Transmission Capacities for Overlaid Wireless Ad Hoc Networks with Outage Constraints

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Abstract-We study the transmission capacities of two coexisting wireless networks (a primary network vs. a secondary network) that operate in the same geographic region and share the same spectrum. We define transmission capacity as the product among the density of transmissions, the transmission rate, and the successful transmission probability (1 minus the outage probability). The primary (PR) network has a higher priority to access the spectrum without particular considerations for the secondary (SR) network, where the SR network limits its interference to the PR network by carefully controlling the density of its transmitters. Assuming that the nodes are distributed according to Poisson point processes and the two networks use different transmission ranges, we quantify the transmission capacities for both of these two networks and discuss their tradeoff based on asymptotic analyses. Our results show that if the PR network permits a small increase of its outage probability, the sum transmission capacity of the two networks (i.e., the overall spectrum efficiency per unit area) will be boosted significantly over that of a single network.

I. INTRODUCTION

Initiated by the seminal work of Gupta and Kumar [1], the studies for understanding the capacities of wireless ad hoc networks have made great progresses. Considering n nodes that are randomly distributed in a unit area and grouped independently into one-to-one source-destination (S-D) pairs, Gupta and Kumar [1] showed that a typical time-slotted multi-hop architecture with a common transmission range and adjacent-neighbor communication can achieve a sum throughput that scales as $\Theta\left(\sqrt{n/\log n}\right)$. By using percolation theory, Franceschetti *et al.* [2] showed that the $\Theta(\sqrt{n})$ sum throughput scaling is achievable. In [3], Grossglauser and Tse showed that by allowing the nodes to move independently and uniformly, a constant throughput scaling $\Theta(1)$ per S-D pair can be achieved. In [4], Baccelli et al. proposed a multi-hop spatial reuse ALOHA protocol. By optimizing the product between the number of simultaneous successful transmissions per unit area and the average transmission range, they showed that the transport capacity is proportional to the square root of the node density, which achieves the upper bound of Gupta and Kumar [1]. Weber et al in [5] derived the upper and lower bounds on transmission capacity of spreadspectrum wireless ad hoc networks, where the transmission capacity is defined as the product between the maximum density of successful transmissions and the corresponding data rate, under a constraint on the outage probability.

All the above results focus on the capacity of a single ad hoc wireless network. In recent years, due to the scarcity and poor utilization of spectrum, the regulation bodies are beginning to consider the possibility of permitting secondary (SR) networks to coexist with licensed primary (PR) networks, which is the main driving force behind the cognitive radio technology [6]. In cognitive radio networks, the PR users have a higher priority to access the spectrum and the SR users need to operate conservatively such that their interference to the PR users is limited below an "acceptable level". In this overlaid regime, the capacity or throughput scaling laws for both of the PR and SR networks are interesting problems. Recently, some preliminary works along this line appeared. In [7], Vu et al. considered the throughput scaling law for a single-hop cognitive radio network, where a linear scaling law is obtained for the SR network with an outage constraint for the PR network. In [8], Jeon et al. considered a multihop cognitive network on top of a PR network and assumed that the SR nodes know the location of each PR node. With an elegant transmission scheme, they showed that by defining a preservation region around each PR node, both networks can achieve the same throughput scaling law as a stand-alone wireless network, while the SR network may suffer from a finite outage probability. In [9], Yin et al. assumed that the SR nodes only konw the locations of PR transmitters (TXs) and proposed a transmission scheme to show that both networks can achieve the same throughput scaling law as a stand-alone wireless network, with zero outage.

In this paper, we study the coexistence of two ad hoc networks with different transmission scales (power and/or transmission range) based on the transmission capacity defined in [5]. We extend the definition of transmission capacity from a single network to two overlaid networks. Different from the approaches in [7], [8], and [9], we resort to stochastic geometry tools to quantify the transmission capacities for both the PR and SR networks without defining any preservation regions. By considering the mutual interferences from the two networks, we discuss the tradeoff of the transmission capacities between them. The results show that if we permit a slight increase over the outage probability of the PR network, the sum transmission capacity (i.e., the overall spectrum efficiency per unit area) of the overlaid networks will be boosted significantly over that of a single network.

The rest of the paper is organized as follows. The network

model and symbol notations are described in Section II. The transmission capacity for a single network case is analyzed in Section III. The transmission capacities for the PR and SR networks and their tradeoff are discussed in Section IV. The numerical results and observations are given in Section V. Finally, Section VI summarizes our conclusions.

II. NETWORK MODEL AND SYSTEM SETUP

Consider the scenario where a network of PR nodes and a network of SR nodes coexist in the same geographic region, and assume that the PR network is the legacy network, which has a higher priority to access the spectrum. The prerequisite condition for introducing a new SR network into the territory of the PR network is that the outage probability increment of the PR network is upper-limited by a target constraint $\Delta \epsilon$, where $\Delta \epsilon$ usually takes a very small value.

We assume that at a certain time instance the distribution of PR TXs follows a homogeneous Poisson point process (PPP) Π_0 of density λ_0 , and the distribution of SR TXs follows another independent homogeneous PPP Π_1 of density λ_1 . Our goal is to evaluate the outage probability of the PR network, \mathcal{P}^0 , and that of the SR network, \mathcal{P}^1 , which are functions of the TX node densities λ_0 and λ_1 . The specific definitions of outage probability will be given in Section III and Section IV. Similar to that in [5], in order to evaluate the outage probabilities, we condition on a typical PR (or SR) RX at the origin, which yields the Palm distribution for PR (or SR) TXs. Following the Slivnyak's theory in stochastic geometry [10], these conditional distributions also follow homogeneous PPPs with the corresponding densities (i.e., λ_0 and λ_1 , respectively). Let $\{X_i \in \mathbb{R}^2, i \in \Pi_0\}$ and $\{Y_j \in \mathbb{R}^2, j \in \Pi_1\}$ denote the locations of the PR TXs and the SR TXs, respectively, $|X_i|$ and $|Y_i|$ denote the distances from PR TX i and SR TX j to the origin, respectively. An attempted transmission is successful if the received signal-to-interference-plus-noise ratio (SINR) at the reference RX is above a threshold, β ; otherwise, the transmission fails, i.e., an outage occurs. We use β_0 and β_1 to represent the SINR thresholds for the PR network and the SR network, respectively.

For simplicity, we limit our discussion to single-hop transmissions, and assume that all PR TXs use the same transmission power ρ_0 , and all PR transmissions are over the same distance r_0 . Similarly, all SR TXs use the same transmission power ρ_1 over the same transmission distance r_1 . For the wireless channel, we only consider the large-scale path-loss, and ignore the effects of shadowing and small-scale multipath fading. As such, the normalized channel power gain g(d) is given as

$$g(d) = \frac{A}{d^{\alpha}},\tag{1}$$

where A is a system-dependent constant, d is the distance between the TX and the corresponding RX, and $\alpha > 2$ denotes the path-loss exponent. In the following discussion, we normalize A to be unity for simplicity. The ambient noise is assumed to be additive white Gaussian noise (AWGN) with an average power η . We assume that all the PR TXs and the SR TXs use the same spectrum with bandwidth normalized to be unity.

As in [5], we define transmission capacity as follows.

Definition 1: Transmission capacity C^{ϵ} of a randomlydeployed wireless network is defined as the product among the maximum density λ^{ϵ} of transmissions, the common transmission data rate R, and $(1-\epsilon)$ with ϵ an asymptotically small outage probability. Therefore, we have

$$C^{\epsilon} = R\lambda^{\epsilon}(1-\epsilon). \tag{2}$$

As noted in [5], C^{ϵ} also represents the unit-area spectral efficiency of the successful transmissions.

III. ASYMPTOTIC ANALYSIS OF THE TRANSMISSION CAPACITY: SINGLE NETWORK CASE

In this section, we derive the asymptotic result (asymptotic over vanishingly-small outage probability values) for the transmission capacity of the PR network when the SR network is absent. As an example, we focus on the case when the pathloss exponent $\alpha = 4$, over which we build an asymptotic analysis framework that is useful for the future study over the cases of general α values.

When the SR network is absent, denote the target outage probability of the PR network over per-link SINR as ϵ_0 . Then we have

$$\mathcal{P}^{0} = \operatorname{Prob}\left(\frac{\rho_{0}r_{0}^{-\alpha}}{\eta + \sum_{i \in \Pi_{0}}\rho_{0}|X_{i}|^{-\alpha}} \le \beta_{0}\right) = \epsilon_{0}.$$
 (3)

Rewrite (3) as

$$\operatorname{Prob}\left(X \ge T_0\right) = \epsilon_0,\tag{4}$$

where $X = \sum_{i \in \Pi_0} \rho_0 |X_i|^{-\alpha}$ and $T_0 = \frac{\rho_0 r_0^{-\alpha}}{\beta_0} - \eta$. The moment generating function (MGF) of X is given by [11]

$$\Phi_X(s) = \exp\left[-\pi\lambda_0\rho_0^{\frac{2}{\alpha}}s^{\frac{2}{\alpha}}\Gamma\left(1-\frac{2}{\alpha}\right)\right].$$
 (5)

When $\alpha = 4$, we have

$$\Phi_X(s) = \exp\left[-\pi^{\frac{3}{2}}\lambda_0\rho_0^{\frac{1}{2}}s^{\frac{1}{2}}\right].$$
 (6)

Via the inverse Laplace transform, we obtain the probability density function (PDF) of X as

$$f_X(x) = \frac{\pi}{2} \lambda_0 \sqrt{\rho_0} x^{-\frac{3}{2}} \exp\left(-\frac{\pi^3}{4x} \lambda_0^2 \rho_0\right),$$
 (7)

and the corresponding cumulative density function (CDF) of \boldsymbol{X} as

$$F_X(x) = 2Q\left(\frac{\pi^{\frac{3}{2}}\lambda_0\sqrt{\rho_0}}{\sqrt{2x}}\right).$$
 (8)

From (8), we have

Prob
$$(X \ge T_0) = 1 - 2Q\left(\frac{\pi^{\frac{3}{2}}\lambda_0\sqrt{\rho_0}}{\sqrt{2T_0}}\right).$$
 (9)

Combined (4) and (9), it is clear that the following condition has to be satisfied:

$$Q\left(\frac{\pi^{\frac{3}{2}}\lambda_0\sqrt{\rho_0}}{\sqrt{2T_0}}\right) = \frac{1-\epsilon_0}{2}.$$
 (10)

When $\epsilon_0 \to 0$ such that $\frac{\pi^{\frac{3}{2}}\lambda_0\sqrt{\rho_0}}{\sqrt{2T_0}} \to 0$, with Taylor series expansion, we obtain the maximum allowable value (via the monotonicity of the Q function) of λ_0 asymptotically for $\alpha = 4$ as

$$\lambda_0^{\epsilon_0} = \frac{\epsilon_0}{\pi} \left(\frac{T_0}{\rho_0}\right)^{\frac{1}{2}} = \frac{\epsilon_0}{\pi} \left(\frac{T_0^{-4}}{\beta_0} - \frac{\eta}{\rho_0}\right)^{\frac{1}{2}}.$$
 (11)

As we can see from (11) that when the outage probability ϵ_0 is very small, the density of TXs is a linear function of ϵ_0 . Therefore, the transmission capacity of the PR network is given by

$$C_0^{\epsilon_0} = R_0 \lambda_0^{\epsilon_0} (1 - \epsilon_0), \tag{12}$$

where R_0 is the data rate when the transmission between the TX and its associated RX is successful, which is set to be same for all the links.

IV. ASYMPTOTIC ANALYSIS OF THE TRANSMISSION CAPACITY: OVERLAID NETWORK CASE

A. Transmission Capacity of the PR Network

When the SR network is present, it introduces interference to the PR network and the outage probability of the PR network will be increased. If we set the target outage probability increment of the PR network as $\Delta \epsilon$, we have

$$\mathcal{P}^{0} = \operatorname{Prob}\left(\frac{\rho_{0}r_{0}^{-\alpha}}{\eta + \sum_{i \in \Pi_{0}}\rho_{0}|X_{i}|^{-\alpha} + \sum_{j \in \Pi_{1}}\rho_{1}|Y_{j}|^{-\alpha}} \leq \beta_{0}\right)$$
$$= \epsilon_{0} + \Delta\epsilon.$$
(13)

With $Y = \sum_{j \in \Pi_1} \rho_1 |Y_j|^{-\alpha}$, (13) can be rewritten as

$$\operatorname{Prob}\left(X+Y \ge T_0\right) = \epsilon_0 + \Delta\epsilon. \tag{14}$$

The MGF of Y is given by

$$\Phi_Y(s) = \exp\left[-\pi\lambda_1 \rho_1^{\frac{2}{\alpha}} s^{\frac{2}{\alpha}} \Gamma\left(1 - \frac{2}{\alpha}\right)\right].$$
(15)

Define Z = X + Y such that the MGF of Z is given by

$$\Phi_Z(s) = \Phi_X(s)\Phi_Y(s)$$

= $\exp\left[-\pi s^{\frac{2}{\alpha}}\Gamma\left(1-\frac{2}{\alpha}\right)\left(\lambda_0\rho_0^{\frac{2}{\alpha}}+\lambda_1\rho_1^{\frac{2}{\alpha}}\right)\right].$

For $\alpha = 4$, we have

$$\Phi_Z(s) = \exp\left[-\pi^{\frac{3}{2}}s^{\frac{1}{2}}\left(\lambda_0\sqrt{\rho_0} + \lambda_1\sqrt{\rho_1}\right)\right],\qquad(16)$$

and the PDF of Z is given by

$$f_Z(z) = \frac{\pi}{2} \left(\lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right) z^{-\frac{3}{2}} \\ \times \exp\left[-\frac{\pi^3}{4z} \left(\lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right)^2 \right].$$
(17)

Applying (17) in (14), we have

$$1 - 2Q\left(\frac{\pi^{\frac{3}{2}}\left(\lambda_0\sqrt{\rho_0} + \lambda_1\sqrt{\rho_1}\right)}{\sqrt{2T_0}}\right) = \epsilon_0 + \Delta\epsilon, \qquad (18)$$

i.e.,

$$Q\left(\frac{\pi^{\frac{3}{2}}\left(\lambda_0\sqrt{\rho_0}+\lambda_1\sqrt{\rho_1}\right)}{\sqrt{2T_0}}\right) = \frac{1-\epsilon_0-\Delta\epsilon}{2}.$$
 (19)

When $\epsilon_0 \rightarrow 0$ and $\Delta \epsilon \rightarrow 0$, with bivariate Taylor series expansion, we obtain

$$\frac{1}{2} - \frac{\pi\lambda_0\sqrt{\rho_0}}{2\sqrt{T_0}} - \frac{\pi\lambda_1\sqrt{\rho_1}}{2\sqrt{T_0}} = \frac{1 - \epsilon_0 - \triangle\epsilon}{2}.$$
 (20)

If we choose $\lambda_0 = \lambda_0^{\epsilon_0}$ as in (11), the maximum allowable value of λ_1 corresponding to a target outage probability increment $\Delta \epsilon$ is given by

$$\lambda_1^{\Delta\epsilon} = \frac{1}{\pi} \left(\frac{T_0}{\rho_1} \right)^{\frac{1}{2}} \Delta\epsilon = \frac{1}{\pi} \left(\frac{\rho_0}{\rho_1} \cdot \frac{r_0^{-4}}{\beta_0} - \frac{\eta}{\rho_1} \right)^{\frac{1}{2}} \Delta\epsilon, \quad (21)$$

and the transmission capacity of the PR network is given by

$$C_0^{\epsilon} = R_0 \lambda_0^{\epsilon_0} \left(1 - \epsilon_0 - \Delta \epsilon \right).$$
⁽²²⁾

As shown in (20), when the SR network is presented, the outage probability of the PR network can be approximated by an affine function of λ_0 and λ_1 over asymptotically small ϵ_0 's and $\Delta \epsilon_0$'s.

B. Transmission Capacity of the SR Network

Denote the outage probability of the SR network as ϵ_1 , the outage probability of the SR network is given by

$$\mathcal{P}^{1} = \operatorname{Prob}\left(\frac{\rho_{1}r_{1}^{-\alpha}}{\eta + \sum_{i \in \Pi_{0}}\rho_{0}|X_{i}|^{-\alpha} + \sum_{j \in \Pi_{1}}\rho_{1}|Y_{j}|^{-\alpha}} \le \beta_{1}\right) = \epsilon_{1}.$$
(23)

Rewrite (23) as

$$\operatorname{Prob}\left(Z \ge \rho_1 \frac{r_1^{-\alpha}}{\beta_1} - \eta\right) = \epsilon_1.$$
(24)

Define $T_1 = \rho_1 \frac{r_1^{-\alpha}}{\beta_1} - \eta$, and we have $\operatorname{Prob} (Z \ge T_1) = \epsilon_1.$ (25)

Similar to (19), we obtain

$$Q\left(\frac{\pi^{\frac{3}{2}}\left(\lambda_0\sqrt{\rho_0}+\lambda_1\sqrt{\rho_1}\right)}{\sqrt{2T_1}}\right) = \frac{1-\epsilon_1}{2}.$$
 (26)

When $\epsilon_1 \rightarrow 0$, with bivariate Taylor series expansion, we have

$$\frac{1}{2} - \frac{\pi\lambda_0\sqrt{\rho_0}}{2\sqrt{T_1}} - \frac{\pi\lambda_1\sqrt{\rho_1}}{2\sqrt{T_1}} = \frac{1-\epsilon_1}{2}.$$
 (27)

Therefore, the outage probability of the SR network is given by

$$\epsilon_1 = \frac{\pi}{\sqrt{T_1}} \left(\lambda_0 \sqrt{\rho_0} + \lambda_1 \sqrt{\rho_1} \right), \tag{28}$$

and the transmission capacity of the SR network is given by

$$C_1^{\epsilon} = R_1 \lambda_1^{\epsilon} \left(1 - \epsilon_1 \right), \tag{29}$$

where R_1 is the data rate adopted by successful SR links.

On the other hand, if we set the target outage probability of the PR network to be $\epsilon_0 + \Delta \epsilon$, and set the target outage probability of the SR network to be ϵ_1 simultaneously, we could choose the value of λ_1^{ϵ} in (29) as follows

$$\lambda_1^{\epsilon} = \min\left(\lambda_1^{\Delta\epsilon}, \lambda_1^{\epsilon_1}\right),\tag{30}$$

where $\lambda_1^{\epsilon_1}$ is given by (via (28))

$$\lambda_1^{\epsilon_1} = \frac{\epsilon_1}{\pi} \left(\frac{r_1^{-\alpha}}{\beta_1} - \frac{\eta}{\rho_1} \right)^{\frac{1}{2}} - \lambda_0^{\epsilon_0} \sqrt{\frac{\rho_0}{\rho_1}}.$$
 (31)

C. Sum Transmission Capacity of the Overlaid Network

When the SR network is present, based on the above analyses, the sum transmission capacity of the overlaid networks is given by

$$C_{s}^{\epsilon} = C_{0}^{\epsilon} + C_{1}^{\epsilon}$$

$$= R_{0}\lambda_{0}^{\epsilon_{0}}\left(1 - \epsilon_{0} - \Delta\epsilon\right) + R_{1}\lambda_{1}^{\epsilon}(1 - \epsilon_{1})$$

$$= R_{0}\lambda_{0}^{\epsilon_{0}}\left(1 - \frac{\pi}{\sqrt{T_{0}}}\left(\lambda_{0}^{\epsilon_{0}}\sqrt{\rho_{0}} + \lambda_{1}^{\epsilon}\sqrt{\rho_{1}}\right)\right)$$

$$+ R_{1}\lambda_{1}^{\epsilon}\left(1 - \frac{\pi}{\sqrt{T_{1}}}\left(\lambda_{0}^{\epsilon_{0}}\sqrt{\rho_{0}} + \lambda_{1}^{\epsilon}\sqrt{\rho_{1}}\right)\right)$$

$$= \left(R_{0}\lambda_{0}^{\epsilon_{0}} + R_{1}\lambda_{1}^{\epsilon}\right) - \pi\left(\lambda_{0}^{\epsilon_{0}}\sqrt{\rho_{0}} + \lambda_{1}^{\epsilon}\sqrt{\rho_{1}}\right)$$

$$\times \left(\frac{R_{0}}{\sqrt{T_{0}}}\lambda_{0}^{\epsilon_{0}} + \frac{R_{1}}{\sqrt{T_{1}}}\lambda_{1}^{\epsilon}\right). \quad (32)$$

Compared to the single network case, the gain of the transmission capacity (i.e., the overall spectrum efficiency) of the overlaid networks over that of a single network is given by

$$K_g = \frac{C_s^{\epsilon}}{C_0^{\epsilon_0}} \approx 1 + \frac{C_1^{\epsilon}}{C_0^{\epsilon}}.$$
(33)

D. Tradeoff of the Transmission Capacities

Here we consider two setups to study the tradeoff between the transmission capacities of the PR network and the SR network. The first setup is that we change the value of $\Delta \epsilon$ only, and fix other parameters (ρ_0 , ρ_1 , r_0 , r_1 , β_0 , β_1 , η , and ϵ_0). The second setup is that we change the value of ρ_1 , and let other parameters (ρ_0 , r_0 , r_1 , β_0 , β_1 , η , ϵ_0 , and λ_1) be fixed.

Let us consider the first setup. When ϵ_0 is fixed, λ_0 is also fixed, see (11). From (22), we can see that C_0^{ϵ} is a linear function of $\Delta \epsilon$. As such, when $\Delta \epsilon$ is increased, C_0^{ϵ} is reduced. Rewrite (29) as

$$C_1^{\epsilon} = \frac{R_1}{\pi} \sqrt{\frac{T_0}{\rho_1}} \Delta \epsilon \left(1 - \sqrt{\frac{T_0}{T_1}} \epsilon_0 - \sqrt{\frac{T_0}{T_1}} \Delta \epsilon \right).$$
(34)

From (34), we can easily verify that when $\sqrt{T_1/T_0} > \epsilon_0$, C_1^{ϵ} is a convex function of $\Delta \epsilon$, and when $\Delta \epsilon < \frac{1}{2}(\sqrt{T_1/T_0} - \epsilon_0)$, C_1^{ϵ} increases monotonically over $\Delta \epsilon$.

Table I NETWORK PARAMETERS.

Symbol	Description	Value
$ ho_0$	Transmission power of PR TXs	20 W
ρ_1	Transmission power of SR TXs	0.1 W
r_0	Transmission range of PR TXs	20 m
r_1	Transmission range of SR TXs	5 m
η	Average power of ambient noise	$10^{-6} { m W}$
β_0	Target SINR for PR network	10 dB
β_1	Target SINR for SR network	10 dB

Now, we consider the second setup. Rewrite (22) and (29) as follows,

 $C_0^{\epsilon} = R_0 \lambda_0^{\epsilon_0} (1 - \epsilon_0 - \frac{\pi}{\sqrt{T_0}} \lambda_1^{\epsilon} \sqrt{\rho_1})$

(35)

and

$$C_1^{\epsilon} = R_1 \lambda_1^{\epsilon} \left(1 - \frac{\pi \lambda_0^{\epsilon_0}}{\sqrt{\frac{\rho_1}{\rho_0} \frac{r_1^{-\alpha}}{\beta_1} - \frac{\eta}{\rho_0}}} + \frac{\pi \lambda_1^{\epsilon}}{\sqrt{\frac{r_1^{-\alpha}}{\beta_1} - \frac{\eta}{\rho_1}}} \right). \quad (36)$$

We can easily show that when ρ_1 increases, C_0^{ϵ} decreases and C_1^{ϵ} increases.

V. NUMERICAL RESULTS AND INTERPRETATIONS

In this section, we present some numerical results based on our previous analyses and give some interpretations. We set the values of the network parameters as in Table I unless otherwise specified.

A. Single Network Case

In Fig. 1, we show the normalized transmission capacity $C_{0}^{\epsilon_0}/R_0$ as a function of the outage probability ϵ_0 , as well as the density of PR TXs λ_0 vs. the outage probability ϵ_0 . Note that these are exact results (not asymptotic ones) by using (2) and (10). We could see from this figure that when ϵ_0 is about 0.55, $C_0^{\epsilon_0}$ is maximized, and when $\epsilon_0 < 0.4$, λ_0 is nearly a linear function of ϵ_0 , which verifies the asymptotic result in (11).



Figure 1. Normalized transmission capacity/density of PR TXs vs. outage probability for the PR network when the SR network is absent.

In Fig. 2, we show the normalized asymptotic transmission capacity $C_0^{\epsilon_0}/R_0$ as a function of the outage probability ϵ_0 , and the upper and lower bounds of the transmission capacity based on the results derived in [5], which verifies the tightness of the upper bound.



Figure 2. Normalized transmission capacity vs. outage probability for the PR network when SR network is absent.

B. Overlaid Network Case

The normalized transmission capacity of the PR network C_0^{ϵ}/R_0 vs. the increment of the outage probability $\Delta \epsilon$ of the PR network is shown in Fig. 3. As expected, C_0^{ϵ}/R_0 is inversely proportional to $\Delta \epsilon$. On the other hand, since C_0^{ϵ} is a convex function of ϵ_0 ; and when $\epsilon_0 < \frac{1-\Delta\epsilon}{2}$, C_0^{ϵ} increases over ϵ_0 monotonically for a fixed $\Delta \epsilon$.



Figure 3. Normalized transmission capacity of the PR network vs. increment of the outage probability of the PR network.

In Fig. 4, we show the normalized transmission capacity of the SR network C_1^{ϵ}/R_1 as a function of $\Delta\epsilon$, see (29). As shown in the figure, we see that C_1^{ϵ} increases monotonically over $\Delta\epsilon$, since the larger $\Delta\epsilon$ is, the larger the values of λ_1^{ϵ} and ϵ_1 are, but the effect of λ_1^{ϵ} on C_1^{ϵ} is dominant when ϵ_1 is small.



Figure 4. Normalized transmission capacity of the SR network vs. increment of the outage probability of the PR network.

Assuming that $R_0 = R_1$, the capacity gain K_g of the overlaid networks (i.e., the sum transmission capacity) over that of a single network is shown in Fig. 5, see (33). We see that K_g increases over $\Delta \epsilon$ since the extra capacity contribution from the secondary network increases over $\Delta \epsilon$.



Figure 5. Gain of the transmission capacity of the overlaid network over that of the PR network.

In Fig. 6, we show the tradeoff between the normalized transmission capacity of the PR network C_0^{ϵ}/R_0 and that of the SR network C_1^{ϵ}/R_1 when $\Delta\epsilon$ changes as an intermediate variable. We see that C_0^{ϵ} decreases over C_1^{ϵ} , which verifies the result in Section IV.

VI. CONCLUSIONS

In this paper, we extended the concept of transmission capacity defined for the single network case to overlaid network case. By considering the mutual interference effect across two overlaid networks, i.e., the PR network vs. the SR network, we derived the transmission capacities for these two networks and studied their tradeoffs. Different from the previous approach for the single network case, we resorted to obtain the asymptotic solutions for these capacities. The results



Figure 6. Tradeoff of the transmission capacities of the PR and the SR networks when the value of $\Delta\epsilon$ is changed.

showed that by letting a SR network coexist with a legacy PR network, the spectrum efficiency per unit area could be increased significantly. Although we focused on a simple path-loss channel model with single-hop transmissions, the results are meaningful and motivating us to study more complex cases in the future work.

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