Adaptive Minimum Bit Error Rate Beamforming Assisted QPSK Receiver

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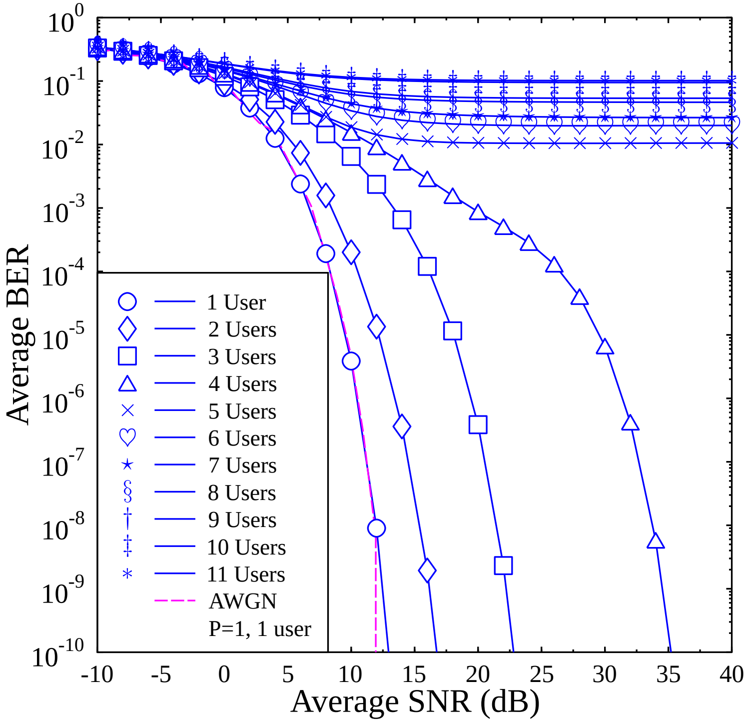
Overview

Adaptive beamforming assisted multiuser detection for multiple receive antennas aided SDMA systems with QPSK modulation scheme

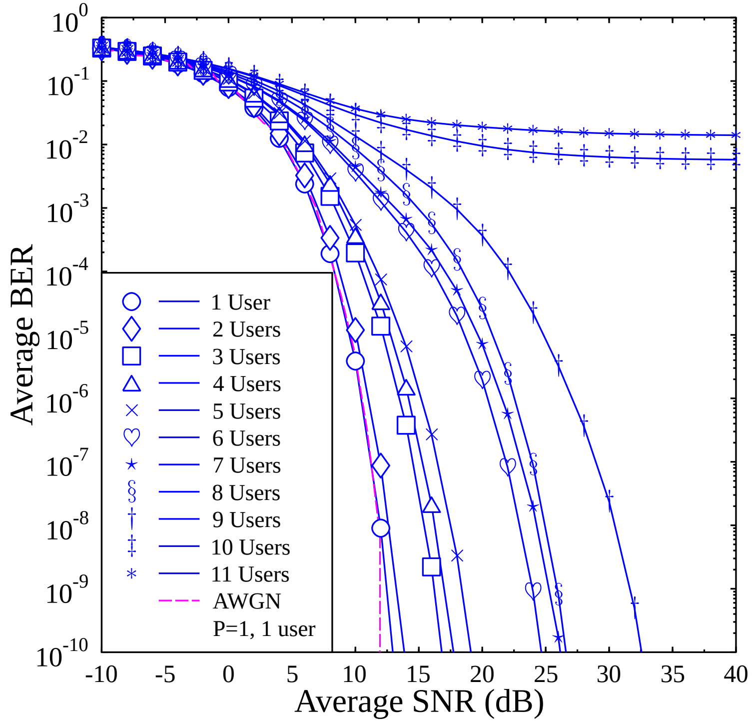
- Motivation for minimum bit error rate design
- System model and standard minimum mean square error solution
- Minimum bit error rate beamforming solution
- Adaptive implementation of minimum bit error rate design
- Simulation results

Motivation

◯ 128-subcarriers OFDM 4-receive-antennas aided SDMA, observing user 1 BER with increasing number of users:



(a) MMSE



(b) MBER

◯ Given number of antennas, capacity is fixed. But changing design from MMSE to MBER ⇒ improve performance or better realizing the capacity

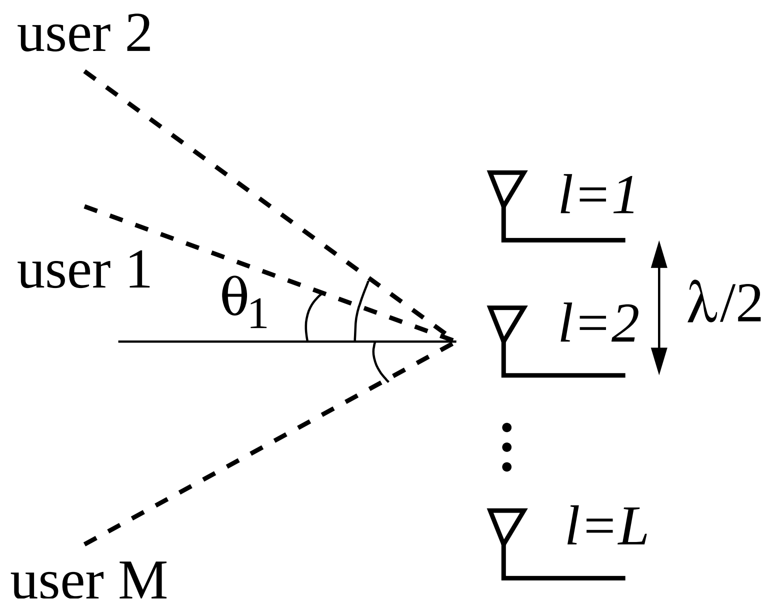
System Model

○ L receive antennas and M users, point-source model with narrow band channels A_i for $1 \leq i \leq M$

○ Received signal model:

$$\mathbf{x}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k)$$

where $\mathbf{n}(k) = [n_1(k) \cdots n_L(k)]^T$ is system noise vector, user QPSK symbol vector $\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^T$, and system matrix



$$\mathbf{P} = [\alpha_1 A_1 \mathbf{s}_1 \; \alpha_2 A_2 \mathbf{s}_2 \cdots \alpha_M A_M \mathbf{s}_M]$$

with $\mathbf{s}_i$, $1 \leq i \leq M$, denoting steering vectors and α_i^2 transmitted signal powers. User 1 is desired user

Beamforming

◯ Linear beamformer:

$$y(k) = \mathbf{w}^H \mathbf{x}(k) = \mathbf{w}^H (\bar{\mathbf{x}}(k) + \mathbf{n}(k)) = \bar{y}(k) + e(k)$$

with beamformer weight vector $\mathbf{w} = [w_1 \ w_2 \cdots w_L]^T$, and the decision for $b_1(k)$:

$$\hat{b}_1(k) = \text{sgn}(y_R(k)) + j\text{sgn}(y_I(k))$$

◯ Let $\mathbf{w}^H \mathbf{P} = \mathbf{w}^H [\mathbf{p}_1 \ \mathbf{p}_2 \cdots \mathbf{p}_M] = [c_1 \ c_2 \cdots c_M]$. Then

$$y(k) = c_1 b_1(k) + \sum_{i=2}^{M} c_i b_i(k) + e(k)$$

To make sure c_1 being real and positive, weight vector rotation:

$$\mathbf{w}^{\text{new}} = \frac{c_1^{\text{old}}}{|c_1^{\text{old}}|} \mathbf{w}^{\text{old}}$$

◯ Minimum mean square solution: $\mathbf{w}_{\text{MMSE}} = \left(\mathbf{P}\mathbf{P}^H + \sigma_n^2 \mathbf{I}_L\right)^{-1} \mathbf{p}_1$, σ_n^2 being system noise variance and $\mathbf{I}_L$ identity matrix

Bit Error Rate

- Conditional PDF of $y(k)$ given $b_1(k) = +1 + j$:

$$p(y| + 1 + j) = \frac{1}{N_{sb}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} \frac{1}{2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}} \exp\left(-\frac{|y - \bar{y}^{(q)}|^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}} \right)$$

where $\bar{y}^{(q)} \in \mathcal{Y}_{+,+}$ are points of $\bar{y}(k)$ conditioned on $b_1(k) = +1 + j$ and N_{sb} is number of points in $\mathcal{Y}_{+,+}$

- BER:

$$P_E(\mathbf{w}) = \frac{1}{2} \left(P_{E_R}(\mathbf{w}) + P_{E_I}(\mathbf{w}) \right)$$

with

$$P_{E_R}(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} Q\left(g_R^{(q)}(\mathbf{w}) \right) \qquad P_{E_I}(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} Q\left(g_I^{(q)}(\mathbf{w}) \right)$$

$$g_R^{(q)}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{R,1}^{(q)}) \bar{y}_R^{(q)}}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}} \qquad g_I^{(q)}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{I,1}^{(q)}) \bar{y}_I^{(q)}}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}}$$

Minimum Bit Error Rate

◯ MBER solution:

$$\mathbf{w}_{\mathrm{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w})$$

◯ No closed-from solution, but it can be obtained via gradient-based optimization, with gradient for normalized $\mathbf{w}$ given by

$$\nabla P_E(\mathbf{w}) = \frac{1}{2}\left(\nabla P_{E_R}(\mathbf{w}) + \nabla P_{E_I}(\mathbf{w})\right)$$

$$\nabla P_{E_R}(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n} \sum_{\bar{y}^{(q)}\in\mathcal{Y}_{+,+}} \exp\left(-\frac{\left(\bar{y}_R^{(q)}\right)^2}{2\sigma_n^2}\right) \operatorname{sgn}\left(b_{R,1}^{(q)}\right)\left(\bar{y}_R^{(q)}\mathbf{w} - \bar{\mathbf{x}}^{(q)}\right)$$

$$\nabla P_{E_I}(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n} \sum_{\bar{y}^{(q)}\in\mathcal{Y}_{+,+}} \exp\left(-\frac{\left(\bar{y}_I^{(q)}\right)^2}{2\sigma_n^2}\right) \operatorname{sgn}\left(b_{I,1}^{(q)}\right)\left(\bar{y}_I^{(q)}\mathbf{w} + j\bar{\mathbf{x}}^{(q)}\right)$$

Adaptive Implementation

○ Given a block of training data $\{\mathbf{x}(k), b_1(k)\}_{k=1}^{K}$, a Parzen window estimate for the PDF of $y(k)$, $p(y)$, is given by

$$\hat{p}(y) = \frac{1}{K 2\pi \rho_n^2} \sum_{k=1}^{K} \exp\left(-\frac{|y - y(k)|^2}{2\rho_n^2} \right)$$

where ρ_n is kernel width

○ From the estimated PDF $\hat{p}(y)$, one obtains the estimated BER $\hat{P}_E(\mathbf{w})$

○ Block-data based adaptive MBER solution: minimizing $\hat{P}_E(\mathbf{w})$ using a gradient-based optimization

○ To derive sample-by-sample adaptation, consider one-sample PDF "estimate":

$$\hat{p}(y, k) = \frac{1}{2\pi \rho_n^2} \exp\left(-\frac{|y - y(k)|^2}{2\rho_n^2} \right)$$

Least Bit Error Rate

- Conceptually, from one-sample estimate $\hat{p}(y,k)$, one has instantaneous BER $\hat{P}_E(\mathbf{w},k)$

- Minimizing this instantaneous BER with stochastic gradient

$$\nabla \hat{P}_E(\mathbf{w},k) = \frac{\left(-\mathrm{sgn}(b_{R,1}(k))\exp\left(-\frac{y_R^2(k)}{2\rho_n^2}\right) + j\mathrm{sgn}(b_{I,1}(k))\exp\left(-\frac{y_I^2(k)}{2\rho_n^2}\right)\right)}{4\sqrt{2\pi}\rho_n}\mathbf{x}(k)$$

leads to the LBER:

$$\begin{aligned}
\mathbf{w}(k+1) &= \mathbf{w}(k) + \mu\left(-\nabla \hat{P}_E(\mathbf{w}(k),k)\right) \\
c_1 &= \mathbf{w}^H(k+1)\mathbf{p}_1 \\
\mathbf{w}(k+1) &= \frac{c_1}{|c_1|}\mathbf{w}(k+1)
\end{aligned}$$

Comparison with Least Mean Square

- Compared with LMS:

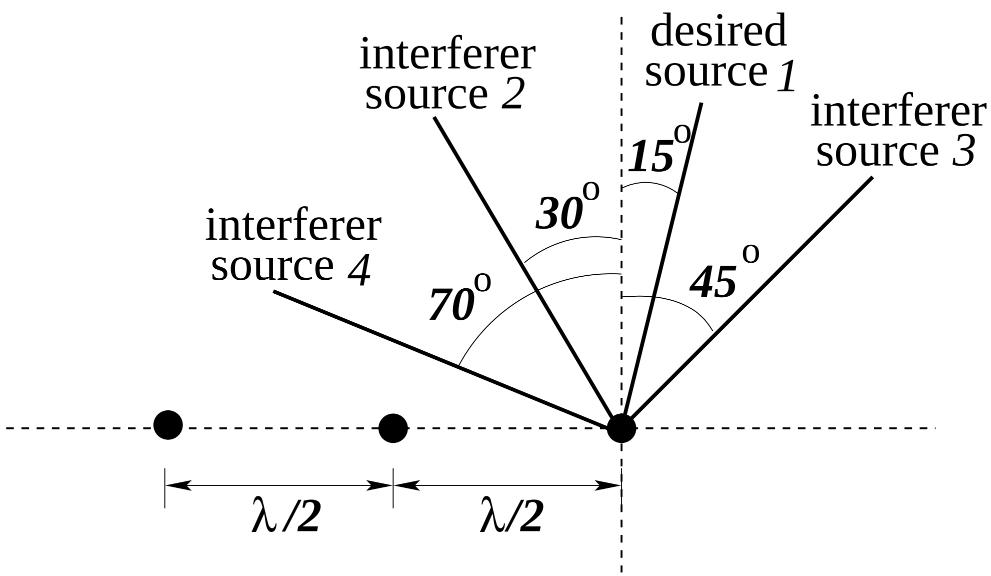
$$\begin{aligned}\mathbf{w}(k+1) &= \mathbf{w}(k)+\mu\left(b_1(k)-y(k)\right)^*\mathbf{x}(k)\\ c_1 &= \mathbf{w}^H(k+1)\mathbf{p}_1\\ \mathbf{w}(k+1) &= \frac{c_1}{|c_1|}\mathbf{w}(k+1)\end{aligned}$$

LBER has a similarly low complexity:

algorithm	multiplications	additions	square root	$\exp(\bullet)$
LMS	$16\times L+6$	$14\times L-2$	1	–
LBER	$16\times L+10$	$14\times L-4$	1	2

Simulation (Fixed Channels)

3-element antenna array, 4 users, and fixed channel conditions: $A_i = 1 + j0$ for $1 \le i \le 4$



Desired user signal to noise ratio:

$$\text{SNR} = \frac{1}{\sigma_n^2}$$

Desired user signal to interferer i ratio:

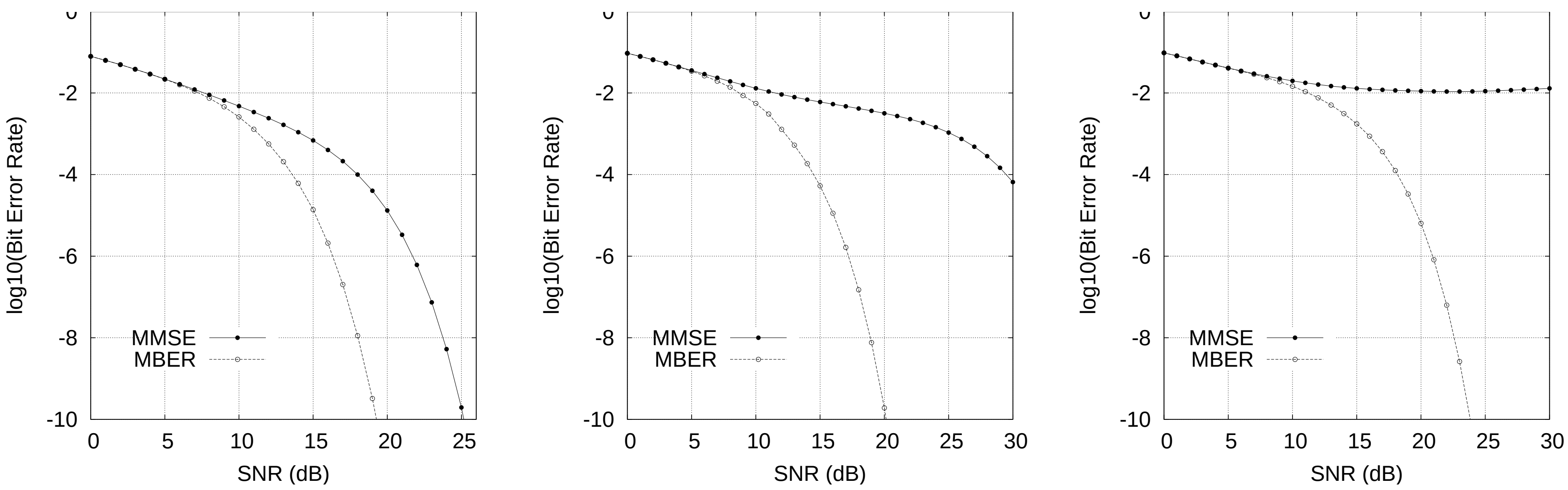
$$\text{SIR}_i = \frac{\alpha_1^2}{\alpha_i^2}, \quad i = 2, 3, 4$$

Comparison of BERs

(**a**) desired user and all three interfering users had equal power: $\text{SIR}_i = 0$ dB, $i = 2, 3, 4$

(**b**) desired user and interfering users 2 and 3 had equal power but interfering user 4 had 6 dB more power than desired user: $\text{SIR}_2 = 0$ dB, $\text{SIR}_3 = 0$ dB, $\text{SIR}_4 = -6$ dB

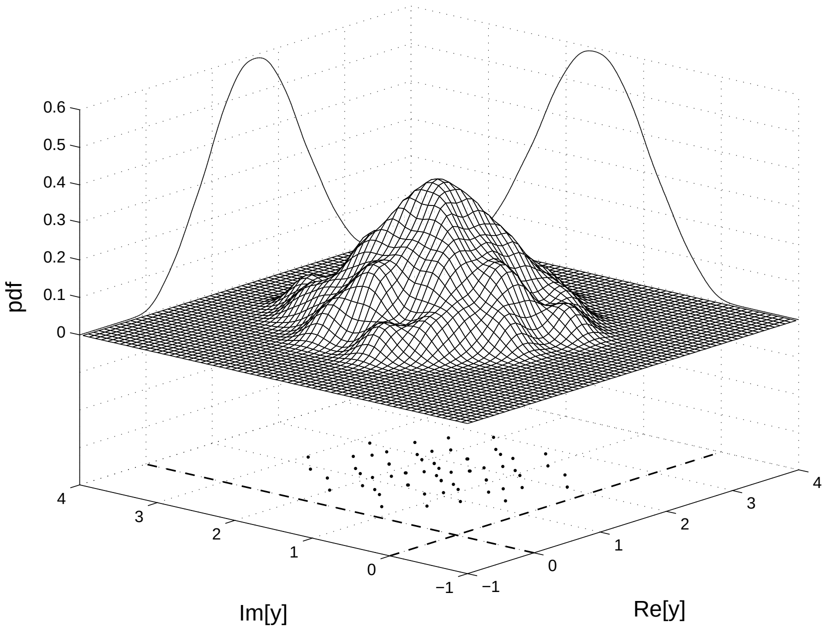
(**c**) all three interfering users had 2 dB more power than desired user: $\text{SIR}_i = -2$ dB, $i = 2, 3, 4$



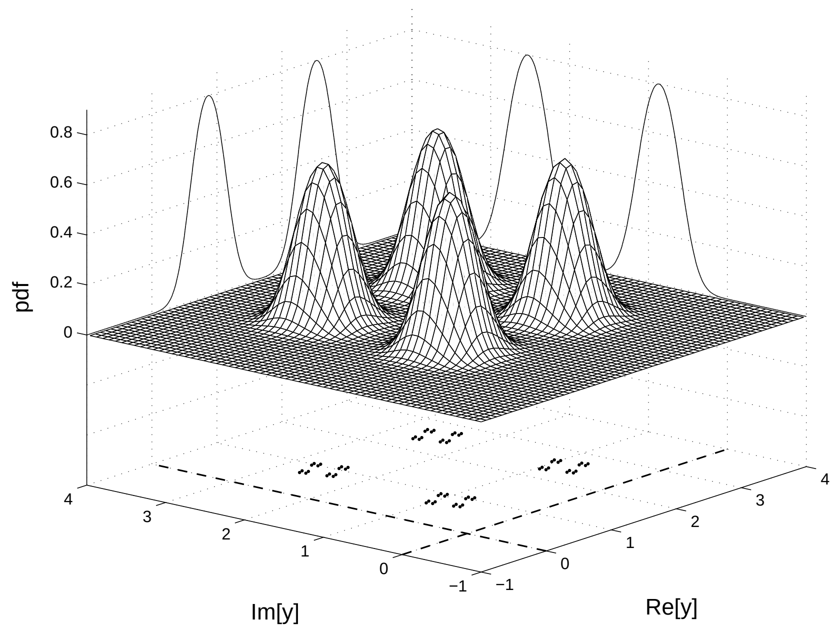
(a) (b) (c)

Comparison of PDFs

Case (**a**) with SNR$= 15$ dB: conditional PDFs, marginal conditional PDFs, and signal subsets $\mathcal{Y}_{+,+}$



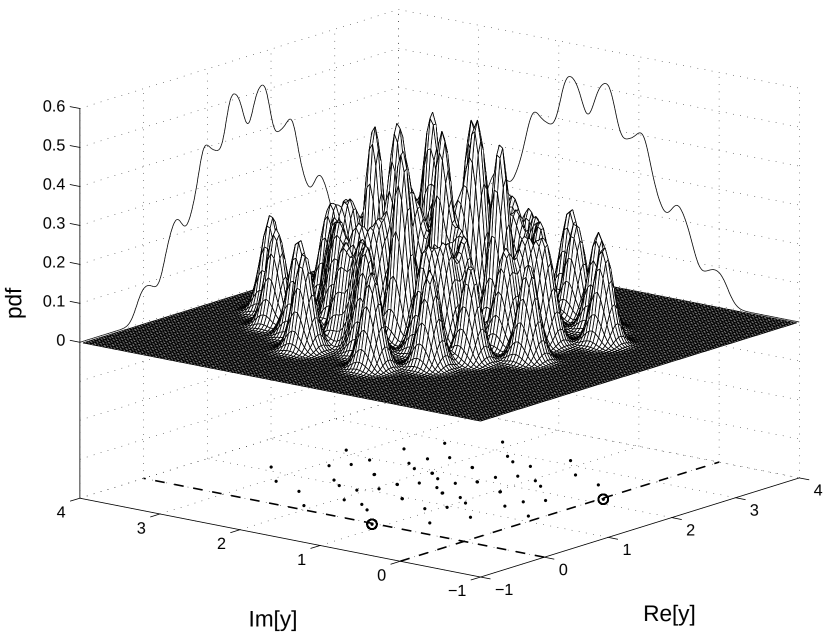
MMSE



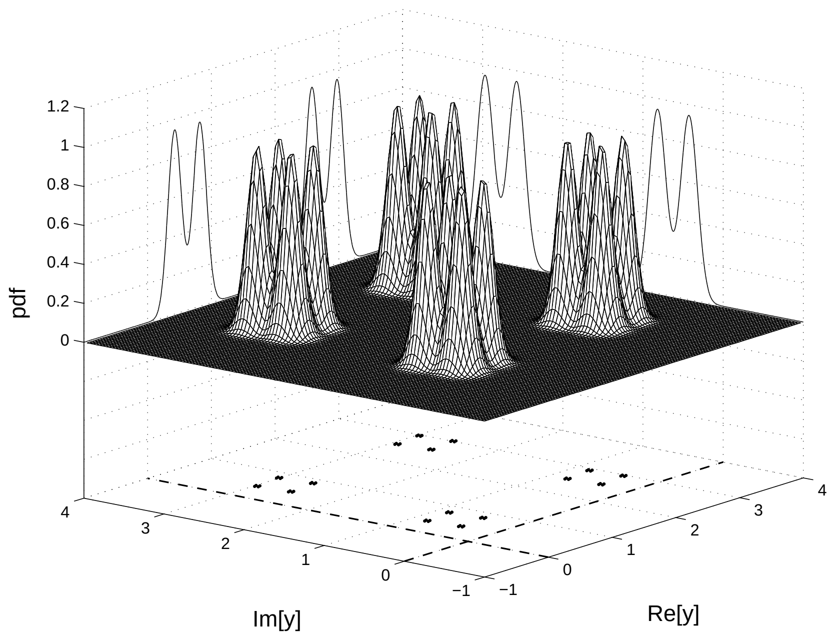
MBER

Comparison of PDFs

Case (**c**) with SNR$= 20$ dB: conditional PDFs, marginal conditional PDFs, and signal subsets $\mathcal{Y}_{+,+}$



MMSE

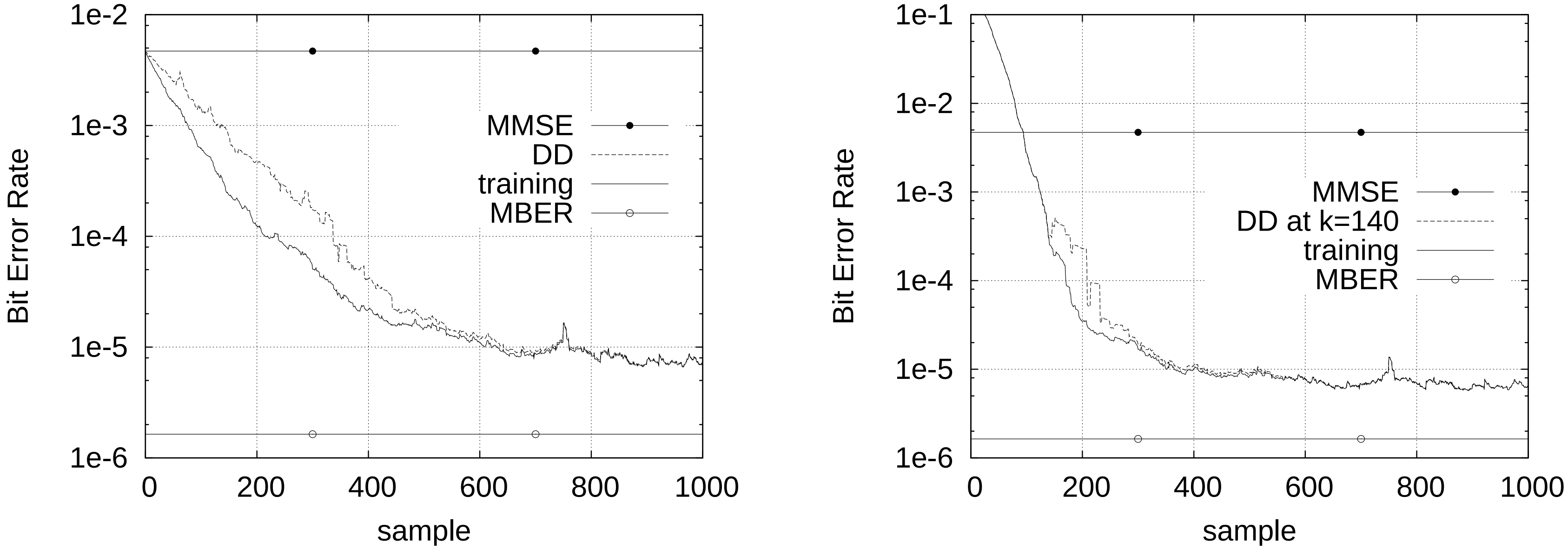


MBER

Learning Curves of LBER

Case (**b**) with SNR$= 17$ dB: (i) $\mathbf{w}(0) = \mathbf{w}_{\text{MMSE}}$, and
(ii) $\mathbf{w}(0) = [0.0 + j0.1\ 0.1 + j0.0\ 0.1 + j0.0]^T$

DD: decision-directed adaptation with $\hat{b}_1(k)$ substituting $b_1(k)$



(i) (ii)

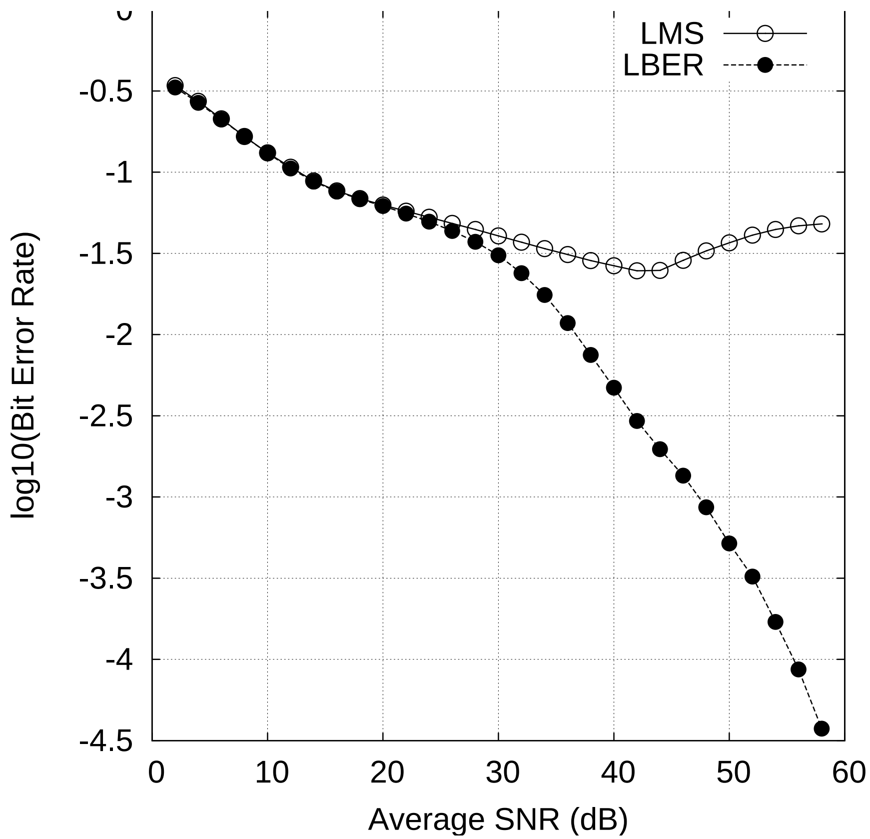
Simulation (Fading Channels)

Same 3-element antenna array and 4 users, but magnitudes of channels A_i, $1 \leq i \leq 4$, were Rayleigh processes, each with root mean power $\sqrt{0.5} + j\sqrt{0.5}$

Fading was continuous, yielding different channel magnitude and phase for each transmitted symbol

Fading is slow at normalized Doppler frequency 10^{-6}

Frame structure: 40 training symbols followed by 400 data symbols



Conclusions

- An adaptive beamforming assisted multiuser detection scheme based on the minimum bit error rate design has been derived for multiple receive antennas aided SDMA systems

- The minimum bit error rate design provides better performance and improves system capacity, compared with the standard minimum mean square error design

- Sample-by-sample adaptation has been realized using the least bit error rate algorithm, which has a similarly low complexity as the least mean square algorithm, for the QPSK modulation

- Our approach can be extended to space-time multiuser detection scheme for generic SDMA systems with wideband channels