On the Achievable Throughput Region of Multiple-Access Fading Channels with QoS Constraints

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Abstract $-^{1}$ Effective capacity, which provides the maximum constant arrival rate that a given service process can support while satisfying statistical delay constraints, is analyzed in a multiuser scenario. In particular, we study the achievable effective capacity region of the users in multiaccess fading channels (MAC) in the presence of quality of service (QoS) constraints. We assume that channel side information (CSI) is available at both the transmitters and the receiver, and superposition coding technique with successive decoding is used. When the power is fixed at the transmitters, we show that varying the decoding order with respect to the channel state can significantly increase the achievable throughput region. For a two-user case, we obtain the optimal decoding strategy when the users have the same OoS constraints. Meanwhile, it is shown that time-division multiple-access (TDMA) can achieve better performance than superposition coding with fixed successive decoding order at the receiver side for certain QoS constraints. For power and rate adaptation, we determine the optimal power allocation policy with fixed decoding order at the receiver side. Numerical results are provided to demonstrate our results.

I. INTRODUCTION

Multiaccess fading channels have been extensively studied over the years from an information-theoretic point of view [1]-[6]. For instance, Tse and Hanly [3] have characterized the capacity region and determined the optimal resource allocation policies. It has been shown that the boundary surface points are achieved by successive decoding techniques, and each boundary point is associated with a weighted maximization of the sum rate. Vishawanath et al. [5] derived the explicit optimal power and rate allocation schemes (similar to waterfilling) by considering that the users are successively decoded in the same order for all channel states. For the convex capacity region, the unique decoding order was shown to be the reverse order of the priority weight. Caire et al. proved that TDMA is always suboptimal in low-SNR case [6]. On the other hand, these information theoretical studies have not addressed the delay and QoS constraints.

In this paper, we consider statistical QoS constraints and study the achievable rate region under such constraints in multiaccess fading channels. For this analysis, we employ the concept of effective capacity [7], which can be seen as the maximum constant arrival rate that a given time-varying

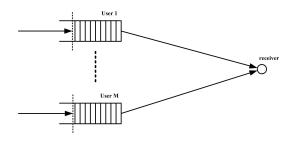


Fig. 1. The system model.

service process can support while satisfying statistical QoS guarantees. Effective capacity formulation uses the large deviations theory and incorporates the statistical QoS constraints by capturing the rate of decay of the buffer occupancy probability for large queue lengths. The analysis and application of effective capacity in various settings has attracted much interest recently (see e.g., [8]-[11] and references therein). We here consider the scenario in which both the transmitters and the receiver have the channel side information (CSI). First, we characterize the rate regions when the transmitters work at fixed power. Unlike the results obtained in [1], varying the decoding order is shown to significantly increase the achievable rate region under QoS constraints. Also, it is demonstrated that time sharing strategies among the vertex of the rate regions can no longer achieve the boundary surface. If we take the sum-rate throughput, or the sum effective capacity, as a measure, TDMA can even achieve better performance than superposition coding with fixed decoding order in certain cases. When power adaptation is considered, we provide the optimal power allocation policy when the users are being decoded in a fixed order at the receiver side.

The paper is organized as follows. Section II describes the system model. In Section III, effective capacity as a measure of the performance under statistical QoS constraints is briefly discussed, and the *throughput region* under QoS constraints is defined. Section IV includes our main results and presents numerical results. Finally, Section V concludes the paper.

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II. SYSTEM MODEL

As shown in Figure 1, we consider an uplink scenario where M users with individual power constraints and QoS constraints communicate with a single receiver. It is assumed that the transmitters generate data sequences which are divided into frames of duration T. These data frames are initially stored in the buffers before they are transmitted over the wireless channel. The discrete-time signal at the receiver in the i^{th} symbol duration is given by

$$Y[i] = \sum_{j=1}^{M} h_j[i] X_j[i] + n[i], \quad i = 1, 2, \dots$$
 (1)

where M is the number of users, $X_j[i]$ and $h_j[i]$ denote the complex-valued channel input and the fading coefficient of the *j*th user, respectively. We assume that $\{h_j[i]\}$'s are jointly stationary and ergodic discrete-time processes, and we denote the magnitude-square of the fading coefficients by $z_j[i] = |h_j[i]|^2$. The channel input of user *j* is subject to an average power constraint $\mathbb{E}\{|x_j[i]|^2\} \leq \bar{P}_j$ for all *j*, and we assume that the bandwidth available in the system is *B*. Y[i]is the channel output. Above, n[i] is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\{|n[i]|^2\} = N_0$. The additive Gaussian noise samples $\{n[i]\}$ are assumed to form an independent and identically distributed (i.i.d.) sequence.

A. Fixed Power and Variable Rate

First, we consider the case in which the transmitters operate at fixed power. The capacity region of this channel is given by [1]:

$$\mathcal{R}_{\text{MAC}} = \left\{ (R_1, \dots, R_M) : \mathbf{R}(S) \le B\mathbb{E}_{\mathbf{z}} \left\{ \log_2 \left(1 + \sum_{j \in S} \text{SNR}_j z_j \right) \right\}, \forall S \subset \{1, \dots, M\} \right\}$$
(2)

where $\text{SNR}_j = P_j/(N_0B)$ denotes the average transmitted signal-to-noise ratio of user j, $\mathbf{z} = (z_1, \dots, z_M)$ is a random vector comprised of the channel coefficients. As is known, there are M! vertices for the polyhedron defined in (2). The vertex $\mathbf{R}_{\pi} = (R_{\pi(1)}, \dots, R_{\pi(M)})$ corresponds to a permutation π , or the successive decoding order at the receiver, i.e., users are decoded in the order given by $\pi(1), \dots, \pi(M)$. The vertex is given by :

$$R_{\pi(k)} = B\mathbb{E}_{\mathbf{z}} \left\{ \log_2 \left(1 + \frac{\mathrm{SNR}_{\pi(k)} z_{\pi(k)}}{1 + \sum_{i=k+1}^{M} \mathrm{SNR}_{\pi(i)} z_{\pi(i)}} \right) \right\}$$

bits/s, $k = 1, \cdots, M.$ (3)

which also defines the maximum instantaneous service rate for user $\pi(k)$ at the given decoding order π . Time sharing among these M! permutations yields any point on the boundary surface [12]. As can be easily verified, due to the log term in the expression for the capacity region (2), varying decoding order according to the channel state does not provide any improvement for the achievable capacity region.

B. Variable Power and Variable Rate

Now, we suppose that dynamic power and rate allocation is performed according to time-variations in the channels. For a given power allocation policy $\mathcal{U} = \{\mu_1, \dots, \mu_M\}$, where $\mu_j \ge 0 \forall j$ can be viewed as a function of \mathbf{z} . The achievable rates are defined as

$$\mathcal{R}(\mathcal{U}) = \left\{ \mathbf{R} : \mathbf{R}(S) \le \mathbb{E}_{\mathbf{z}} \left\{ B \log_2 \left(1 + \sum_{j \in S} \mu_j(\mathbf{z}) z_j \right) \right\}, \\ \forall S \subset \{1, \cdots, M\} \right\}.$$
(4)

The instantaneous rate at a given decoding order can be obtained similar to (3) with SNR replaced by μ . Then, the rate region is given by

$$\mathcal{R}_{MAC} = \bigcup_{\mathcal{U} \in \mathcal{F}} \mathcal{R}(\mathcal{U})$$
(5)

where \mathcal{F} is the set of all feasible power control policies satisfying the average power constraint

$$\mathcal{F} \equiv \{ \mathcal{U} : \mathbb{E}_{\mathbf{z}} \{ \mu_j(\mathbf{z}) \le \text{SNR}_j, \mu_j \ge 0, \forall j \} \}$$
(6)

where $SNR_j = \bar{P}_j/(N_0B)$ denotes the average transmitted signal-to-noise ratio of user j.

C. TDMA

For simplicity, we assume that the time division strategy should be fixed prior to transmission. Let δ_j denote the fraction of time allocated to user j. Note that we have $\sum_{j=1}^{M} \delta_j = 1$. In each frame, each user occupies the entire bandwidth to transmit the signal in the corresponding fraction of time. Then, the instantaneous service rate for user j is given by

$$R_j(\text{SNR}_j) = B \log_2\left(1 + \frac{\text{SNR}_j}{\delta_j} z_j\right) \text{ bits/s}$$
(7)

III. PRELIMINARIES

A. Effective Capacity

In [7], Wu and Negi defined the effective capacity as the maximum constant arrival rate² that a given service process can support in order to guarantee a statistical QoS requirement specified by the QoS exponent θ . If we define Q as the stationary queue length, then θ is the decay rate of the tail distribution of the queue length Q:

$$\lim_{q \to \infty} \frac{\log P(Q \ge q)}{q} = -\theta.$$
 (8)

Therefore, for large q_{\max} , we have the following approximation for the buffer violation probability: $P(Q \ge q_{\max}) \approx e^{-\theta q_{\max}}$. Hence, while larger θ corresponds to more strict QoS constraints, smaller θ implies looser QoS guarantees. Similarly, if D denotes the steady-state delay experienced in the buffer, then $P(D \ge d_{\max}) \approx e^{-\theta \delta d_{\max}}$ for large d_{\max} , where δ is determined by the arrival and service processes [9].

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²For time-varying arrival rates, effective capacity specifies the effective bandwidth of the arrival process that can be supported by the channel.

The effective capacity is given by

$$C(\theta) = -\frac{\Lambda(-\theta)}{\theta} = -\lim_{t \to \infty} \frac{1}{\theta t} \log_e \mathbb{E}\{e^{-\theta S[t]}\} \quad \text{bits/s, (9)}$$

where the expectation is with respect to $S[t] = \sum_{i=1}^{t} s[i]$, which is the time-accumulated service process. $\{s[i], i = 1, 2, ...\}$ denote the discrete-time stationary and ergodic stochastic service process.

In this paper, in order to simplify the analysis while considering general fading distributions, we assume that the fading coefficients stay constant over the frame duration T and vary independently for each frame and each user. In this scenario, s[i] = TR[i], where R[i] is the instantaneous service rate in the *i*th frame duration [iT; (i+1)T]. Then, (9) can be written as

$$C(\theta) = -\frac{1}{\theta T} \log_e \mathbb{E}_{\mathbf{z}} \{ e^{-\theta T R[i]} \} \quad \text{bits/s}, \qquad (10)$$

where R[i] denotes the instantaneous rate sequence with respect to z. (10) is obtained using the fact that instantaneous rates $\{R[i]\}$ vary independently. The effective capacity normalized by bandwidth B is

$$C(\theta) = \frac{C(\theta)}{B}$$
 bits/s/Hz. (11)

B. Throughput Region

Suppose that $\Theta = (\theta_1, \dots, \theta_M)$ is a vector composed of the QoS constraints of M users. Let $C(\Theta) = (C_1(\theta_1), \dots, C_M(\theta_M))$ denote the vector of the normalized effective capacities. We first have the following characterization.

Proposition 1: The instantaneous *throughput region* can be defined as

$$\mathcal{C}_{\text{MAC}}(\Theta, \mathbf{SNR}) = \left\{ \mathsf{C}(\Theta) \ge \mathbf{0} : \mathsf{C}_{j}(\theta_{j}) \le -\frac{1}{\theta_{j}TB} \log_{e} \mathbb{E}_{\mathbf{z}} \left\{ e^{-\theta TR_{j}[i]} \right\}, \\ \text{subject to:} \forall \mathbb{E} \{ \mathbf{R}[\mathbf{i}] \} \in \mathcal{R}_{\text{MAC}} \right\}.$$
(12)

where $\mathbf{R}[i] = \{R_1[i], R_2[i], \dots, R_M[i]\}$ represents vector composed of the instantaneous rate of M users.

Remark: The *throughput region* defined in Proposition 1 represents the set of all vectors of constant arrival rates such that there exists a possible instantaneous rate adaptation $\mathbf{R}[i]$ among the M users, which can guarantee the QoS constraints $\Theta = (\theta_1, \dots, \theta_M)$.

Corollary: The *throughput region* for TDMA can be deemed as the achievable vectors of arrival rates with each component bounded by the effective capacity obtained for the instantaneous service rate given in (7). The effective capacity for user j on the boundary surface becomes

$$\mathsf{C}_{j}^{\mathrm{TD}}(\theta_{j}) = -\frac{1}{\theta_{j}TB} \log_{e} \mathbb{E} \left\{ e^{-\delta_{j}\theta_{j}TB \log_{2}\left(1 + \frac{\mathsf{SNR}_{j}}{\delta_{j}}z_{j}\right)} \right\}$$
(13)

We assume that $\mathbb{E}{\mathbf{R}[i]}$ can take any possible values defined in the \mathcal{R}_{MAC} . We have the following premilinary result. Theorem 1: The throughput region $C_{MAC}(\Theta, SNR)$ is convex.

Proof: Let $C_1(\Theta)$ and $C_2(\Theta)$ belong to $\mathcal{C}_{MAC}(\Theta, SNR)$. Therefore, there exists some $\mathbf{R}[i]$ and $\mathbf{R}'[i]$ for $C_1(\Theta)$ and $C_2(\Theta)$, respectively. By a time sharing strategy, for any $\alpha \in (0, 1)$, we know that $\mathbb{E}\{\alpha \mathbf{R}[i] + (1 - \alpha)\mathbf{R}'[i]\} \in \mathcal{R}_{MAC}$.

$$\begin{aligned} \alpha \mathsf{C}_{1} + (1 - \alpha) \mathsf{C}_{2} \\ &= -\frac{1}{\Theta TB} \log_{e} \left(\mathbb{E} \left\{ e^{-\Theta T \mathbf{R}[i]} \right\} \right)^{\alpha} \left(\mathbb{E} \left\{ e^{-\Theta T \mathbf{R}'[i]} \right\} \right)^{1 - \alpha} \\ &= -\frac{1}{\Theta TB} \log_{e} \left(\mathbb{E} \left\{ \left(e^{-\Theta T \alpha \mathbf{R}[i]} \right)^{\frac{1}{\alpha}} \right\} \right)^{\alpha} \\ &\quad \cdot \left(\mathbb{E} \left\{ \left(e^{-\Theta T (1 - \alpha) \mathbf{R}'[i]} \right)^{\frac{1}{1 - \alpha}} \right\} \right)^{1 - \alpha} \\ &\leq -\frac{1}{\Theta TB} \log_{e} \mathbb{E} \left\{ e^{-\Theta T \left(\alpha \mathbf{R}[i] + (1 - \alpha) \mathbf{R}'[i] \right)} \right\} \end{aligned}$$
(14)

where the vector operation is with respect to each component, and Holder's inequality is used. Hence, $\alpha C_1 + (1 - \alpha)C_2$ still lies in the *throughput region*.

We are interested in the boundary of the region $C_{MAC}(\Theta, SNR)$. Now that $C_{MAC}(\Theta, SNR)$ is convex, we can characterize the boundary surface by considering the following optimization problem [3]:

$$\max \lambda \cdot \mathsf{C}(\Theta)$$
 subject to: $\mathsf{C}(\Theta) \in \mathcal{C}_{MAC}(\Theta, \mathsf{SNR}).$ (15)

for all priority vectors $\lambda = (\lambda_1, \dots, \lambda_M)$ in \mathfrak{R}^M_+ with $\sum_{j=1}^M \lambda_j = 1$.

IV. MULTIPLE-ACCESS CHANNELS WITH QOS CONSTRAINTS

A. MAC without Power Control

If we assume that the receiver decodes the users at a fixed order, it is obvious that only the vertices can be achievable. Suppose that time sharing technique is employed. Moreover, assume that the time fraction for each order π_m is τ_m , such that $\tau_m \ge 0$ and $\sum_{m=1}^{M!} \tau_m = 1$. Then, the effective capacity for each user is

$$\mathsf{C}_{j}(\theta_{j}) = -\frac{1}{\theta_{j}TB} \log_{e} \mathbb{E}_{\mathbf{z}} \left\{ e^{-\theta_{j}T \sum_{m=1}^{M!} \tau_{m} R_{\pi_{m}^{-1}(j)}} \right\}$$
(16)

where $R_{\pi_m^{-1}(j)}[j]$ represents the instantaneous service rate of user j at a given decoding order π_m , which is given by

$$R_{\pi_m^{-1}(j)} = B \log_2 \left(1 + \frac{\text{SNR}_j z_j}{1 + \sum_{\pi_m^{-1}(i) > \pi_m^{-1}(j)} \text{SNR}_i z_i} \right)$$
(17)

where π_m^{-1} is the inverse trace function of π_m .

If the receiver has the freedom to change the decoding order according to the estimated channel state, we suppose there exists a rate allocation policy $\mathbf{R}[i]$ for any $\lambda \in \mathfrak{R}^M_+$. In this paper, we consider a class of successive decoding techniques $\mathcal{F}(\mathbf{z})$ parameterized as a function of the channel states \mathbf{z} . More specifically, the vector space \mathfrak{R}^M_+ for \mathbf{z} is divided into disjoint $\mathcal{Z}_m, m \in \{1, 2, \ldots, M!\}$ regions with respect to each π_m^3 . For instance, when $\mathbf{z} \in \mathcal{Z}_1$, the base station will decode the

³Each region corresponds to a unique π .

information in the order M, M - 1, ..., 1. Then, the effective capacity for each user is

$$C_{j}(\theta_{j}) = -\frac{1}{\theta_{j}TB} \log_{e} \mathbb{E}_{\mathbf{z}} \left\{ e^{-\theta_{j}TR_{j}} \right\}$$
$$= -\frac{1}{\theta_{j}TB} \log_{e} \left(\sum_{m=1}^{M!} \int_{\mathbf{z} \in \mathcal{Z}_{m}} e^{-\theta_{j}TR_{\pi_{m}^{-1}(j)}} d\mathbf{z} \right)$$
(18)

Considering the expression for effective capacity and the optimization problem in (15), the optimal rate adaptation with respect to the channel state seems intractable. In this paper, we consider a simplified scenario in which all users have the same QoS constraint described by θ . This case arises, for instance, if users do not have priorities over others in terms of buffer limitations or delay constraints.

1) Two-user MAC: Similar to the discussion in [13], finding an optimal scheduling scheme can be reduced to finding a function $z_2 = g(z_1)$ in the state space such that users are decoded in the order 1,2 if $z_2 < g(z_1)$ and users are decoded in the order 2,1 if $z_2 > g(z_1)$. The problem in (15) becomes

$$\max \lambda_1 \mathsf{C}_1(\theta, g(z_1)) + (1 - \lambda_1) \mathsf{C}_2(\theta, g(z_1))$$
(19)

where $C_1(\theta, g(z_1))$ and $C_2(\theta, g(z_1))$ are expressed in (20) and (21) at the top of the next page. Implicitly, $g(z_1)$ should always be larger than 0 in (20) and (21) in order for the integral to hold, which may not be guaranteed due to the complexity of the problem. In that case, we need to find a function $z_1 = g(z_2)$ instead, as will be indicated later.

Proposition 2: The optimal scheduling scheme for a specific common QoS constraint θ in the two-user case is given by

$$g(z_1) = \frac{(1 + \text{SNR}_1 z_1) K^{\frac{1}{\beta}} - 1}{\text{SNR}_2}, \quad K \in [1, \infty)$$
(22)

$$g(z_2) = \frac{(1 + \mathrm{SNR}_2 z_2)K^{-\frac{1}{\beta}} - 1}{\mathrm{SNR}_1}, \quad K \in [0, 1)$$
(23)

where $\beta = \frac{\theta TB}{\log_e 2}$, $K \in [0, \infty)$ is some constant.

Proof: Suppose that the optimal scheduling is given by $z_2 = g(z_1)$. We denote

$$\mathcal{J}(g_1(z_1)) = \lambda_1 \mathsf{C}_1(\theta, g_1(z_1)) + (1 - \lambda_1) \mathsf{C}_2(\theta, g_1(z_1))$$
(24)

where $g_1(z_1) = g(z_1) + s\eta(z_1)$. $g(z_1)$ is the optimal scheduling function, s is any constant, and $\eta(z_1)$ represents arbitrary variation. A necessary condition that needs to be satisfied is [14]

$$\left. \frac{d}{ds} \left(\mathcal{J}(g_1(z_1)) \right) \right|_{s=0} = 0.$$
(25)

Define the following (for i=1,2):

....

$$\phi_{1} = \int_{0}^{\infty} \int_{g(z_{1})}^{\infty} e^{-\theta TB \log_{2}(1 + \text{SNR}_{1}z_{1})} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1} + \int_{0}^{\infty} \int_{0}^{g(z_{1})} e^{-\theta TB \log_{2}\left(1 + \frac{\text{SNR}_{1}z_{1}}{1 + \text{SNR}_{2}z_{2}}\right)} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1}$$
(26)

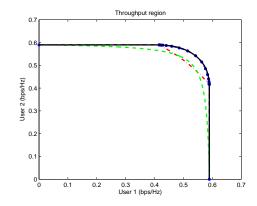


Fig. 2. The throughput region of two-user MAC case. $SNR_1 = SNR_2 = 0$ dB. $\theta_1 = \theta_2 = 0.01$. The solid, dot-dashed, dashed and dotted lines represent the regions achieved by optimal scheduling, suboptimal scheduling, fixed decoding with time sharing, and the TDMA respectively.

$$\phi_{2} = \int_{0}^{\infty} \int_{0}^{g(z_{1})} e^{-\theta T B \log_{2}(1 + \text{SNR}_{2}z_{2})} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1}$$

$$+ \int_{0}^{\infty} \int_{g(z_{1})}^{\infty} e^{-\theta T B \log_{2}\left(1 + \frac{\text{SNR}_{2}z_{2}}{1 + \text{SNR}_{1}z_{1}}\right)} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1}$$
(27)

By noting that $\frac{dg_1(z_1)}{ds} = \eta(z_1)$, and from (25)-(27), we can derive

$$\int_{0}^{\infty} \left(-\frac{\lambda_{1}}{\theta T B \phi_{1}} \left(\left(1 + \frac{\mathrm{SNR}_{1} z_{1}}{1 + \mathrm{SNR}_{2} g(z_{1})} \right)^{-\beta} - (1 + \mathrm{SNR}_{1} z_{1})^{-\beta} \right) \\ -\frac{1 - \lambda_{1}}{\theta T B \phi_{2}} \left((1 + \mathrm{SNR}_{2} g(z_{1}))^{-\beta} - \left(1 + \frac{\mathrm{SNR}_{2} g(z_{1})}{1 + \mathrm{SNR}_{1} z_{1}} \right)^{-\beta} \right) \right) \\ \cdot p_{z_{2}}(g(z_{1})) p_{z_{1}}(z_{1}) \eta(z_{1}) dz_{1} = 0$$

$$(28)$$

Since the above equation holds for any $\eta(z_1)$, it follows that

$$-\frac{\lambda_1}{\theta T B \phi_1} \left(\left(1 + \frac{\mathrm{SNR}_1 z_1}{1 + \mathrm{SNR}_2 g(z_1)} \right)^{-\beta} - \left(1 + \mathrm{SNR}_1 z_1 \right)^{-\beta} \right) \\ -\frac{1 - \lambda_1}{\theta T B \phi_2} \left(\left(1 + \mathrm{SNR}_2 g(z_1) \right)^{-\beta} - \left(1 + \frac{\mathrm{SNR}_2 g(z_1)}{1 + \mathrm{SNR}_1 z_1} \right)^{-\beta} \right) = 0$$
(29)

which after rearranging and defining K as follows yields

$$\frac{\left(1 + \frac{\text{SNR}_{1}z_{1}}{1 + \text{SNR}_{2}g(z_{1})}\right)^{-\beta} - (1 + \text{SNR}_{1}z_{1})^{-\beta}}{\left(1 + \frac{\text{SNR}_{2}g(z_{1})}{1 + \text{SNR}_{1}z_{1}}\right)^{-\beta} - (1 + \text{SNR}_{2}g(z_{1}))^{-\beta}} = \frac{(1 - \lambda_{1})\phi_{1}}{\lambda_{1}\phi_{2}} = K.$$
(30)

Obviously, $K \ge 0$. Notice that after simple computation, (30) becomes

$$\left(\frac{1+\operatorname{SNR}_1 z_1}{1+\operatorname{SNR}_2 g(z_1)}\right)^{-\beta} = K \tag{31}$$

which is (22). Note here that if K < 1, $g(z_1) < 0$ for $z_1 < \frac{K^{-\frac{1}{\beta}}-1}{SNR_1}$. Then the expressions in (20) and (21) cannot hold. In this case, we denote the optimal scheduling as $z_1 = g(z_2)$ instead. Following a similar approach as shown from (20)-(31) will give us (23).

$$C_{1}(\theta, g(z_{1})) = -\frac{1}{\theta T B} \log_{e} \left(\int_{0}^{\infty} \int_{g(z_{1})}^{\infty} e^{-\theta T B \log_{2}(1 + \text{SNR}_{1}z_{1})} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1} + \int_{0}^{\infty} \int_{0}^{g(z_{1})} e^{-\theta T B \log_{2}\left(1 + \frac{\text{SNR}_{1}z_{1}}{1 + \text{SNR}_{2}z_{2}}\right)} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1} \right)$$
(20)

$$C_{2}(\theta, g(z_{1})) = -\frac{1}{\theta TB} \log_{e} \left(\int_{0}^{\infty} \int_{0}^{g(z_{1})} e^{-\theta TB \log_{2}(1 + \text{SNR}_{2}z_{2})} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1} \right. \\ \left. + \int_{0}^{\infty} \int_{g(z_{1})}^{\infty} e^{-\theta TB \log_{2} \left(1 + \frac{\text{SNR}_{2}z_{2}}{1 + \text{SNR}_{1}z_{1}} \right)} p_{z_{2}}(z_{2}) p_{z_{1}}(z_{1}) dz_{2} dz_{1} \right)$$
(21)

2) Suboptimal Scheduling: When all users have the same QoS constraint specified by θ , we propose a suboptimal decoding order given by

$$\frac{\lambda_{\pi(1)}}{z_{\pi(1)}} \le \frac{\lambda_{\pi(2)}}{z_{\pi(2)}} \dots \le \frac{\lambda_{\pi(M)}}{z_{\pi(M)}},\tag{32}$$

due to the observation that whichever λ_j approaches 1, it should be decoded last. Considering a two-user example, we can express the points on the boundary surface as

$$C_{1}(\theta) = -\frac{1}{\theta T B} \log_{e} \left(\int_{0}^{\infty} \int_{\frac{\lambda_{2} z_{1}}{\lambda_{1}}}^{\infty} e^{-\theta T B \log_{2}\left(1 + \text{SNR}_{1} z_{1}\right)} dz_{2} dz_{1} + \int_{0}^{\infty} \int_{0}^{\frac{\lambda_{2} z_{1}}{\lambda_{1}}} e^{-\theta T B \log_{2}\left(1 + \frac{\text{SNR}_{1} z_{1}}{1 + \text{SNR}_{2} z_{2}}\right)} dz_{2} dz_{1} \right)$$
(33)

$$C_{2}(\theta) = -\frac{1}{\theta T B} \log_{e} \left(\int_{0}^{\infty} \int_{0}^{\frac{\lambda_{2} z_{1}}{\lambda_{1}}} e^{-\theta T B \log_{2}\left(1 + \text{SNR}_{2} z_{2}\right)} dz_{2} dz_{1} + \int_{0}^{\infty} \int_{\frac{\lambda_{2} z_{1}}{\lambda_{1}}}^{\infty} e^{-\theta T B \log_{2}\left(1 + \frac{\text{SNR}_{2} z_{2}}{1 + \text{SNR}_{1} z_{1}}\right)} dz_{2} dz_{1} \right).$$
(34)

We have performed numerical analysis over Rayleigh fading channels with $\mathbb{E}{z} = 1$. In Fig. 2 where the throughput region of a two-user MAC is plotted, we observe that varying the decoding order can significantly increase the achievable rate region. Moreover, we see that the suboptimal strategy can achieve almost the same rate region as the optimal strategy. This can be attributed to the fact that with the optimal strategy, the receiver can choose the decoding order according to the channel state such that the weighted sum of effective capacities, i.e., summation of log-moment generate functions, is maximized. Meanwhile, TDMA can achieve some points outside of the throughput region with fixed decoding order at the receiver side. If sum-rate throughput, i.e. the summation of the effective capacities, is considered, we note in Fig. 3 that as θ increases, the curves of different strategies converge, and as θ approaches to 0, TDMA again becomes suboptimal. This may be in large due to the fact that the transmitted energy is concentrated in the corresponding fraction of time for each user, which will introduce considerable weighted sum of throughput as QoS constraints become more stringent, i.e., the supported throughput becomes smaller. As QoS constraints approach 0, this phenomenon can be nicely captured by previous work on the Shannon ergodic capacity.

B. MAC with Power Control

In this part, we consider the power control policies with fixed decoding order at the receiver side. Due to the convexity of C_{MAC} , there exist Lagrange multipliers $\kappa \in \mathfrak{R}^M_+$ such that $C^*(\Theta)$ on the boundary surface is a solution to the optimization problem

$$\max_{\boldsymbol{\omega}} \lambda \cdot \mathsf{C}(\boldsymbol{\Theta}) + \kappa \cdot \mathbb{E}\{\boldsymbol{\mu}\}.$$
 (35)

For a given permutation π , $C_i(\theta_i)$ is given by

$$\mathsf{C}_{j}(\theta_{j}) = -\frac{1}{\theta_{j}TB}\log_{e}\mathbb{E}\left\{e^{-\theta_{j}TB\log_{2}\left(1+\frac{\mu_{j}z_{j}}{1+\sum_{\pi^{-1}(i)>\pi^{-1}(j)}\mu_{i}z_{i}}\right)}\right\}.$$
(36)

Now, the optimization problem (35) is equivalent to

$$\max_{\mu} \sum_{j=1}^{M} -\lambda_{j} \frac{1}{\theta_{j}TB} \log_{e} \mathbb{E} \left\{ e^{-\theta_{j}TB \log_{2} \left(1 + \frac{\mu_{j}z_{j}}{1 + \sum_{\pi^{-1}(i) > \pi^{-1}(j)} \mu_{i}z_{i}} \right)} \right\}$$
$$+ \sum_{j=1}^{M} \kappa_{j} \mathbb{E} \{\mu_{j}\}.$$
(37)

Note that with a fixed decoding order, the user $\pi(M)$ sees no interference from the other users, and hence the derivative of (37) with respect to $\mu_{\pi(M)}$ will only be related to the effective capacity formulation of user $\pi(M)$. Therefore, we can solve an equivalent problem by maximizing $C_{\pi(M)}$ instead. After we derive $\mu_{\pi(M)}$, the derivative of (37) with respect to $\mu_{\pi(M-1)}$ will only be related to the effective capacity formulation of user $\pi(M-1)$. By repeated application of this procedure, with given λ , (37) can be further decomposed into the following M sequential optimization problems

$$\max_{\mu} -\lambda_{j} \frac{1}{\theta_{j}TB} \log_{e} \mathbb{E} \left\{ e^{-\theta_{j}TB \log_{2} \left(1 + \frac{\mu_{j}z_{j}}{1 + \sum_{\pi} - 1_{(i)} > \pi^{-1}(j)} \right)^{\mu_{i}z_{i}}} \right\}$$
$$+ \kappa_{j} \mathbb{E} \{\mu_{j}\} \quad j \in \{1, \cdots, M\}.$$
(38)

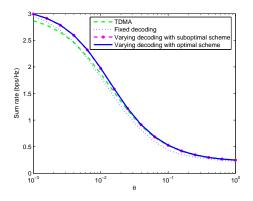


Fig. 3. The sum-rate throughput as a function of θ . SNR₁ = 10 dB; SNR₂ = 0 dB.

in the inverse order of π . Similar to [8], solving the above M parallel optimizations is the same as solving

$$\min_{\mu} \mathbb{E} \left\{ e^{-\theta_j T B \log_2 \left(1 + \frac{\mu_j z_j}{1 + \sum_{\pi^{-1}(i) > \pi^{-1}(j)} \mu_i z_i} \right)} \right\} + \kappa_j \mathbb{E} \{ \mu_j \} \quad j \in \{1, \cdots, M\}. \quad (39)$$

Differentiating the Lagrangians with respect to μ_j respectively and setting the derivatives to zero yields

$$\mu_{j} = \left(\frac{\left(1 + \sum_{\pi^{-1}(i) > \pi^{-1}(j)} \mu_{i} z_{i}\right)^{\frac{\beta_{j}}{\beta_{j}+1}}}{\alpha_{j}^{\frac{1}{\beta_{j}+1}} z_{j}^{\frac{\beta_{j}}{\beta_{j}+1}}} - \frac{1 + \sum_{\pi^{-1}(i) > \pi^{-1}(j)} \mu_{i} z_{i}}{z_{j}}\right)^{+}$$
(40)

where $\beta_j = \frac{\theta_j TB}{\log_e 2}$ is the normalized QoS exponent, $(x)^+ = \max\{x, 0\}$ and $(\alpha_1, \cdots, \alpha_M)$ satisfy the average power constraints. Exploiting the result in (40), we can find that instead of adapting power according to its channel state as in [8], the user adapts power according to its channel state normalized by the interference and the noise observed. Depending on whether each user is transmitting or not, the vector space \Re^M_+ for z can be divided into 2^M disjoint regions $\mathcal{Z}_m, m \in \{1, \cdots, 2^M\}$.

To give an explicit idea of the power control policy, we consider a two-user example where the decoding order is given by 2, 1. For this case, we have

$$\mu_{1} = \begin{cases} \frac{1}{\alpha_{1}^{\frac{1}{\beta_{1}+1}} z_{1}^{\frac{\beta_{1}}{\beta_{1}+1}}} & z_{1} > \alpha_{1}, \\ 0 & \text{otherwise.} \end{cases}$$
(41)

and

$$\mu_{2} = \begin{cases} \frac{1}{\alpha_{2}^{\frac{1}{\beta_{2}+1}} z_{2}^{\frac{\beta_{2}}{\beta_{2}+1}}} & z_{1} \leq \alpha_{1} \& z_{2} > \alpha_{2}, \\ \frac{\beta_{2}}{\alpha_{2}^{\frac{\beta_{2}}{\beta_{2}+1}} z_{2}^{\frac{\beta_{2}}{\beta_{2}+1}}}{\frac{1}{\alpha_{2}^{\frac{1}{\beta_{2}+1}} z_{2}^{\frac{\beta_{2}}{\beta_{2}+1}}}{0} - \frac{\left(\frac{z_{1}}{\alpha_{1}}\right)^{\frac{1}{\beta_{1}+1}}}{z_{2}} & z_{1} > \alpha_{1} \& \frac{z_{2}}{\alpha_{2}} > \left(\frac{z_{1}}{\alpha_{1}}\right)^{\frac{1}{\beta_{1}+1}} \\ 0 & \text{otherwise.} \end{cases}$$

where α_1 and α_2 are chosen to satisfy the average power constraints of the two users.

V. CONCLUSION

In this paper, we have studied the achievable rate regions in multi-access fading channels when users operate under QoS constraints. With the assumption that both the transmitters and the receiver have CSI, we have considered different scenarios under which we have investigated the achievable rate regions. Without power control, varying the decoding order is shown to significantly increase the achievable rate region. We have also shown that TDMA can perform better than superposition coding with fixed decoding order for certain QoS constraints. For a two-user case with same OoS constraints, the optimal strategy for varying decoding order is derived, and a simpler suboptimal decoding rule is proposed which can almost perfectly match the optimal throughput region. Numerical results are provided as well. Furthermore, we have derived the optimal power control policies for any given fixed decoding order.

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