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Asymptotic Performance of DS-CDMA with linear MMSE Receiver and Limited Feedback

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***Abstract*—Signature quantization for reverse-link Direct Sequence (DS)- Code Division Multiple Access (CDMA) is considered. A receiver assumed to have perfect estimates of channel and interference covariance, selects the signature that maximizes signal-to-interference plus noise ratio (SINR) for a desired user from a signature codebook. The codebook index corresponding to the optimal signature is relayed to the user with finite number of bits via a feedback channel. Previously, it was shown that a Random Vector Quantization (RVQ) codebook, which contains independent isotropically distributed vectors, is optimal (i.e., maximizes SINR) in a large system limit in which number of interfering users, processing gain, and feedback bits tend to infinity with fixed ratios. Here we derive exact expressions for a large system SINR for the user whose signature is selected from RVQ codebook. We assume that the receiver is linear minimum mean squared error (MMSE) and consider both ideal and multipath fading channels.**

I. Introduction

To improve user performance in Direct Sequence (DS)-Code Division Multiple Access (CDMA), a signature code of the user can be adapted to avoid interference from other users. Several work in the literature [1]–[7] have investigated a joint transmitter-receiver signature optimization and showed that a performance gap between optimized and random signatures can be substantial. However, knowledge of channel and interference covariance is required at both the transmitter and receiver. All of the work mentioned assume that perfect estimates of channel and interference covariance are available. This assumption, especially at the transmitter, is not very practical.

In a wireless system, a receiver typically estimates channel coefficients and interference covariance from pilot signals. The accuracy of the estimation increases with amount of available pilots. The transmitter, on the other hand, is usually unable to directly estimate the forward channel. However, channel information may be obtained from the receiver via a feedback channel. In recent years, many researchers [8]–[15] have proposed feedback schemes in which the receiver computes and quantizes the optimal signature and relays the quantized coefficients to the transmitter via a rate-limited feedback channel. Reference [10]–[15] consider multiantenna systems where spatial signatures are optimized and quantized. Here our interest is signature quantization in DS-CDMA and its performance, which depends largely on quantization codebook and available feedback rate.

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The signature codebook is known *a priori* at both the transmitter and receiver. With B feedback bits, the receiver selects the signature vector, which maximizes the instantaneous signal-to-interference plus noise ratio (SINR), from 2^B-signature codebook and relays the corresponding index to the transmitter via an error-free feedback channel. Reference [8] proposed a Random Vector Quantization (RVQ) codebook, which consists of independent isotropically distributed vectors and showed that the RVQ codebook is optimal (i.e., maximize the SINR over all codebooks) in a large system limit in which number of users K, processing gain N, and feedback bits B tend to infinity with fixed $\bar{K} = K/N$ and $\bar{B} = B/N$. The approximation for a large system SINR was derived in [8] for the minimum mean squared error (MMSE) receiver and was shown to predict the performance of a finite-size system well for small $\bar{B}$.

Recently, [9] derives the exact expression of a large system SINR for RVQ with a matched filter. (Similar results for the performance of RVQ in multiantenna system were derived in [12].) We apply similar techniques used in [9], [12] to derive expressions for asymptotic SINR for RVQ signature with linear MMSE receiver. We first consider a nonfading channel and derive an exact expression for a large system SINR, which is a function of $\bar{K}$ and $\bar{B}$. Comparison between the large system SINR and the approximation derived in [8], which over-estimates the performance for large $\bar{B}$, is shown. Numerical examples show that the large system results predict the performance of the finite-size system well. We also extend the results to flat and frequency-selective fading channels and the case in which users are assigned different transmit powers.

II. System Model

We consider a discrete-time reverse-link synchronous DS-CDMA in which there are K users and processing gain N. The $N \times 1$ received vector is given by

$$\boldsymbol{r} = \sum_{k=1}^{K} \sqrt{A_k} \boldsymbol{H}_k \boldsymbol{s}_k b_k + \boldsymbol{n} \tag{1}$$

where $\sqrt{A_k}$ is the amplitude of user k, $\boldsymbol{H}_k$ is the $N \times N$ channel matrix for user k, $\boldsymbol{s}_k$ is the $N \times 1$ signature vector for user k, b_k is the transmitted symbol for user k, and $\boldsymbol{n}$ is the additive white Gaussian noise with zero mean and covariance $\sigma_n^2\boldsymbol{I}$. For ideal nonfading channel, $\boldsymbol{H}_k = \boldsymbol{I}$. For frequency-selective channel, we assume that the symbol duration is much longer than the delay spread and, thus, we discard any inter-symbol interference. Assuming that each user traverses L fading paths, we have

$$\boldsymbol{H}_k = \begin{bmatrix} h_{k,1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & h_{k,1} & & \vdots & 0 & & \vdots \\ h_{k,L} & \vdots & \ddots & 0 & \vdots & & 0 \\ 0 & h_{k,L} & & h_{k,1} & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots & h_{k,1} & & 0 \\ 0 & \vdots & & h_{k,L} & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & h_{k,L} & \cdots & h_{k,1} \end{bmatrix} \quad (2)$$

where fading gains for user k, $h_{k,1}, \ldots, h_{k,L}$, are independent complex Gaussian random variables with zero mean and variances $E|h_{k,1}|^2, \ldots, E|h_{k,l}|^2$, respectively. For flat fading channel ($L = 1$), $\boldsymbol{H}_k = h_{k,1}\boldsymbol{I}$.

We consider a linear MMSE receiver, which is shown to be robust in suppressing multiple-access interference [16]. Without loss of generality, we consider user 1 whose MMSE receive filter is given by

$$\boldsymbol{c}_1 = \boldsymbol{R}^{-1}\tilde{\boldsymbol{s}}_1 \quad (3)$$

where we let $\tilde{\boldsymbol{s}}_k \triangleq \boldsymbol{H}_k\boldsymbol{s}_k$ and the received covariance

$$\boldsymbol{R} = E[\boldsymbol{r}\boldsymbol{r}^\dagger] = \sum_{k=1}^{K} A_k \tilde{\boldsymbol{s}}_k \tilde{\boldsymbol{s}}_k^\dagger + \sigma_n^2\boldsymbol{I}, \quad (4)$$

assuming that b_k's are independent and identically distributed *(i.i.d.)* with zero mean and unit variance. The associated SINR for user 1 is given by

$$\beta_1 = \frac{|\sqrt{A_1}\boldsymbol{c}_1^\dagger \tilde{\boldsymbol{s}}_1|^2}{\boldsymbol{c}_1^\dagger \boldsymbol{R}_1 \boldsymbol{c}_1} = \frac{A_1 \boldsymbol{s}_1^\dagger \boldsymbol{H}_1^\dagger \boldsymbol{R}_1^{-1} \boldsymbol{H}_1 \boldsymbol{s}_1}{\boldsymbol{s}_1^\dagger \boldsymbol{s}_1} \quad (5)$$

where the interference-plus-noise covariance

$$\boldsymbol{R}_1 = \tilde{\boldsymbol{S}}_1 \boldsymbol{A}_1 \tilde{\boldsymbol{S}}_1^\dagger + \sigma_n^2\boldsymbol{I}, \quad (6)$$

$\tilde{\boldsymbol{S}}_1$ is the $N\times(K-1)$ effective signature matrix whose columns consist of $\tilde{\boldsymbol{s}}_k$, $\forall k \neq 1$ and $\boldsymbol{A}_1$ is the $(K-1)\times(K-1)$ diagonal matrix whose diagonal entries are $A_2, \ldots, A_K$. We note that, for given $\boldsymbol{R}_1$ and $\boldsymbol{H}_1$, the SINR for user 1 is a function of the signature $\boldsymbol{s}_1$.

The receiver, which is assumed to have a perfect estimate of the interference covariance $\boldsymbol{R}_1$, can optimize the signature for the desired user to avoid interference from other users. The optimal $\boldsymbol{s}_1$, which maximizes (5), is the eigenvector of $\boldsymbol{H}_1^\dagger \boldsymbol{R}_1^{-1} \boldsymbol{H}_1$ corresponding to the maximum eigenvalue. Ideally, the receiver sends the optimal signature back to user 1 via a feedback channel and the user changes the signature, accordingly. Practically, a feedback channel has limited rate. Thus, the receiver can only relay finite number of feedback bits to the user. (We assume that the feedback does not incur any errors.) With B bits, the receiver selects the signature from a signature set or codebook containing 2^B signatures. This codebook is designed *a priori*, and is known at both the user and receiver. The performance of the optimized user depends on the codebook. Several work [8], [10]–[14], [17] focused on codebook design and analyzed the associated performance. (All of work previously mentioned except [8] are in context of spatial signature in a multiantenna channel.) In this work, we analyze the performance of a Random Vector Quantization (RVQ) codebook proposed by [8]. RVQ codebook $\mathcal{V} = \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{2^B}\}$ in which $\boldsymbol{v}_j$'s are independent isotropically distributed with unit norm ($\|\boldsymbol{v}_j\| = 1$). In other words, signature vectors in RVQ codebook are uniformly distributed on a surface of an N-dimensional unit sphere. In [8], [9], [12], RVQ was shown to maximize SINR over all quantization codebooks in a large system limit to be defined.

Given the codebook $\mathcal{V}$, the receiver selects

$$\boldsymbol{s}_1 = \arg\max_{\boldsymbol{v}_j \in \mathcal{V}} \left\{ \beta_1(\boldsymbol{v}_j) = \frac{A_1 \boldsymbol{v}_j^\dagger \boldsymbol{H}_1^\dagger \boldsymbol{R}_1^{-1} \boldsymbol{H}_1 \boldsymbol{v}_j}{\boldsymbol{v}_j^\dagger \boldsymbol{v}_j} \right\}. \quad (7)$$

The index of the optimal signature vector is relayed to user 1 via a feedback channel. The corresponding SINR for user 1 averaged over interfering signatures, channel matrices, and codebook is given by

$$\beta_{\mathsf{rvq}} = E[\max_{1\le j\le 2^B} \beta_1(\boldsymbol{v}_j)]. \quad (8)$$

We are interested in analyzing β_{rvq} and how β_{rvq} relates to other system parameters (e.g., feedback bits B, number of users K, and number of fading paths L).

III. Large System SINR

Since $\boldsymbol{v}_j$'s in RVQ codebook are *i.i.d.*, the corresponding $\beta_1(\boldsymbol{v}_j)$'s are also *i.i.d.* for given $\boldsymbol{R}_1$, and $\boldsymbol{H}_1$. Let $f_{\beta|\boldsymbol{R}_1,\boldsymbol{H}_1}(\cdot)$ and $F_{\beta|\boldsymbol{R}_1,\boldsymbol{H}_1}(\cdot)$ be probability density function (pdf) and cumulative distribution function (cdf) for $\beta_1(\boldsymbol{v}_j)$, respectively. The SINR with the optimal signature averaged over the codebook is given by

$$\begin{aligned} &E_{\mathcal{V}}[\max\{\beta(\boldsymbol{v}_1), \ldots, \beta(\boldsymbol{v}_{2^B})\}|\boldsymbol{R}_1, \boldsymbol{H}_1] \\ &\quad = 2^B \int_0^\infty x[F_{\beta|\boldsymbol{R}_1,\boldsymbol{H}_1}(x)]^{2^B-1} f_{\beta|\boldsymbol{R}_1,\boldsymbol{H}_1}(x)\,\mathrm{d}x. \quad (9) \end{aligned}$$

It is difficult to evaluate (9) for any finite N, K, L, and B. For an ideal channel, it was shown that the SINR converges to a deterministic value in a large system limit in which K, N, and B all tend to infinity with fixed normalized load $\bar{K} = K/N$ and normalized feedback bits $\bar{B} = B/N$ [8], [9]. Applying theory of extreme order statistics [18] similar to [8], the large system SINR with fading channel is given by

$$\begin{aligned} \beta_{\mathsf{rvq}}^\infty &= \lim_{(N,K,B,L)\to\infty} E_{\mathcal{V}}[\max\{\beta_1, \ldots, \beta_{2^B}\}|\boldsymbol{R}_1, \boldsymbol{H}_1] \quad (10) \\ &= \lim_{(N,K,B,L)\to\infty} F^{-1}_{\beta|\boldsymbol{R}_1,\boldsymbol{H}_1}(1 - 2^{-B}) \quad (11) \end{aligned}$$

where we assume that the empirical eigenvalue distribution of $\boldsymbol{R}_1$ converges almost surely to a nonrandom limit and $\lim_{L\to\infty} \sum_{l=1}^{L} E|h_{k,l}|^2 < \infty$ for all k.

Although RVQ is optimal in a large system limit [8], it was shown to perform close to the optimal codebook designed for a finite-size system [19]. Reference [8] derived the approximation for $\beta_{\mathsf{rvq}}^{\infty}$ by approximating cdf for $\beta_1(\boldsymbol{v}_j)$ to be Gaussian. The approximation is a function of $\bar{K}$, $\bar{B}$, and σ_n^2 and is good for small $\bar{B}$. For large $\bar{B}$, it over-estimates the actual performance. In this work, we derive exact expressions for $\beta_{\mathsf{rvq}}^{\infty}$.

We first consider the ideal channel ($\boldsymbol{H}_k = \boldsymbol{I}, \forall k$). We rearrange (11) to obtain

$$\lim_{\substack{(N,K,B)\to\infty \\ z\to\beta_{\mathsf{rvq}}^{\infty}}} [1 - F_{\beta|\boldsymbol{R}_1}(z)]^{\frac{1}{N}} = 2^{-\bar{B}}. \tag{12}$$

Similar to [9], [12], it can be shown that the left-hand side of (12) is evaluated to

$$\lim_{\substack{(N,K,B)\to\infty \\ z\to\beta_{\mathsf{rvq}}^{\infty}}} [1 - F_{\beta|\boldsymbol{R}_1}(z)]^{\frac{1}{N}} = \exp\{-\Phi(\rho^*, \beta_{\mathsf{rvq}}^{\infty})\} \tag{13}$$

where

$$\Phi(\rho, \beta_{\mathsf{rvq}}^{\infty}) = \int \log(1 + \rho(\beta_{\mathsf{rvq}}^{\infty} - \frac{A_1}{\tau + \sigma_n^2})) f_{\boldsymbol{S}_1\boldsymbol{A}_1\boldsymbol{S}_1^\dagger}(\tau)\, \mathrm{d}\tau, \tag{14}$$

$$\rho^* = \arg \max_{0<\rho<\frac{1}{\beta_{\max}^{\infty} - \beta_{\mathsf{rvq}}^{\infty}}} \Phi(\rho, \beta_{\mathsf{rvq}}^{\infty}), \tag{15}$$

$f_{\boldsymbol{S}_1\boldsymbol{A}_1\boldsymbol{S}_1^\dagger}(\cdot)$ is the asymptotic eigenvalue density for $\boldsymbol{S}_1\boldsymbol{A}_1\boldsymbol{S}_1^\dagger$, $\boldsymbol{S}_1$ is the $N \times (K-1)$ signature matrix whose columns are $\boldsymbol{s}_2, \ldots, \boldsymbol{s}_K$, and $\beta_{\max}^{\infty}$ is the asymptotic maximum eigenvalue of $A_1\boldsymbol{R}_1^{-1}$ and corresponds to the SINR with infinite feedback ($\bar{B} \to \infty$).

Combining (12) and (13), $\beta_{\mathsf{rvq}}^{\infty}$ satisfies the following fixed-point equation

$$\Phi(\rho^*, \beta_{\mathsf{rvq}}^{\infty}) = \bar{B}\log(2). \tag{16}$$

Suppose $\boldsymbol{s}_k$ has independent complex Gaussian entries with zero mean and variance $1/N$ ($\|\boldsymbol{s}_k\| \to 1$) and empirical distribution of A_k converges to a limit. We can express (14) as follows

$$\begin{aligned}\Phi(\rho, \beta_{\mathsf{rvq}}^{\infty}) = \log(1 + \rho(\beta_{\mathsf{rvq}}^{\infty} - \frac{A_1}{\sigma_n^2})) + \bar{K}\nu_{\boldsymbol{A}_1}(\zeta\Theta(\zeta)) \\ -\bar{K}\nu_{\boldsymbol{A}_1}(\sigma_n^{-2}\Theta(\sigma_n^{-2})) - \log(\Theta(\zeta)) + \log(\Theta(\sigma_n^{-2})) \\ +\Theta(\zeta) - \Theta(\sigma_n^{-2})\end{aligned} \tag{17}$$

where

$$\zeta = \frac{1 + \rho\beta_{\mathsf{rvq}}^{\infty}}{\sigma_n^2 + \rho\beta_{\mathsf{rvq}}^{\infty}\sigma_n^2 - \rho A_1}, \tag{18}$$

$\Theta(x)$ is the solution of the following fixed-point equation

$$\bar{K} = \frac{1 - \Theta(x)}{1 - \eta_{\boldsymbol{A}_1}(x\Theta(x))}, \tag{19}$$

and η- and Shannon transforms of a random variable X with pdf $f_X(\cdot)$ are defined as follows [20]:

$$\eta_X(\gamma) = \int \frac{1}{1 + \gamma x} f_X(x)\, \mathrm{d}x \tag{20}$$

$$\nu_X(\gamma) = \int \log(1 + \gamma X) f_X(x)\, \mathrm{d}x. \tag{21}$$

For an equal-power ($A_1 = \ldots = A_K = 1$) system, $f_{\boldsymbol{A}_1}(a) = \delta(a-1)$ and, thus,

$$\eta_{\boldsymbol{A}_1}(\gamma) = \frac{1}{1+\gamma}, \tag{22}$$

$$\nu_{\boldsymbol{A}_1}(\gamma) = \log(1+\gamma). \tag{23}$$

Substitute (22) and (23) into (17) and (19), respectively and simplify. We can express (16) as follows.

Theorem 1: For $\bar{K} \leq 1$, $\beta_{\mathsf{rvq}}^{\infty}$ satisfies the following equation

$$\begin{aligned}\log(\frac{\bar{K}}{1 - \beta_{\mathsf{rvq}}^{\infty}\sigma_n^2} - \frac{1}{\beta_{\mathsf{rvq}}^{\infty}}) + (1 - \bar{K})\log(\frac{p}{\sigma_n^2}) \\ + \bar{K}\log(\frac{w(p)}{w(\sigma_n^2)}) - (1 - \bar{K})\log(\frac{1 - v(p)}{1 - v(\sigma_n^2)}) \\ - v(p) + v(\sigma_n^2) = \bar{B}\log(2)\end{aligned} \tag{24}$$

where

$$w(x) = \frac{1}{2}(1 + \bar{K} + x + \sqrt{(1 + \bar{K} + x)^2 - 4\bar{K}}) \tag{25}$$

$$v(x) = \frac{1}{2}(1 + \bar{K} + x - \sqrt{(1 + \bar{K} + x)^2 - 4\bar{K}}) \tag{26}$$

and

$$p = \frac{1 - \beta_{\mathsf{rvq}}^{\infty}\sigma_n^2}{\bar{K}\beta_{\mathsf{rvq}}^{\infty} - 1 + \beta_{\mathsf{rvq}}^{\infty}\sigma_n^2} - \frac{1}{\beta_{\mathsf{rvq}}^{\infty}} + \sigma_n^2. \tag{27}$$

For $\bar{K} > 1$ and $\bar{B} \leq \bar{B}^*$, $\beta_{\mathsf{rvq}}^{\infty}$ satisfies the following equation

$$\begin{aligned}\log(\frac{\bar{K}}{1 - \beta_{\mathsf{rvq}}^{\infty}\sigma_n^2} - \frac{1}{\beta_{\mathsf{rvq}}^{\infty}}) + \log(\frac{w(p)}{w(\sigma_n^2)}) \\ - (\bar{K} - 1)\log(\frac{\bar{K} - v(p)}{\bar{K} - v(\sigma_n^2)}) - v(p) + v(\sigma_n^2) = \bar{B}\log(2)\end{aligned} \tag{28}$$

where

$$\begin{aligned}\bar{B}^* = \frac{1}{\log(2)}(\log(\bar{K} - \sqrt{\bar{K}} + \sigma_n^2) + \bar{K}\log(\sqrt{\bar{K}}) \\ - \bar{K}\log(\sqrt{\bar{K}} - 1) - \sqrt{\bar{K}} - \log(w(\sigma_n^2)) \\ + (\bar{K} - 1)\log(1 - \frac{v(\sigma_n^2)}{\bar{K}}) + v(\sigma_n^2)).\end{aligned} \tag{29}$$

For $\bar{K} > 1$ and $\bar{B} > \bar{B}^*$,

$$\begin{aligned}\beta_{\mathsf{rvq}}^{\infty} = \beta_{\max}^{\infty}(1 - 2^{-\bar{B}}[\exp\{\frac{1}{2}\bar{K}\log(\bar{K}) \\ - (\bar{K} - 1)\log(\frac{\bar{K}\sqrt{\bar{K}} - \bar{K}}{\bar{K} - v(\sigma_n^2)}) - \log(w(\sigma_n^2)) \\ + v(\sigma_n^2) - \sqrt{\bar{K}}\}]).\end{aligned} \tag{30}$$

Due to limited space, the derivation is not shown here.

For a flat-fading channel ($L = 1$), we can combine the channel gain $|h_{k,1}|^2$ for user k with its transmit power A_k. That is the diagonal matrix $\boldsymbol{A}_1 = \mathrm{diag}\{|h_{2,1}|^2 A_2, \ldots, |h_{K,1}|^2 A_K\}$ whose empirical distribution converges to a nonrandom limit. With an asymptotic distribution for diagonal entries of $\boldsymbol{A}_1$, we can apply (16) - (19) to obtain the output SINR $\beta_{\mathsf{rvq}}^{\infty}$.

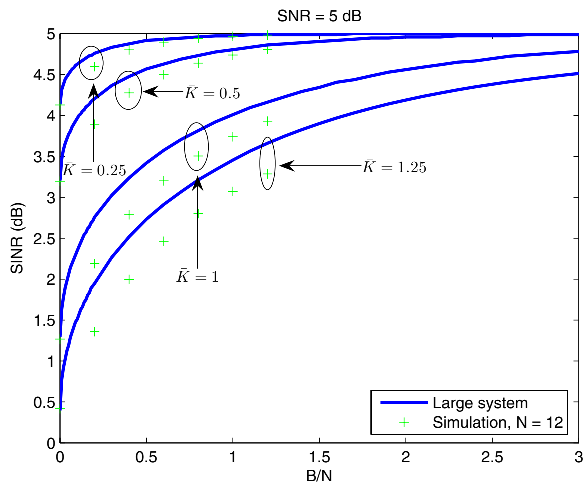

Fig. 1. Shown is a large system SINR versus normalized feedback bit $\bar{B}$ with different normalized loads $\bar{K} = 0.25, 0.5, 1, 1.25$ and SNR $= 5$ dB.

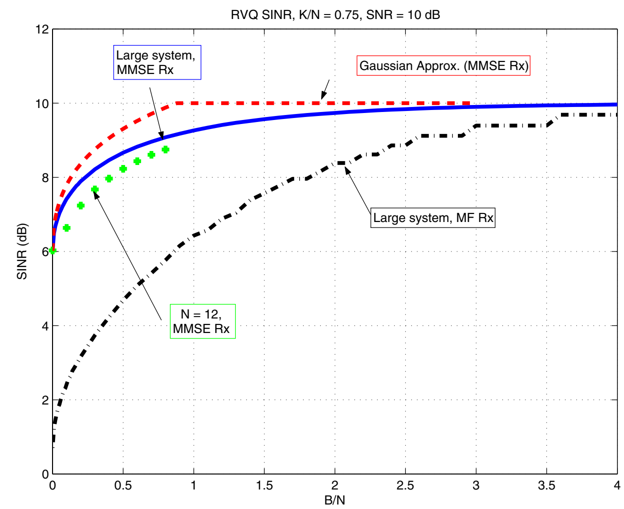

Fig. 2. The large system SINR for MMSE receiver is compared with the approximation derived in [8] and the large system SINR for matched filter derived in [9]. Also shown is the simulation result for $N = 12$, $\bar{K} = 0.75$ and SNR $= 10$ dB.

For frequency-selective fading, the signal of each users is assumed to propagate L discrete chip-spaced paths. The channel matrix for user k is shown in (2). First, we assume that L is finite and does not grow with N. Thus, the number of paths per processing gain $\bar{L} = L/N \to 0$. To compute $\beta_{\text{rvq}}^{\infty}$, we require the asymptotic eigenvalue distribution of $\boldsymbol{R}_1$ (6). Reference [21] showed that the asymptotic eigenvalue distribution of $\boldsymbol{R}_1$ with L-path channels (2) equals that of $\boldsymbol{R}_1$ with flat-fading channels and $\boldsymbol{A}_1 = \text{diag}\{A_2(\sum_{l=1}^{L} |h_{2,l}|^2), \ldots, A_K(\sum_{l=1}^{L} |h_{K,l}|^2)\}$. Thus, a multipath interferer is asymptotically equivalent to a single-path interferer with combined gain of $\sum_{l=1}^{L} |h_{k,l}|^2$. For $L \to \infty$ with fixed L/N, the same result applies as long as $\lim_{L\to\infty} \sum_{l=1}^{L} E|h_{k,l}|^2 < \infty$, for all k.

IV. Numerical Results

Fig. 1 shows the asymptotic SINR in Theorem 1 versus normalized feedback bit $\bar{B}$ with different normalized loads $\bar{K} = 0.25, 0.5, 1, 1.25$. As expected, the SINR increases with normalized feedback and decreases with normalized load. For $\bar{K} = 0.25$, RVQ achieves close to the single-user performance with approximately $\bar{B} = 0.5$ (0.5 bits per processing gain or degree of freedom). As number of interfering users increases, amount of feedback required also increases to achieve a target SINR. For example, $\bar{B} = 3$ is needed for system with $\bar{K} = 1$ to achieve close to the single-user performance. We also compare the asymptotic results with simulation results marked by pluses in Fig. 1. We note that the large system results predict the performance of finite-size systems ($N = 12$) well. As N increases, the gap between the simulation and analytical results is expected to be closing.

RVQ codebook requires an exhaustive search to locate the optimal signature. The search complexity increases exponentially with feedback bits B. (For $\bar{B} = 3$, number of entries in RVQ codebook is 2^{36}.) Thus, we do not have simulation results for a large B.

In Fig. 2, we compare the asymptotic SINR in Theorem 1 with the approximation derived in [8] for $\bar{K} = 0.75$ and SNR $= 10$ dB. Also shown is the simulation results with $N = 12$. The large system SINR is closer to the simulated performance than the approximation. We also show the RVQ performance of a matched filter derived in [9] with that of MMSE receiver derived here. The performance difference can be substantial for small to moderate $\bar{B}$. With 1 feedback bit per degree of freedom, the MMSE receiver outperforms a matched filter by as much as 30%. However, an MMSE filter is more complex than a matched filter. Therefore, there is a performance tradeoff between feedback and receiver complexity.

We also simulated a multipath fading channel in which each user's signal transverses 2 paths with different gains ($E|h_{k,1}|^2 = 0.9$ and $E|h_{k,2}|^2 = 0.1, \forall k$). Furthermore, K interfering users are divided into 2 groups. K_1 users transmit signal with $A_k = P_1$ while K_2 users with $A_k = P_2$. This scenario may follow from a system with differentiated quality of service. We obtain the large system SINR from (17)-(21) with the asymptotic distribution of $\boldsymbol{A}_1$

$$f_{\boldsymbol{A}_1}(a) = \bar{K}_1 \delta(a - P_1) + \bar{K}_2 \delta(a - P_2) \tag{31}$$

where normalized loads $\bar{K}_1 = K_1/N$ and $\bar{K}_2 = K_2/N$. Both the large system and corresponding simulated results with $\bar{K}_1 = \bar{K}_2 = 0.25$ and different sets of P_1 and P_2 are shown in Fig. 3. The large system performance closely approximates the performance of the system with $N = 32$. As N grows, the performance of a finite-size system will converge to that of the large system. In this example, reducing the transmit power of one group of users by 20 dB (P_2 from 10 to 0.1) decreases the required feedback to achieve 0.5 dB away from the single-user performance by $\bar{B} = 0.4$.

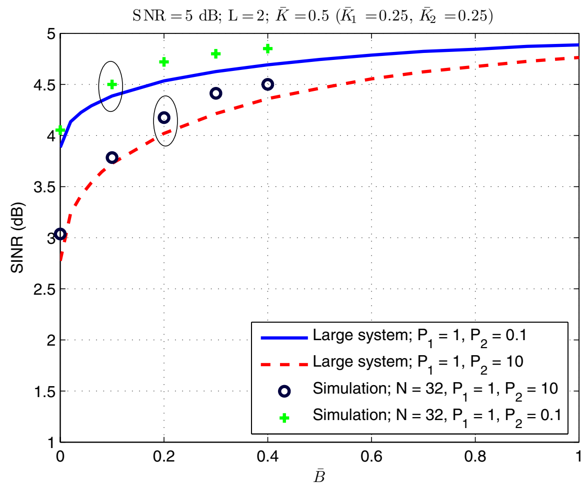

Fig. 3. The large system SINR for multipath-fading CDMA with two groups of users is shown with simulation results. SNR $= 5\,$dB, number of paths $L = 2$ for all users, and $\bar{K} = 0.5$.

V. Conclusions

We have shown expressions for a large system SINR for RVQ with MMSE receiver, which is a function of normalized load (number of users per degree of freedom) and normalized feedback bit (number of feedback bit per degree of freedom). Both nonfading ideal channel and multipath fading channel were considered. The SINR of the quantized signature increases with $\bar{B}$. For a small load, RVQ achieves close to the single-user performance with only fraction of feedback bit per quantized signature coefficient. We compared performance of the MMSE receiver with that of matched filter derived in [9] and showed that the performance gap is large for small $\bar{B}$. The matched filter requires more feedback to achieve a target SINR than the MMSE receiver does. However, the matched filter is simpler.

In this work, we assume that the receiver can estimate channel and interference covariances perfectly. In practice, a very accurate channel estimation is achieved by a large amount of training. How the performance of RVQ is affected by imperfect channel estimate at the receiver (or limited training) is studied by [22]. Here we considered signature quantization for a *single* user. Performance of group of users with RVQ-quantized signatures is more difficult to analyze and is an interesting research problem.

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