition of practical pulse-shaping, fractional delay-Doppler effects, and channel estimation errors.

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